EL-MOASSER

In Mathematics
The Main Book

For 2nd Prep
First Term

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By
A group of supervisors

CONTENTS

First Algebra and Statistics

Second Geometry

First
Algebra and Statistics

Revision

The sets of numbers

You had studied before the following sets of numbers :

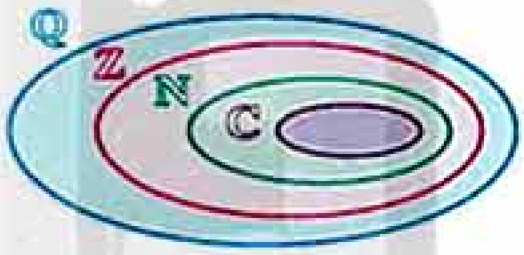
- The set of counting numbers : $\mathbb{C} = \{1, 2, 3, 4, ...\}$
- The set of natural numbers : $\mathbb{N} = \{0, 1, 2, 3, ...\} = \mathbb{C} \cup \{0\}$
- The set of integers : $\mathbb{Z} = \{..., 3, 2, 1, 0, -1, -2, -3, ...\}$
- The set of positive integers : $\mathbb{Z}_+ = \{1, 2, 3, ...\} = \mathbb{C}$
- The set of negative integers : $\mathbb{Z}_- = \{-1, -2, -3, ...\}$
- The set of rational numbers : $\mathbb{Q} = \{\frac{a}{b} : a \in \mathbb{Z}, b \in \mathbb{Z}, b \neq 0\}$

Examples of rational numbers : $\frac{2}{3}$, $-\frac{1}{2}$, zero, 3, −5, 0.2, 25%, ...

Notice that :

$\mathbb{C} \subset \mathbb{N} \subset \mathbb{Z} \subset \mathbb{Q}$

The opposite figure shows that.



Writing a rational number in its simplest form

To put the rational number $\frac{a}{b}$ in its simplest form, divide each of its terms by the highest common factor (H.C.F.) between them if it exists.

For example:

To put the rational number $\frac{8}{12}$ in its simplest form.

Divide each of its terms 8 and 12 by the highest common factor (H.C.F.) between them which is 4 as follows:

$$\frac{8}{12} = \frac{8 \div 4}{12 \div 4} = \frac{2}{3}$$

, then $\frac{2}{3}$ is the simplest form of the rational number $\frac{8}{12}$

The absolute value of a rational number

We denote the absolute value of the number a by $|a|$ where $|a| \geq 0$

For example:

- $|4| = 4$
- $|-4| = 4$
- $|0| = 0$

> **Remark**
>
> If $|x| = a$, then $x = a$ *or* $x = -a$

For example:

If $|x| = 5$, then $x = 5$ or $x = -5$

Indices

If a and b are two rational numbers , m and n are two integers then :

	For example :
① $a^{-n} = \frac{1}{a^n}$	$5^{-1} = \frac{1}{5}$
② $a^m \times a^n = a^{m+n}$	$2^3 \times 2^2 = 2^{3+2} = 2^5 = 32$
③ $\frac{a^m}{a^n} = a^{m-n}$	$\frac{3^2}{3^{-1}} = 3^{2-(-1)} = 3^{2+1} = 3^3 = 27$
④ $(ab)^n = a^n b^n$	$(5 \times 10)^2 = 5^2 \times (10)^2 = 25 \times 100 = 2500$
⑤ $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$	$\left(\frac{3}{4}\right)^2 = \frac{3^2}{4^2} = \frac{9}{16}$
⑥ $(a^m)^n = a^{mn}$	$(2^2)^3 = 2^{2\times3} = 2^6 = 64$

The standard form of the rational number

The number is written in its standard form (scientific notation) if it is in the form :

$a \times 10^n$ where $n \in \mathbb{Z}$, $1 \le |a| < 10$

For example:

- The standard form of the number 32.4×10^5 is 3.24×10^6
- The standard form of the number 0.000423 is 4.23×10^{-4}

The perfect square rational number

It is the rational number that can be written in the form of a square of a rational number i.e. in the form (rational number)^2

For example:

The number 9 is a perfect square rational number because it can be written in the form $(3)^2$ or $(-3)^2$

Examples of prefect square rational numbers : zero , 1 , 4 , $\frac{9}{25}$, $\frac{16}{49}$, 2.25 , ...

Revision

The square root of the perfect square rational number

The square root of the perfect square rational number (a) is the rational number whose square equals (a)

For example:

- 25 has two square roots which are 5 and -5

  Because : $(5)^2 = 25$, $(-5)^2 = 25$

- $\frac{16}{49}$ has two square roots which are $\frac{4}{7}$ and $-\frac{4}{7}$

Notice that :

The two square roots of the rational number , each of them is the additive inverse of the other and their sum = zero.

Remark

The symbol $\sqrt{}$ means the positive square root of a number , then we find that :

- $\sqrt{16} = 4$, $-\sqrt{16} = -4$, $\pm\sqrt{16} = \pm 4$
- $\sqrt{0} = 0$
- $\sqrt{\text{negative number}}$ is meaningless
- $\sqrt{a^2} = |a|$

For example:

$\sqrt{3^2} = |3| = 3$, $\sqrt{(-6)^2} = |-6| = 6$, $\sqrt{\left(-\frac{2}{5}\right)^2} = \left|-\frac{2}{5}\right| = \frac{2}{5}$

- Sometimes , you need to factorize a number to its prime factors to facilitate finding its square root , then you take a factor from each two equal factors , then the product of these taken factors is the square root of this number.

For example:

$\because 441 = \underbrace{3 \times 3}_{3} \times \underbrace{7 \times 7}_{7}$

$\therefore \sqrt{441} = 3 \times 7$

$= 21$

441	3
147	3
49	7
7	7
1	

3 , 7

You can use your calculator to check your answer.

Solving equations

Example Find the solution set of each of the following equations :

1. $x+2=|-2|$, $x\in\mathbb{N}$
2. $2x-5=13$, $x\in\mathbb{Q}$
3. $x^2-4=5$, $x\in\mathbb{Q}$
4. $x^2+25=0$, $x\in\mathbb{Q}$

Solution

1. $\because x+2=2$
   $\therefore x=2-2$
   $\therefore x=0$
   $\therefore$ The S.S. $=\{0\}$

2. $\because 2x-5=13$
   $\therefore 2x=13+5$
   $\therefore 2x=18$
   $\therefore x=\frac{18}{2}$
   $\therefore x=9$
   $\therefore$ The S.S. $=\{9\}$

3. $\because x^2-4=5$
   $\therefore x^2=5+4$
   $\therefore x^2=9$
   $\therefore x=\pm\sqrt{9}$
   $\therefore x=\pm 3$
   $\therefore$ The S.S. $=\{3,-3\}$

Notice that :
We used the concept of the square root to find the value of x according to the following remark :
If $x^2=a$, then $x=\pm\sqrt{a}$

4. $\because x^2+25=0$
   $\therefore x^2=-25$
   $\therefore x=\pm\sqrt{-25}\notin\mathbb{Q}$
   (There is no square root for a negative rational number in $\mathbb{Q}$)
   $\therefore$ The S.S. $=\varnothing$

UNIT 1

Real Numbers

Lessons of the unit :

1. The cube root of a rational number.
2. The set of irrational numbers $\acute{\mathbb{Q}}$
3. The set of real numbers $\mathbb{R}$ – Ordering numbers in $\mathbb{R}$
4. Intervals.
5. Operations on the real numbers.
6. Operations on the square roots.
7. The two conjugate numbers.
8. Operations on the cube roots.
9. Applications on the real numbers.
10. Solving equations and inequalities of the first degree in one variable in $\mathbb{R}$

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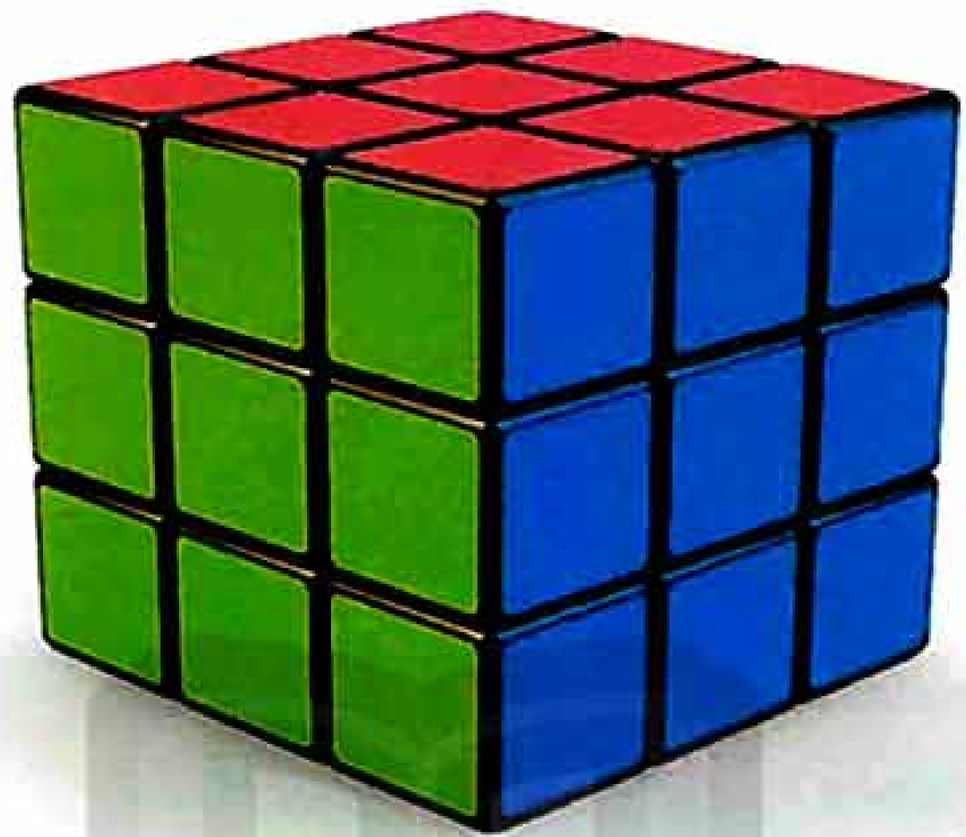
Unit Objectives :

By the end of this unit, student should be able to :

- recognize the cube root of a rational number.
- find the cube root of a rational number.
- recognize the set of irrational numbers.
- represent the irrational number on the number line.
- recognize the set of real numbers.
- perform the operations on the intervals.
- perform the arithmetic operations on the real numbers.
- solve equations and inequalities of the first degree in one variable in $\mathbb{R}$
- perform the operations on the square roots and the cube roots.
- recognize two conjugate numbers.
- apply what he studied in the real numbers to find the volumes and the areas of some of the solids.



Lesson 1

The cube root of a rational number

- The product of a number by itself three times is the cube of that number.

For example: 64 is the cube of 4 because $4 \times 4 \times 4 = 64$

- The reverse of finding the cube is finding the cube root.
- Finding the cube root of a number is finding another number if multiplied by itself three times , we get the first number.

For example: 4 is the cube root of 64 because $64 = 4 \times 4 \times 4$

Definition

The cube root of the number "a" is the number whose cube equals a

- The symbol $\sqrt[3]{\ }$ (read as "the cube root of ") is used to designate the cube root.

For example: $\sqrt[3]{64}$ designates the cube root of 64

- The cube root of a positive number is positive , and the cube root of a negative number is negative.

For example: $\sqrt[3]{64} = 4$ and $\sqrt[3]{-64} = -4$

i.e. The cube root of any number has the same sign of this number.

Finding the cube root of a rational number (representing a perfect cube)

- The perfect cube rational number is the number which can be written as a cube of a rational number **i.e.** (rational number)^3 as the numbers : $8 = 2^3$, $-27 = (-3)^3$
- The cube root of a perfect cube rational number is also a rational number.

  For example: $\sqrt[3]{8} = 2$, $\sqrt[3]{-27} = -3$
- If a number is not a perfect cube , then you indicate its cube root by using the cube root symbol.

  For example: The cube root of 4 is $\sqrt[3]{4}$ because 4 is not a perfect cube.
- You can use factorization to find the cube root of a perfect cube number , as in the following example.

Example 1 **Find each of the following :**

1. $\sqrt[3]{216}$
2. $\sqrt[3]{\frac{-8}{125}}$
3. $\sqrt[3]{0.064}$

Solution

1. $\sqrt[3]{216} = 2 \times 3 = 6$

216	2	
108	2	2
54	2	
27	3	
9	3	3
3	3	
1		

2. $\sqrt[3]{-\frac{8}{125}} = -\frac{2}{5}$

8	2	
4	2	2
2	2	
1		

125	5	
25	5	5
5	5	
1		

3. $\sqrt[3]{0.064} = \sqrt[3]{\frac{64}{1000}} = \frac{2 \times 2}{2 \times 5}$

$= \frac{4}{10} = 0.4$

64	2	
32	2	2
16	2	
8	2	
4	2	2
2	2	
1		

1000	2	
500	2	2
250	2	
125	5	
25	5	5
5	5	
1		

TRY 1 by yourself

Find each of the following :

1. $\sqrt[3]{1728}$
2. $\sqrt[3]{-0.216}$
3. $\sqrt[3]{3\frac{3}{8}}$

Final answers of try by yourself question are at the end of each lesson to check your answer.

Unit 1

Remarks

- $\sqrt[3]{a^3} = a$ For example: $\sqrt[3]{5^3} = 5$, $\sqrt[3]{(-5)^3} = -5$
- $\sqrt[3]{a^n} = a^{\frac{n}{3}}$ where $n \in \mathbb{Z}$ For example: $\sqrt[3]{a^6} = a^{\frac{6}{3}} = a^2$
- If "a" is a perfect cube number ,
  then the equation : $x^3 = a$ has a unique solution in $\mathbb{Q}$, which is $\sqrt[3]{a}$

For example :

* The equation :
  $x^3 = 8$ has a unique solution in $\mathbb{Q}$ which is $\sqrt[3]{8} = 2$
* The equation :
  $x^3 = 9$ has no solution in $\mathbb{Q}$ because 9 is not a perfect cube.

Example 2 Solve each of the following equations in $\mathbb{Q}$:

1. $40x^3 - 1 = -136$
2. $(y-2)^3 = -343$

Solution

1. $\because 40x^3 - 1 = -136$ $\therefore 40x^3 = -136 + 1$

$\therefore 40x^3 = -135$ $\therefore x^3 = -\frac{135}{40}$

$\therefore x^3 = -\frac{27}{8}$ $\therefore x = \sqrt[3]{-\frac{27}{8}}$

$\therefore x = -\frac{3}{2}$

2. $\because (y-2)^3 = -343$

Taking the cube root of each side :

$\therefore \sqrt[3]{(y-2)^3} = \sqrt[3]{-343}$ $\therefore y - 2 = -7$

$\therefore y = -7 + 2$ $\therefore y = -5$

TRY 2 by yourself

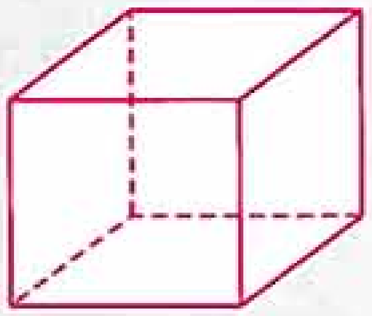
Find in $\mathbb{Q}$ the S.S. of each of the following equations :

1. $27x^3 - 2 = 62$
2. $(5x-3)^3 - 2 = 6$

Applications

Remember that

- The volume of a cube = the edge length × itself × itself
- The area of one face of a cube = the edge length × itself
- The lateral area of a cube = the area of one face × 4
- The total area of a cube = the area of one face × 6



For example: If the volume of a cube is 8 cm^3, then :

- The edge length = $\sqrt[3]{8} = 2$ cm.
- The area of one face = $2 \times 2 = 4$ cm^2.
- The lateral area = $4 \times 4 = 16$ cm^2.
- The total area = $4 \times 6 = 24$ cm^2.

Example 3 **Find each of the following :**

1. The length of the inner edge of a vessel in the shape of a cube if its capacity = 8 litres.
2. The radius length of a sphere of volume $\frac{36}{125}\pi$ cm^3.
   Knowing that : the volume of the sphere = $\frac{4}{3}\pi r^3$
   where r is the radius length of the sphere , π is the ratio between the circumference of the circle and its diameter length.
3. The diameter length of a sphere of volume equals 38808 cm^3. $(\pi \approx \frac{22}{7})$

Solution

1. $\because$ The capacity of the vessel = 8 litres = $8 \times 1000 = 8000$ cm^3.
   $\therefore$ The inner edge length = $\sqrt[3]{8000}$
   = 20 cm.

Remember that
1 litre = 1000 cm^3.

2. $\because$ The volume of the sphere = $\frac{4}{3}\pi r^3$
   $\therefore \frac{4}{3}\cancel{\pi} r^3 = \frac{36}{125}\cancel{\pi}$ $\qquad \therefore \frac{4}{3} r^3 = \frac{36}{125}$
   $\therefore r^3 = \frac{36}{125} \times \frac{3}{4}$ $\qquad \therefore r^3 = \frac{27}{125}$
   $\therefore r = \sqrt[3]{\frac{27}{125}} = \frac{3}{5}$ cm. $\qquad \therefore$ The radius length of the sphere = $\frac{3}{5}$ cm.

Unit 1

3 $\because$ The volume of the sphere $= \frac{4}{3} \pi r^3$

$\therefore \frac{4}{3} \pi r^3 = 38808$ $\therefore \frac{4}{3} \times \frac{22}{7} r^3 = 38808$

$\therefore \frac{88}{21} r^3 = 38808$ $\therefore r^3 = 38808 \times \frac{21}{88}$

$\therefore r^3 = 9261$ $\therefore r = \sqrt[3]{9261}$

9261	3
3087	3
1029	3
343	7
49	7
7	7
1	

$\therefore r = 3 \times 7 = 21$ cm.

$\therefore$ The diameter length $= 21 \times 2 = 42$ cm.

Notice that : You can use the calculator to find $\sqrt[3]{9261}$ directly.

TRY 3 by yourself

1. Find the length of the inner edge of a vessel in the shape of a cube with capacity equals 27 litres.
2. Find the length of the diameter of a sphere of volume 36π cm^3. (Knowing that : the volume of the sphere $= \frac{4}{3} \pi r^3$)

At the end of each lesson, you will find the final answers of try by yourself questions in the same form.

Answers of try by yourself

1. (1) 12 (2) -0.6 (3) $\frac{3}{2}$
2. (1) $\{\frac{4}{3}\}$ (2) $\{1\}$
3. (1) 30 cm. (2) 6 cm.

Lesson 2 The set of irrational numbers $\bar{\mathbb{Q}}$

Prelude

* You studied before that a rational number is the number that can be written as $\frac{a}{b}$ where a and b are integers and $b \neq 0$, and the set of rational numbers is denoted by $\mathbb{Q}$

* **Based on the previous , you know that :**

All integers are rational numbers	**All decimals** are rational numbers	**All percentages** are rational numbers
For example: 3 is a rational number because it can be expressed as $\frac{3}{1}$ or $\frac{6}{2}$ or ...	For example: 2.5 is a rational number because it can be expressed as $\frac{25}{10}$ or $\frac{5}{2}$ or ...	For example: 15 % is a rational number because it can be expressed as $\frac{15}{100}$ or $\frac{150}{1000}$ or ...

The square root of a perfect square rational number is a rational number	**The cube root of a perfect cube rational number** is a rational number
For example: $\sqrt{36}$, $\sqrt{\frac{4}{25}}$, $\sqrt{0.09}$ are all rational numbers where $\sqrt{36}=6$, $\sqrt{\frac{4}{25}}=\frac{2}{5}$, $\sqrt{0.09}=\sqrt{\frac{9}{100}}=\frac{3}{10}$	For example: $\sqrt[3]{8}$, $\sqrt[3]{-64}$, $\sqrt[3]{\frac{27}{1000}}$ are all rational numbers where $\sqrt[3]{8}=2$, $\sqrt[3]{-64}=-4$, $\sqrt[3]{\frac{27}{1000}}=\frac{3}{10}$

Unit 1

Irrational numbers

The square root of a rational number which is not a perfect square is not a rational number	The cube root of a rational number which is not a perfect cube is not a rational number
For example: $\sqrt{2} \notin \mathbb{Q}$ because there is no rational number whose square is 2 , so $\sqrt{2}$ cannot be written as $\frac{a}{b}$ where a and b are integers , $b \neq 0$	For example: $\sqrt[3]{4} \notin \mathbb{Q}$ because there is no rational number whose cube is 4 , so $\sqrt[3]{4}$ cannot be written as $\frac{a}{b}$ where a and b are integers , $b \neq 0$
π is not a rational number	Other examples of numbers not rational
(However $\frac{22}{7}$, 3.14 and 3.142 are rational numbers , each of them represents an approximating value of π)	$\sqrt{5}+1$, $1-\sqrt[3]{7}$, $2\sqrt{7}$, $-\frac{\sqrt[3]{9}}{5}$

The set of irrational numbers is denoted by $\grave{\mathbb{Q}}$

Notice that : $\mathbb{Q}$ and $\grave{\mathbb{Q}}$ are disjoint sets. **i.e.** $\mathbb{Q} \cap \grave{\mathbb{Q}} = \varnothing$

Example 1 **Show which of the following numbers belongs to $\mathbb{Q}$ and which of them belongs to $\grave{\mathbb{Q}}$:**

1. $\sqrt{0.49}$
2. $\sqrt[3]{-0.064}$
3. $\sqrt{\frac{25}{49}}$
4. $\sqrt[3]{\frac{25}{49}}$
5. $\sqrt{25}+\sqrt[3]{16}$

Solution

1. $\because \sqrt{0.49} = 0.7 = \frac{7}{10}$ $\therefore \sqrt{0.49} \in \mathbb{Q}$
2. $\because \sqrt[3]{-0.064} = -0.4 = -\frac{4}{10}$ $\therefore \sqrt[3]{-0.064} \in \mathbb{Q}$
3. $\because \sqrt{\frac{25}{49}} = \sqrt{\left(\frac{5}{7}\right)^2} = \frac{5}{7}$ $\therefore \sqrt{\frac{25}{49}} \in \mathbb{Q}$
4. $\because \sqrt[3]{\frac{25}{49}} \notin \mathbb{Q}$ because there is no rational number whose cube is $\frac{25}{49}$
   $\therefore \sqrt[3]{\frac{25}{49}} \in \grave{\mathbb{Q}}$
5. $\because \sqrt{25}+\sqrt[3]{16} = 5+\sqrt[3]{16}$ $\because$ There is no rational number whose cube is 16
   $\therefore \sqrt[3]{16} \notin \mathbb{Q}$ $\therefore (5+\sqrt[3]{16}) \notin \mathbb{Q}$ $\therefore (\sqrt{25}+\sqrt[3]{16}) \in \grave{\mathbb{Q}}$

Remark

- $(\sqrt{a})^2 = \sqrt{a} \times \sqrt{a} = a$, where $a \geq 0$ For example: $(\sqrt{2})^2 = \sqrt{2} \times \sqrt{2} = 2$
- $(\sqrt[3]{a})^3 = \sqrt[3]{a} \times \sqrt[3]{a} \times \sqrt[3]{a} = a$, where $a \in \mathbb{Q}$ For example: $(\sqrt[3]{-7})^3 = \sqrt[3]{-7} \times \sqrt[3]{-7} \times \sqrt[3]{-7} = -7$

Example 2 If $x \in \grave{\mathbb{Q}}$, find the S.S. of each of the following equations :

1. $x^2 = 5$
2. $x^3 = 7$
3. $\frac{2}{5} x^2 = \frac{4}{25}$
4. $64 x^3 - 2 = -29$
5. $(x^2 - 10)(x^3 - 4) = 0$

Solution

1. $\because x^2 = 5$ $\quad \therefore x = \pm\sqrt{5}$ $\quad \therefore$ The S.S. $= \{\sqrt{5}, -\sqrt{5}\}$
2. $\because x^3 = 7$ $\quad \therefore x = \sqrt[3]{7}$ $\quad \therefore$ The S.S. $= \{\sqrt[3]{7}\}$
3. $\because \frac{2}{5} x^2 = \frac{4}{25}$ $\quad \therefore x^2 = \frac{4}{25} \times \frac{5}{2}$ $\quad \therefore x^2 = \frac{2}{5}$

   $\therefore x = \pm\sqrt{\frac{2}{5}}$ $\quad \therefore$ The S.S. $= \left\{\sqrt{\frac{2}{5}}, -\sqrt{\frac{2}{5}}\right\}$
4. $\because 64 x^3 - 2 = -29$ $\quad \therefore 64 x^3 = -29 + 2$ $\quad \therefore 64 x^3 = -27$

   $\therefore x^3 = -\frac{27}{64}$ $\quad \therefore x = \sqrt[3]{-\frac{27}{64}}$ $\quad \therefore x = -\frac{3}{4}$

   $\because -\frac{3}{4} \in \mathbb{Q}$ $\quad \therefore -\frac{3}{4} \notin \grave{\mathbb{Q}}$ $\quad \therefore$ The S.S. $= \varnothing$
5. $\because (x^2 - 10)(x^3 - 4) = 0$

   $\therefore x^2 - 10 = 0$ or $x^3 - 4 = 0$

   $\therefore x^2 = 10$ $\quad \therefore x^3 = 4$

   $\therefore x = \pm\sqrt{10}$ $\quad \therefore x = \sqrt[3]{4}$

   $\therefore$ The S.S. $= \{\sqrt{10}, -\sqrt{10}, \sqrt[3]{4}\}$

Remember that

For any two numbers x , y :
If xy = zero , then
x = zero or y = zero

TRY 1 by yourself

1 *Complete using one of the symbols $\mathbb{Q}$ or $\grave{\mathbb{Q}}$:*

1. $3 \in$ ....
2. $\sqrt{3} \in$ ....
3. $9 \in$ ....
4. $\sqrt{9} \in$ ....
5. $-8 \in$ ....
6. $\sqrt[3]{-8} \in$ ....
7. $5 \in$ ....
8. $\sqrt[3]{5} \in$ ....
9. $\sqrt[3]{-9} \in$ ....

2 *Find the S.S. in $\grave{\mathbb{Q}}$ for each of the following :*

1. $2 x^3 - 7 = 3$
2. $\frac{1}{2} x^2 - 5 = 3$

Unit 1

Finding an approximated value of an irrational number

If you use the calculator to find the values of some irrational numbers , you will find that :

$\sqrt{2} \simeq 1.4142 \ldots$, $\sqrt{3} \simeq 1.73205 \ldots$, $\sqrt{5} \simeq 2.236 \ldots$

i.e. The irrational number is represented by an infinite decimal and not recurring.

And you can deduce an approximated value of the irrational number without using the calculator.

For example:

You can deduce an approximated value of the irrational number $\sqrt{5}$ as follows :

$\because 4 < 5 < 9$ (notic that we chose 4 and 9 because each of them is a perfect square , and the number 5 includes between them) and by taking the square root for all the terms.

$\therefore \sqrt{4} < \sqrt{5} < \sqrt{9}$ $\therefore 2 < \sqrt{5} < 3$

i.e. $\sqrt{5} = 2 +$ decimal less than 1

To find an approximated value of the number $\sqrt{5}$, you search for the values of the following numbers : $(2.1)^2$, $(2.2)^2$ and $(2.3)^2$

, then you find that $(2.1)^2 = 4.41$, $(2.2)^2 = 4.84$, $(2.3)^2 = 5.29$

$\because 4.84 < 5 < 5.29$ $\therefore \sqrt{4.84} < \sqrt{5} < \sqrt{5.29}$ $\therefore 2.2 < \sqrt{5} < 2.3$

We can say that 2.2 and 2.3 are approximated values of $\sqrt{5}$ and thus we can get more accurate values for the irrational number $\sqrt{5}$ and we can use the calculator to check the approximated value of the number $\sqrt{5}$

Remark

Each irrational number lies between two rational numbers.

Example 3 **Prove that :**

1. $\sqrt{3}$ lies between 1.7 and 1.8
2. $\sqrt[3]{12}$ lies between 2.2 and 2.3

Solution

1. $\because (\sqrt{3})^2 = \sqrt{3} \times \sqrt{3} = 3$, $(1.7)^2 = 2.89$, $(1.8)^2 = 3.24$

$\because 2.89 < 3 < 3.24$ $\therefore \sqrt{2.89} < \sqrt{3} < \sqrt{3.24}$ $\therefore 1.7 < \sqrt{3} < 1.8$

i.e. $\sqrt{3}$ lies between 1.7 and 1.8

You can solve the problem using the calculator as follows :

$\because \sqrt{3} \simeq 1.73$, $\because 1.7 < 1.73 < 1.8$

$\therefore 1.7 < \sqrt{3} < 1.8$ $\therefore \sqrt{3}$ lies between 1.7 and 1.8

2 $\because (\sqrt[3]{12})^3 = \sqrt[3]{12} \times \sqrt[3]{12} \times \sqrt[3]{12} = 12$, $(2.2)^3 = 10.648$, $(2.3)^3 = 12.167$

, $\because 10.648 < 12 < 12.167$ $\quad \therefore \sqrt[3]{10.648} < \sqrt[3]{12} < \sqrt[3]{12.167}$

$\therefore 2.2 < \sqrt[3]{12} < 2.3$

i.e. $\sqrt[3]{12}$ lies between 2.2 and 2.3

You can solve the problem using the calculator as follows :

$\sqrt[3]{12} \simeq 2.289$ $\quad \because 2.2 < 2.289 < 2.3$

$\therefore 2.2 < \sqrt[3]{12} < 2.3$ $\quad \therefore \sqrt[3]{12}$ lies between 2.2 and 2.3

TRY 2 by yourself

1. Find two consecutive integers such that $\sqrt{13}$ lies between them.
2. Prove that : $\sqrt{7}$ lies between 2.6 and 2.7

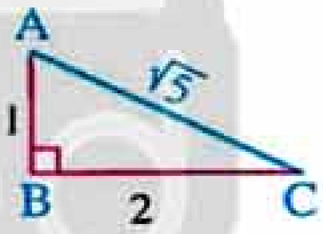
Representing an irrational number on the number line

- If you draw the right-angled triangle ABC at B such that

AB = 1 length unit , BC = 2 length units , then according to Pythagoras' theorem you find :

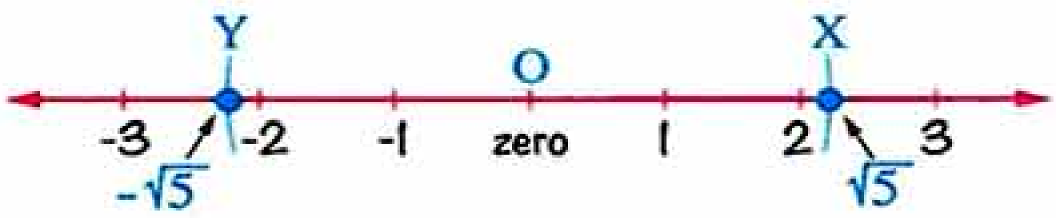
$(AC)^2 = (AB)^2 + (BC)^2 = (1)^2 + (2)^2 = 1 + 4 = 5$

$\therefore AC = \sqrt{5}$ length unit



i.e. The length of $\overline{AC}$ represents the irrational number $\sqrt{5}$

- If you draw the number line and you open the compasses with a distance equal to the length of $\overline{AC}$ and using O which represents zero as a centre and draw an arc cutting the number line at the point X on the right of the point O , then the point X represents the number $\sqrt{5}$ on the number line.



- And with the same length of $\overline{AC}$, if you use O as a centre and draw an arc cutting the number line at the point Y on the left side of O , then the point Y represents the number $-\sqrt{5}$ on the number line.

Generally

Each irrational number can be represented by a point on the number line.

Unit 1

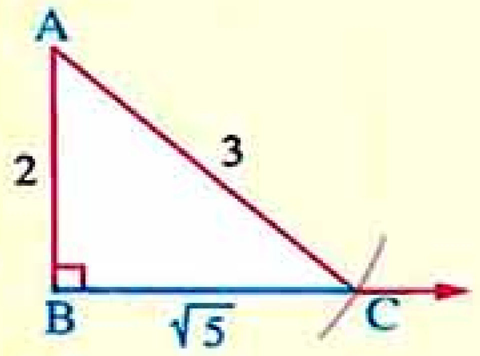
Remark

If you draw the right-angled triangle ABC at B such that

AB = 2 length units , AC = 3 length units ,

then $(BC)^2 = (AC)^2 - (AB)^2 = (3)^2 - (2)^2 = 9 - 4 = 5$

i.e. $BC = \sqrt{5}$ length unit, then you can use the length of $\overline{BC}$ to determine the point which represents $\sqrt{5}$ or $-\sqrt{5}$



From the previous , we deduce that :

To get a line segment with length that equals the irrational number $\sqrt{a}$, we search for two numbers , the sum of their squares or the difference between their squares = a , then we use them to draw a right-angled triangle.

The following figures can help you to get two numbers such that the difference between their squares equals the square of the irrational number.

- **To draw a line segment with length $\sqrt{3}$ length unit ,**

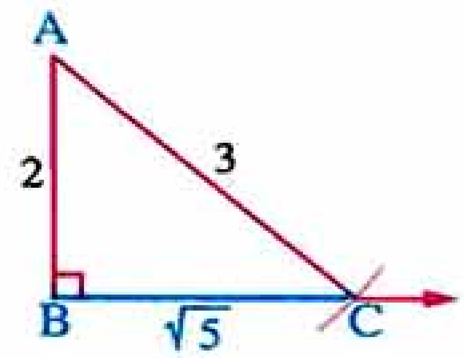
Then the length of one of the two sides of the right-angle = $\frac{3-1}{2} = 1$ length unit

and the length of the hypotenuse = $\frac{3+1}{2} = 2$ length units.

- **To draw a line segment with length $\sqrt{5}$ length unit ,**

Then the length of one of the two sides of the right-angle = $\frac{5-1}{2} = 2$ length units

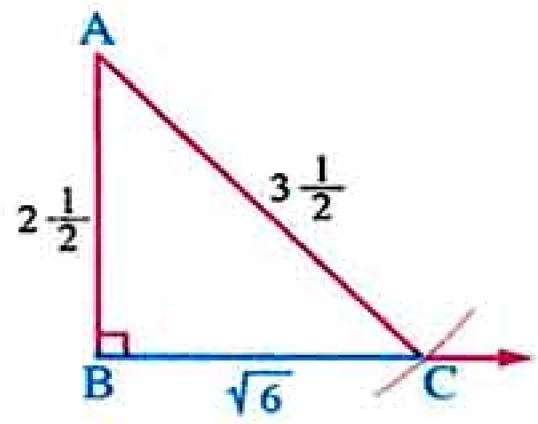
and the length of the hypotenuse = $\frac{5+1}{2} = 3$ length units.



- **To draw a line segment with length $\sqrt{6}$ length unit ,**

Then the length of one side of the right-angle = $\frac{6-1}{2} = 2\frac{1}{2}$ length units

and the length of the hypotenuse = $\frac{6+1}{2} = 3\frac{1}{2}$ length units.



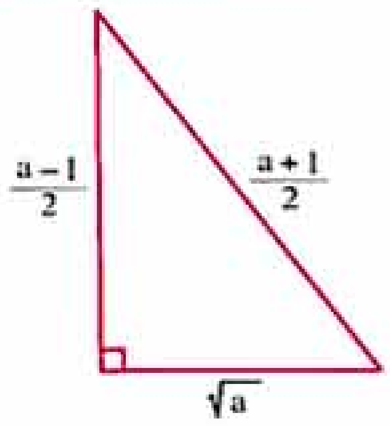
Generally

To draw a line segment with length $\sqrt{a}$ length unit where $a > 1$,

draw a right-angled triangle in which

the length of one side of the right-angle = $\frac{a-1}{2}$ length unit.

and the length of the hypotenuse = $\frac{a+1}{2}$ length unit.



Example 4 **Draw a line segment with length $\sqrt{7}$ length unit , then use it to determine the points which represent the following numbers on the number line :**

1. $\sqrt{7}$
2. $-\sqrt{7}$
3. $1+\sqrt{7}$
4. $2-\sqrt{7}$
5. $2\sqrt{7}$

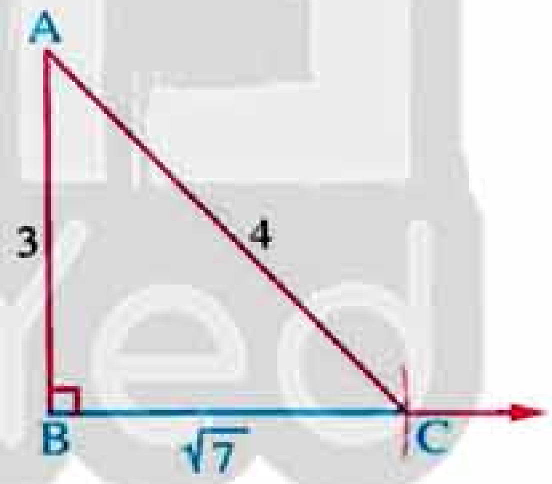
Solution **Draw the right-angled triangle ABC at B such that :**

$AB = \frac{7-1}{2} = 3$ length units

, $AC = \frac{7+1}{2} = 4$ length units.

, then $(BC)^2 = (AC)^2 - (AB)^2 = 16 - 9 = 7$

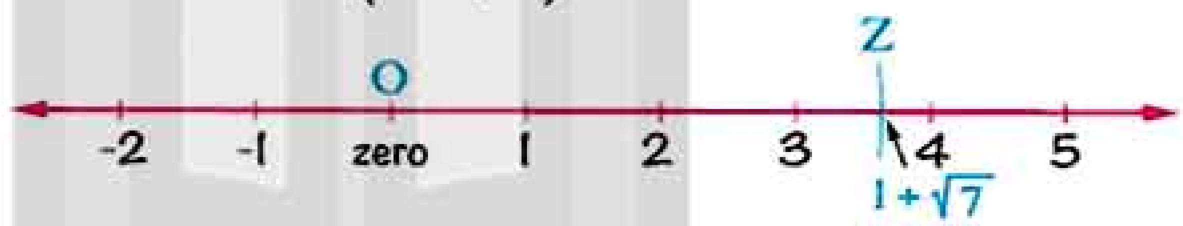
$\therefore BC = \sqrt{7}$ length unit.



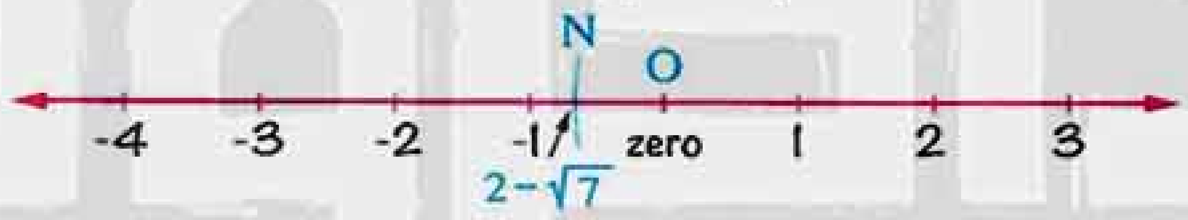
1 Using the compasses with a distance equal to the length of $\overline{BC}$ taking O as a centre , draw an arc to cut the number line on the right side of O at the point X , then X is the point which represents $\sqrt{7}$

2 Using the same previous distance and taking O as a centre , draw an arc to cut the number line on the left side of O at the point Y , then Y is the point which represents the number $-\sqrt{7}$

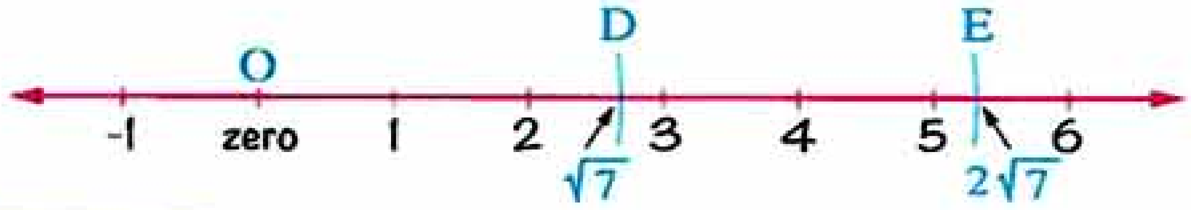
3 Using the same previous distance and taking the point which represents the number 1 on the number line as a centre , draw an arc to cut the number line on the right side of the previous point at Z , then Z represents the number $(1+\sqrt{7})$



4 Using the same previous distance and taking the point which represents the number 2 on the number line as a centre , draw an arc to cut the number line on the left side of this point at the point N , then N is the point which represents the number $(2-\sqrt{7})$

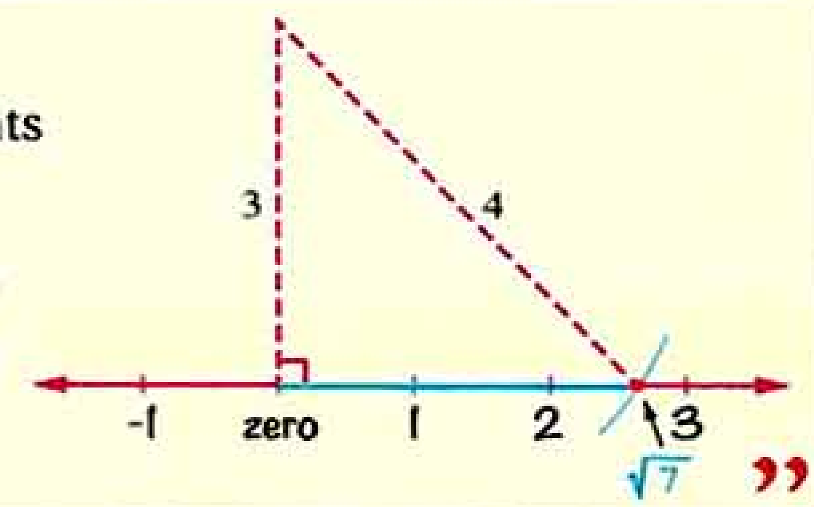


5 Using the same previous distance and taking the point O on the number line as a centre , draw an arc to cut the number line on the right side of O at the point D , then taking D as a centre and with the same previous distance in the same direction , draw an arc to cut the number line at E , then E is the point which represents the number $2\sqrt{7}$



Remark

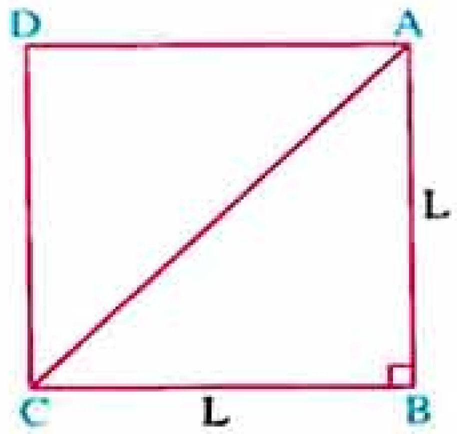
In the previous example we can determine the point which represents the number $\sqrt{7}$ on the number line by drawing the right-angled triangle directly on the number line as in the opposite figure.



Example 5 **Find the length of the diagonal of a square whose area = 5 cm^2.**

Solution

Let the side length of the square be L cm.



$\therefore$ The area of the square $= L \times L = L^2$ cm^2.

$\therefore L^2 = 5$ $\qquad\qquad$ $\therefore L = \sqrt{5}$ cm.

(**Notice that :** the square side length must be positive , so L equals $\sqrt{5}$ not $-\sqrt{5}$)

$\because \Delta ABC$ is a right-angled triangle at B ,

(from the properties of the square ABCD)

$\therefore (AC)^2 = (AB)^2 + (BC)^2 = \left(\sqrt{5}\right)^2 + \left(\sqrt{5}\right)^2 = 5 + 5 = 10$

$\therefore AC = \sqrt{10}$ cm.

$\therefore$ The length of the diagonal of the square $= \sqrt{10}$ cm.

TRY 3 by yourself

Determine the point which represents the number $\sqrt{11}$ on the number line.

Answers *of try by yourself*

1 1 (1) $\mathbb{Q}$ $\qquad$ (2) $\mathbb{Q}'$ $\qquad$ (3) $\mathbb{Q}$

(4) $\mathbb{Q}$ $\qquad$ (5) $\mathbb{Q}$ $\qquad$ (6) $\mathbb{Q}$

(7) $\mathbb{Q}$ $\qquad$ (8) $\mathbb{Q}'$ $\qquad$ (9) $\mathbb{Q}'$

2 (1) $\{\sqrt[3]{5}\}$ $\qquad$ (2) $\varnothing$

2 1 3 , 4 $\qquad$ 2 prove by yourself.

3 determine by yourself.

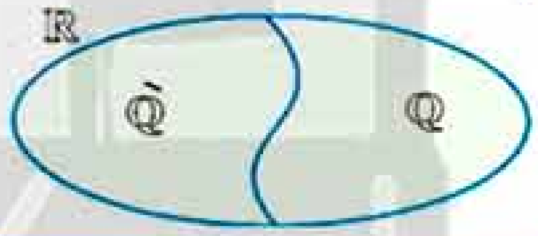
Lesson 3

The set of real numbers $\mathbb{R}$ and ordering numbers in $\mathbb{R}$

The set of real numbers

It is the set obtained from the union of the set of rational numbers and the set of irrational numbers. It is denoted by $\mathbb{R}$

i.e. $\mathbb{R} = \mathbb{Q} \cup \grave{\mathbb{Q}}$ (as shown in the opposite figure)

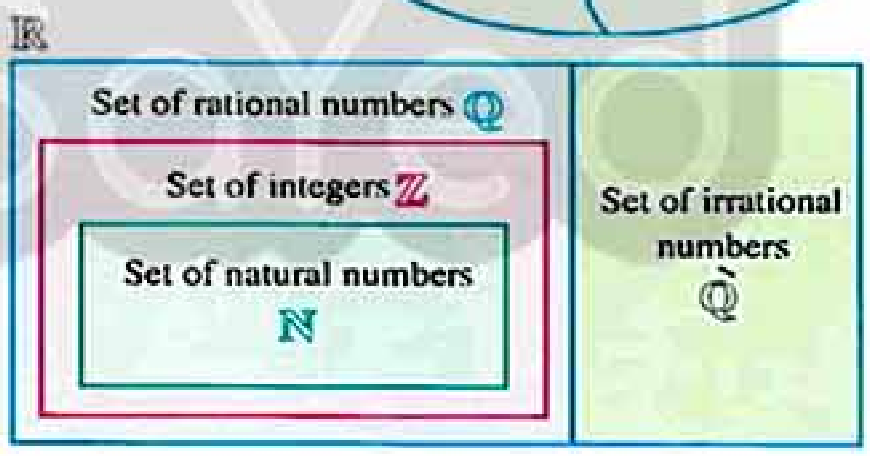


Noticing that : $\mathbb{Q} \cap \grave{\mathbb{Q}} = \varnothing$

- **The opposite Venn diagram shows that :**

$\mathbb{N} \subset \mathbb{Z} \subset \mathbb{Q} \subset \mathbb{R}$

and $\grave{\mathbb{Q}} \subset \mathbb{R}$



The following diagram shows the relation among the sets of numbers that we studied till now :

- Real numbers $\mathbb{R}$
  - Irrational numbers $\grave{\mathbb{Q}}$
  - Rational numbers $\mathbb{Q}$
    - Integers $\mathbb{Z}$
      - Natural numbers $\mathbb{N}$
        - $\{0\}$
        - Positive integers $\mathbb{Z}_+$
      - Negative integers $\mathbb{Z}_-$
    - Non integers

Ordering numbers in $\mathbb{R}$

- Each real number is represented by a unique point on the number line.
- The set of real numbers is an ordered set.
- If the point representing the number x on the number line lies on the left of the point representing the number y as shown in the figure , then $x < y$ or $y > x$

- Each real number represented by a point lying on the right side of the origin O is greater than zero , and all these numbers form a set called "the set of the positive real numbers" denoted by $\mathbb{R}_+$

$$\mathbb{R}_+ = \{x : x \in \mathbb{R} , x > \text{zero}\}$$

- Each real number represented by a point lying on the left side of the origin O is less than zero , and all these numbers form a set called "the set of the negative real numbers" denoted by $\mathbb{R}_-$

$$\mathbb{R}_- = \{x : x \in \mathbb{R} , x < \text{zero}\}$$

Remarks

- $\mathbb{R}_+ \cap \mathbb{R}_- = \varnothing$
- $\mathbb{R} = \mathbb{R}_+ \cup \{0\} \cup \mathbb{R}_-$
- The number zero is neither positive nor negative.
- $\mathbb{R}_+ \cup \{0\} = \{x : x \in \mathbb{R} , x \geq 0\}$
  and it is called the set of the non-negative real numbers.
- $\mathbb{R}_- \cup \{0\} = \{x : x \in \mathbb{R} , x \leq 0\}$
  and it is called the set of the non-positive real numbers.
- The set of real numbers without zero (The non-zero real numbers) is denoted by $\mathbb{R}^*$
  i.e. $\mathbb{R}^* = \mathbb{R} - \{0\} = \mathbb{R}_+ \cup \mathbb{R}_-$

Unit 1

Example 1 **Arrange the following numbers ascendingly :**

$\sqrt{75}$, $\sqrt{68}$, $-\sqrt{45}$, -8 , 7 and $-\sqrt{32}$

Solution

- **Arrange the positive numbers which are** $\sqrt{75}$, $\sqrt{68}$ and 7

$\because 7 = \sqrt{49}$

, $\because 49 < 68 < 75$ $\quad \therefore \sqrt{49} < \sqrt{68} < \sqrt{75}$

i.e. $7 < \sqrt{68} < \sqrt{75}$

- **Arrange the negative numbers which are** $-\sqrt{45}$, -8 and $-\sqrt{32}$

$\because 8 = \sqrt{64}$

, $\because 64 > 45 > 32$ $\quad \therefore \sqrt{64} > \sqrt{45} > \sqrt{32}$

$\therefore -\sqrt{64} < -\sqrt{45} < -\sqrt{32}$

i.e. $-8 < -\sqrt{45} < -\sqrt{32}$

$\therefore$ The ascending order is : -8 , $-\sqrt{45}$, $-\sqrt{32}$, 7 , $\sqrt{68}$ and $\sqrt{75}$

Remark

You can use the calculator to get the solution by finding approximated values of the roots.

Example 2 **Write three irrational numbers included between 11 and 12**

Solution

$\because (11)^2 = 121$, $(12)^2 = 144$

$\therefore$ 125 , 126 and 130 are three integers included between 121 and 144

$\therefore 121 < 125 < 126 < 130 < 144$

$\therefore \sqrt{121} < \sqrt{125} < \sqrt{126} < \sqrt{130} < \sqrt{144}$

$\therefore$ The required irrational numbers are : $\sqrt{125}$, $\sqrt{126}$ and $\sqrt{130}$

(Notice that : there are other irrational numbers included between 11 and 12)

Example 3 Find the S.S. in $\mathbb{R}$ for each of the following equations :

1. $3x^2 + 125 = 221$
2. $\frac{1}{6}x^3 - 8 = 28$
3. $2x^2 + 6 = 4$

Solution

1. $\because 3x^2 + 125 = 221$ $\quad \therefore 3x^2 = 221 - 125$ $\quad \therefore 3x^2 = 96$

   $\therefore x^2 = \frac{96}{3}$ $\quad \therefore x^2 = 32$ $\quad \therefore x = \pm\sqrt{32}$

   $\therefore$ The S.S. $= \{\sqrt{32}, -\sqrt{32}\}$

2. $\because \frac{1}{6}x^3 - 8 = 28$ $\quad \therefore \frac{1}{6}x^3 = 36$ $\quad \therefore x^3 = 6 \times 36$

   $\therefore x^3 = 216$ $\quad \therefore x = \sqrt[3]{216}$ $\quad \therefore x = 6$

   $\therefore$ The S.S. $= \{6\}$

3. $\because 2x^2 + 6 = 4$ $\quad \therefore 2x^2 = 4 - 6$ $\quad \therefore 2x^2 = -2$

   $\therefore x^2 = -\frac{2}{2}$ $\quad \therefore x^2 = -1$ $\quad \therefore x = \pm\sqrt{-1}$

   $, \because \sqrt{-1} \notin \mathbb{R}, -\sqrt{-1} \notin \mathbb{R}$ $\quad \therefore$ The S.S. $= \varnothing$

TRY by yourself

Complete each of the following using one of the symbols > or < :

1. $\sqrt{2}$ 1
2. $-\sqrt{3}$ .... -1
3. $\sqrt[3]{9}$ 3
4. $-\sqrt[3]{7}$ .... -2
5. $\sqrt{7}$ 2.6
6. $-\sqrt[3]{16}$ .... -2.52

Answers of try by yourself

1. >
2. <
3. <
4. >
5. >
6. >

Lesson 4

Intervals

Prelude • Through your previous study , you knew different methods to express a subset of the set of natural numbers and a subset of the set of integers and you learnt how to represent them on the number line.

For example:

If X = the set of integers which are greater than or equal to -3 and less than 2

* Then you can express the set X by the description method as follows : → $X = \{a : a \in \mathbb{Z} , -3 \leq a < 2\}$
* You can also express it by listing method as follows : → $X = \{-3 , -2 , -1 , 0 , 1\}$
* The set X is represented on the number line as shown in the figure : →

• **And now the question is : Is it possible to use the same previous methods to express a subset of the set of real numbers and represent it on the number line ?**

Assuming that : K = the set of real numbers that are greater than or equal to -3 and less than 2

* You can express the set K by the description method as follows : → $K = \{a : a \in \mathbb{R} , -3 \leq a < 2\}$
* But it is impossible to express the set K by listing method because there are an infinity of real numbers existing between -3 and 2
* For the same reason , it is impossible to represent this set K by separate points on the number line as shown in the previous figure therefore we use another method to express a subset of the set of real numbers , which is the intervals.

• **In the following , we will show the types of intervals :**

First Limited intervals

A Closed interval

- The set $\{x : x \in \mathbb{R} , -3 \le x \le 2\}$ expresses the set of real numbers which consists of the two numbers -3 and 2 and all the real numbers included between them.

  We denote it by $[-3 , 2]$ and it is called a «closed interval».

Notice that :
The smaller number must be written first when you write the interval.

- It is represented on the number line as shown in the figure :

Notice that : $-3 \in [-3 , 2]$, $2 \in [-3 , 2]$

We express this by drawing two shaded circles at the two points representing the two numbers -3 and 2

B Opened interval

- The set $\{x : x \in \mathbb{R} , -3 < x < 2\}$ expresses the set of real numbers included between the two numbers -3 and 2 such that the two numbers -3 and 2 are not contained in this set.

  We denote this set by $]-3 , 2[$ and it is called an «opened interval».

- It is represented on the number line as in the figure :

Notice that : $-3 \notin]-3 , 2[$ and $2 \notin]-3 , 2[$

We express this by drawing two unshaded circles at the two points representing the two numbers -3 and 2

C Half opened interval (Half closed interval)

1 • The set $\{x : x \in \mathbb{R} , -3 \le x < 2\}$ expresses the number -3 and all the real numbers included between -3 and 2 without the number 2 , we denote it by $[-3 , 2[$ and it is called a «half opened interval» or «half closed interval».

- It is represented on the number line as in the figure :

Notice that : $-3 \in [-3 , 2[$, $2 \notin [-3 , 2[$

2 • The set $\{x : x \in \mathbb{R} , -3 < x \le 2\}$ expresses the number 2 and all the real numbers included between -3 and 2 without the number -3 , we denote it by $]-3 , 2]$ and it is called a «half opened interval» or «half closed interval».

- It is represented on the number line as in the figure :

Notice that : $-3 \notin]-3 , 2]$, $2 \in]-3 , 2]$

Unit 1

Second Unlimited intervals

1 • The set $\{x : x \in \mathbb{R} , x \geq 2\}$ expresses the set of real numbers which consists of the number 2 and all the real numbers which are greater than 2 with no end.

It is denoted by $[2 , \infty[$ where the symbol «∞» is read as positive infinity and it doesn't represent a real number

• It is represented on the number line as shown in the figure :

Notice that : $2 \in [2 , \infty[$

2 • The set $\{x : x \in \mathbb{R} , x > 2\}$ expresses the set of all real numbers which are greater than the number 2 with no end. It is denoted by $]2 , \infty[$

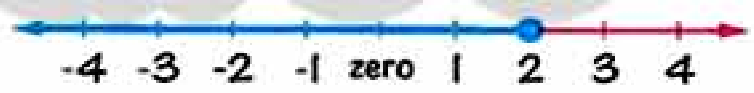
• It is represented on the number line as shown in the figure :

Notice that : $2 \notin]2 , \infty[$

3 • The set $\{x : x \in \mathbb{R} , x \leq 2\}$ expresses the set of real numbers which consists of the number 2 and all the real numbers which are smaller than the number 2 with no end.

It is denoted by $]-\infty , 2]$ where the symbol «$-\infty$» is read as negative infinity and it doesn't represent a real number.

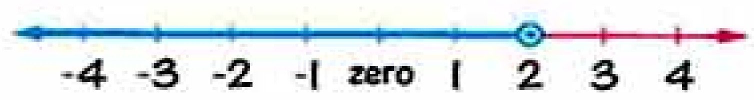
• It is represented on the number line as shown in the figure :



Notice that : $2 \in]-\infty , 2]$

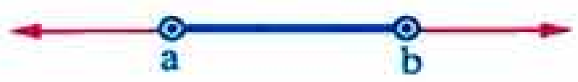
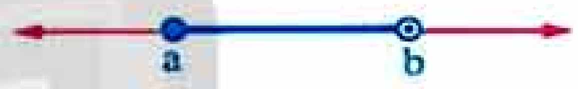
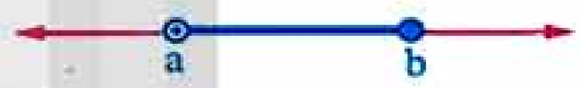
4 • The set $\{x : x \in \mathbb{R} , x < 2\}$ expresses the set of all real numbers which are smaller than the number 2 with no end. It is denoted by $]-\infty , 2[$

• It is represented on the number line as shown in the figure :



Notice that : $2 \notin]-\infty , 2[$

• We can express the previous symbolically in the following table assuming that :
$a \in \mathbb{R}$, $b \in \mathbb{R}$ and $a < b$

Types of intervals		The interval	Expression by distinguished property	Representation on the number line	Notice that
The limited intervals	Closed	$[a , b]$	$\{x : x \in \mathbb{R} , a \leq x \leq b\}$		• $a \in [a , b]$ • $b \in [a , b]$
	Opened	$]a , b[$	$\{x : x \in \mathbb{R} , a < x < b\}$		• $a \notin]a , b[$ • $b \notin]a , b[$
	half opened (half closed)	$[a , b[$	$\{x : x \in \mathbb{R} , a \leq x < b\}$		• $a \in [a , b[$ • $b \notin [a , b[$
		$]a , b]$	$\{x : x \in \mathbb{R} , a < x \leq b\}$		• $a \notin]a , b]$ • $b \in]a , b]$
The unlimited intervals		$[a , \infty[$	$\{x : x \in \mathbb{R} , x \geq a\}$		$a \in [a , \infty[$
		$]a , \infty[$	$\{x : x \in \mathbb{R} , x > a\}$		$a \notin]a , \infty[$
		$]-\infty , a]$	$\{x : x \in \mathbb{R} , x \leq a\}$		$a \in]-\infty , a]$
		$]-\infty , a[$	$\{x : x \in \mathbb{R} , x < a\}$		$a \notin]-\infty , a[$

Remarks

1. $\mathbb{R} =]-\infty , \infty[$
2. $\mathbb{R}_+ =]0 , \infty[$
3. $\mathbb{R}_- =]-\infty , 0[$
4. The set of non-negative real numbers $= \mathbb{R}_+ \cup \{0\} = [0 , \infty[$
5. The set of non-positive real numbers $= \mathbb{R}_- \cup \{0\} =]-\infty , 0]$

Example 1 Write each of the following sets in the form of an interval , then represent it on the number line :

1. $\{x : x \in \mathbb{R} , -3 < x \leq 0\}$
2. $\{a : a \in \mathbb{R} , 1 \geq a \geq -2\}$
3. $\{x : x \in \mathbb{R} , x > 0\}$
4. $\{y : y \in \mathbb{R} , -1 \geq y\}$

Unit 1

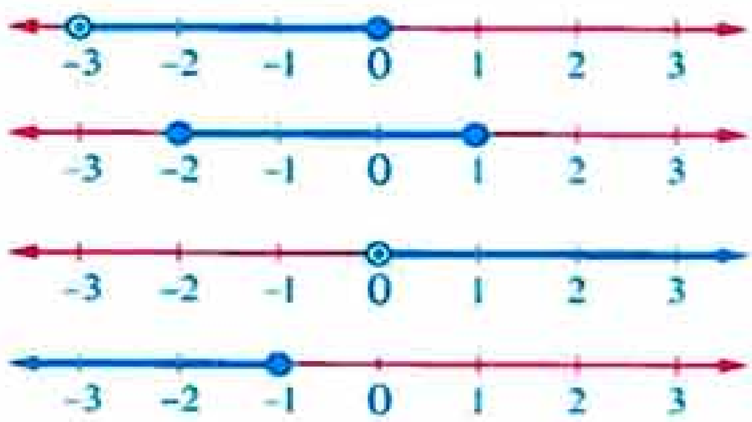
Solution

1 $]-3 , 0]$

2 $[-2 , 1]$

3 $]0 , \infty[$

4 $]-\infty , -1]$



TRY 1 by yourself

1 ***Write each of the following in an interval form , then represent it on the number line :***

1 $\{x : x \in \mathbb{R} , -4 < x \leq 2\}$ 2 $\{y : y \in \mathbb{R} , y \geq -5\}$

2 ***Represent each of the following on the number line and express it by the description method :***

1 $]-3 , 0[$ 2 $]-\infty , 2]$

Operations on intervals

You studied before the sets and how to carry out the operations of intersection , union , difference and complement on them.

For example:

If $X = \{1 , 2 , 3 , 4\}$, $Y = \{3 , 4 , 5 , 6\}$, then :

- $X \cap Y$ = the set of elements which are common in X and Y $= \{3 , 4\}$
- $X \cup Y$ = the set of all elements in X or Y without repeating $= \{1 , 2 , 3 , 4 , 5 , 6\}$
- $X - Y$ = the set of elements which are in X and not in Y $= \{1 , 2\}$
- $Y - X$ = the set of elements which are in Y and not in X $= \{5 , 6\}$
- If the universal set $U = \{1 , 2 , 3 , 4 , 5 , 6 , 7\}$,

then the complement of X which is denoted by $\grave{X} = U - X$

i.e. $\grave{X}$ = the set of elements which are in U and not in X $= \{5 , 6 , 7\}$

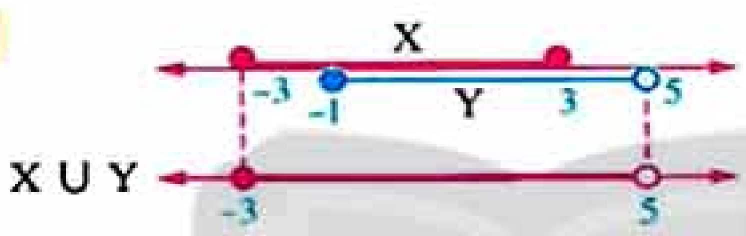
The following examples show how to carry out the operations of intersection , union and difference on intervals :

Example 2 If $X = [-3\ ,\ 3]$ and $Y = [-1\ ,5\ [$, find using the number line :

1. $X \cup Y$
2. $X \cap Y$
3. $X - Y$
4. $Y - X$

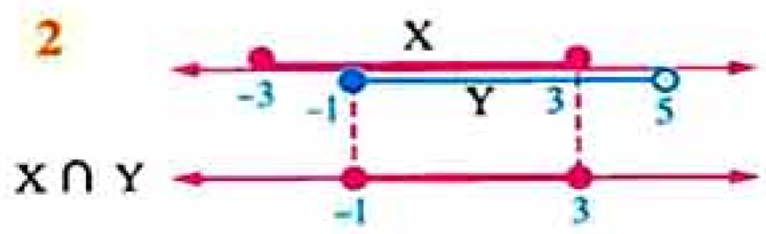
Solution

1



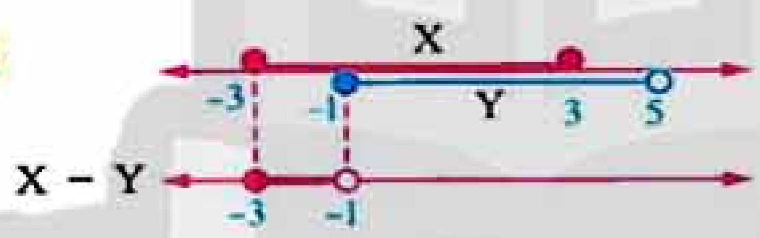
$X \cup Y = [-3\ ,5[$

2



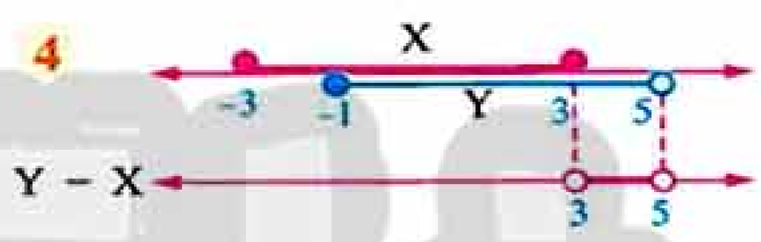
$X \cap Y = [-1\ ,3]$

3



$X - Y = [-3\ ,-1[$

4



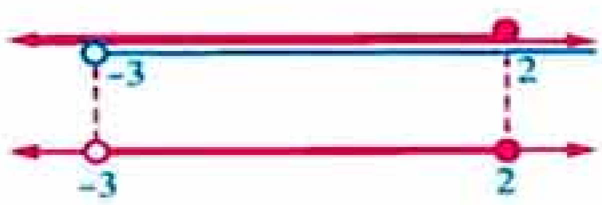
$Y - X =]3\ ,5[$

Example 3 Find each of the following :

1. $]-\infty, 2\,] \cap\,]-3\ ,\infty\,[$
2. $]-\infty, 3\,] \cup [-2\ ,5\,[$
3. $[5\ ,\infty\,[\, -\,]\,5\ ,\infty\,[$
4. $[2\ ,\infty\,[\, \cap\,]-\infty\ ,2\,[$

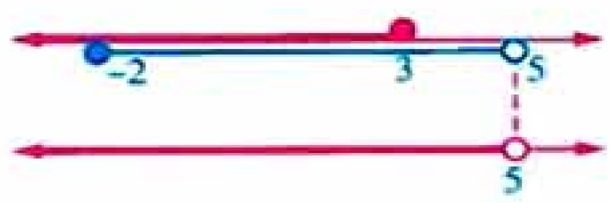
Solution

1



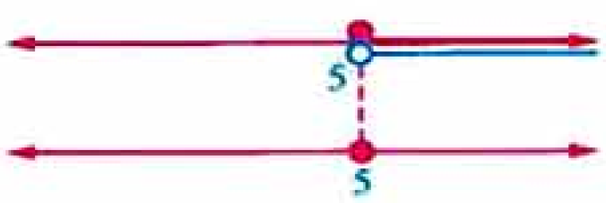
$]-\infty\ ,2\,] \cap\,]-3\ ,\infty[\, =\,]-3\ ,2\,]$

2



$]-\infty\ ,3\,] \cup [-2\ ,5\,[\, =\,]-\infty\ ,5\,[$

3



$[5\ ,\infty[\, -\,]\,5\ ,\infty\,[\, = \{5\}$

4

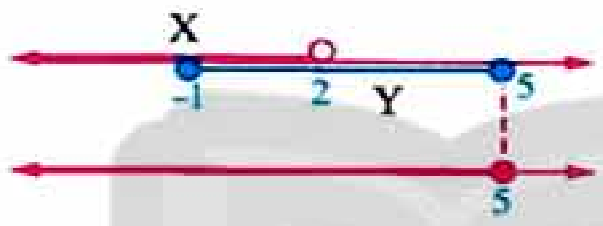
$[2\ ,\infty[\, \cap\,]-\infty\ ,2[\, = \varnothing$

Unit 1

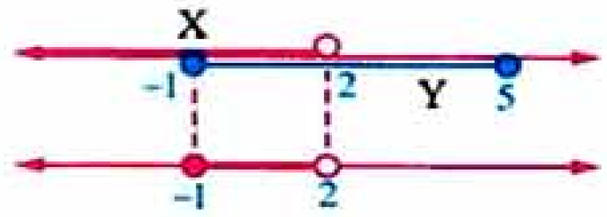
Example 4 If $X =]-\infty , 2[$ and $Y = [-1 , 5]$, find using the number line :

1. $X \cup Y$
2. $X \cap Y$
3. $X - Y$
4. $Y - X$
5. $\grave{X}$
6. $\grave{Y}$

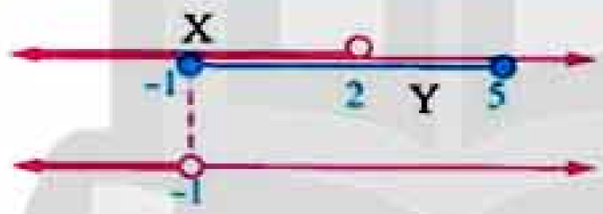
Solution

1 

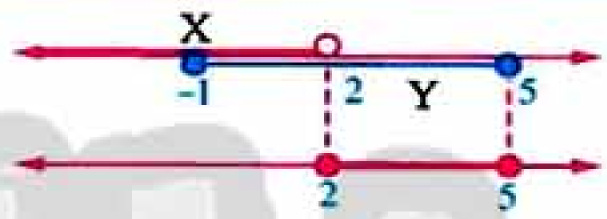
$X \cup Y =]-\infty , 5]$

2 

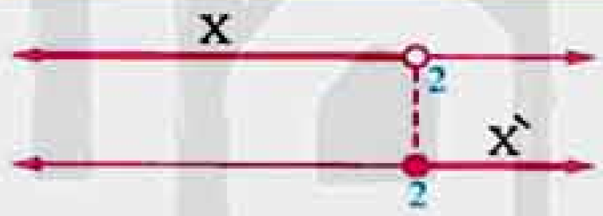
$X \cap Y = [-1 , 2[$

3 

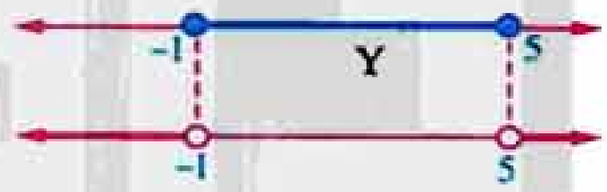
$X - Y =]-\infty , -1[$

4 

$Y - X = [2 , 5]$

5 

$\grave{X} = [2 , \infty[$

6 

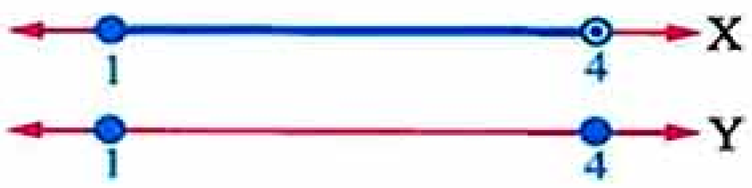
$\grave{Y} =]-\infty , -1[\cup]5 , \infty[$
$= \mathbb{R} - [-1 , 5]$

Example 5 If $X = [1 , 4[$ and $Y = \{1 , 4\}$, find :

1. $X \cap Y$
2. $X \cup Y$
3. $X - Y$
4. $Y - X$

Solution

1. $X \cap Y = \{1\}$
2. $X \cup Y = [1 , 4]$
3. $X - Y =]1 , 4[$
4. $Y - X = \{4\}$



TRY 2 by yourself

If $X = [-1 , 3[$ and $Y =]0 , 4]$, find using the number line :

1. $X \cap Y$
2. $X \cup Y$
3. $X - [0 , \infty[$
4. $]0 , \infty[- Y$
5. $\grave{X}$
6. $X \cap \{-2 , -1 , 0 , 1 , 2 , 3\}$

Answers of try by yourself

1 1 (1) $]-4 , 2]$

(2) $[-5 , \infty[$

2 (1)

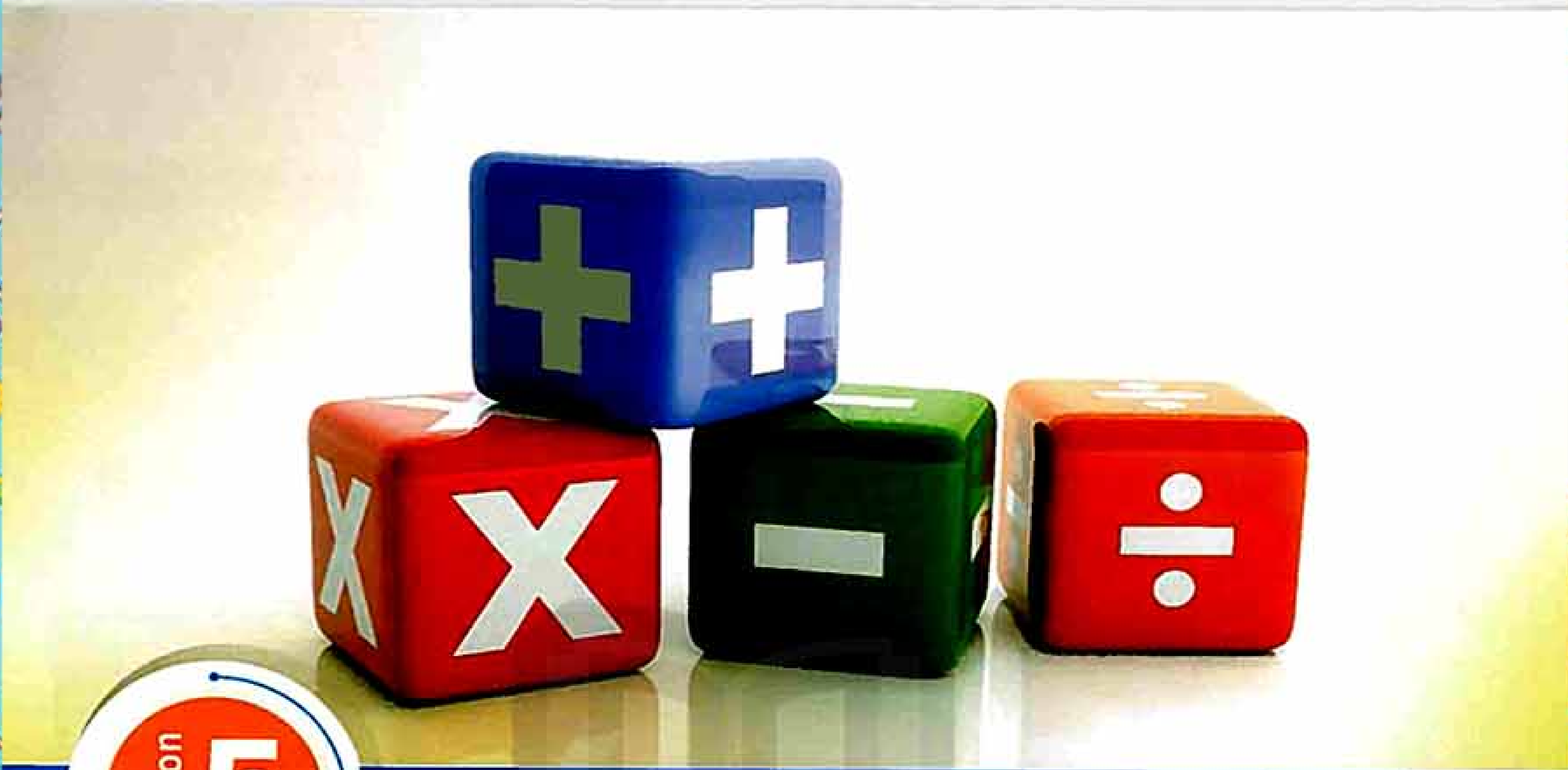
$\{x : x \in \mathbb{R} , -3 < x < 0\}$

(2)

$\{x : x \in \mathbb{R} , x \leq 2\}$

2 1 $]0 , 3[$ 2 $[-1 , 4]$ 3 $[-1 , 0[$

4 $]4 , \infty[$ 5 $]-\infty , -1[\cup [3 , \infty[= \mathbb{R} - [-1 , 3[$ 6 $\{-1 , 0 , 1 , 2\}$



Lesson 5 Operations on the real numbers

First Addition

- We know that $2x$ and $3x$ are two like algebraic terms and their sum is an algebraic term like them.

Where $2x + 3x = (2 + 3)\,x = 5\,x$

Then we deduce that : $2\sqrt{5} + 3\sqrt{5} = (2 + 3)\sqrt{5}$

$= 5\sqrt{5}$

Remember that

The real number $2\sqrt{5}$ is produced by multiplying the rational number 2 by the irrational number $\sqrt{5}$

- We know that $2x$ and 3 y are two unlike algebraic terms and we express their sum by an algebraic expression whose simplest form is $2x + 3y$

Therefore we deduce that :

The two real numbers $2\sqrt{3}$ and $3\sqrt{2}$, their sum is expressed by a real number whose simplest form is $2\sqrt{3} + 3\sqrt{2}$

Example 1 Find the result of :

$-3\sqrt{5} + 4\sqrt{5} + \left(-2\sqrt{5}\right)$

Solution

$-3\sqrt{5} + 4\sqrt{5} + \left(-2\sqrt{5}\right) = \left(-3 + 4 + (-2)\right)\sqrt{5}$

$= -\sqrt{5}$

Properties of addition of real numbers

Closure

For every $a \in \mathbb{R}$ and $b \in \mathbb{R}$ we find that $(a + b) \in \mathbb{R}$

i.e. The sum of any two real numbers is a real number, therefore we say $\mathbb{R}$ is closed under addition.

For example: $\sqrt{5} \in \mathbb{R}$ and $2\sqrt{5} \in \mathbb{R}$, we find that : $\sqrt{5} + 2\sqrt{5} = 3\sqrt{5} \in \mathbb{R}$

Commutative property

For every $a \in \mathbb{R}$ and $b \in \mathbb{R}$ it will be $a + b = b + a$

For example: $5\sqrt[3]{2} + 4\sqrt[3]{2} = 9\sqrt[3]{2}$, $4\sqrt[3]{2} + 5\sqrt[3]{2} = 9\sqrt[3]{2}$

i.e. $5\sqrt[3]{2} + 4\sqrt[3]{2} = 4\sqrt[3]{2} + 5\sqrt[3]{2}$

Associative property

For every $a \in \mathbb{R}$, $b \in \mathbb{R}$ and $c \in \mathbb{R}$ it will be $(a + b) + c = a + (b + c) = a + b + c$

For example: $\left(\sqrt{3} + 2\sqrt{3}\right) + 5\sqrt{3} = 3\sqrt{3} + 5\sqrt{3} = 8\sqrt{3}$,

$\sqrt{3} + \left(2\sqrt{3} + 5\sqrt{3}\right) = \sqrt{3} + 7\sqrt{3} = 8\sqrt{3}$

i.e. $\left(\sqrt{3} + 2\sqrt{3}\right) + 5\sqrt{3} = \sqrt{3} + \left(2\sqrt{3} + 5\sqrt{3}\right)$

The additive neutral

For every $a \in \mathbb{R}$ it will be $a + 0 = 0 + a = a$

i.e. Zero is the additive neutral.

For example: $\sqrt{2} + 0 = 0 + \sqrt{2} = \sqrt{2}$, $-\sqrt[3]{5} + 0 = 0 + \left(-\sqrt[3]{5}\right) = -\sqrt[3]{5}$

The additive inverse of every real number

For every $a \in \mathbb{R}$ there is $(-a) \in \mathbb{R}$ where $a + (-a) =$ zero (the additive neutral)

For example:
- The additive inverse of $\sqrt{3}$ is $-\sqrt{3}$ and vice versa because $\sqrt{3} + \left(-\sqrt{3}\right) = 0$
- The additive inverse of $2 + \sqrt{5}$ is $-\left(2 + \sqrt{5}\right)$ and equals $-2 - \sqrt{5}$
- The additive inverse of $3 - \sqrt{2}$ is $-\left(3 - \sqrt{2}\right)$ and equals $\sqrt{2} - 3$
- The additive inverse of zero is itself.

Unit 1

Remark

Since every real number has an additive inverse , then the subtraction operation is possible entirely in $\mathbb{R}$, and it is defined as follows :

For every $a \in \mathbb{R}$ and $b \in \mathbb{R}$ it will be $a - b = a + (-b)$

i.e. The subtraction operation $(a - b)$ means adding the number a to the additive inverse of the number b

And we can deduce that :

Subtraction operation in $\mathbb{R}$ is not commutative and it is not associative.

Example 2 Simplify to the simplest form : $4 + \sqrt{3} - 7 - \sqrt{3}$

Solution

$4 + \sqrt{3} - 7 - \sqrt{3} = (4 - 7) + \left(\sqrt{3} - \sqrt{3}\right) = -3 + 0 = -3$

TRY by yourself **1**

1 *Write the additive inverse for each of the following numbers :*

$\sqrt{2}$, $-\sqrt[3]{5}$, $\sqrt{2} + \sqrt{7}$, $\sqrt[3]{5} - 3$, $-\sqrt{6} - \sqrt[3]{7}$

2 *Simplify to the simplest form :*

1 $2 + 2\sqrt{7} - 1 - 5\sqrt{7}$

2 $3\sqrt{5} + \sqrt{3} - 3\sqrt{5} + 5\sqrt{3}$

Second Multiplication

- We know that : $3 \times 2x = (3 \times 2)x = 6x$

  Therefore we find that : $3 \times 2\sqrt{3} = (3 \times 2)\sqrt{3} = 6\sqrt{3}$

- We know also $2x \times 5x = (2 \times 5)(x \times x) = 10x^2$

  Therefore we find that : $2\sqrt{3} \times 5\sqrt{3} = (2 \times 5) \times \left(\sqrt{3} \times \sqrt{3}\right) = 10\left(\sqrt{3}\right)^2 = 10 \times 3 = 30$

Example 3 Find the result of each of the following :

1 $-2 \times 3\sqrt{5}$

2 $4\sqrt{2} \times \sqrt{2}$

3 $-2\sqrt{7} \times 4\sqrt{7}$

Solution

1 $-2 \times 3\sqrt{5} = (-2 \times 3)\sqrt{5} = -6\sqrt{5}$

2 $4\sqrt{2} \times \sqrt{2} = 4\left(\sqrt{2}\right)^2 = 4 \times 2 = 8$

3 $-2\sqrt{7} \times 4\sqrt{7} = (-2 \times 4) \times \left(\sqrt{7}\right)^2 = -8 \times 7 = -56$

Properties of multiplication of real numbers

Closure

For every $a \in \mathbb{R}$ and $b \in \mathbb{R}$ it will be $a \times b \in \mathbb{R}$

i.e. The product of any two real numbers is a real number therefore we say :

$\mathbb{R}$ is closed under multiplication.

For example: $\sqrt{3} \in \mathbb{R}$ and $2\sqrt{3} \in \mathbb{R}$

We find that : $\sqrt{3} \times 2\sqrt{3} = 2 \times 3 = 6 \in \mathbb{R}$

Commutative property

For every $a \in \mathbb{R}$ and $b \in \mathbb{R}$ it will be $a \times b = b \times a$

For example: $2\sqrt{5} \times 3\sqrt{5} = 6 \times 5 = 30$, $3\sqrt{5} \times 2\sqrt{5} = 6 \times 5 = 30$

i.e. $2\sqrt{5} \times 3\sqrt{5} = 3\sqrt{5} \times 2\sqrt{5}$

Associative property

For every $a \in \mathbb{R}$, $b \in \mathbb{R}$ and $c \in \mathbb{R}$ it will be $(a \times b) \times c = a \times (b \times c) = a \times b \times c$

For example: $\left(2\sqrt{7} \times 4\sqrt{7}\right) \times \sqrt{7} = 56 \times \sqrt{7} = 56\sqrt{7}$,

$2\sqrt{7} \times \left(4\sqrt{7} \times \sqrt{7}\right) = 2\sqrt{7} \times 28 = 56\sqrt{7}$

i.e. $\left(2\sqrt{7} \times 4\sqrt{7}\right) \times \sqrt{7} = 2\sqrt{7} \times \left(4\sqrt{7} \times \sqrt{7}\right)$

The multiplicative neutral

For every $a \in \mathbb{R}$ it will be $a \times 1 = 1 \times a = a$

i.e. One is the multiplicative neutral in $\mathbb{R}$

For example: $\sqrt[3]{5} \times 1 = 1 \times \sqrt[3]{5} = \sqrt[3]{5}$

The multiplicative inverse of any non-zero real number

For every real number $a \neq 0$, there is a real number $\frac{1}{a}$ where $a \times \frac{1}{a} = 1$ which is the multiplicative neutral.

Unit 1

For example:

- The multiplicative inverse of $\sqrt{3}$ is $\frac{1}{\sqrt{3}}$ because $\sqrt{3} \times \frac{1}{\sqrt{3}} = 1$
- The multiplicative inverse of $-\frac{\sqrt{2}}{5}$ is $-\frac{5}{\sqrt{2}}$
- The multiplicative inverse of 1 is itself and also the multiplicative inverse of -1 is itself.

Notice that :

- Both the number and its multiplicative inverse have the same sign.
- There is no multiplicative inverse for zero because $\frac{1}{\text{zero}}$ is meaningless (i.e. undefined)

Remark

Since each non-zero real number has a multiplicative inverse , then the division operation by any real number does not equal zero is possible in $\mathbb{R}$ and it is defined as follows :

For every $a \in \mathbb{R}$ and $b \in \mathbb{R}^*$ it will be $a \div b = a \times \frac{1}{b}$

i.e. The division operation $(a \div b)$ means multiplying the number a by the multiplicative inverse of the number b such that $b \neq 0$

And we can deduce that :

Division operation in $\mathbb{R}$ is not commutative and it is not associative.

Example 4 Find the result of : $\frac{\sqrt{5}}{5} \times \frac{4\sqrt{5}}{12\sqrt{2}} \div \frac{1}{3\sqrt{2}}$

Solution

$$\left(\frac{\sqrt{5}}{5} \times \frac{4\sqrt{5}}{12\sqrt{2}}\right) \div \frac{1}{3\sqrt{2}} = \frac{5}{15\sqrt{2}} \div \frac{1}{3\sqrt{2}} = \frac{1}{3\sqrt{2}} \times 3\sqrt{2} = 1$$

Example 5 Write each of the following such that the denominator is an integer :

1. $\frac{9}{\sqrt{3}}$
2. $-\frac{3}{\sqrt{2}}$
3. $\frac{5}{3\sqrt{5}}$

Solution

1 Multiplying the two terms of $\frac{9}{\sqrt{3}}$ by $\sqrt{3}$

, we get $\frac{9}{\sqrt{3}} = \frac{9}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{9\sqrt{3}}{3} = 3\sqrt{3}$

Notice that : $\frac{\sqrt{3}}{\sqrt{3}} = 1$ "The multiplicative neutral"

2 $-\frac{3}{\sqrt{2}} = -\frac{3}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{-3\sqrt{2}}{2}$

3 $\frac{5}{3\sqrt{5}} = \frac{5}{3\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{5\sqrt{5}}{3 \times 5} = \frac{\sqrt{5}}{3}$

Another solution :

$\because \sqrt{5} \times \sqrt{5} = 5$ $\quad \therefore \frac{5}{3\sqrt{5}} = \frac{\sqrt{5} \times \sqrt{5}}{3\sqrt{5}} = \frac{\sqrt{5}}{3}$

TRY 2 by yourself

1 ***Find each of the following :***

1 $\sqrt{5} \times \frac{1}{\sqrt{5}} \times \sqrt{5}$ $\quad$ 2 $\frac{\sqrt{3}}{3} \times \frac{4\sqrt{5}}{20} \times \frac{5\sqrt{3}}{\sqrt{5}}$

2 ***Make the denominator an integer :***

1 $\frac{3}{\sqrt{7}}$ $\quad$ 2 $\frac{9}{2\sqrt{6}}$

Distributing multiplication on addition and subtraction

For any three real numbers a , b and c it will be :

- $a(b \pm c) = ab \pm ac$
- $(b \pm c)a = ba \pm ca$

Example 6 **Find each of the following :**

1 $2\sqrt{3}\left(5\sqrt{3} - 4\right)$ $\quad$ 2 $\left(2 + \sqrt{3}\right)\left(\sqrt{3} + 7\right)$

3 $\left(7\sqrt{2} - 5\right)\left(7\sqrt{2} + 5\right)$ $\quad$ 4 $\left(5\sqrt{3} - 2\right)^2$

Solution

1 $2\sqrt{3}\left(5\sqrt{3} - 4\right) = 2\sqrt{3} \times 5\sqrt{3} + 2\sqrt{3} \times (-4)$

$= 10 \times 3 - 8 \times \sqrt{3} = 30 - 8\sqrt{3}$

2 $\left(2 + \sqrt{3}\right)\left(\sqrt{3} + 7\right) = 2\left(\sqrt{3} + 7\right) + \sqrt{3}\left(\sqrt{3} + 7\right)$

$= 2 \times \sqrt{3} + 2 \times 7 + \sqrt{3} \times \sqrt{3} + \sqrt{3} \times 7$

$= 2\sqrt{3} + 14 + 3 + 7\sqrt{3}$

$= \left(2\sqrt{3} + 7\sqrt{3}\right) + (14 + 3) = 9\sqrt{3} + 17$

3 $\left(7\sqrt{2}-5\right)\left(7\sqrt{2}+5\right)=98+35\sqrt{2}-35\sqrt{2}-25=73$

Another solution by multiplying by inspection :

$$\left(7\sqrt{2}-5\right)\left(7\sqrt{2}+5\right)=\left(7\sqrt{2}\right)^2-(5)^2$$
$$=7^2\times\left(\sqrt{2}\right)^2-5^2$$
$$=49\times 2-25=98-25=73$$

Notice that :
$(a+b)(a-b)=a^2-b^2$

4 Multiplying by inspection

$$\therefore \left(5\sqrt{3}-2\right)^2=\left(5\sqrt{3}\right)^2-2\times 5\sqrt{3}\times 2+(-2)^2$$
$$=5^2\times\left(\sqrt{3}\right)^2-20\sqrt{3}+4$$
$$=25\times 3-20\sqrt{3}+4$$
$$=75-20\sqrt{3}+4=79-20\sqrt{3}$$

Notice that :
- $(a+b)^2=a^2+2ab+b^2$
- $(a-b)^2=a^2-2ab+b^2$

Example 7 If $x=5\sqrt{3}-2$, $y=5\sqrt{3}+2$

, find the value of the expression : $x^2+2xy+y^2$

Solution **From multiplying by inspection , we find that :**

$(x+y)^2=x^2+2xy+y^2$

$$\therefore x^2+2xy+y^2=\left(5\sqrt{3}-2+5\sqrt{3}+2\right)^2$$
$$=\left(10\sqrt{3}\right)^2=(10)^2\times\left(\sqrt{3}\right)^2=100\times 3=300$$

Example 8 **Give an estimation for the result of :**

$\left(5+\sqrt{10}\right)\left(3-\sqrt[3]{7}\right)$, then check your answer using the calculator.

Solution

First : The estimation of $\sqrt{10}$ is 3 $\left(\text{because } \sqrt{9}=3\right)$

$\therefore$ The estimation of $\left(5+\sqrt{10}\right)$ is $5+3=8$

, the estimation of $\sqrt[3]{7}$ is 2 $\left(\text{because } \sqrt[3]{8}=2\right)$

$\therefore$ The estimation of $\left(3-\sqrt[3]{7}\right)$ is $3-2=1$

$\therefore$ The estimation of $\left(5+\sqrt{10}\right)\left(3-\sqrt[3]{7}\right)$ is $8\times 1=8$

Second : By using the calculator , we find that the result approximated to the nearest thousandths is 8.873

i.e. The estimation is accepted.

TRY 3 *by yourself*

1 ***Find the result of each of the following in the simplest form :***

1. $5\sqrt{2}\left(3\sqrt{2}-2\right)$
2. $\left(2\sqrt{3}-3\right)\left(2\sqrt{3}+3\right)$

2 If $x = 2\sqrt{3} - 1$ and $y = 2\sqrt{3} + 1$

, find the value of the expression : $x^2 - 2xy + y^2$

3 ***Give an estimation for the result of :*** $\left(1+\sqrt{15}\right)\left(4-\sqrt{8}\right)$

, then check your answer by using the calculator.

Answers *of try by yourself*

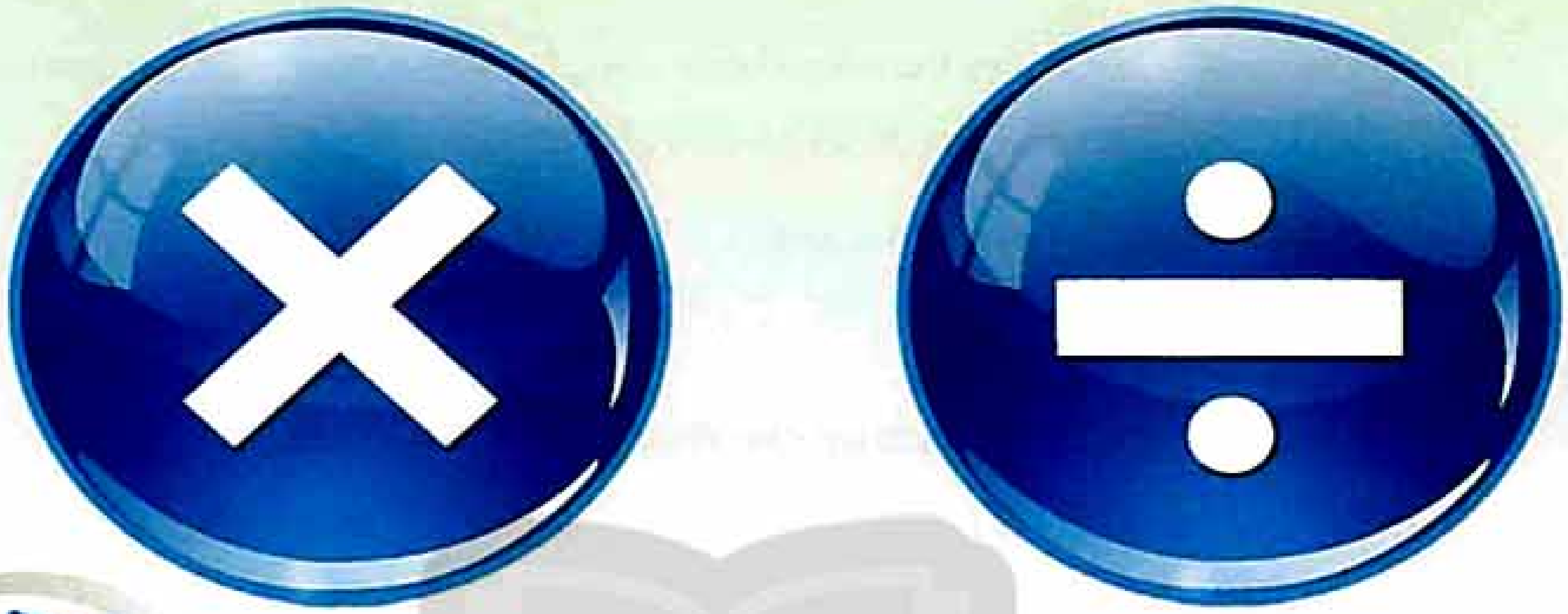
1 1. $-\sqrt{2}$, $\sqrt[3]{5}$, $-\sqrt{2}-\sqrt{7}$, $-\sqrt[3]{5}+3$, $\sqrt{6}+\sqrt[3]{7}$

2. (1) $1-3\sqrt{7}$ (2) $6\sqrt{3}$

2 1. (1) $\sqrt{5}$ (2) 1

2. (1) $\frac{3\sqrt{7}}{7}$ (2) $\frac{3\sqrt{6}}{4}$

3 1. (1) $30-10\sqrt{2}$ (2) 3 2. 4 3. 5 (Check by yourself)



Lesson 6 Operations on the square roots

If a and b are two non negative real numbers , then :

1. $\sqrt{a} \times \sqrt{b} = \sqrt{a\,b}$

For example: • $\sqrt{3} \times \sqrt{12} = \sqrt{36} = 6$ • $\sqrt{50} = \sqrt{25 \times 2} = \sqrt{25} \times \sqrt{2} = 5\sqrt{2}$

2. $\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$ (where $b \neq 0$)

For example: • $\frac{\sqrt{8}}{\sqrt{2}} = \sqrt{\frac{8}{2}} = \sqrt{4} = 2$ • $\sqrt{\frac{16}{49}} = \frac{\sqrt{16}}{\sqrt{49}} = \frac{4}{7}$

3. $\frac{\sqrt{a}}{\sqrt{b}} = \frac{\sqrt{a}}{\sqrt{b}} \times \frac{\sqrt{b}}{\sqrt{b}} = \frac{\sqrt{ab}}{b}$ (where $b \neq 0$)

This operation is carried out to make the denominator an integer.

For example: • $\frac{\sqrt{2}}{\sqrt{5}} = \frac{\sqrt{2}}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{\sqrt{10}}{5}$ • $\sqrt{\frac{3}{2}} = \frac{\sqrt{3}}{\sqrt{2}} = \frac{\sqrt{3}}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{6}}{2}$

Remarks

1. $\sqrt{a^2+b^2} \neq a+b$, $\sqrt{a^2-b^2} \neq a-b$

For example:

- $\sqrt{6^2+8^2} \neq 6+8$ because $\sqrt{6^2+8^2} = \sqrt{100} = 10$
- $\sqrt{25-9} \neq 5-3$ because $\sqrt{25-9} = \sqrt{16} = 4$

2. $a\sqrt{b} = \sqrt{a^2 b}$

For example:

- $2\sqrt{\frac{1}{2}} = \sqrt{4 \times \frac{1}{2}} = \sqrt{2}$
- $15\sqrt{\frac{1}{3}} = 5 \times 3\sqrt{\frac{1}{3}} = 5\sqrt{9 \times \frac{1}{3}} = 5\sqrt{3}$

Example 1 Write each of the following in the form $a\sqrt{b}$ where a and b are two integers , b is the least possible value :

1. $\sqrt{27}$
2. $5\sqrt{54}$
3. $3\sqrt{\frac{2}{3}}$
4. $\frac{\sqrt{84}}{\sqrt{7}}$

Solution

1. $\sqrt{27} = \sqrt{9 \times 3}$

$= \sqrt{9} \times \sqrt{3} = 3\sqrt{3}$

2. $5\sqrt{54} = 5\sqrt{9 \times 6} = 5 \times \sqrt{9} \times \sqrt{6}$

$= 5 \times 3 \times \sqrt{6} = 15\sqrt{6}$

3. $3\sqrt{\frac{2}{3}} = 3 \times \frac{\sqrt{2}}{\sqrt{3}} = 3 \times \frac{\sqrt{2}}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = 3 \times \frac{\sqrt{6}}{3} = \sqrt{6}$

Another solution :

$3\sqrt{\frac{2}{3}} = \sqrt{3^2 \times \frac{2}{3}} = \sqrt{3 \times 2} = \sqrt{6}$

4. $\frac{\sqrt{84}}{\sqrt{7}} = \sqrt{\frac{84}{7}} = \sqrt{12} = \sqrt{4 \times 3} = \sqrt{4} \times \sqrt{3} = 2\sqrt{3}$

Unit 1

Example 2 Simplify to the simplest form :

1. $\sqrt{45}-2\sqrt{20}+2\sqrt{5}$
2. $2\sqrt{18}+\sqrt{50}-42\sqrt{\frac{1}{2}}$
3. $2\sqrt{27}-3\sqrt{\frac{1}{3}}-\frac{6}{\sqrt{3}}$

Solution

1. $\sqrt{45}-2\sqrt{20}+2\sqrt{5}=\sqrt{9\times5}-2\sqrt{4\times5}+2\sqrt{5}$

$=\sqrt{9}\times\sqrt{5}-2\times\sqrt{4}\times\sqrt{5}+2\sqrt{5}$

$=3\sqrt{5}-2\times2\sqrt{5}+2\sqrt{5}$

$=3\sqrt{5}-4\sqrt{5}+2\sqrt{5}=\sqrt{5}$

2. $2\sqrt{18}+\sqrt{50}-42\sqrt{\frac{1}{2}}=2\sqrt{9\times2}+\sqrt{25\times2}-42\times\frac{\sqrt{1}}{\sqrt{2}}$

$=2\times\sqrt{9}\times\sqrt{2}+\sqrt{25}\times\sqrt{2}-42\times\frac{1}{\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}}$

$=2\times3\sqrt{2}+5\sqrt{2}-21\sqrt{2}=-10\sqrt{2}$

3. $2\sqrt{27}-3\sqrt{\frac{1}{3}}-\frac{6}{\sqrt{3}}=2\sqrt{9\times3}-3\times\frac{\sqrt{1}}{\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}}-\frac{6}{\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}}$

$=6\sqrt{3}-\sqrt{3}-\frac{6\sqrt{3}}{3}$

$=6\sqrt{3}-\sqrt{3}-2\sqrt{3}=3\sqrt{3}$

Example 3 Find the result of each of the following :

1. $2\sqrt{3}\left(\sqrt{6}+5\right)$
2. $\left(3\sqrt{2}-5\right)\left(3\sqrt{2}+5\right)$
3. $\left(\sqrt{2}+\sqrt{6}\right)^2$

Solution

1. $2\sqrt{3}\left(\sqrt{6}+5\right)=2\sqrt{3}\times\sqrt{6}+2\sqrt{3}\times5$

$=2\sqrt{18}+10\sqrt{3}$

$=2\sqrt{9\times2}+10\sqrt{3}$

$=6\sqrt{2}+10\sqrt{3}$

2. $\left(3\sqrt{2}-5\right)\left(3\sqrt{2}+5\right)=\left(3\sqrt{2}\right)^2-(5)^2$

$=3^2\times\left(\sqrt{2}\right)^2-(5)^2$

$=9\times2-25$

$=18-25=-7$

Remember that

$(a-b)(a+b)=a^2-b^2$

3 $\left(\sqrt{2}+\sqrt{6}\right)^2=\left(\sqrt{2}\right)^2+2\times\sqrt{2}\times\sqrt{6}+\left(\sqrt{6}\right)^2$

$=2+2\sqrt{12}+6$

$=8+2\sqrt{4\times3}=8+4\sqrt{3}$

Remember that

- $(a+b)^2=a^2+2\,ab+b^2$
- $(a-b)^2=a^2-2\,ab+b^2$

Example 4 If $a=\dfrac{\sqrt{6}-\sqrt{2}}{\sqrt{2}}$, find the value of $a^2+2\sqrt{3}$

Solution To facilitate the solution , we will make the denominator an integer by multiplying both the numerator and the denominator by $\sqrt{2}$

$\therefore\ a=\dfrac{\sqrt{6}-\sqrt{2}}{\sqrt{2}}\times\dfrac{\sqrt{2}}{\sqrt{2}}=\dfrac{\sqrt{6}\times\sqrt{2}-\sqrt{2}\times\sqrt{2}}{\sqrt{2}\times\sqrt{2}}=\dfrac{\sqrt{12}-2}{2}$

$=\dfrac{\sqrt{4\times3}-2}{2}=\dfrac{2\sqrt{3}-2}{2}=\dfrac{2\left(\sqrt{3}-1\right)}{2}=\sqrt{3}-1$

$\therefore\ a^2=\left(\sqrt{3}-1\right)^2=\left(\sqrt{3}\right)^2-2\times\sqrt{3}\times1+1=3-2\sqrt{3}+1=4-2\sqrt{3}$

$\therefore\ a^2+2\sqrt{3}=4-2\sqrt{3}+2\sqrt{3}=4$

Another method to simplify a :

$\because\ a=\dfrac{\sqrt{6}-\sqrt{2}}{\sqrt{2}}$ $\qquad\therefore\ a=\dfrac{\sqrt{6}}{\sqrt{2}}-\dfrac{\sqrt{2}}{\sqrt{2}}=\sqrt{\dfrac{6}{2}}-1=\sqrt{3}-1$

TRY by yourself

1 ***Simplify to the simplest form :***

(1) $\sqrt{75}-2\sqrt{27}+\sqrt{3}$ $\qquad$ (2) $2\sqrt{50}-3\sqrt{2}-4\sqrt{\frac{1}{8}}$

2 ***Write each of the following such that the denominator is an integer :***

(1) $\dfrac{5\sqrt{3}}{2\sqrt{5}}$ $\qquad$ (2) $\dfrac{1+\sqrt{3}}{3\sqrt{3}}$

Answers of try by yourself

1 (1) 0 $\qquad$ (2) $6\sqrt{2}$

2 (1) $\dfrac{\sqrt{15}}{2}$ $\qquad$ (2) $\dfrac{3+\sqrt{3}}{9}$

Lesson 7 The two conjugate numbers

If a and b are two positive rational numbers

Then each of the two numbers $\left(\sqrt{a}+\sqrt{b}\right)$ and $\left(\sqrt{a}-\sqrt{b}\right)$ is conjugate to the other one and we find that

- **Their sum =** $\left(\sqrt{a}+\sqrt{b}\right)+\left(\sqrt{a}-\sqrt{b}\right)=2\sqrt{a}=$ twice the first term.

- **Their product =** $\left(\sqrt{a}+\sqrt{b}\right)\left(\sqrt{a}-\sqrt{b}\right)=\underbrace{\left(\sqrt{a}\right)^2}_{\text{the square of } 1^{st} \text{ term}} - \underbrace{\left(\sqrt{b}\right)^2}_{\text{the square of } 2^{nd} \text{ term}} = a-b$

For example: $\left(\sqrt{3}-\sqrt{2}\right)$ its conjugate is $\left(\sqrt{3}+\sqrt{2}\right)$, then we find that

- Their sum $=2\sqrt{3}$
- Their product $=3-2=1$

Remark

"The product of the two conjugate numbers is always a rational number."

Remark

"If we have a real number whose denominator is written in the form $\left(\sqrt{a}+\sqrt{b}\right)$ or $\left(\sqrt{a}-\sqrt{b}\right)$, we should put it in the simplest form by multiplying both the numerator and denominator by the conjugate of the denominator."

Example 1 Write the number $\frac{4}{\sqrt{7}-\sqrt{3}}$ in the simplest form.

Solution Multiplying the two terms of the number by the conjugate of the denominator which is $\left(\sqrt{7}+\sqrt{3}\right)$

$$\therefore \frac{4}{\sqrt{7}-\sqrt{3}} = \frac{4}{\sqrt{7}-\sqrt{3}} \times \frac{\sqrt{7}+\sqrt{3}}{\sqrt{7}+\sqrt{3}} = \frac{4\left(\sqrt{7}+\sqrt{3}\right)}{\left(\sqrt{7}\right)^2-\left(\sqrt{3}\right)^2}$$

$$= \frac{4\left(\sqrt{7}+\sqrt{3}\right)}{7-3} = \sqrt{7}+\sqrt{3}$$

Example 2 If $x = \frac{4}{2-\sqrt{2}}$ and $y = \frac{3-2\sqrt{2}}{3+2\sqrt{2}}$, write each of x and y such that its denominator is a rational number, then find $x+y$

Solution

$$\because x = \frac{4}{2-\sqrt{2}} \times \frac{2+\sqrt{2}}{2+\sqrt{2}} = \frac{4\left(2+\sqrt{2}\right)}{4-2} = \frac{4\left(2+\sqrt{2}\right)}{2}$$

$$= 2\left(2+\sqrt{2}\right) = 4+2\sqrt{2}$$

$$, y = \frac{3-2\sqrt{2}}{3+2\sqrt{2}} \times \frac{3-2\sqrt{2}}{3-2\sqrt{2}}$$

$$= \frac{\left(3-2\sqrt{2}\right)^2}{9-8} = \frac{9-12\sqrt{2}+8}{1} = 17-12\sqrt{2}$$

$$\therefore x+y = 4+2\sqrt{2}+17-12\sqrt{2} = 21-10\sqrt{2}$$

TRY 1 by yourself

Write each of the following such that the denominator is a rational number :

1. $\frac{12}{\sqrt{6}-\sqrt{2}}$
2. $\frac{\sqrt{8}}{3+2\sqrt{2}}$

Important remarks from direct product (multiplying by inspection)

- **We know that :** $(x - y)(x + y) = x^2 - y^2$
- **And we know also :**

$(x + y)^2 = x^2 + 2xy + y^2$

Then

- $x^2 + xy + y^2 = (x + y)^2 - xy$
- $x^2 + y^2 = (x + y)^2 - 2xy$

$(x - y)^2 = x^2 - 2xy + y^2$

Then

- $x^2 - xy + y^2 = (x - y)^2 + xy$
- $x^2 + y^2 = (x - y)^2 + 2xy$

Example 3 If $x = \frac{2}{\sqrt{5} - \sqrt{3}}$ and $y = \sqrt{5} - \sqrt{3}$, prove that x and y are conjugate numbers , then find the value of each of :

1. $x^2 + 2xy + y^2$
2. $x^2 + xy + y^2$

Solution

$\because x = \frac{2}{\sqrt{5} - \sqrt{3}} \times \frac{\sqrt{5} + \sqrt{3}}{\sqrt{5} + \sqrt{3}} = \frac{2(\sqrt{5} + \sqrt{3})}{5 - 3}$

$= \frac{2(\sqrt{5} + \sqrt{3})}{2} = \sqrt{5} + \sqrt{3}$

$, \because y = \sqrt{5} - \sqrt{3}$

$\therefore x$ and y are conjugate numbers.

1. $x^2 + 2xy + y^2 = (\sqrt{5} + \sqrt{3})^2 + 2(\sqrt{5} + \sqrt{3})(\sqrt{5} - \sqrt{3}) + (\sqrt{5} - \sqrt{3})^2$

$= (5 + 2\sqrt{15} + 3) + 2(5 - 3) + (5 - 2\sqrt{15} + 3)$

$= 8 + 2\sqrt{15} + 4 + 8 - 2\sqrt{15} = 20$

Another solution using the previous remarks :

since $x^2 + 2xy + y^2 = (x + y)^2$

$\therefore x^2 + 2xy + y^2 = [(\sqrt{5} + \sqrt{3}) + (\sqrt{5} - \sqrt{3})]^2$

$= (2\sqrt{5})^2 = 4 \times 5 = 20$

2 $x^2 + xy + y^2 = \left(\sqrt{5}+\sqrt{3}\right)^2 + \left(\sqrt{5}+\sqrt{3}\right)\left(\sqrt{5}-\sqrt{3}\right) + \left(\sqrt{5}-\sqrt{3}\right)^2$

$$= \left(5+3+2\sqrt{15}\right) + (2) + \left(5+3-2\sqrt{15}\right) = 18$$

Another solution using the previous remarks :

$$x^2 + xy + y^2 = (x+y)^2 - xy = \left(\sqrt{5}+\sqrt{3}+\sqrt{5}-\sqrt{3}\right)^2 - \left(\sqrt{5}+\sqrt{3}\right)\left(\sqrt{5}-\sqrt{3}\right)$$

$$= \left(2\sqrt{5}\right)^2 - 2 = 20 - 2 = 18$$

TRY 2 by yourself

If $x = \dfrac{3}{2\sqrt{2}-\sqrt{5}}$ and $y = 2\sqrt{2}-\sqrt{5}$, find the value of the expression : $x^2 - y^2$

Answers of try by yourself

1 (1) $3\sqrt{6}+3\sqrt{2}$ (2) $-8+6\sqrt{2}$

2 $8\sqrt{10}$

Lesson 8 Operations on the cube roots

If a and b are two real numbers , then :

1 $\sqrt[3]{a} \times \sqrt[3]{b} = \sqrt[3]{ab}$

For example:

- $\sqrt[3]{3} \times \sqrt[3]{9} = \sqrt[3]{3 \times 9} = \sqrt[3]{27} = 3$
- $\sqrt[3]{2} \times \sqrt[3]{-4} = \sqrt[3]{2 \times -4} = \sqrt[3]{-8} = -2$
- $\sqrt[3]{16} = \sqrt[3]{8 \times 2} = \sqrt[3]{8} \times \sqrt[3]{2} = 2\sqrt[3]{2}$
- $\sqrt[3]{-54} = \sqrt[3]{-27 \times 2} = \sqrt[3]{-27} \times \sqrt[3]{2} = -3\sqrt[3]{2}$

2 $\dfrac{\sqrt[3]{a}}{\sqrt[3]{b}} = \sqrt[3]{\dfrac{a}{b}}$ (where $b \neq 0$)

For example:

- $\dfrac{\sqrt[3]{32}}{\sqrt[3]{4}} = \sqrt[3]{\dfrac{32}{4}} = \sqrt[3]{8} = 2$
- $\dfrac{\sqrt[3]{54}}{\sqrt[3]{-2}} = \sqrt[3]{-\dfrac{54}{2}} = \sqrt[3]{-27} = -3$
- $\sqrt[3]{\dfrac{8}{125}} = \dfrac{\sqrt[3]{8}}{\sqrt[3]{125}} = \dfrac{2}{5}$
- $\sqrt[3]{-\dfrac{27}{64}} = \dfrac{\sqrt[3]{-27}}{\sqrt[3]{64}} = \dfrac{-3}{4}$

Example 1 Find the result of each of the following in its simplest form :

1. $\sqrt[3]{\frac{2}{3}} \times \sqrt[3]{\frac{4}{9}}$

2. $\sqrt[3]{\frac{5}{4}} \div \sqrt[3]{\frac{2}{25}}$

Solution

1. $\sqrt[3]{\frac{2}{3}} \times \sqrt[3]{\frac{4}{9}} = \sqrt[3]{\frac{2}{3} \times \frac{4}{9}} = \sqrt[3]{\frac{8}{27}} = \frac{\sqrt[3]{8}}{\sqrt[3]{27}} = \frac{2}{3}$

2. $\sqrt[3]{\frac{5}{4}} \div \sqrt[3]{\frac{2}{25}} = \sqrt[3]{\frac{5}{4} \div \frac{2}{25}} = \sqrt[3]{\frac{5}{4} \times \frac{25}{2}} = \sqrt[3]{\frac{125}{8}} = \frac{\sqrt[3]{125}}{\sqrt[3]{8}} = \frac{5}{2}$

Remarks

If a and b are two real numbers , then :

1. $\sqrt[3]{a^3 + b^3} \neq a + b$, $\sqrt[3]{a^3 - b^3} \neq a - b$

2. $\sqrt[3]{-a} = -\sqrt[3]{a}$

3. $a\sqrt[3]{b} = \sqrt[3]{a^3 b}$

   For example: • $3\sqrt[3]{\frac{1}{9}} = \sqrt[3]{27 \times \frac{1}{9}} = \sqrt[3]{3}$

   • $8\sqrt[3]{\frac{1}{4}} = 4 \times 2\sqrt[3]{\frac{1}{4}} = 4\sqrt[3]{8 \times \frac{1}{4}} = 4\sqrt[3]{2}$

4. $\sqrt[3]{\frac{a}{b}} = \sqrt[3]{\frac{a}{b} \times \frac{b^2}{b^2}} = \sqrt[3]{\frac{a\,b^2}{b^3}} = \frac{1}{b}\sqrt[3]{a\,b^2}$ (Where $b \neq 0$)

   For example: $\sqrt[3]{\frac{1}{3}} = \sqrt[3]{\frac{1}{3} \times \frac{9}{9}} = \sqrt[3]{\frac{9}{27}} = \frac{1}{3}\sqrt[3]{9}$

Example 2 Put each of the following in its simplest form :

1. $\sqrt[3]{24} + \sqrt[3]{3} - \sqrt[3]{81}$

2. $\sqrt[3]{54} + 6\sqrt[3]{16} - 6\sqrt[3]{\frac{1}{4}}$

3. $\sqrt[3]{81} + \sqrt{12} - 2\sqrt[3]{3} - 2\sqrt{3}$

Unit 1

Solution

1 $\sqrt[3]{24}+\sqrt[3]{3}-\sqrt[3]{81}=\sqrt[3]{8\times 3}+\sqrt[3]{3}-\sqrt[3]{27\times 3}$

$=\sqrt[3]{8}\times\sqrt[3]{3}+\sqrt[3]{3}-\sqrt[3]{27}\times\sqrt[3]{3}$

$=2\sqrt[3]{3}+\sqrt[3]{3}-3\sqrt[3]{3}=\text{zero}$

2 $\sqrt[3]{54}+6\sqrt[3]{16}-6\sqrt[3]{\frac{1}{4}}=\sqrt[3]{27\times 2}+6\sqrt[3]{8\times 2}-3\times 2\sqrt[3]{\frac{1}{4}}$

$=\sqrt[3]{27}\times\sqrt[3]{2}+6\times\sqrt[3]{8}\times\sqrt[3]{2}-3\times\sqrt[3]{8\times\frac{1}{4}}$

$=3\times\sqrt[3]{2}+6\times 2\times\sqrt[3]{2}-3\times\sqrt[3]{2}$

$=3\sqrt[3]{2}+12\sqrt[3]{2}-3\sqrt[3]{2}=12\sqrt[3]{2}$

Another solution :

$\because \sqrt[3]{\frac{1}{4}}=\sqrt[3]{\frac{1}{4}\times\frac{16}{16}}=\sqrt[3]{\frac{16}{64}}=\frac{\sqrt[3]{16}}{\sqrt[3]{64}}=\frac{1}{4}\sqrt[3]{16}$

$=\frac{1}{4}\sqrt[3]{8\times 2}=\frac{1}{4}\times 2\sqrt[3]{2}=\frac{1}{2}\sqrt[3]{2}$

$\therefore \sqrt[3]{54}+6\sqrt[3]{16}-6\sqrt[3]{\frac{1}{4}}=3\sqrt[3]{2}+6\times 2\sqrt[3]{2}-6\times\frac{1}{2}\sqrt[3]{2}$

$=3\sqrt[3]{2}+12\sqrt[3]{2}-3\sqrt[3]{2}=12\sqrt[3]{2}$

One more solution :

$\because \sqrt[3]{\frac{1}{4}}=\sqrt[3]{\frac{1}{4}\times\frac{2}{2}}=\sqrt[3]{\frac{2}{8}}=\frac{\sqrt[3]{2}}{\sqrt[3]{8}}=\frac{\sqrt[3]{2}}{2}$

$\therefore \sqrt[3]{54}+6\sqrt[3]{16}-6\sqrt[3]{\frac{1}{4}}=3\sqrt[3]{2}+12\sqrt[3]{2}-6\times\frac{\sqrt[3]{2}}{2}$

$=3\sqrt[3]{2}+12\sqrt[3]{2}-3\sqrt[3]{2}=12\sqrt[3]{2}$

3 $\sqrt[3]{81}+\sqrt{12}-2\sqrt[3]{3}-2\sqrt{3}=\sqrt[3]{27\times 3}+\sqrt{4\times 3}-2\sqrt[3]{3}-2\sqrt{3}$

$=\sqrt[3]{27}\times\sqrt[3]{3}+\sqrt{4}\times\sqrt{3}-2\sqrt[3]{3}-2\sqrt{3}$

$=3\sqrt[3]{3}+2\sqrt{3}-2\sqrt[3]{3}-2\sqrt{3}=\sqrt[3]{3}$

Example 3 Find in the simplest form : $2\sqrt[3]{4}\left(5\sqrt[3]{\frac{1}{2}}-\sqrt[3]{32}\right)$

Solution

$$2\sqrt[3]{4}\left(5\sqrt[3]{\frac{1}{2}}-\sqrt[3]{32}\right)=2\times5\sqrt[3]{4\times\frac{1}{2}}-2\times\sqrt[3]{4\times32}$$

$$=10\sqrt[3]{2}-2\times\sqrt[3]{128}=10\sqrt[3]{2}-2\times\sqrt[3]{64\times2}$$

$$=10\sqrt[3]{2}-2\times4\sqrt[3]{2}=10\sqrt[3]{2}-8\sqrt[3]{2}=2\sqrt[3]{2}$$

Example 4 If $x=\sqrt[3]{5}+2$ and $y=\sqrt[3]{5}-2$, find the value of $(x+y)^3-(x-y)^3$

Solution

$\because x+y=\sqrt[3]{5}+2+\sqrt[3]{5}-2=2\sqrt[3]{5}$

$, x-y=\sqrt[3]{5}+2-\left(\sqrt[3]{5}-2\right)=\sqrt[3]{5}+2-\sqrt[3]{5}+2=4$

$\therefore (x+y)^3-(x-y)^3=\left(2\sqrt[3]{5}\right)^3-(4)^3=2^3\times\left(\sqrt[3]{5}\right)^3-4^3$

$=8\times5-64=40-64=-24$

TRY by yourself

Simplify each of the following to the simplest form :

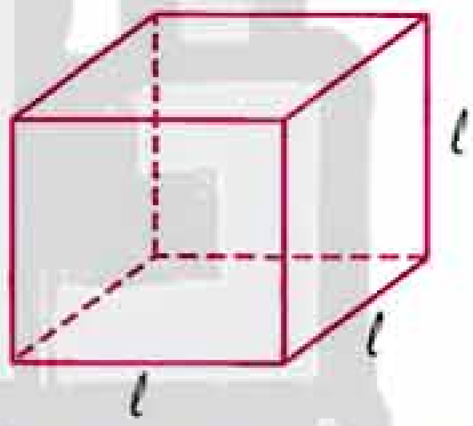
1. $5\sqrt[3]{2}-\sqrt[3]{16}+\sqrt[3]{-54}$
2. $\sqrt[3]{72}+\sqrt[3]{\frac{1}{3}}+\sqrt[3]{-9}$

Answers of try by yourself

1. 0
2. $\frac{4}{3}\sqrt[3]{9}$

Lesson 9 Applications on the real numbers

The cube



It is a solid whose six faces are congruent squares.

i.e. all its edges are equal in length.

Assuming that the edge length of the cube $= \ell$ length unit , then :

1. The area of each face $= \ell^2$ square unit.
2. Its lateral area $= 4\ \ell^2$ square unit.
3. Its total area (the area of its 6 faces) $= 6\ \ell^2$ square unit.
4. Its volume $= \ell^3$ cube unit.

Example 1 A cube with volume 125 cm^3. Find its total area and its lateral area.

Solution

Assuming that the edge length of the cube $= \ell$ cm.

$\therefore$ Its volume $= \ell^3\ cm^3$.

$\therefore \ell^3 = 125$ $\qquad \therefore \ell = \sqrt[3]{125} = 5$ cm.

$\therefore$ The total area of the cube $= 6\ \ell^2 = 6\ (5)^2 = 150\ cm^2$.

, the lateral area of the cube $= 4\ \ell^2 = 4\ (5)^2 = 100\ cm^2$.

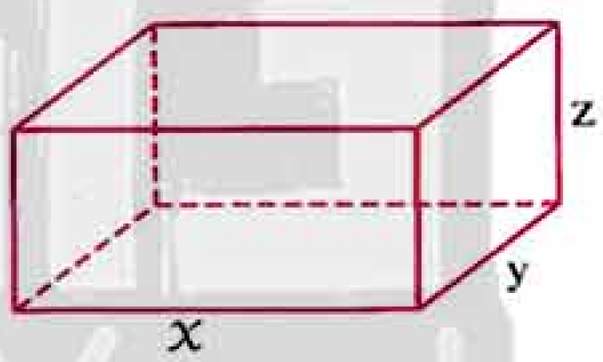
TRY 1 by yourself

Complete the following table :

	Edge length of the cube	Area of one face	Lateral area	Total area	Volume
1	3 cm.	....	....	....	....
2	....	49 cm^2.	....	....	....
3	....	....	144 cm^2.	....	....
4	....	....	....	150 cm^2.	....
5	....	....	....	....	64 cm^3.

The cuboid

It is a solid that contains 6 faces , each of them is a rectangle and each two opposite faces are congruent. Assuming that the lengths of the edges of the cuboid are x , y and z length unit , then :



1. Its lateral area = the perimeter of the base × height = $2(x + y) \times z$ square unit.
2. Its total area (the area of its six faces) = the lateral area + twice the area of the base
$$= 2(x + y) \times z + 2xy$$
$$= 2(xy + yz + zx) \text{ square unit.}$$
3. Its volume = the area of the base × the height
$$= x \times y \times z \text{ cube unit.}$$

Remarks

- The cuboid may contain two opposite faces , each of them is a square.
- The cube is a special case of the cuboid.
  i.e. the cube is a cuboid with edges having the same length.

Unit 1

Example 2 The height of a cuboid is 4 cm. and its base is a square of side length 5 cm. Find :

1 Its volume. 2 Its lateral area. 3 Its total area.

Solution

1 The volume of the cuboid = the area of the base × the height

$= 5 \times 5 \times 4 = 100 \text{ cm}^3.$

2 The lateral area of the cuboid = the perimeter of the base × the height

$= 4 \times 5 \times 4 = 80 \text{ cm}^2.$

3 The total area of the cuboid

= the lateral area + twice the area of the base $= 80 + 2 \times 25 = 130 \text{ cm}^2.$

TRY 2 by yourself

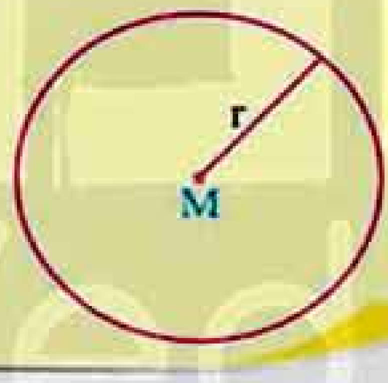
The dimensions of a cuboid are 3 cm. , 4 cm. and 5 cm. Calculate its volume and its total area.

The circle

If M is a circle with radius length r , then :

1 The circumference of the circle $= 2\pi r$ length unit.

2 The area of the circle $= \pi r^2$ square unit.



Example 3 The area of a circle is $25\pi \text{ cm}^2.$ Calculate its circumference in terms of π

Solution

$\because$ The area of the circle $= \pi r^2$ $\therefore \cancel{\pi} r^2 = 25 \cancel{\pi}$

$\therefore r^2 = 25$ $\therefore r = \sqrt{25} = 5$ cm.

$\therefore$ The circumference of the circle $= 2\pi r$

$= 2 \times 5 \times \pi = 10\pi$ cm.

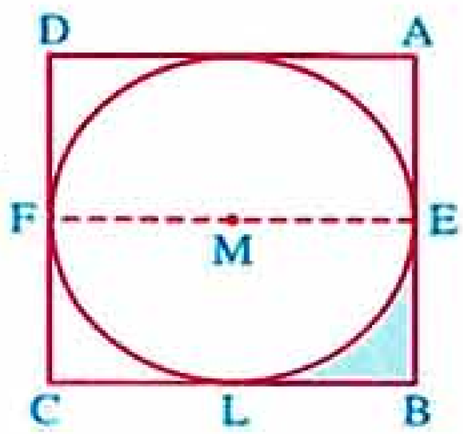
Example 4 **In the opposite figure :**

A circle M is drawn inside a square (touching its sides).

If the area of the square $= 196 \text{ cm}^2.$, find :

1 The area of the shaded part.

2 The perimeter of the shaded part.



Solution $\because$ The area of the square = 196 cm².

$\therefore$ The side length of the square $= \sqrt{196} = 14$ cm.

, $\because$ the side length of the square = 2 r

$\therefore 14 = 2r$ $\qquad\qquad$ $\therefore r = 7$ cm.

1 The area of the shaded part

= (the area of the square − the area of the circle) ÷ 4

$= \left(196 - \frac{22}{\cancel{7}} \times \cancel{7} \times 7\right) \div 4 = 42 \div 4 = 10.5$ cm².

2 The perimeter of the shaded part

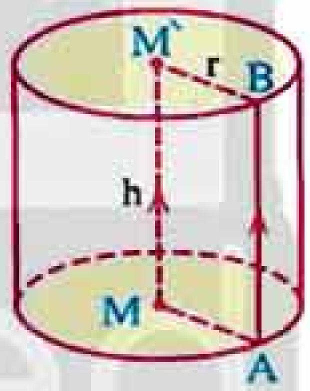
$= BE + BL + \frac{1}{4}$ circumference of the circle $= 7 + 7 + \left(\frac{1}{\cancel{4}_2} \times \cancel{2} \times \frac{22}{\cancel{7}} \times \cancel{7}\right)$

$= 14 + 11 = 25$ cm.

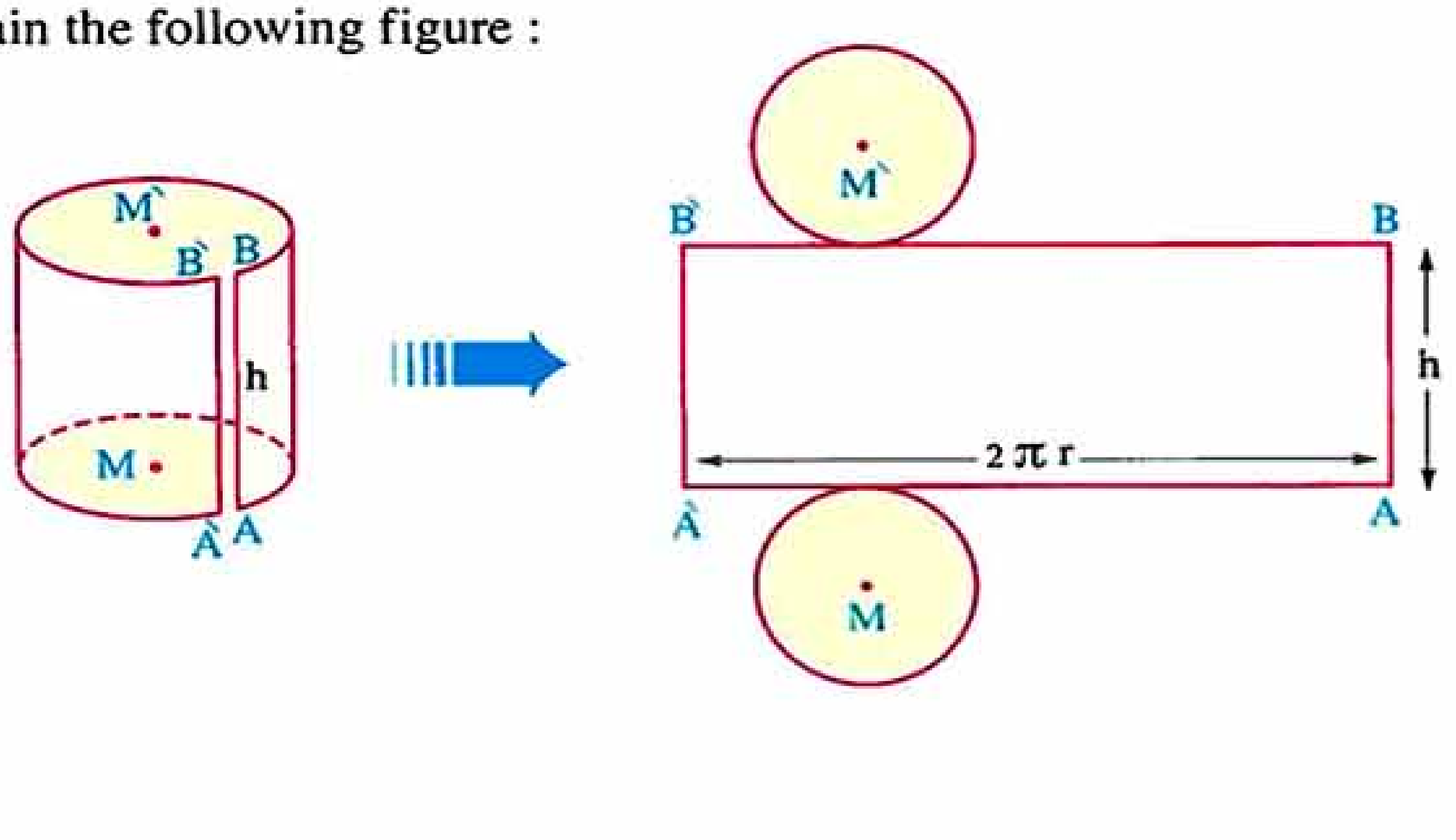
TRY 3 by yourself

The circumference of a circle is 88 cm. Find its area. $\left(\pi = \frac{22}{7}\right)$

The right circular cylinder



- It is a solid having two parallel congruent bases , each of them is a circular-shaped surface while its lateral surface is a curved surface which is called cylindrical surface.
- The line segment $\overline{MM`}$ drawn between the two centres of the two bases is perpendicular to each plane of the two bases and it is called the height of the cylinder.
- If we draw $\overline{AB}$ on the cylindrical surface such that A ∈ the circle M , B ∈ the circle M` , $\overline{AB} \parallel \overline{MM`}$ and if we cut the lateral surface of the cylinder at $\overline{AB}$ and flattened it out, then we will obtain the following figure :



Unit 1

This figure consists of the surface of the rectangle ABB̀À and it is the same cylindrical surface of the cylinder in addition to the two surfaces of two circles which represent the two bases of the cylinder , then we find :

AB = the height of the cylinder.

AÀ = the circumference of the base of the cylinder.

∴ The lateral area of the cylinder = the area of the rectangle ABB̀À = AÀ × AB

= the circumference of the base of the cylinder × its height

and if we assume that the length of the radius of the base = r and its height = h , then :

1. The lateral area of the cylinder = $2 \pi r h$ square unit.
2. The total area of the cylinder = the lateral area of the cylinder + twice the area of the base $= 2 \pi r h + 2 \pi r^2$ square unit.
3. The volume of the cylinder = the area of the base × height = $\pi r^2 h$ cube unit.

Example 5 A right circular cylinder is of height 10 cm. and its volume is 1540 cm^3. Find its total area $\left(\pi = \frac{22}{7}\right)$

Solution

∵ The volume of the cylinder = $\pi r^2 h$

∴ $1540 = \frac{22}{7} \times r^2 \times 10$

∴ $1540 = \frac{220}{7} r^2$

∴ $r^2 = 1540 \times \frac{7}{220} = 49$

∴ $r = \sqrt{49} = 7$ cm.

∴ The total area of the cylinder $= 2 \pi r h + 2 \pi r^2$

$= 2 \times \frac{22}{7} \times 7 \times 10 + 2 \times \frac{22}{7} \times 7^2$

$= 440 + 308 = 748$ cm^2.

TRY 4 by yourself

A right circular cylinder is of volume 90π cm^3 and its height is 10 cm. Find the diameter length of its base.

The sphere

- It is a solid with a curved surface whose all points are equidistant from a fixed point inside the sphere.
- The equal distances are called the radius length of the sphere.
- The fixed point is called the centre of the sphere.
- If we cut the sphere by a plane passing through its centre , then the resulted section is a circle having the same centre of the sphere and its radius length is the same of the sphere.

Assuming that the radius length of the sphere = r , then :

1. The area of the sphere = $4\pi r^2$ square unit.
2. The volume of the sphere = $\frac{4}{3}\pi r^3$ cube unit.

Example 6 The volume of a sphere = $\frac{500}{3}\pi$ cm³. Find the length of its diameter.

Solution

$\because$ The volume of the sphere = $\frac{4}{3}\pi r^3$ $\qquad \therefore \frac{500}{3}\cancel{\pi} = \frac{4}{3}\cancel{\pi} r^3$

$\therefore r^3 = \frac{500}{3} \times \frac{3}{4} = 125$ $\qquad \therefore r = \sqrt[3]{125} = 5$ cm.

$\therefore$ The diameter length of the sphere = $2 \times 5 = 10$ cm.

Example 7 A right circular cylinder is of height 6 cm. and its volume = $\frac{2}{3}$ the volume of a sphere whose radius length is 3 cm. Find the radius length of the base of the cylinder.

Solution Let the radius length of the sphere be r_1 cm. and the radius length of the base of the cylinder be r_2 cm.

$\because$ The volume of the sphere = $\frac{4}{3}\pi r_1^3 = \frac{4}{3}\pi (3)^3 = 36\pi$ cm³.

$\because$ The volume of the cylinder = $\frac{2}{3}$ the volume of the sphere.

$\therefore \cancel{\pi} r_2^2 h = \frac{2}{3} \times 36\cancel{\pi}$ $\qquad \therefore r_2^2 \times 6 = 24$

$\therefore r_2^2 = 4$ $\qquad \therefore r_2 = \sqrt{4} = 2$ cm.

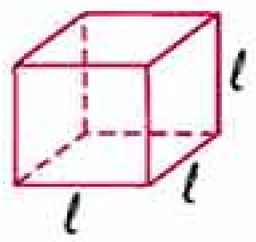
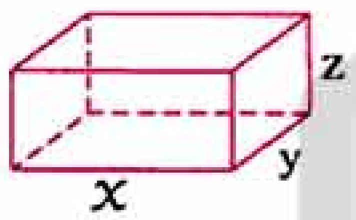
$\therefore$ The radius length of the base of the cylinder = 2 cm.

TRY 5 by yourself

The area of a sphere is 36π cm². Find its volume in terms of π

Unit 1

In the following , we will summarize the previous rules of areas and volumes of some solids :

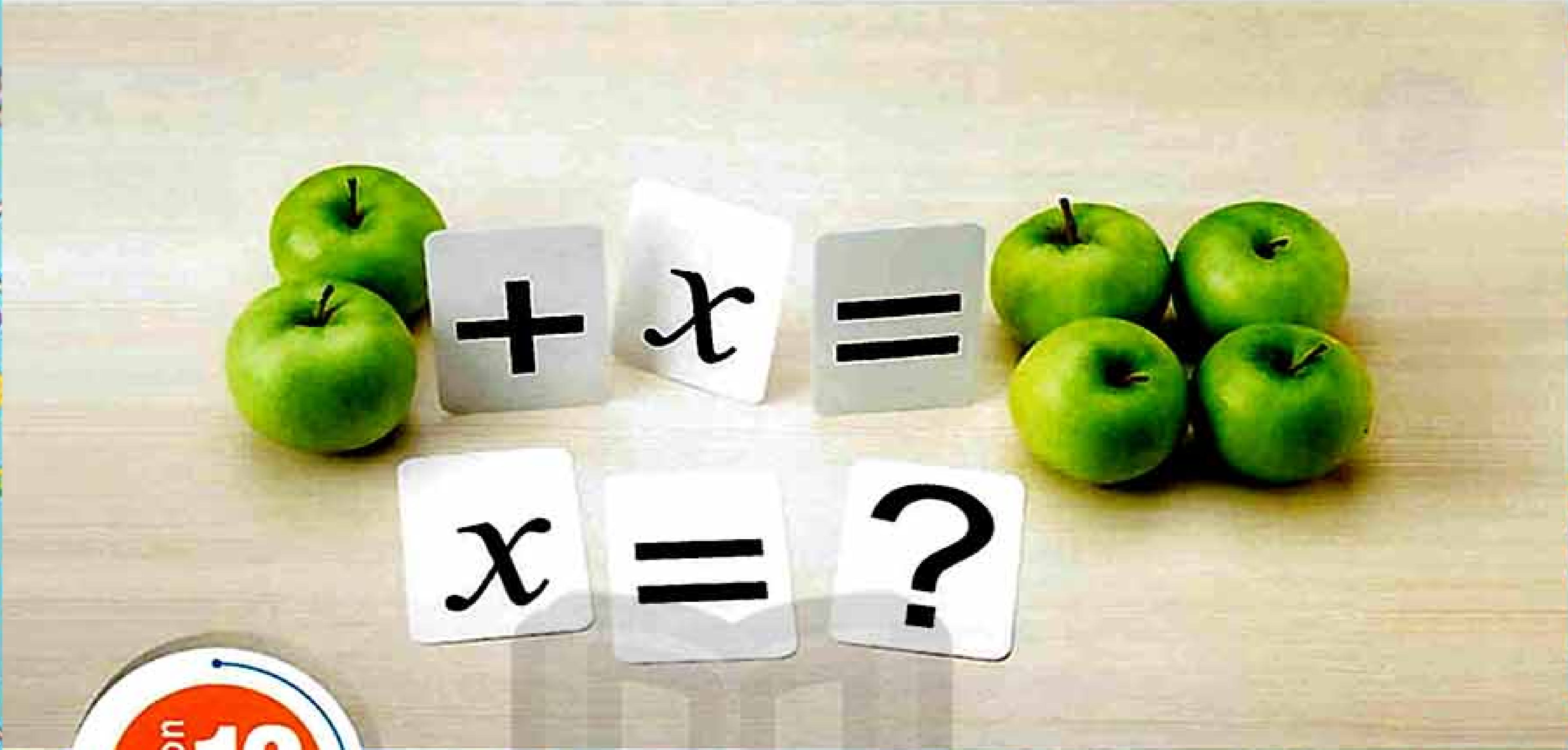
The solid		The lateral area	The total area	The volume
The cube		$4\ell^2$	$6\ell^2$	ℓ^3
The cuboid		$2(x+y)\times z$	$2(xy+yz+zx)$	xyz
The cylinder		$2\pi rh$	$2\pi rh+2\pi r^2$ $=2\pi r(h+r)$	$\pi r^2 h$
The sphere		–	$4\pi r^2$	$\frac{4}{3}\pi r^3$

Answers of try by yourself

1 [1] 9 cm², 36 cm², 54 cm², 27 cm³
[2] 7 cm., 196 cm², 294 cm², 343 cm³
[3] 6 cm., 36 cm², 216 cm², 216 cm³
[4] 5 cm., 25 cm², 100 cm², 125 cm³
[5] 4 cm., 16 cm², 64 cm², 96 cm²

2 The volume = 60 cm³, the total area = 94 cm²

3 616 cm² 4 6 cm. 5 36π cm³



Lesson 10 Solving equations and inequalities of the first degree in one variable in $\mathbb{R}$

First Solving equations of the first degree in one unknown in $\mathbb{R}$

* Each of the equations :
  • $2x - 5 = 3$
  • $\sqrt{3}x - 1 = 8$
  • $\frac{1}{2}x - \sqrt{5} = 0$

  is called an equation of the first degree in one variable (one unknown) which is x because the exponent of the variable x equals one.

* Solving the equation of the first degree in one variable means finding the real number which satisfies this equation.
* The following examples will show how to solve an equation of the first degree in one variable :

Example 1 **Find in $\mathbb{R}$ the S.S. of each of the following equations , then represent the solution on the number line :**

1 $3x + 2 = 1$ **2** $\sqrt{3}x - 1 = 2$ **3** $7x - \sqrt{7} = 6\sqrt{7}$ **4** $x - \sqrt{5} = 1$

Solution

1 $\because 3x + 2 = 1$ (adding -2 to both sides)

$\therefore 3x + 2 - 2 = 1 - 2$ $\therefore 3x = -1$

(multiplying both sides by $\frac{1}{3}$ the multiplicative inverse of the coefficient of x)

$\therefore 3x \times \frac{1}{3} = -1 \times \frac{1}{3}$ $\therefore x = -\frac{1}{3}$ $\therefore$ The S.S. $= \left\{-\frac{1}{3}\right\}$

• We can represent the number $-\frac{1}{3}$ on the number line as follows :

Unit 1

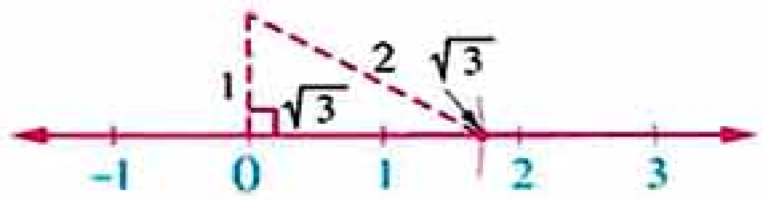
2 $\because \sqrt{3}\,x - 1 = 2$ $\therefore \sqrt{3}\,x = 2 + 1$

$\therefore \sqrt{3}\,x = 3$ $\therefore x = \frac{3}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$

$\therefore x = \frac{3\sqrt{3}}{3}$ $\therefore x = \sqrt{3}$

$\therefore$ The S.S. $= \{\sqrt{3}\}$

- We can represent the number $\sqrt{3}$ on the number line as follows :



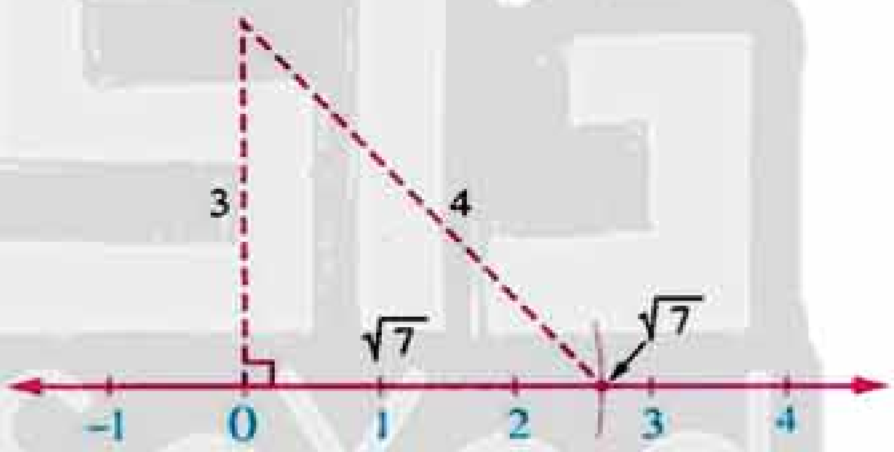
3 $\because 7\,x - \sqrt{7} = 6\sqrt{7}$ $\therefore 7\,x = 6\sqrt{7} + \sqrt{7}$

$\therefore 7\,x = 7\sqrt{7}$ $\therefore x = \frac{7\sqrt{7}}{7}$

$\therefore x = \sqrt{7}$

$\therefore$ The S.S. $= \{\sqrt{7}\}$

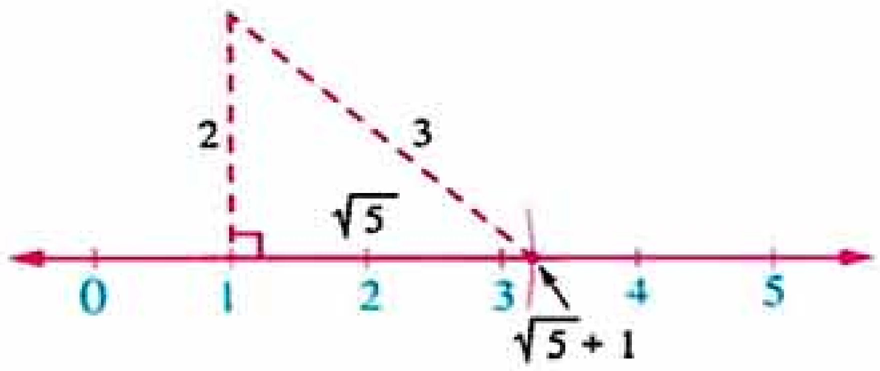
- We can represent the number $\sqrt{7}$ on the number line as follows :



4 $\because x - \sqrt{5} = 1$ $\therefore x = 1 + \sqrt{5}$

$\therefore$ The S.S. $= \{1 + \sqrt{5}\}$

- We can represent the number $(1 + \sqrt{5})$ on the number line as follows :



TRY 1 by yourself

Find in $\mathbb{R}$ the S.S. of each of the following equations , then represent the solution on the number line :

$\boxed{1}$ $2\,x + 5 = 4$ $\boxed{2}$ $\sqrt{5}\,x - 1 = 4$ $\boxed{3}$ $x - \sqrt{3} = 2$

Second — Solving inequalities of the first degree in one unknown in $\mathbb{R}$

- Each of the inequalities :
  - $2x < 5$
  - $3x + 2 \le 1$
  - $5 + x > 2x - 1 \ge 3 + x$

  is called an inequality of the first degree in one unknown denoted by x

- Solving the inequality means finding all values of the unknown (x) which satisfy this inequality.
- The S.S. of the inequality in $\mathbb{R}$ will be written as an interval as will be shown later.

The methods of solving these inequalities in $\mathbb{R}$ depend on the properties of the inequality relation which will be summarized in the following :

Let a , b and c be three real numbers and assuming that $a < b$, then :

$a + c < b + c$ $\xrightarrow{\text{whether}}$ c is **positive** or **negative** (the addition property)

$ac < bc$ $\xrightarrow{\text{if}}$ c is **positive** (the property of multiplying by a positive real number)

$ac > bc$ $\xrightarrow{\text{if}}$ c is **negative** (the property of multiplying by a negative real number)

i.e. when we multiply (or divide) the two sides of an inequality by a negative number , we should change the symbol of the inequality.

Example 2 **Find in $\mathbb{R}$ the S.S. of each of the following inequalities , then represent the solution on the number line :**

1 $2x + 6 < 2$ 2 $5 - 4x \le -3$

Solution

1 $\because 2x + 6 < 2$ (adding the additive inverse of the number 6 (it is -6) to both sides)

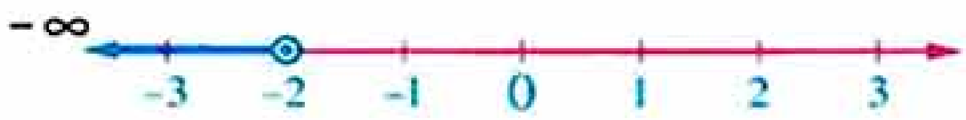
$\therefore 2x + 6 - 6 < 2 - 6$ $\therefore 2x < -4$

(multiplying both sides by the multiplicative inverse of the number 2 (it is $\frac{1}{2}$))

$\therefore 2x \times \frac{1}{2} < -4 \times \frac{1}{2}$ $\therefore x < -2$

$\therefore$ The S.S. is all the real numbers which are less than -2

i.e. The S.S. $=]-\infty , -2[$



2 $\because 5 - 4x \le -3$ (adding -5 to both sides)

$\therefore -4x \le -8$ (dividing both sides by -4) $\therefore x \ge 2$

(Notice the change in the symbol of the inequality because we divided by a negative number)

$\therefore$ The S.S. $= [2 , \infty[$

Unit 1

Example 3 Find in $\mathbb{R}$ the S.S. of each of the following inequalities , then represent the solution on the number line :

1 $-3 < 2x - 1 \leq 5$ **2** $3 < 3 - 5x < 13$

Solution

1 $\because -3 < 2x - 1 \leq 5$ (adding 1 to all sides)

$\therefore -2 < 2x \leq 6$ (dividing all sides by 2)

$\therefore -1 < x \leq 3$

$\therefore$ The S.S. $=]-1 , 3]$

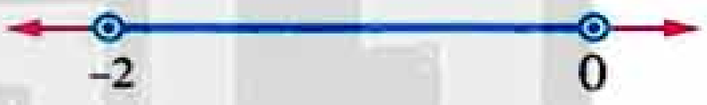
2 $\because 3 < 3 - 5x < 13$ (subtracting 3 from all sides)

$\therefore 0 < -5x < 10$ (dividing all sides by -5)

$\therefore 0 > x > -2$

(Notice the change in the symbols of the inequality because we divided by a negative number).

$\therefore$ The S.S. $=]-2 , 0[$



Example 4 Find in $\mathbb{R}$ the S.S. of each of the following inequalities :

1 $x - 2 \geq 3x - 5$ **2** $x - 1 < 3x - 3 \leq x + 5$

Solution

1 $\because x - 2 \geq 3x - 5$ (adding 2 to both sides)

$\therefore x \geq 3x - 3$ (adding $-3x$ to both sides)

$\therefore -2x \geq -3$ $\left(\text{multiplying both sides by } -\frac{1}{2}\right)$

$\therefore x \leq \frac{3}{2}$ (Notice the change in the symbol of the inequality)

$\therefore$ The S.S. $= \left]-\infty , \frac{3}{2}\right]$

2 $\because x - 1 < 3x - 3 \leq x + 5$ (adding 3 to all sides)

$\therefore x + 2 < 3x \leq x + 8$ (adding $-x$ to all sides)

$\therefore 2 < 2x \leq 8$ $\left(\text{multiplying by } \frac{1}{2}\right)$

$\therefore 1 < x \leq 4$ $\therefore$ The S.S. $=]1 , 4]$

Another solution for number (2) :

We can divide this inequality into two inequalities as follows :

$x - 1 < 3x - 3 \longrightarrow (1)$ and $3x - 3 \leq x + 5 \longrightarrow (2)$

Then the solution set of the origin inequality is the intersection set of the two sets of solutions of the two inequalities (1) and (2)

- **Finding the S.S. of the inequality (1) :**

$\because x - 1 < 3x - 3$ (adding 1 to both sides)

$\therefore x < 3x - 2$ (adding $-3x$ to both sides)

$\therefore -2x < -2$ $\left(\text{multiplying both sides by } -\frac{1}{2}\right)$

$\therefore x > 1$ $\therefore$ The S.S. $=]1, \infty[$

- **Finding the S.S. of the inequality (2) :**

$\because 3x - 3 \leq x + 5$ (adding 3 to both sides)

$\therefore 3x \leq x + 8$ (adding $-x$ to both sides)

$\therefore 2x \leq 8$ $\left(\text{multiplying both sides by } \frac{1}{2}\right)$

$\therefore x \leq 4$ $\therefore$ The S.S. $=]-\infty, 4]$

- The S.S. of the origin inequality $=]1, \infty[\cap]-\infty, 4] =]1, 4]$

TRY 2 by yourself

Find in $\mathbb{R}$ the S.S. of each of the following inequalities :

1. $3x - 1 > 8$
2. $2 - 2x \geq -6$
3. $-16 < 5x + 4 \leq 9$
4. $2x + 1 > 4x - 3 > 2x - 11$

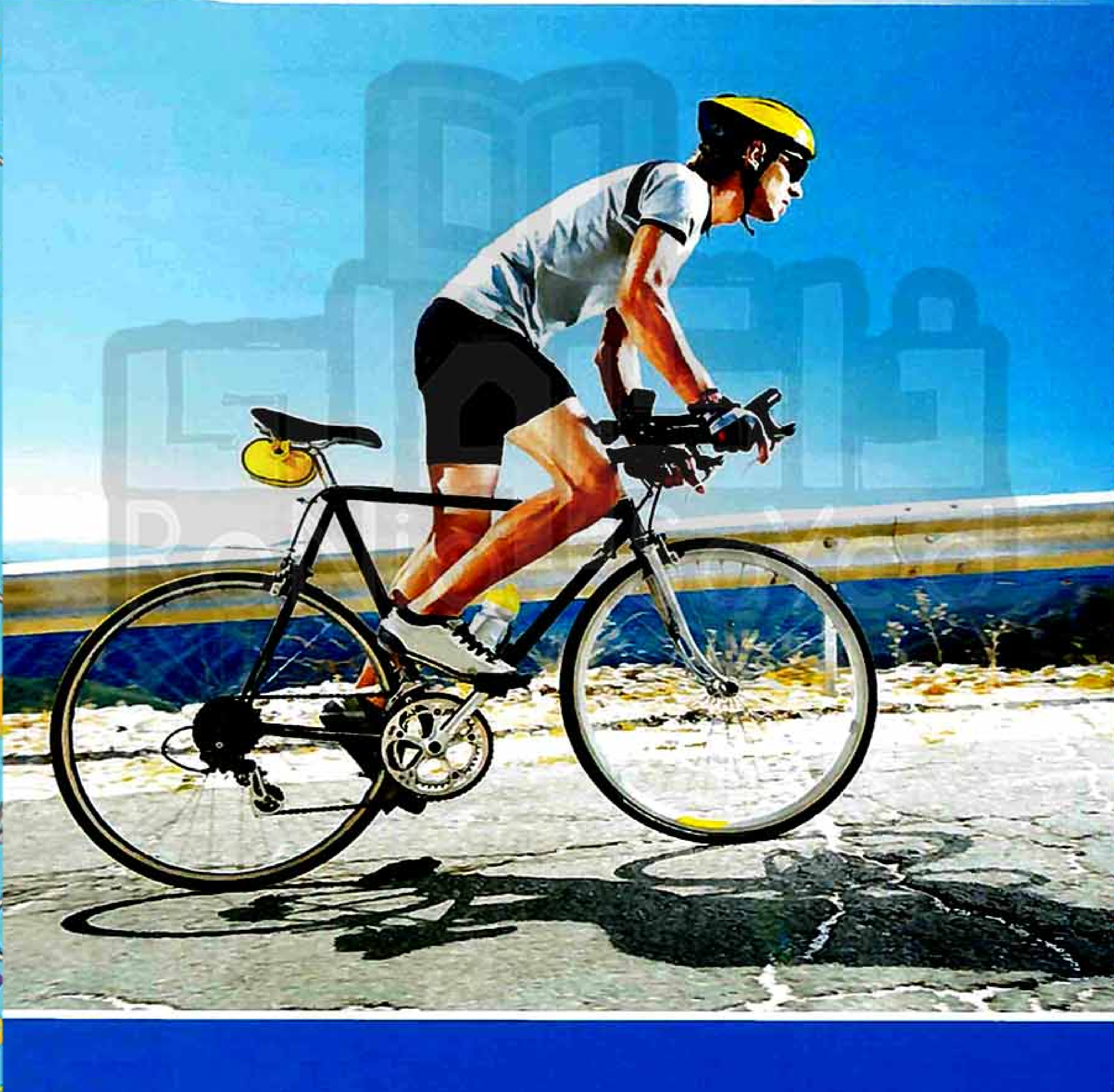
Answers *of try by yourself*

1 (1) $\{-\frac{1}{2}\}$ (2) $\{\sqrt{5}\}$ (3) $\{2 + \sqrt{3}\}$

2 (1) $]3, \infty[$ (2) $]-\infty, 4]$ (3) $]-4, 1]$ (4) $]-4, 2[$

UNIT 2

Relation between Two Variables



Lessons of the unit :

1. Relation between two variables.
2. Slope of straight line.
3. Real life applications on the slope.

▶ Use your smart phone or tablet to scan **the QR Code** and enjoy watching videos.

Unit Objectives :

By the end of this unit, student should be able to :

- recognize the relation between two variables of first degree.
- represent the relation between two variables of first degree graphically.
- recognize the slope of the straight line.
- find the slope of the straight line passing through two given points.
- recognize the slope of the straight line parallel to x-axis and the slope of the straight line parallel to y-axis.
- verify using the slope of the straight line that the three points are collinear or not.
- find the uniform velocity of a car by using the slope of the straight line.
- solve applications on the slope of the straight line.

Lesson 1

Relation between two variables

The concept of the relation between two variables

- **Islam has 50 pounds. If Islam went to the amusement park , he would find two kinds of favourite games :**

The first kind

costs 5 pounds for playing one game.

The second kind

costs 10 pounds for playing one game.

- What are the different possibilities for playing the two kinds such that he spends all his money ?
- **To find all the possibilities :**
  - Assume that he will play x games of the first kind and y games of the second kind
  - Then , the cost of playing the first kind is $5x$ pounds and the cost of playing the second kind is $10y$ pounds.
  - In order to spend all his money , it should be : $5x + 10y = 50$
  - This is an algebraic relation between the two variables x and y and it is called an equation of the first degree in two variables.
- We can simplify the previous relation by dividing all terms by 5 to get an equivalent equation which is : $x + 2y = 10$

It can be written also in the form : $2y = 10 - x$

i.e. $y = \frac{10 - x}{2}$

$5x + 10y = 50$ $\div 5$

$x + 2y = 10$

$2y = 10 - x$ $\div 2$

$y = \frac{10 - x}{2}$

For example:

- If Islam decided that he will not play the first kind.

i.e. $x = 0$, then $y = \frac{10-0}{2} = 5$

i.e. He can spend all his money by playing 5 games of the second kind.

We express that by the ordered pair (0 , 5)

- If he decided to play one game of the first kind.

i.e. $x = 1$, then $y = \frac{10-1}{2} = 4\frac{1}{2}$

but in this case , he cannot play $4\frac{1}{2}$ games of the second kind because the number of games must be a natural number.

- If he decided to play two games of the first kind

i.e. $x = 2$, then $y = \frac{10-2}{2} = 4$

i.e. He can spend all his money by playing 2 games of the first kind and 4 games of the second kind . We express that by the ordered pair (2 , 4)

Thus we can know the different possibilities and put them in a table such as the following :

Number of games of the 1st kind (x)	0	2	4	6	8	10
Number of games of the 2nd kind (y)	5	4	3	2	1	0

Remarks

- There is an infinite number of ordered pairs which satisfy the previous relation but some of them can't represent the possible numbers of each games because the number of games must be a natural number.
  - As we mentioned before $\left(1 , 4\frac{1}{2}\right)$ satisfies the relation but it is not possible to represent the number of games because $4\frac{1}{2} \notin \mathbb{N}$
  - Similarly (– 2 , 6) satisfies the relation but it is not to be used because $-2 \notin \mathbb{N}$
- To find all the possibilities , we write the equation : $x + 2y = 10$ putting y in one hand side as : $y = \frac{10-x}{2}$

  we can also put x in one hand side as : $x = 10 - 2y$

  And the following example shows that.

Unit 2

Example 1 What are the different possibilities for a person to pay L.E. 45 using two kinds of bills (banknotes) of L.E. 5 and L.E. 10 ?

Solution Let the number of bills of L.E. 5 be x , then its value = $5x$ pounds and the number of bills of L.E. 10 be y , then its value = 10 y pounds.

$\therefore 5x + 10y = 45$, dividing the two sides by 5

$\therefore x + 2y = 9$

Putting x in one hand side , then the equation will be in the form :

$x = 9 - 2y$

The following table shows all possibilities to pay the sum of money :

y	x	(x , y)	Number of bills of each kind
0	$9 - 2 \times 0 = 9$	(9 , 0)	9 bills of 5 pounds
1	$9 - 2 \times 1 = 7$	(7 , 1)	7 bills of 5 pounds and 1 bill of 10 pounds
2	$9 - 2 \times 2 = 5$	(5 , 2)	5 bills of 5 pounds and 2 bills of 10 pounds
3	$9 - 2 \times 3 = 3$	(3 , 3)	3 bills of 5 pounds and 3 bills of 10 pounds
4	$9 - 2 \times 4 = 1$	(1 , 4)	1 bill of 5 pounds and 4 bills of 10 pounds

Notice that :

If y = 5 , then $x = -1 \notin \mathbb{N}$, then y = 5 is impossible.

TRY by yourself 1

Find the different possibilities for a person to pay L.E. 65 of bills (banknotes) of L.E. 5 and L.E. 20

The linear relation

- The linear relation is a relation of the first degree between two variables x and y, it is in the form

$a x + b y = c$, where a, b and c are real numbers, a and b are not both equal to zero

- There is an infinite number of ordered pairs which satisfy this relation.
- If we represent it graphically, the graph will be a straight line therefore it is called a linear relation, this will be shown later when we study the graphic representation of the linear relation.

Example 2 Find three ordered pairs satisfying each of the following relations :

1. $3x + y = 5$
2. $3x - 2y = 6$
3. $2x = 3$
4. $y = -2$

Solution

We can find these ordered pairs by setting a value for x and substituting in the relation to get its corresponding value of y or we do the converse :

1
- Set $x = 0$

  $\therefore 3 \times 0 + y = 5$ $\qquad$ $\therefore y = 5$

  $\therefore (0, 5)$ satisfies the relation.

- Set $x = 1$

  $\therefore 3 \times 1 + y = 5$ $\qquad$ $\therefore y = 5 - 3 = 2$

  $\therefore (1, 2)$ satisfies the relation.

- Set $x = -2$

  $\therefore 3 \times (-2) + y = 5$ $\qquad$ $\therefore y = 5 + 6 = 11$

  $\therefore (-2, 11)$ satisfies the relation.

2 By substituting directly as we did in 1 we can get the ordered pairs but we will present another method of solution by putting one of the two variables in one hand side alone.

$\because 3x - 2y = 6$ $\qquad$ $\therefore -2y = 6 - 3x$ (multiply by (-1))

$\therefore 2y = 3x - 6$ $\qquad$ $\therefore y = \frac{3x - 6}{2}$

- Set $x = 0$ $\therefore y = \frac{3 \times 0 - 6}{2} = \frac{-6}{2} = -3$

  $\therefore (0, -3)$ satisfies the relation.

- Set $x = 1$ $\therefore y = \frac{3 \times 1 - 6}{2} = -\frac{3}{2} = -1\frac{1}{2}$

  $\therefore \left(1, -1\frac{1}{2}\right)$ satisfies the relation.

- Set $x = 2$ $\therefore y = \frac{3 \times 2 - 6}{2} = 0$

  $\therefore (2, 0)$ satisfies the relation.

3 $\because 2x = 3$ $\therefore x = \frac{3}{2}$ $\therefore x = 1\frac{1}{2}$

This relation will be satisfied for all ordered pairs (x, y) where $x = 1\frac{1}{2}$ whatever the value of y such as $\left(1\frac{1}{2}, 0\right)$, $\left(1\frac{1}{2}, 1\right)$ and $\left(1\frac{1}{2}, 2\right)$

4 $y = -2$

This relation will be satisfied for all ordered pairs (x, y), where $y = -2$, whatever the value of x such as $(0, -2)$, $(1, -2)$ and $(2, -2)$

Example 3 Show which of the following ordered pairs satisfies the relation $2x - y = 1$:
$(0, 1)$, $(5, 3)$, $(3, 5)$, $(-2, -5)$

Solution

- Set $x = 0$ and $y = 1$

  $\therefore 2x - y = 2 \times 0 - 1 = -1 \neq 1$

  $\therefore (0, 1)$ does not satisfy the relation.

- Set $x = 5$ and $y = 3$

  $\therefore 2x - y = 2 \times 5 - 3 = 7 \neq 1$

  $\therefore (5, 3)$ does not satisfy the relation.

- Set $x = 3$ and $y = 5$

  $\therefore 2x - y = 2 \times 3 - 5 = 1$

  $\therefore (3, 5)$ satisfies the relation.

- Set $x = -2$ and $y = -5$

  $\therefore 2x - y = 2(-2) - (-5) = 1$

  $\therefore (-2, -5)$ satisfies the relation.

Example 4

1. If $(-2, 1)$ satisfies the relation : $3x + by = 1$, find the value of b
2. If $(k, 2k)$ satisfies the relation : $5x - y = 6$, find the value of k

Solution

1. $\because (-2, 1)$ satisfies the relation : $3x + by = 1$

$\therefore 3(-2) + b(1) = 1$ $\quad \therefore -6 + b = 1$ $\quad \therefore b = 1 + 6 = 7$

2. $\because (k, 2k)$ satisfies the relation : $5x - y = 6$

$\therefore 5(k) - (2k) = 6$ $\quad \therefore 5k - 2k = 6$

$\therefore 3k = 6$ $\quad \therefore k = 2$

TRY 2 by yourself

1. Find four ordered pairs satisfying the relation : $3x + y = 2$
2. If $(3k, 2k)$ satisfies the relation : $x - 3y = 9$, find the value of k

The graphic representation of the linear relation

- We mentioned that linear relation between two variables x and y is usually written in the form : $ax + by = c$, where a, b and c are real numbers, a and b are not both equal to zero. This linear relation is represented graphically by a straight line (that is why it is called linear).
- To graph a linear relation, you need to graph at least two ordered pairs satisfying this relation. You can add a third ordered pair to check that the three points lie on the same straight line which is the graphic representation of the relation.

Example 5 **Represent the relation :** $2x - y = 3$ graphically

Solution

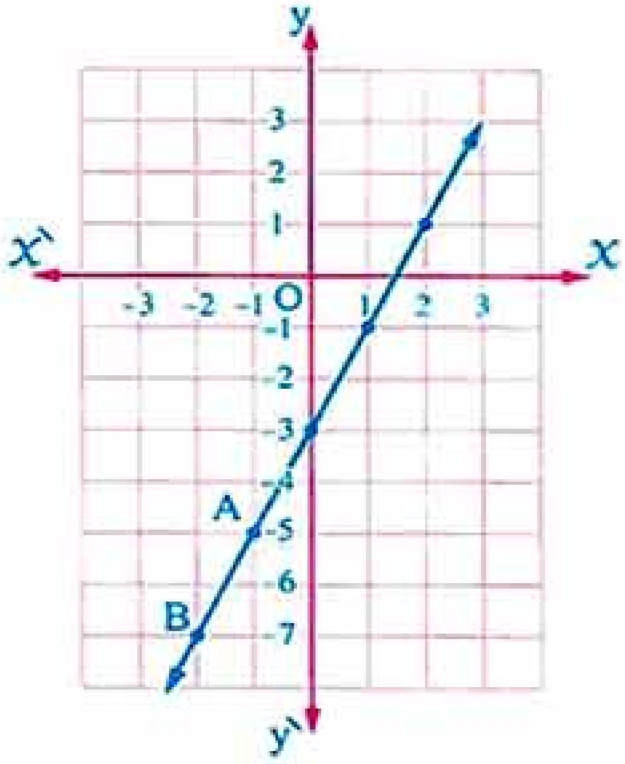
To represent this relation graphically, we should determine three ordered pairs satisfying the relation : $2x - y = 3$, as follows :

- Set $x = 0$ $\quad \therefore 2 \times 0 - y = 3$ $\quad \therefore -y = 3$ $\quad \therefore y = -3$
- Set $x = 1$ $\quad \therefore 2 \times 1 - y = 3$ $\quad \therefore -y = 1$ $\quad \therefore y = -1$
- Set $x = 2$ $\quad \therefore 2 \times 2 - y = 3$ $\quad \therefore -y = -1$ $\quad \therefore y = 1$

It is preferable to put the values of x and y in a table as the following :

x	0	1	2
y	-3	-1	1

Then we determine the points which represent these ordered pairs : $(0 , -3)$, $(1 , -1)$ and $(2 , 1)$ on orthogonal coordinates system , then we draw the straight line passing through these points , it will be the graphic representation of the relation : $2x - y = 3$



Remark

All the points of the straight line which represents the relation determine ordered pairs which satisfy the relation.

For example:

The point A determines the ordered pair $(-1 , -5)$ which satisfies the relation when we put $x = -1$ we find that $2 \times (-1) - y = 3$ i.e. $y = -5$ and also the point B $(-2 , -7)$

TRY 3 by yourself

Represent the relation : $y - 2x = -1$ graphically.

Special cases

We studied before the relation : $a x + b y = c$, where a , b are not both equal to zero and it is called a linear relation and it is represented graphically by a straight line and now we study the following cases :

Examples :

1 If $a = 0$, $b \neq 0$

Then the relation becomes in the form :

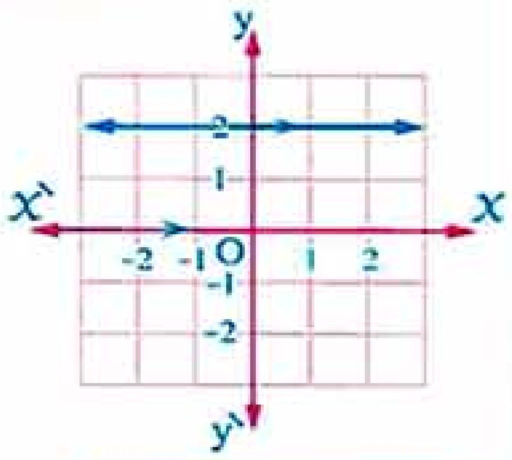
$$b y = c$$

and it is represented graphically by a straight line parallel to x-axis and intersects y-axis at the point $\left(0 , \frac{c}{b}\right)$

For example:

The relation : $2 y = 4$

i.e. $y = 2$ is represented by a straight line parallel to x-axis and intersects y-axis at the point $(0 , 2)$



Notice that :

The relation : $y = 0$ is represented by x-axis

2 **If b = 0 , a ≠ 0**

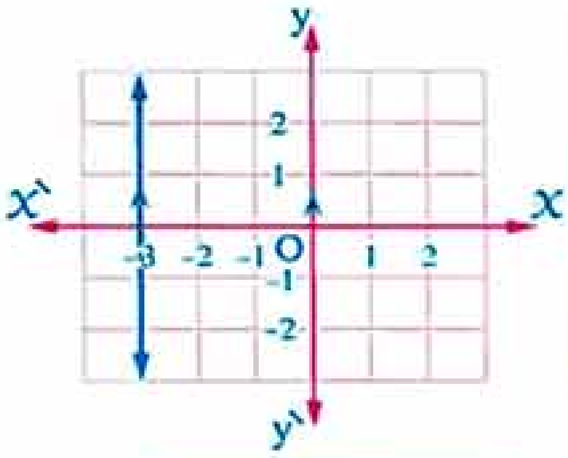
Then the relation becomes in the form :

$$a x = c$$

and it is represented graphically by a straight line parallel to y-axis and intersects x-axis at the point $\left(\frac{c}{a}, 0\right)$

For example:

The relation : $x = -3$ is represented by a straight line parallel to y-axis and intersects x-axis at the point $(-3, 0)$



Notice that :

The relation : $x = 0$ is represented by y-axis

3 **If c = 0**

Then the relation becomes :

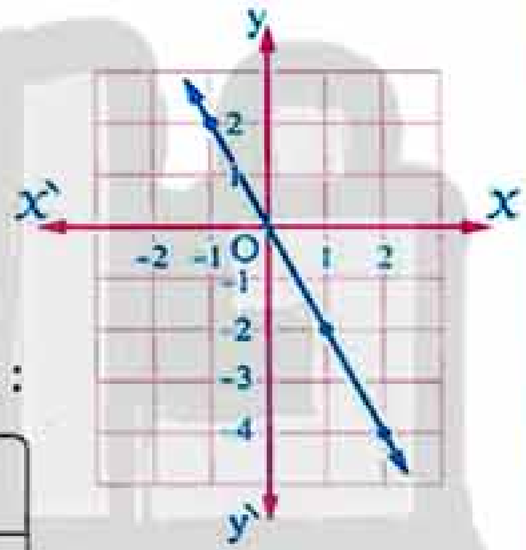
$$a x + b y = 0$$

and it is represented by a straight line passing through the origin point $(0, 0)$

For example:

The relation : $2x + y = 0$ is represented graphically by a straight line passing through the origin point as shown in the opposite graph :

x	1	−1	2
y	−2	2	−4



Example 6 **Graph the straight line which represents the relation : $2x + 5y = 10$ and if this straight line intersects x-axis at the point A and y-axis at the point B , find the area of Δ OAB where O is the origin point.**

Solution

$\because 2x + 5y = 10$ $\therefore 2x = 10 - 5y$

$\therefore x = \frac{10 - 5y}{2}$

- Set y = 0 $\therefore x = \frac{10 - 5(0)}{2} = 5$

  $\therefore (5, 0)$ satisfies the relation

- Set y = 2 $\therefore x = \frac{10 - 5(2)}{2} = 0$

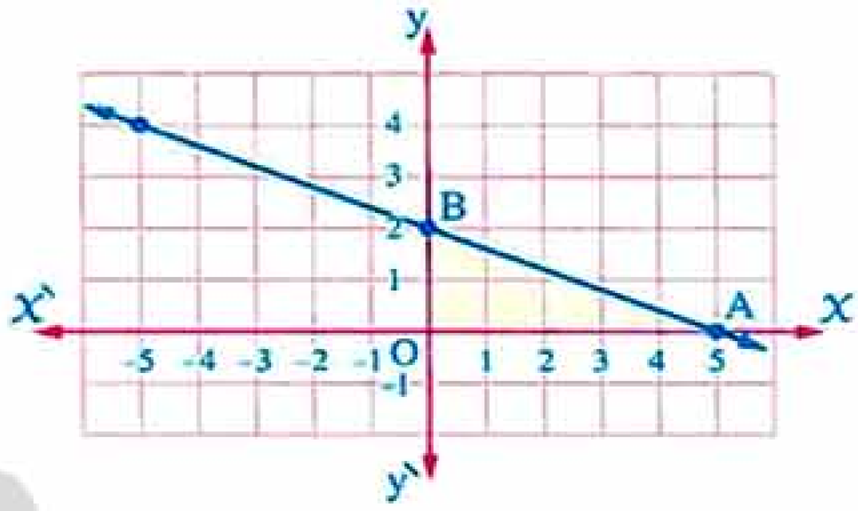
  $\therefore (0, 2)$ satisfies the relation

Unit 2

- Set y = 4 $\therefore \mathcal{X} = \frac{10 - 5\,(4)}{2} = -5$

$\therefore$ (– 5 , 4) satisfies the relation

$\mathcal{X}$	5	0	– 5
y	0	2	4



$\because$ The straight line intersects $\mathcal{X}$-axis at the point (5 , 0)

$\therefore$ OA = 5 length units.

, $\because$ the straight line intersects y-axis at the point (0 , 2)

$\therefore$ OB = 2 length units.

$\therefore$ The area of Δ OAB = $\frac{1}{2}$ OA × OB = $\frac{1}{2}$ × 5 × 2 = 5 square units.

Remark

In the previous example , we can get the points of intersection of the straight line representing the relation : $2\mathcal{X} + 5y = 10$ and the coordinate axes without using the graph as the following :

- Set y = 0 $\therefore 2\mathcal{X} + 5 \times 0 = 10$
  $\therefore 2\mathcal{X} = 10$ $\therefore \mathcal{X} = 5$
  $\therefore$ The point of intersection with $\mathcal{X}$-axis is (5 , 0)
- Set $\mathcal{X} = 0$ $\therefore 2\,(0) + 5y = 10$
  $\therefore 5y = 10$ $\therefore y = 2$
  $\therefore$ The point of intersection with y-axis is (0 , 2)

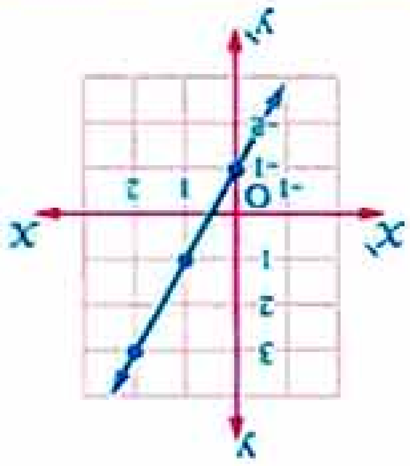
Answers of try by yourself

1 13 bills of 5 pounds - 9 bills of 5 pounds and one bill of 20 pounds - 5 bills of 5 pounds and 2 bills of 20 pounds - One bill of 5 pounds and 3 bills of 20 pounds.

2 1 (–1 , 5) , (0 , 2) , (1 , –1) , (2 , – 4) "There are other solutions"

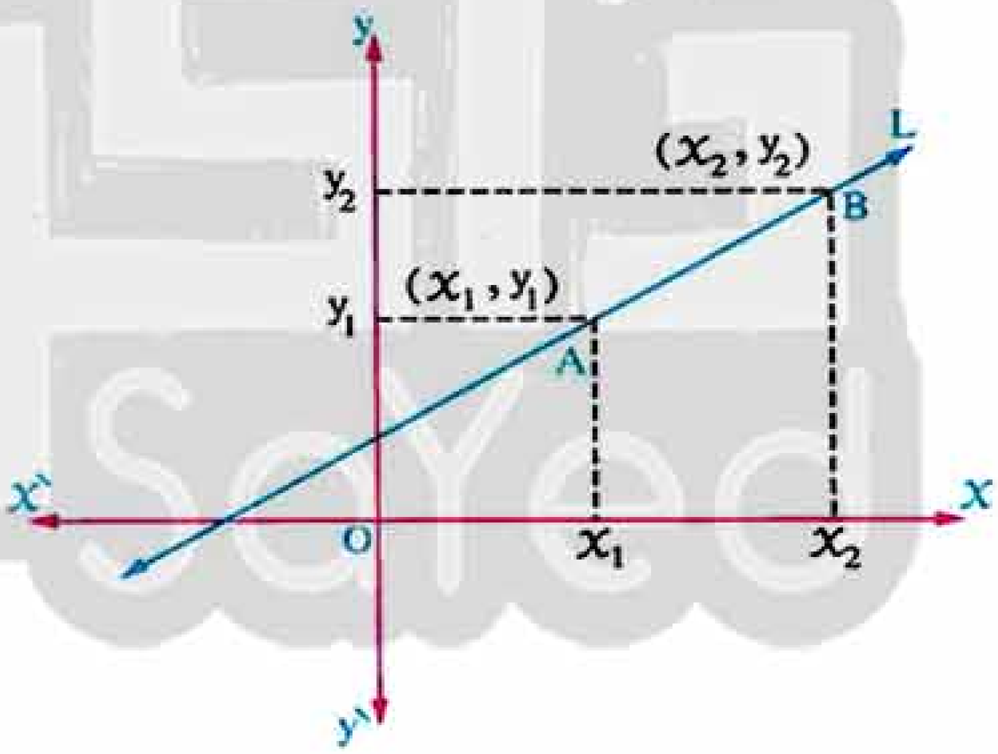
2 –3

3



Lesson 2 Slope of straight line

If a point moves on a straight line L from the location A (x_1 , y_1) to the location B (x_2 , y_2) , then :



– The change in the x-coordinates = $x_2 - x_1$
It is called (the horizontal change).

– The change in the y-coordinates = $y_2 - y_1$
It is called (the vertical change).

The ratio of the change in the y-coordinates to the change in the x-coordinates is called the slope of the straight line (S).

Definition

$$\text{The slope of the straight line} = \frac{\text{the change in y-coordinates}}{\text{the change in } x\text{-coordinates}} = \frac{\text{the vertical change}}{\text{the horizontal change}}$$

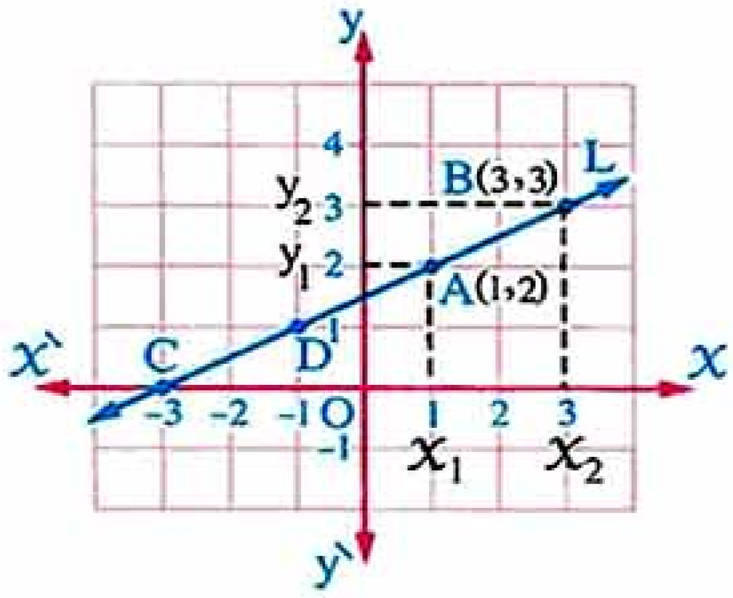
i.e. • $S = \dfrac{y_2 - y_1}{x_2 - x_1}$, where $x_1 \neq x_2$

• S is undefined if $x_1 = x_2$

Unit 2

Example 1 **In the opposite figure :**

Find the slope of the straight line L



Solution We determine two points on the straight line such as A = (1 , 2) and B = (3 , 3)

$\because S = \frac{y_2 - y_1}{x_2 - x_1}$

$\therefore S = \frac{3-2}{3-1} = \frac{1}{2}$

Remark

In the previous example , notice that if we used another two points of the straight line to find its slope as the points C (– 3 , 0) and D (– 1 , 1) we find that :

$S = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1-0}{-1-(-3)} = \frac{1}{2}$ (the same result)

i.e. The slope of the straight line is constant for any two selected points on it.

Example 2 **Find the slope of the straight line passing through each pair of points in the following :**

1. (2 , 4) , (4 , 5)
2. (1 , 3) , (4 , 2)
3. (– 2 , – 3) , (– 4 , 1)
4. (3 , 1) , (– 1 , 0)

Solution

1. $S = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5-4}{4-2} = \frac{1}{2}$
2. $S = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2-3}{4-1} = \frac{-1}{3}$
3. $S = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1-(-3)}{-4-(-2)} = \frac{4}{-2} = -2$
4. $S = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0-1}{-1-3} = \frac{-1}{-4} = \frac{1}{4}$

TRY yourself 1

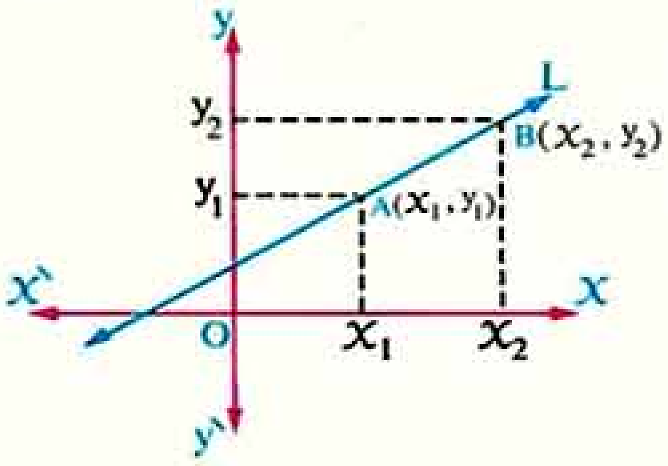
Find the slope of the straight line passing through each pair of points in the following :

1. (2 , 1) , (3 , 4)
2. (3 , – 5) , (– 4 , 2)
3. (– 3 , – 1) , (1 , 0)
4. (– 6 , 3) , (– 4 , 2)

Remarks

- If a point moves on a straight line from the location A (x_1, y_1) to the location B (x_2, y_2), where $x_2 > x_1$, then

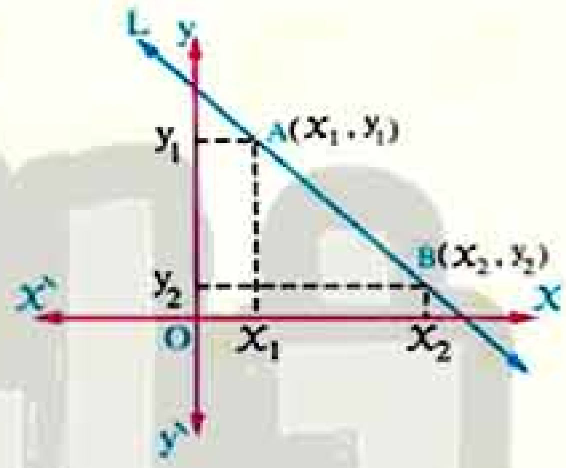
1 If $y_2 > y_1$



i.e. y increases as x increases, then the slope of the straight line is a positive number.

i.e. $S > 0$

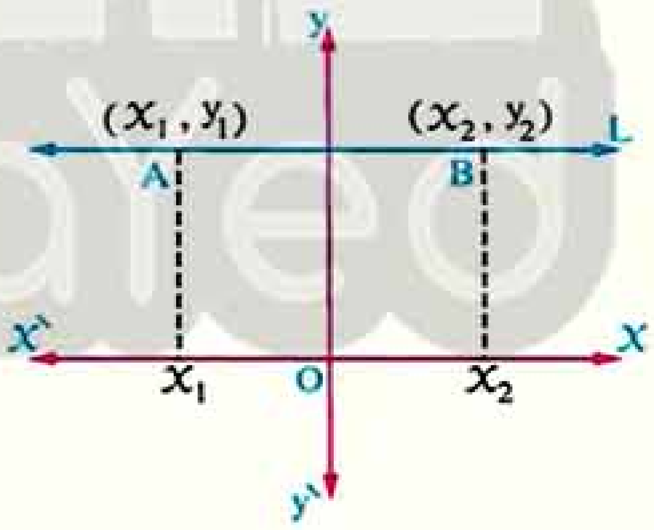
2 If $y_2 < y_1$



i.e. y decreases as x increases, then the slope of the straight line is a negative number.

i.e. $S < 0$

3 If $y_2 = y_1$

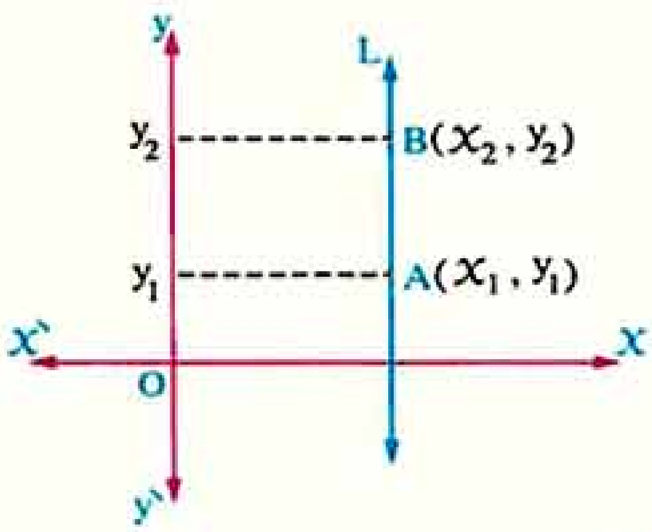


i.e. y is constant as x changes, then the slope of the straight line = zero

i.e. $S = 0$

i.e. The slope of the straight line parallel to x-axis = zero

- If : $x_2 = x_1$,



then the slope of the straight line is undefined because there is no change in the x-axis.

i.e. $x_2 - x_1 = 0$

i.e. The slope of the straight line parallel to y-axis is undefined.

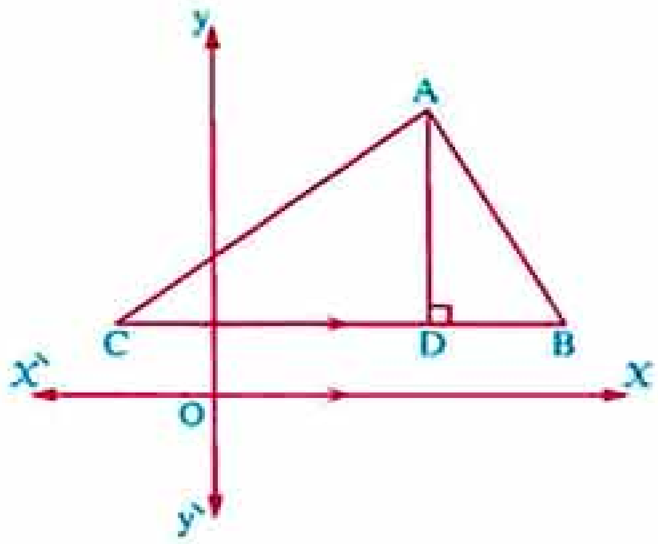
Unit 2

Example 3 **In the opposite figure :**

ABC is a triangle in which

$\overline{BC} \parallel \overleftrightarrow{XX'}$, $\overline{AD} \perp \overline{BC}$

Complete the following using one of the words (positive , negative , zero , undefined) in the spaces :



1. The slope of $\overleftrightarrow{AB}$ is ....
2. The slope of $\overleftrightarrow{BC}$ is ....
3. The slope of $\overleftrightarrow{AC}$ is ....
4. The slope of $\overleftrightarrow{AD}$ is ....

Solution

1. Negative
2. Zero
3. Positive
4. Undefined

Example 4 **If the slope of the straight line passing through the two points (– 3 , 4) and (1 , y) is 2 , find the value of y**

Solution

$\because S = \frac{y_2 - y_1}{x_2 - x_1}$ $\therefore 2 = \frac{y-4}{1-(-3)}$ $\therefore 2 = \frac{y-4}{4}$

$\therefore y - 4 = 2 \times 4$ $\therefore y - 4 = 8$ $\therefore y = 12$

An important remark

"In the previous , we found that the slope of the straight line is constant and it does not change whatever the two selected points on the line , therefore to prove that the three points A , B and C are collinear , then we find the slope of $\overleftrightarrow{AB}$ and the slope of $\overleftrightarrow{BC}$

If the slope of $\overleftrightarrow{AB}$ = the slope of $\overleftrightarrow{BC}$, then A , B and C are collinear."

Example 5 **Prove that the points A (2 , 3) , B (4 , 2) and C (8 , 0) are collinear.**

Solution

$\because S = \frac{y_2 - y_1}{x_2 - x_1}$

$\therefore$ The slope of $\overleftrightarrow{AB} = \frac{2-3}{4-2} = -\frac{1}{2}$, the slope of $\overleftrightarrow{BC} = \frac{0-2}{8-4} = \frac{-2}{4} = -\frac{1}{2}$

, $\because$ the slope of $\overleftrightarrow{AB}$ = the slope of $\overleftrightarrow{BC}$ and the point B is common.

$\therefore$ The points A , B and C are collinear.

Example 6 **If the points A , B and C are collinear where A (3 , 2) , B (5 , – 1) and C (1 , k) , find the value of k**

Solution

$\because S = \dfrac{y_2 - y_1}{x_2 - x_1}$

$\therefore$ The slope of $\overleftrightarrow{AB} = \dfrac{-1-2}{5-3} = \dfrac{-3}{2}$

, the slope of $\overleftrightarrow{BC} = \dfrac{k-(-1)}{1-5} = \dfrac{k+1}{-4}$

, $\because$ A , B and C are collinear , the slope of the straight line is constant for any two points on it.

$\therefore$ The slope of $\overleftrightarrow{AB}$ = the slope of $\overleftrightarrow{BC}$

$\therefore \dfrac{-3}{2} = \dfrac{k+1}{-4}$

$\therefore 2(k+1) = -3 \times (-4)$

$\therefore 2k + 2 = 12$

$\therefore 2k = 10$ $\qquad \therefore k = 5$

TRY 2 by yourself

1. If the slope of the straight line passing through the two points (3 , – 1) , (7 , a) is $\frac{3}{4}$, find the value of a

2. **Prove that :** C (– 1 , 2) $\in \overleftrightarrow{AB}$, where A (1 , 3) and B (3 , 4)

Answers of try by yourself

1. 1) 3 2) – 1 3) $\frac{1}{4}$ 4) $-\frac{1}{2}$

2. 1) 2 2) Prove by yourself [**Hint :** Prove that the slope of $\overleftrightarrow{AC}$ = the slope of $\overleftrightarrow{AB}$].

Lesson 3

Real life applications on the slope

- We studied before that if there is a linear relation between two variables x and y , then :

The slope of the straight line which represents this relation $= \dfrac{\text{the change in y-coordinates}}{\text{the change in } x\text{-coordinates}}$

i.e. The slope of the straight line (S) expresses the rate of change of y with respect to x

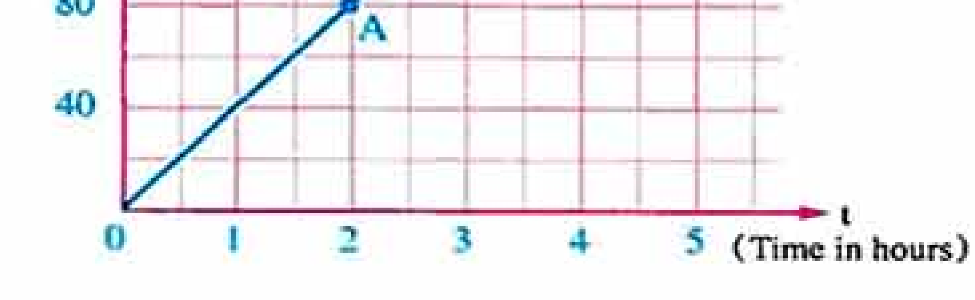
- In our life , there are many applications which we need to know the rate of change in dealing with them.

For example:

1 If the opposite graph represents the motion of a car , then :

The uniform velocity of the car (v)
= the rate of change of the distance (d) with respect to the time (t)

i.e. The uniform velocity of the car (v)
= the slope of the straight line (S)
and by selecting two points on the straight line as A (2 , 80) and B (5 , 200)



$\therefore \ v = \dfrac{d_2 - d_1}{t_2 - t_1} = \dfrac{200 - 80}{5 - 2} = \dfrac{120}{3} = 40$ km./hour

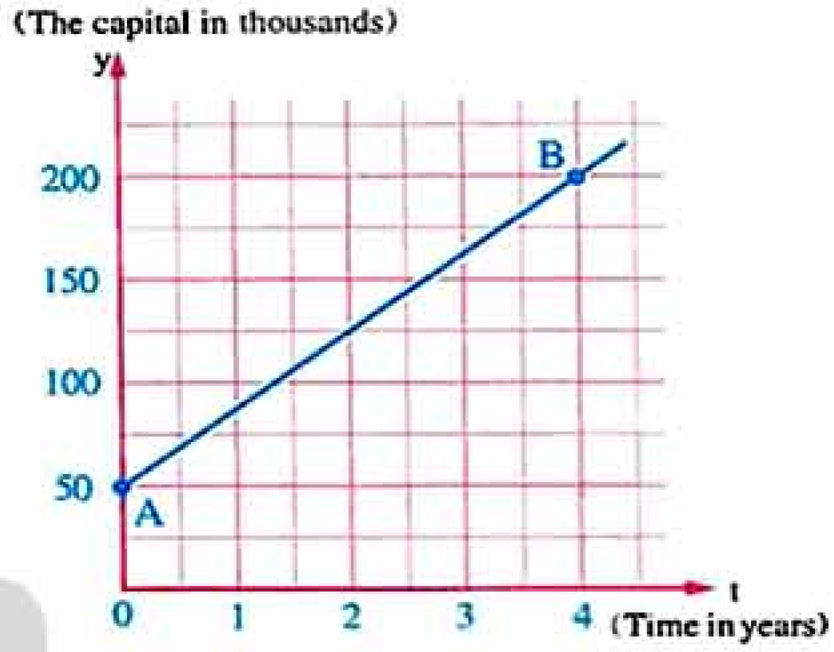
2 If the opposite graph represents the change in the capital of a company (y) within the time (t) , then :

The rate of change in the capital of the company = the slope of the straight line $\overleftrightarrow{AB}$

$\therefore$ The rate of change of the capital of the company

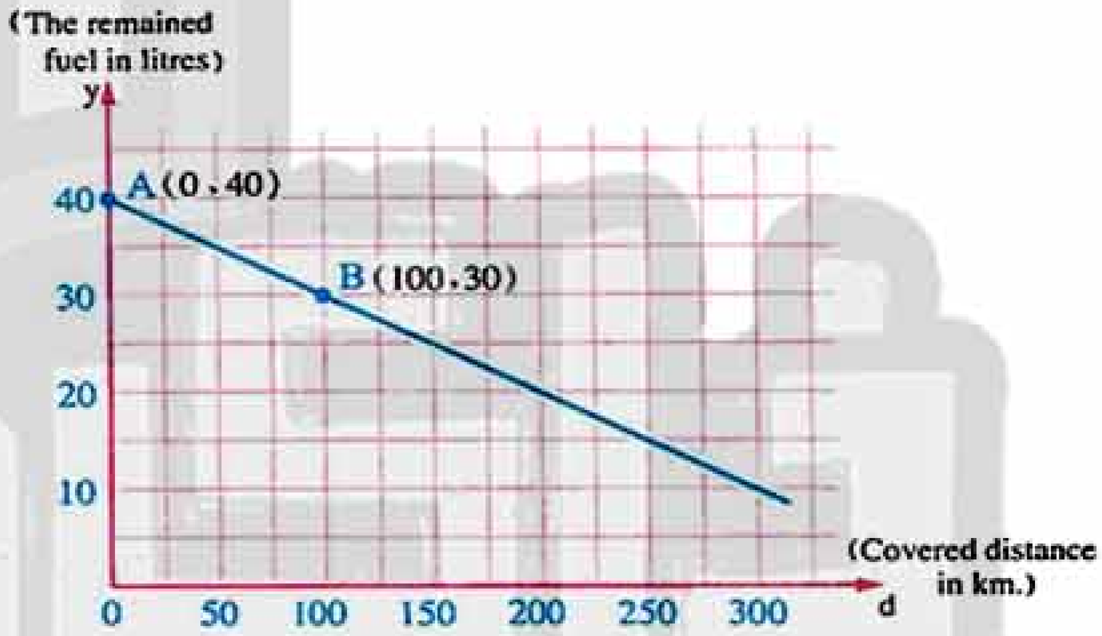
$$= \frac{y_2 - y_1}{t_2 - t_1} = \frac{200 - 50}{4 - 0}$$

$$= \frac{150}{4} = 37.5 \text{ thousand pounds / year.}$$



i.e. The capital of the company increases in the rate = 37.5 × 1000 = 37500 pounds/year.

3 A person filled the tank of his car whose capacity is 40 litres with fuel. After he covered a distance 100 km. , he found that the remained fuel in the tank = 30 litres. The opposite figure shows the relation between the covered distance in km. (d) and the amount of the remained fuel in the tank in litres (y) , then :

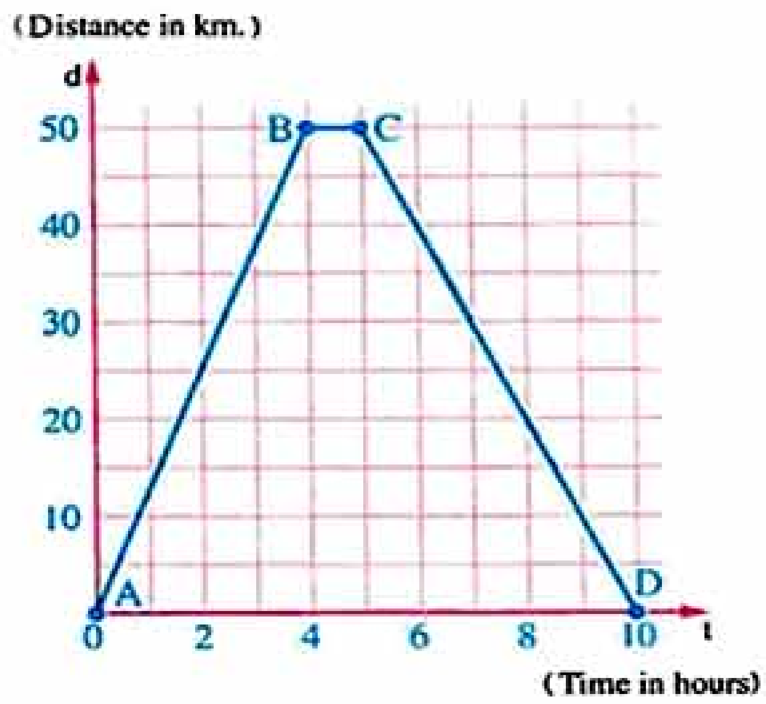


The rate of consumption of fuel = the slope of $\overleftrightarrow{AB}$

i.e. The rate of consumption of fuel $= \frac{y_2 - y_1}{d_2 - d_1} = \frac{30 - 40}{100 - 0} = \frac{-10}{100} = -\frac{1}{10}$ litre/km.

(The negative sign denotes the amount of fuel decreases in the rate of one litre for each 10 km.)

Example 1 **Waleed rode his bicycle from Cairo to Benha , then he returned back to Cairo. The opposite graph represents the bicycle motion during going and returning back :**



1 Find his velocity in going trip.

2 Find his velocity in returning back trip.

3 Find the average velocity during all trips.

4 What do you say about the horizontal line segment in the graph ?

Solution

1 Taking the two points A (0 , 0) and B (4 , 50)

$\therefore$ v (during going trip) $= \frac{50-0}{4-0} = 12.5$ km./hr.

2 Taking the two points C (5 , 50) and D (10 , 0)

$\therefore$ v (during returning back trip) $= \frac{0-50}{10-5} = \frac{-50}{5} = -10$ km./hr.

(The negative sign means that Waleed moved in the opposite direction of his first motion returning back to Cairo with velocity 10 km./hr.)

3 The average velocity $= \frac{\text{the total distance}}{\text{the total time}} = \frac{100}{10} = 10$ km./hr.

4 The horizontal line segment in the graph shows that Waleed stopped for an hour after he covered a distance equal to 50 km. , then he returned back to the start point.

Example 2 **The following graph shows the change of the capital of a company within 10 years :**

1 Find the slope of each of $\overleftrightarrow{AB}$, $\overleftrightarrow{BC}$ and $\overleftrightarrow{CD}$ What is the meaning of each of them ?

2 Calculate the capital of the company at the beginning.

Solution

$\because$ A (0 , 40) , B (3 , 100) , C (5 , 100) and D (10 , 80)

1 • The slope of $\overleftrightarrow{AB} = \frac{100-40}{3-0} = \frac{60}{3} = 20$

It expresses the increase in the capital of the company within the first three years from the beginning in the rate of 20000 pounds/year.

- The slope of $\overleftrightarrow{BC} = \frac{100 - 100}{5 - 3} = \frac{0}{2} = 0$

  It expresses that the capital of the company is still constant without increasing or decreasing within the fourth and the fifth years from the beginning.

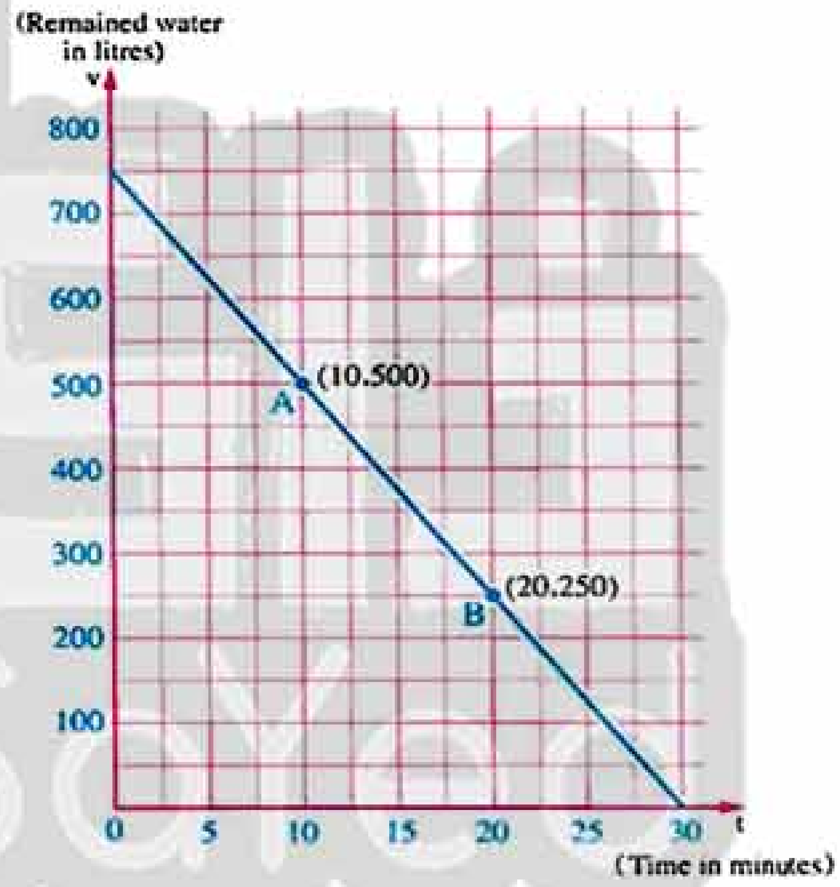
- The slope of $\overleftrightarrow{CD} = \frac{80 - 100}{10 - 5} = \frac{-20}{5} = -4$

  It expresses the decrease in the capital of the company within the last five years in the rate of 4000 pounds / year.

2 $\because$ A (0 , 40)

$\therefore$ The capital of the company in the beginning = 40000 pounds.

Example 3 **A tank of water is filled with water completely. A tap is opened below the tank to empty it. The opposite graph represents the relation between the time (t) in minutes and the amount of water remained in the tank (v) in litres :**



1 What is the greatest capacity of the tank ?

2 What is the time needed to empty the tank ?

3 What is the amount remained in the tank after 20 minutes ?

4 What is the rate of emptying the tank ?

Solution

1 From the graph , we find that $\overleftrightarrow{AB}$ intersects the axis which represents the amount of remained water (v) at the point (0 , 750)

$\therefore$ The greatest capacity of the tank = 750 litres.

2 From the graph , we find that $\overleftrightarrow{AB}$ intersects the axis which represents the time (t) at the point (30 , 0)

$\therefore$ The needed time for emptying the tank is 30 minutes.

3 $\because$ The point $(20 , 250) \in \overleftrightarrow{AB}$

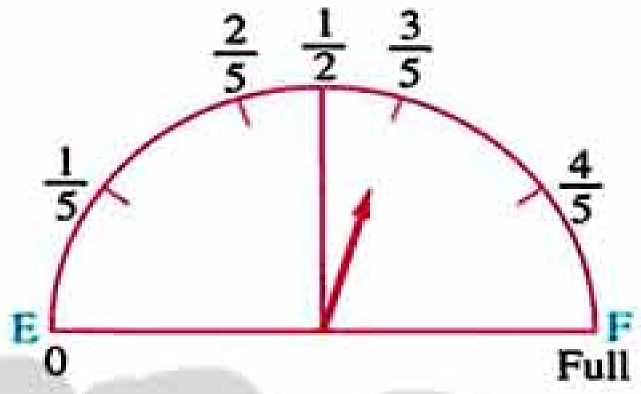
$\therefore$ After 20 minutes , the remained amount of water in the tank is 250 litres.

4 The rate of emptying the tank = the slope of $\overleftrightarrow{AB}$

$$= \frac{v_2 - v_1}{t_2 - t_1} = \frac{250 - 500}{20 - 10} = \frac{-250}{10} = -25$$

$\therefore$ The tank is emptied by the rate 25 litres/minute.

Example 4 **Hossam filled the tank of his car with fuel given that its capacity is 50 litres. After Hossam covered a distance 200 km. , he noticed that fuel meter shows that the tank has fuel = $\frac{3}{5}$ its capacity.**



Graph the relation between the distance covered by the car and the amount of fuel in the tank and calculate the distance covered by the car till the tank becomes empty.

Solution Let the covered distance = d (km.)
and the remained amount of fuel = y (litres)

$\therefore$ In the beginning , the distance = 0 km.

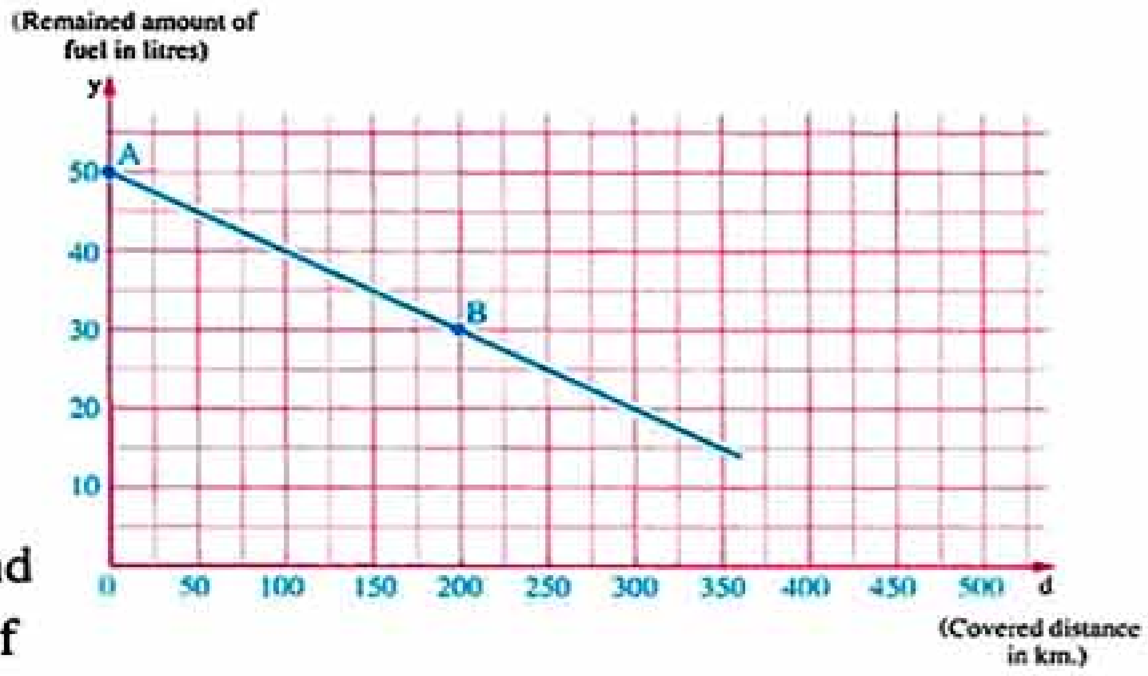
i.e. d = 0 and the amount of fuel in the tank = 50 litres.

i.e. y = 50

$\therefore$ The point A (0 , 50) expresses the amount of fuel in the tank in the beginning of motion.

$\because \frac{3}{5}$ the capacity of the tank $= \frac{3}{5} \times 50 = 30$ litres.

$\therefore$ The point B (200 , 30) expresses the amount of fuel in the tank after a covered distance 200 km. from the beginning.



$\therefore \overleftrightarrow{AB}$ represents the relation between the covered distance (d) and the remained amount of fuel in the tank (y)

$\therefore$ The rate of decrease of fuel = the slope of $\overleftrightarrow{AB}$

$$= \frac{y_2 - y_1}{d_2 - d_1} = \frac{30 - 50}{200 - 0} = \frac{-20}{200} = -\frac{1}{10} \text{ litre/km.}$$

i.e. The amount of fuel in the tank decreases with rate of one litre per 10 km.

$\therefore$ The covered distance from beginning the motion till the tank becomes empty

$$= \frac{\text{the amount of fuel in the beginning}}{\text{rate of decrease of fuel}} = \frac{50}{\frac{1}{10}} = 50 \times 10 = 500 \text{ km.}$$

Remark

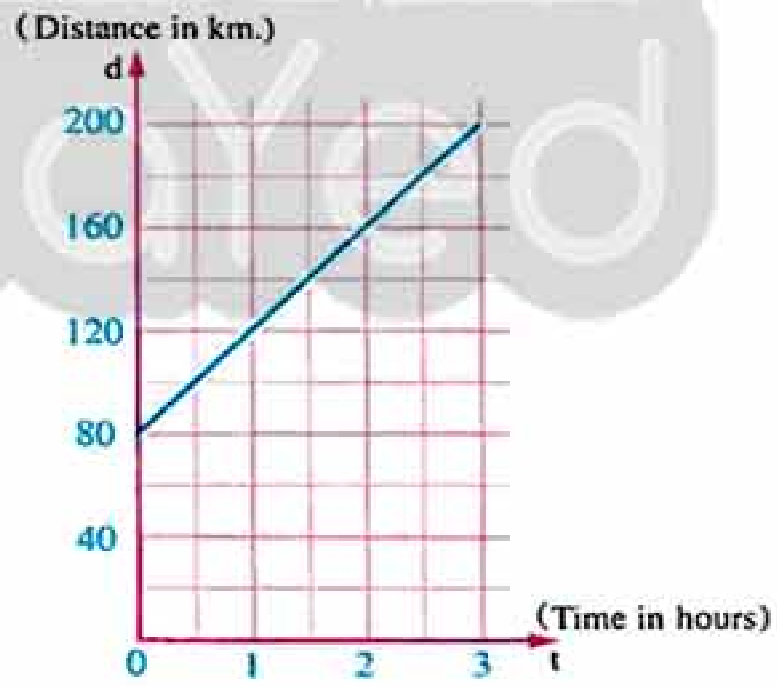
We can find the covered distance from the beginning till the tank becomes empty from the graph by finding the point of intersection of $\overleftrightarrow{AB}$ with the axis which represents the distance d which is (500 , 0)

i.e. The covered distance by the car when the tank becomes empty = 500 km.

TRY *by yourself*

The opposite graph represents the motion of a car measured from a fixed point A :

1. Determine the uniform velocity of the car.
2. Calculate the covered distance after two hours from the beginning of the motion.



Answers *of try by yourself*

1. 40 km. / hour
2. 80 km.

UNIT 3

Statistics

Lessons of the unit :

1. Collecting and organizing data.
2. The ascending and descending cumulative frequency tables and their graphical representation.
3. Mean.
4. Median.
5. Mode.

- Use your smart phone or tablet to scan **the QR Code** and enjoy watching videos.

Unit Objectives :

By the end of this unit, student should be able to :

- organize data in frequency tables with sets.
- form each of the ascending and descending cumulative frequency tables.
- graph each of the ascending and descending cumulative frequency tables.
- find the mean of a set of data organized in a frequency table with sets.
- find the median of a frequency distribution with sets.
- calculate the mode from a frequency table with sets.

Lesson 1

Collecting and organizing data

In the last year , you knew how to organize data and put them in a simple frequency table , but when summarizing large masses of data , it is useful to distribute them into sets , and determine the number of individuals belonging to each set.

The table consisting of sets and their corresponding frequencies is called "frequency table with sets"

The following example shows how to organize data into a frequency table with sets.

Example **In the following table , these are the marks of 54 students in one of the classes in grade two preparatory in a school , which they took in an exam in mathematics where the full mark is 60**

42	54	36	46	34	45	51	40	48
48	40	47	25	48	45	36	56	44
38	47	30	37.5	40	20	42	28	50
47	55	27	45	30	42	51	43	46
29	43	59	35	44.5	32	24	39	54
41	36	45	39	42	58	35	50	45

The required is forming the frequency table with sets.

Solution

1 Determine the range

(it is the difference between the greatest mark and the smallest mark)

$\because$ The smallest mark is 20 and the greatest mark is 59

$\therefore$ The range $= 59 - 20 = 39$

2 Divide these data into a suitable number of sets of marks , say 10 disjoint sets , the length of each of them is 4 , then you obtain the following sets :

- **The first set :**
  The students who obtain 20 marks till less than 24 marks , which is written as (20 –)

- **The second set :**
  The students who obtain 24 marks till less than 28 marks , it is written as (24 –)

- **The third set :**
  The students who obtain 28 marks till less than 32 marks , it is written as (28 –) and so on till you reach the tenth set.

- **The tenth set :**
  The students who obtain 56 marks till less than 60 , it is written as (56 –)

3 **Form the tally table as follows :**

Sets	Tallies	Frequency
20 –	/	1
24 –	///	3
28 –	////	4
32 –	////	4
36 –	~~////~~ //	7
40 –	~~////~~ ~~////~~	10
44 –	~~////~~ ~~////~~ //	12
48 –	~~////~~ //	7
52 –	///	3
56 –	///	3
Total		54

(The tally table)

4 Omit the tallies column from the table to get the final form of the frequency table with sets. It can be written vertically or horizontally.

The following is the horizontal form of the frequency table :

Sets	20-	24-	28-	32-	36-	40-	44-	48-	52-	56-	Total
Frequency	1	3	4	4	7	10	12	7	3	3	54

From the previous table , we deduce that :

- The set that has the greatest frequency is 44 –
- The set that has the least frequency is 20 –

TRY by yourself

The following is the weights of 50 persons :

52	35	40	57	43	40	36	49	43	58
47	48	51	30	59	36	45	41	44	37
42	54	38	55	42	47	46	34	53	44
47	32	41	62	50	39	58	46	43	49
40	41	64	44	54	45	38	40	48	41

Form the frequency table with sets.

Answers of try by yourself

Sets	30 –	35 –	40 –	45 –	50 –	55 –	60 –
Frequency	3	7	16	11	6	5	2

Lesson 2 The ascending and descending cumulative frequency tables and their graphical representation

Prelude

- In the previous lesson , you learnt how to form a frequency table with sets and how to get some information from it as the following table which represents the distribution of weekly wages of 50 workers in one factory :

Sets of wages	54 –	58 –	62 –	66 –	70 –	Total
No. of workers (Frequency)	5	12	22	7	4	50

From this table , you can know the number of workers (the frequency) in each set.

For example:

– The number of workers whose wages lie between 58 and less than 62 pounds is 12 workers.

– The number of workers whose wages lie between 66 and less than 70 pounds is 7 workers.

- **But some other information cannot be obtained directly from this table such as :**
  – The number of workers who obtain wages less than 62 pounds.
  – The number of workers who obtain wages equal to 58 pounds or more.
- **In order to be able to know such information , you need to study how to form another type of tables called cumulative frequency tables (ascending and descending) , and this what will be shown in the following examples :**

Unit 3

Example 1

The following frequency table shows the weekly wages in pounds of 50 workers in one factory :

Sets of wages	54 –	58 –	62 –	66 –	70 –	Total
No. of workers (Frequency)	5	12	22	7	4	50

Form the ascending cumulative frequency table and represent it graphically , then find :

1 The number of workers whose weekly wages are less than 60 pounds.

2 The percentage of the number of workers whose weekly wages are less than 60 pounds.

Solution

• Form the ascending cumulative frequency table as follows :

Sets of wages	54 –	58 –	62 –	66 –	70 –
Number of workers (Frequency)	5	12	22	7	4

Less than 54 = 0
Less than 58 = 5 + 0 = 5
Less than 62 = 5 + 12 = 17
Less than 66 = 5 + 12 + 22 = 39
Less than 70 = 5 + 12 + 22 + 7 = 46
Less than 74 = 5 + 12 + 22 + 7 + 4 = 50

The upper boundaries of sets	Frequency
Less than 54	zero
Less than 58	5
Less than 62	17
Less than 66	39
Less than 70	46
Less than 74	50

The ascending cumulative frequency table.

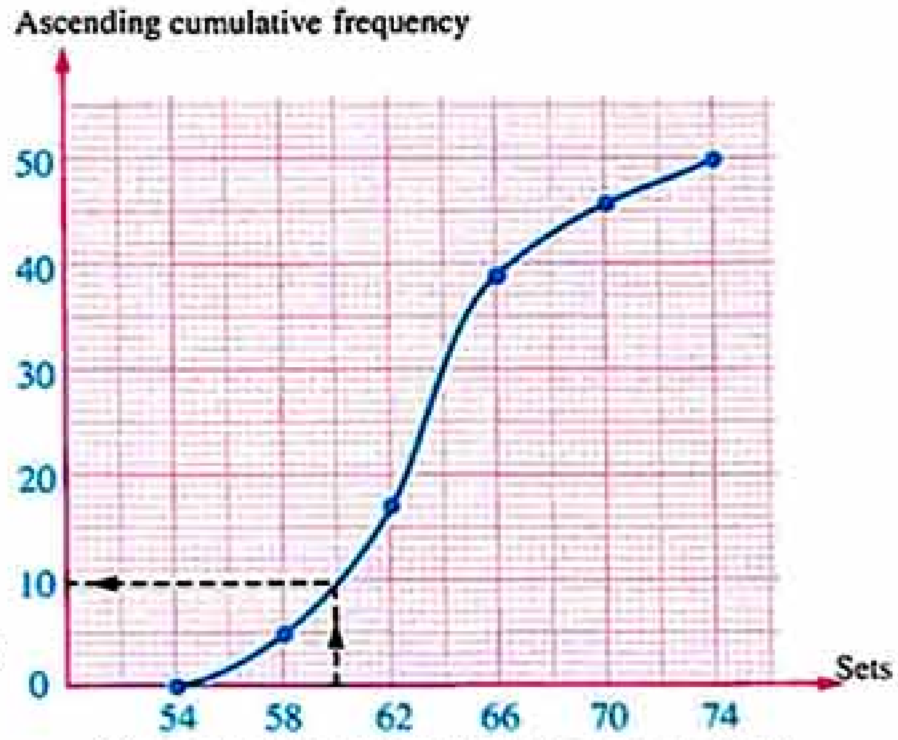
Notice that : The ascending cumulative frequency begins with zero and ends at the total frequency.

To represent the ascending cumulative frequency table graphically , do as follows :

1 Specialize the horizontal axis for sets and the vertical axis for the ascending cumulative frequency.

2 Choose a suitable scale to represent data on the vertical axis so that it contains the ascending cumulative frequency easily.

3 Represent the ascending cumulative frequency of each set , then draw the graph (the curve) such that it passes through the points which we located as shown in the opposite figure.



The assending cumulative frequency curve

• **From the graph , we find that :**

1 The number of workers whose weekly wages are less than 60 pounds = 10 workers.

2 The percentage of the number of workers whose weekly wages are less than 60 pounds $= \frac{10}{50} \times 100\%$
$= 20\%$

Example 2

The following frequency table shows the weekly wages of 50 workers in one factory :

Sets of wages	54 –	58 –	62 –	66 –	70 –	Total
No. of workers (Frequency)	5	12	22	7	4	50

Form the descending cumulative frequency table and represent it graphically , then find :

1. The number of workers whose weekly wages are 60 pounds or more.
2. The percentage of the number of workers whose weekly wages are 60 pounds or more.

Solution

• Form the descending cumulative frequency table as follows :

Sets of wages	54 –	58 –	62 –	66 –	70 –
Number of workers (Frequency)	5	12	22	7	4

54 and more = 5 + 12 + 22 + 7 + 4 = 50 → 54 and more

58 and more = 12 + 22 + 7 + 4 = 45 → 58 and more

62 and more = 22 + 7 + 4 = 33 → 62 and more

66 and more = 7 + 4 = 11 → 66 and more

70 and more = 4 → 70 and more

74 and more = 0 → 74 and more

The lower boundaries of sets	Frequency
54 and more	50
58 and more	45
62 and more	33
66 and more	11
70 and more	4
74 and more	zero

The descending cumulative frequency table

Notice that : The descending cumulative frequency begins with the total frequency and ends with zero.

• To represent this table graphically , follow the same previous steps in the ascending cumulative frequency table to get the opposite graph.

The descending cumulative frequency curve

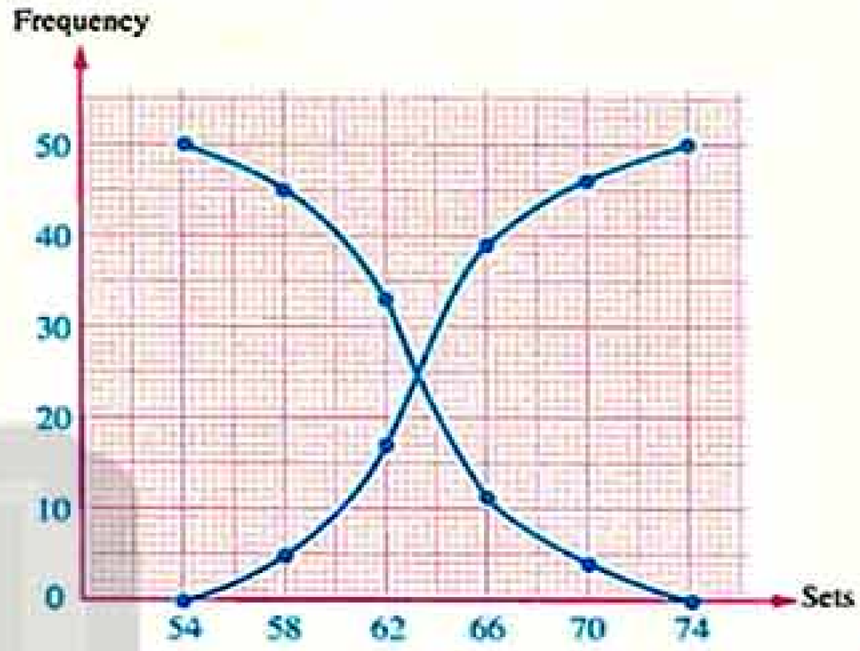
• **From the graph , we find that :**

1. The number of workers whose weekly wages are 60 pounds or more = 40 workers.
2. The percentage of those workers $= \frac{40}{50} \times 100\% = 80\%$

Unit 3

Remark

You can graph the two curves of the ascending and descending cumulative frequency of a frequency distribution in one sketch as shown in the opposite graph.



TRY by yourself

The following table shows the frequency distribution of marks of 40 students in math exam :

Sets	10 –	20 –	30 –	40 –	50 –	Total
Frequency	4	8	12	10	6	**40**

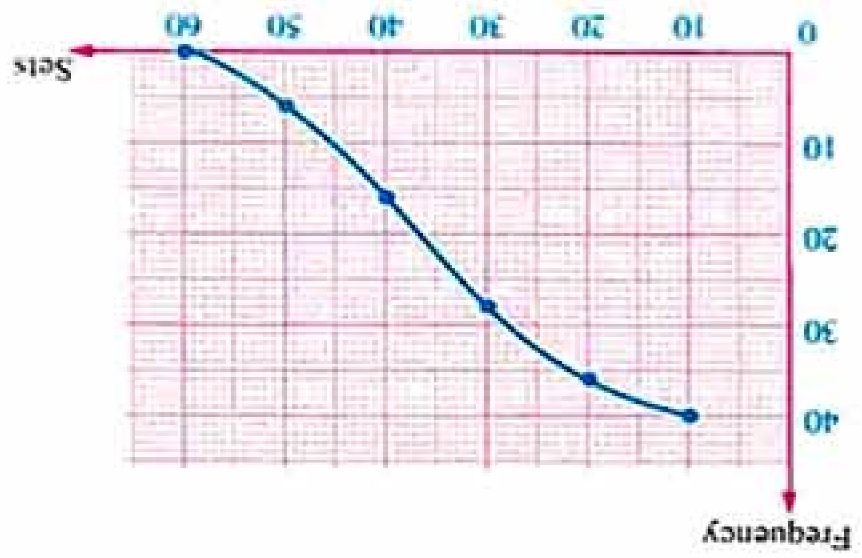
Graph each of :

1. The ascending cumulative frequency curve.
2. The descending cumulative frequency curve.

Answers of try by yourself

1

2



Lesson 3

Mean

You studied last year some of the measures of central tendency of a set of values which are the mean , the median and the mode.

Now you will study how you can find these three measures of a set of data organized in a frequency table with sets.

Remember that

To calculate the mean of a set of values , do as follows :

1. Find the sum of these values.
2. Divide this sum by the number of these values

i.e. The mean of a set of values $= \dfrac{\text{The sum of values}}{\text{Number of values}}$

For example:

If the marks of 5 students are 25 , 23 , 21 , 22 , 24

, then the mean of marks $= \dfrac{25 + 23 + 21 + 22 + 24}{5} = 23$ marks.

Notice that : $23 \times 5 = 115$

, the sum of marks of the 5 students $= 25 + 23 + 21 + 22 + 24 = 115$

i.e. The mean is the value which is given to each item of a set , then the sum of these new values is the same sum of the original values.

Unit 3

Finding the mean of data from the frequency table with sets

Example **The following table shows the distribution of the marks of 50 students in mathematics :**

Sets	10 –	20 –	30 –	40 –	50 –	Total
Frequency	8	12	14	9	7	50

Find the mean of these marks.

Solution

1 **Determine the centres of sets according to the rule :**

$$\text{The centre of a set} = \frac{\text{the lower limit} + \text{the upper limit}}{2}$$

, then the centre of the first set $= \frac{10+20}{2} = 15$

, the centre of the second set $= \frac{20+30}{2} = 25$ ... and so on.

Since the lengths of the subsets are equal and each of them = 10 therefore we consider the upper limit of the last set = 60

, then its centre $= \frac{50+60}{2} = 55$

2 **Form the vertical table :**

Set	Centre of the set « X »	Frequency « f »	$X \times f$
10 –	15	8	120
20 –	25	12	300
30 –	35	14	490
40 –	45	9	405
50 –	55	7	385
Total		50	1700

3 The mean $= \frac{\text{The sum of } (X \times f)}{\text{The sum of } f} = \frac{1700}{50} = 34$ marks.

TRY by yourself

The following table shows the daily wages in pounds of 50 workers in a factory :

Sets	5 –	15 –	25 –	35 –	45 –	Total
Frequency	7	10	12	13	8	50

Find the mean of the wage of the worker in pounds.

Answers of try by yourself

31 Pounds.

Lesson **4**

Median

Remember that

The median is the middle value in a set of values after arranging it ascendingly or descendingly such that the number of values which are less than it is equal to the number of values which are greater than it.

- To find the median of a set of values , do as follows :

Arrange the values ascendingly or descendingly

then

If the values number is odd

Then :

The median is the value lying in the middle exactly.

For example:

- **If the values are :**

42 , 23 , 17 , 30 , 20

- **We arrange them ascendingly as follows**

17 , 20 , $\boxed{23}$, 30 , 42

The median = 23

If the values number is even

Then :

$$\text{The median} = \frac{\text{The sum of the two values lying in the middle}}{2}$$

For example:

- **If the values are :**

27 , 13 , 23 , 24 , 13 , 21

- **We arrange them ascendingly as follows**

13 , 13 , $\boxed{21 , 23}$, 24 , 27

$$\text{The median} = \frac{21 + 23}{2} = 22$$

Unit 3

Finding the median of a frequency distribution with sets graphically

To find the median of a frequency distribution with sets graphically , do the following steps :

1. Form the ascending or the descending cumulative frequency table , then draw the cumulative frequency curve of it.
2. Find the order of the median = $\frac{\text{The total of frequency}}{2}$
3. Determine the point which represents the order of the median on the vertical axis, from this point , draw a horizontal straight line to intersect the curve at a point , then from this point , draw a perpendicular to the horizontal axis to intersect it at a point which represents the median.

The following example shows how to find the median using the two curves (the ascending or the descending cumulative frequency curve).

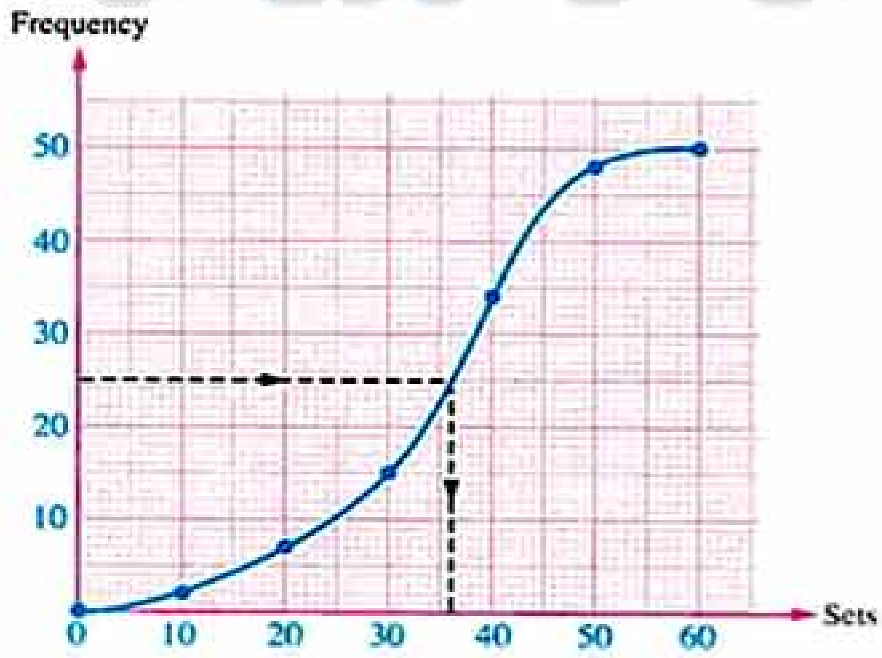
Example **The following table shows the frequency distribution of marks of 50 students in math exam :**

Sets of marks	0 –	10 –	20 –	30 –	40 –	50 –	Total
Number of students	2	5	8	19	14	2	50

Find the median mark of the students.

Solution * First : Using the ascending cumulative frequency curve :

The upper boundaries of sets	Frequency
Less than 0	0
Less than 10	2
Less than 20	7
Less than 30	15
Less than 40	34
Less than 50	48
Less than 60	50

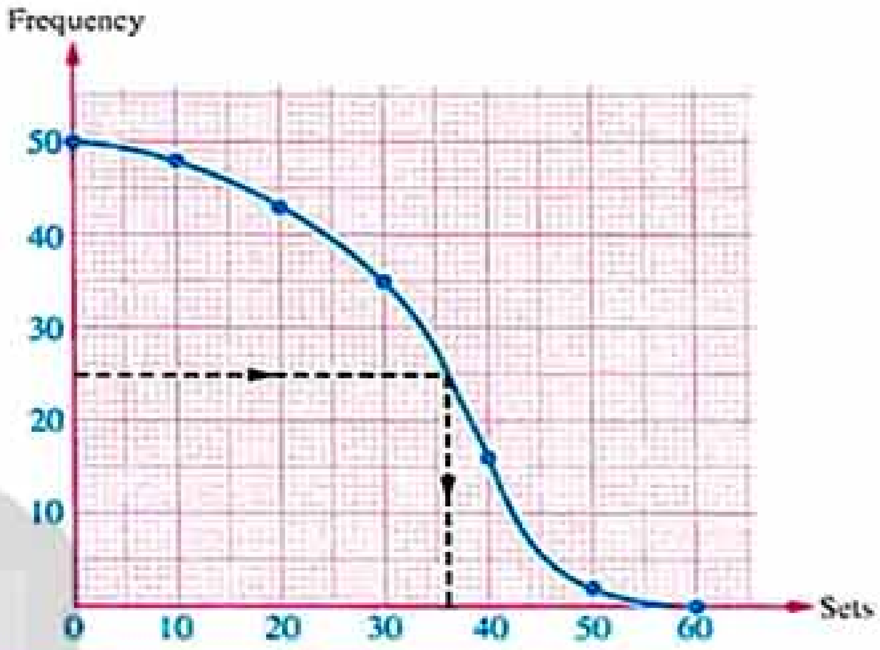


$\because$ The order of the median $= \frac{50}{2} = 25$

$\therefore$ From the graph , the median = 36 approximately

*** Second : Using the descending cumulative frequency curve :**

The lower boundaries of sets	Frequency
0 and more	50
10 and more	48
20 and more	43
30 and more	35
40 and more	16
50 and more	2
60 and more	0



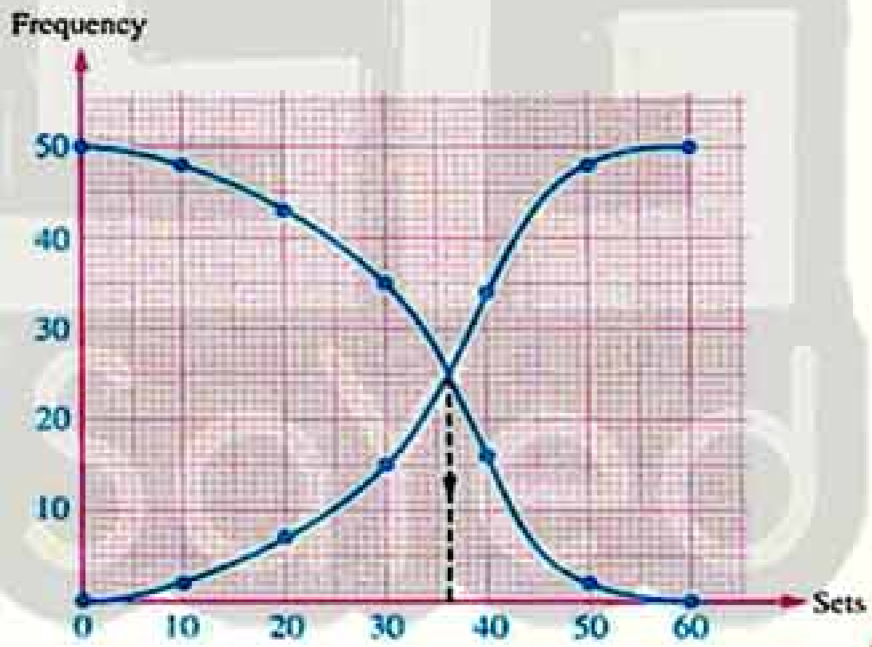
$\because$ The order of the median $= \frac{50}{2} = 25$

$\therefore$ From the graph , the median = 36 approximately

Remark

"You can find the median by more accurate method , this by drawing the two curves (the ascending and descending cumulative frequency curves) together in one graph to intersect at one point.

From this point , draw a vertical straight line to meet the horizontal axis at a point which represents the median as shown in the opposite graph to get the median = 36 approximately."



TRY by yourself

Using the ascending or descending cumulative frequency curve , find the median of the following frequency distribution :

Sets	4 –	8 –	12 –	16 –	20 –	Total
Frequency	2	4	8	6	4	24

Answers of try by yourself

15 approximately.

Lesson 5

Mode

Remember that

The mode of a set of values is the most common value in the set , or in other words , it is the value which is repeated more than any other values.

For example: The mode of the set of the values
7 , 3 , 4 , 1 , 7 , 9 , 7 , 4 is 7

Finding the mode for a frequency distribution with sets

The following example shows how to find the mode of a frequency distribution with sets :

Example **The following is the frequency distribution of marks of 100 students in an exam :**

Sets of marks	10 –	20 –	30 –	40 –	50 –	Total
Number of students	16	24	30	20	10	**100**

Find the mode mark for these students.

Solution **You can find the mode of that distribution graphically using the histogram as follows :**

1 Draw two orthogonal axes , one of them is horizontal and the other is vertical to represent the frequency of each set.

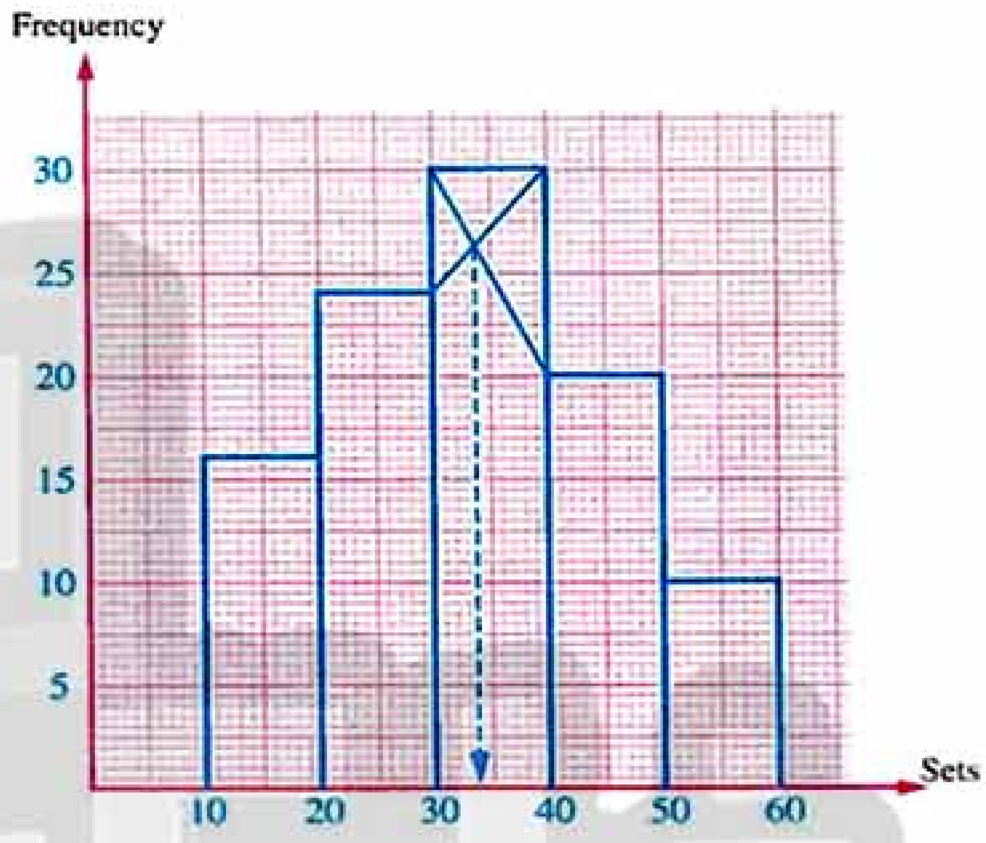
2 Divide the horizontal axis into a number of equal parts with a suitable drawing scale to represent the sets.

3 Divide the vertical axis into a number of equal parts with a suitable drawing scale to represent the greatest frequency in the sets.

4 Draw a rectangle whose base is the set (10 –) and its height equals the frequency (16)

5 Draw a second rectangle adjacent to the first one whose base is the set (20 –) and its height equals the frequency (24)

6 Repeat drawing the remained adjacent rectangles till the last set (50 –)



7 Determine the set which has the greatest frequency then draw two lines as shown in the histogram to intersect at a point.

From this point , draw a vertical line to intersect the horizontal axis at a point which represents the value of the mode.

i.e. The mode mark is 34 approximately.

TRY by yourself

Find the mode for the following frequency distribution :

Sets	2 –	4 –	6 –	8 –	10 –	Total
Frequency	3	10	12	10	5	40

Answers of try by yourself
7 approximately.

Second

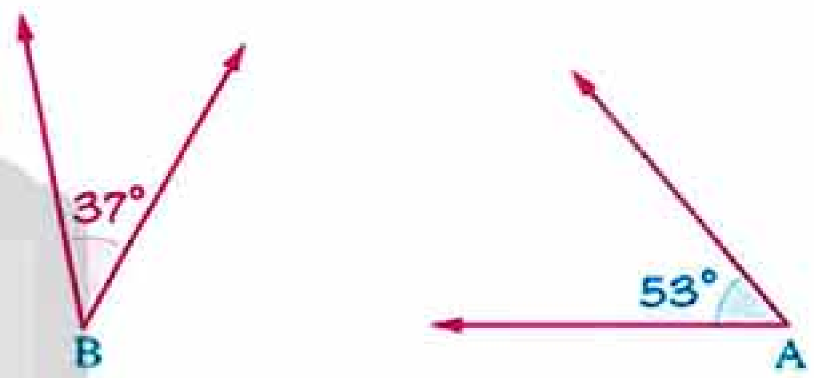
Geometry

Revision

1 Some relations between angles

Complementary angles :

Two angles are said to be complementary, if the sum of their measures is 90°

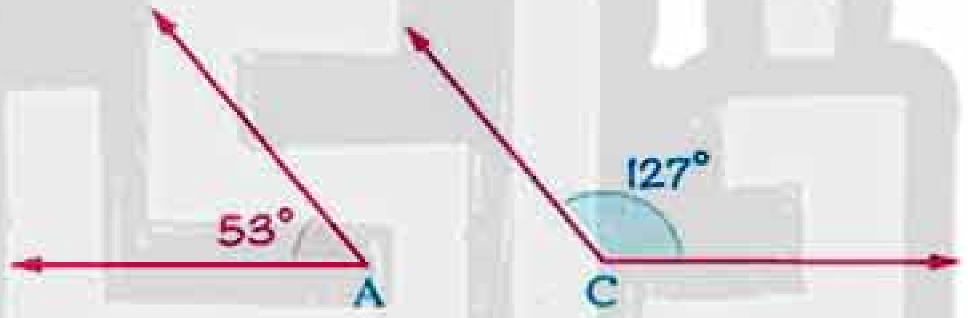


In the opposite figure :

∠ A and ∠ B are complementary angles.

Supplementary angles :

Two angles are said to be supplementary, if the sum of their measures is 180°

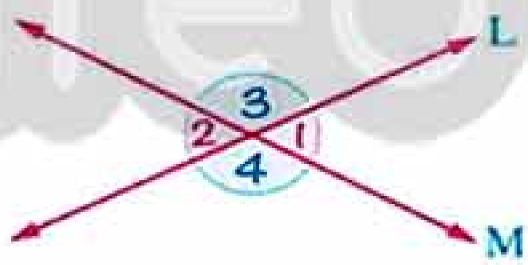


In the opposite figure :

∠ A and ∠ C are supplementary angles.

Vertically opposite angles (V.O.A.) :

If two straight lines intersect, then each two vertically opposite angles are equal in measure.

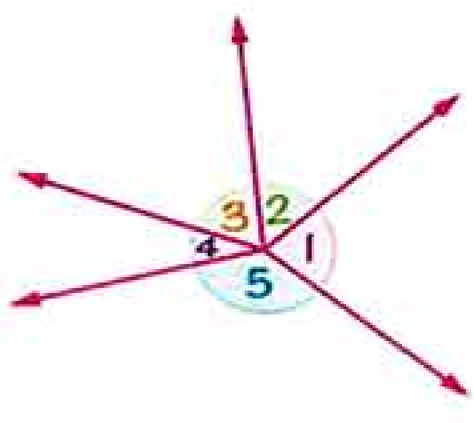


In the opposite figure :

m (∠ 1) = m (∠ 2) , m (∠ 3) = m (∠ 4)

Accumulative angles at a point :

The sum of measures of the accumulative angles at a point is 360°



In the opposite figure :

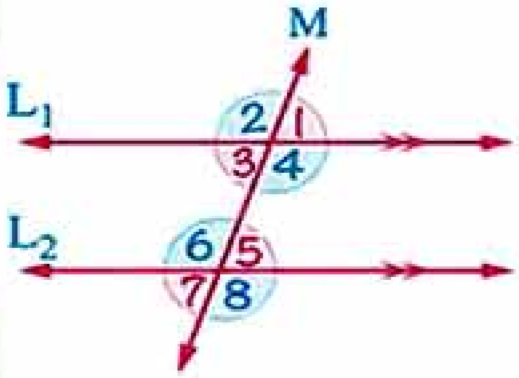
m (∠ 1) + m (∠ 2) + m (∠ 3) + m (∠ 4) + m (∠ 5) = 360°

Revision

2 Parallelism

If a straight line intersects two parallel straight lines , then :

1 Each two alternate angles are equal in measure.



- $m(\angle 3) = m(\angle 5)$ "Alternate angles"
- $m(\angle 4) = m(\angle 6)$ "Alternate angles"

2 Each two corresponding angles are equal in measure.

- $m(\angle 1) = m(\angle 5)$ "Corresponding angles"
- $m(\angle 2) = m(\angle 6)$ "Corresponding angles"
- $m(\angle 3) = m(\angle 7)$ "Corresponding angles"
- $m(\angle 4) = m(\angle 8)$ "Corresponding angles"

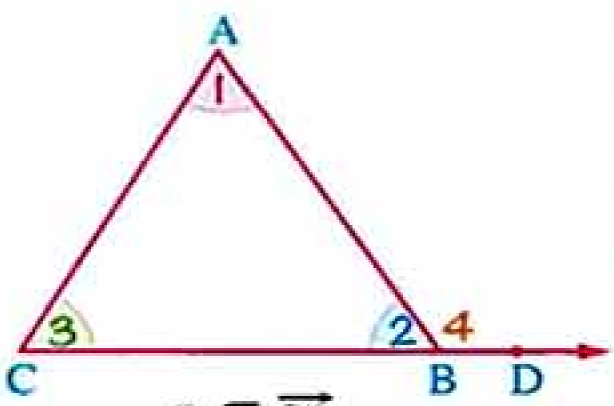
3 Each two interior angles in the same side of the transversal are supplementary.

- $m(\angle 3) + m(\angle 6) = 180^\circ$ "Interior angles in the same side of the transversal"
- $m(\angle 4) + m(\angle 5) = 180^\circ$ "Interior angles in the same side of the transversal"

3 The triangle

- **The sum of measures of the interior angles of a triangle = 180°**

$m(\angle 1) + m(\angle 2) + m(\angle 3) = 180^\circ$



$D \in \overrightarrow{CB}$

- **The measure of the exterior angle of a triangle equals the sum of measures of its non-adjacent interior angles.**

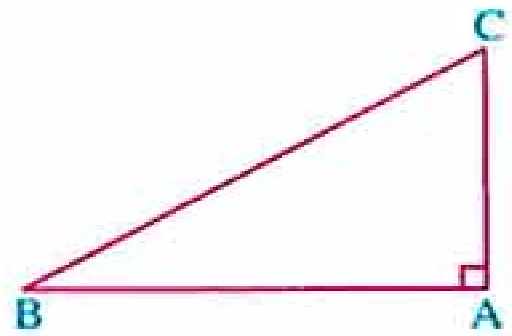
$m(\angle 4) = m(\angle 1) + m(\angle 3)$

Pythagoras' Theorem :

In a right-angled triangle , the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the other two sides.

In Δ ABC which is right-angled at A :

- $(BC)^2 = (AB)^2 + (AC)^2$
- $(AB)^2 = (BC)^2 - (AC)^2$
- $(AC)^2 = (BC)^2 - (AB)^2$

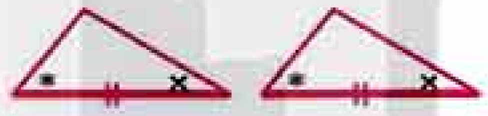


Cases of congruence of two triangles :

Two triangles are congruent if one of the following cases is satisfied :

1. Congruence of two sides and the included angle of one triangle to the corresponding parts of the other triangle.

2. Congruence of two angles and the side drawn between their vertices of one triangle to the corresponding parts of the other triangle.



3. Congruence of each side of one triangle to the corresponding side of the other triangle.

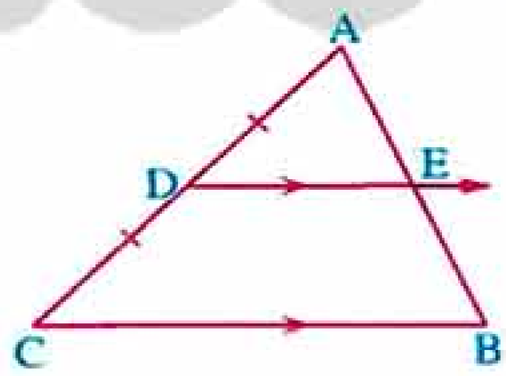
4. Two right-angled triangles are congruent , if the hypotenuse and a side of one triangle are congruent to the corresponding parts of the other triangle.

- **The ray drawn from the midpoint of a side of a triangle parallel to another side bisects the third side.**

In the opposite figure :

If D is the midpoint of $\overline{AC}$, $\overrightarrow{DE} \parallel \overline{BC}$ such that E $\in \overline{AB}$

, then E is the midpoint of $\overline{AB}$ (i.e. AE = EB)

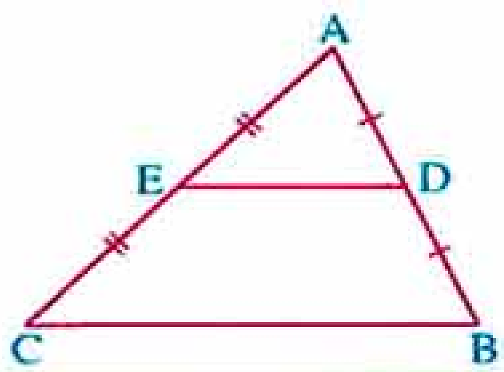


- **The line segment joining the midpoints of two sides in a triangle is parallel to the third side and its length equals half the length of this side.**

In the opposite figure :

If D is the midpoint of $\overline{AB}$ and E is the midpoint of $\overline{AC}$, then :

1. $\overline{DE} \parallel \overline{BC}$
2. DE = $\frac{1}{2}$ BC



Revision

4 The polygon

- **The sum of measures of the interior angles of a polygon with n sides equals $(n - 2) \times 180°$**

For example:

– The sum of measures of the interior angles of the quadrilateral $= (4 - 2) \times 180° = 360°$

– The sum of measures of the interior angles of the pentagon $= (5 - 2) \times 180° = 540°$

- **The measure of each interior angle in a regular polygon with n sides $= \frac{(n - 2) \times 180°}{n}$**

For example:

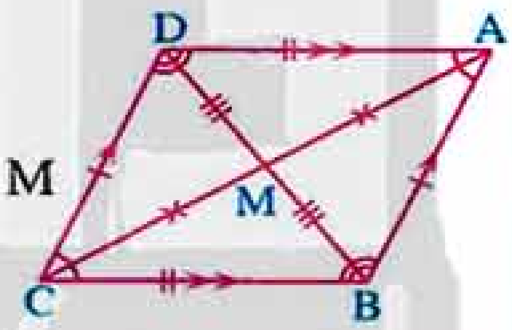
– The measure of the interior angle of the equilateral triangle $= \frac{(3 - 2) \times 180°}{3} = 60°$

– The measure of the interior angle of the regular hexagon $= \frac{(6 - 2) \times 180°}{6} = 120°$

5 The parallelogram and its special cases

Properties of a parallelogram :

In the opposite figure :



ABCD is a parallelogram whose diagonals $\overline{AC}$ and $\overline{BD}$ intersect at M

We can deduce the following properties :

1 The sum of measures of each two consecutive angles in a parallelogram is 180°

i.e. • $m(\angle A) + m(\angle B) = 180°$ • $m(\angle B) + m(\angle C) = 180°$

• $m(\angle C) + m(\angle D) = 180°$ • $m(\angle D) + m(\angle A) = 180°$

2 In a parallelogram , each two opposite angles are equal in measure.

i.e. • $m(\angle A) = m(\angle C)$ • $m(\angle B) = m(\angle D)$

3 In a parallelogram , each two opposite sides are equal in length and parallel.

i.e. • $AB = CD$, $\overline{AB} \parallel \overline{CD}$ • $AD = BC$, $\overline{AD} \parallel \overline{BC}$

4 The two diagonals in a parallelogram bisect each other.

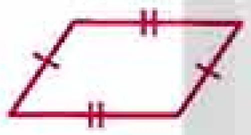
i.e. • $AM = CM$ • $BM = DM$

- The perimeter of the parallelogram = The sum of two consecutive side lengths × 2

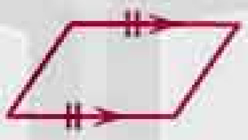
A quadrilateral is a parallelogram if one of the following cases is satisfied

- Each two opposite sides are parallel

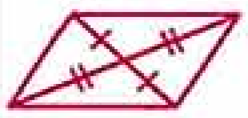
- Each two opposite sides are equal in length



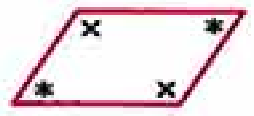
- Two opposite sides are parallel and equal in length



- The two diagonals bisect each other



- Each two opposite angles are equal in measure



A parallelogram is a

Rectangle

If :
One of its angles is right.
or
The diagonals are equal in length.

Rhombus

If :
Two adjacent sides are equal in length.
or
Its diagonals are perpendicular.

Square

If :
One of its angles is right and two adjacent sides are equal in length.
or
One of its angles is right and its diagonals are perpendicular.
or
The diagonals are equal in length and perpendicular.
or
Two adjacent sides are equal in length and its diagonals are equal in length.

Notice that :

- A square is a rectangle with two adjacent sides equal in length.
- A square is a rhombus with a right angle , or a rhombus with two diagonals equal in length.
- To prove that a quadrilateral is a rectangle , a rhombus or a square , you must first prove that it is a parallelogram.

UNIT 4

Medians of Triangle – Isosceles Triangle

Lessons of the unit :

1. Medians of triangle.
2. Medians of triangle "follow".
3. The isosceles triangle.
4. The converse of the isosceles triangle theorem.
5. Corollaries of the isosceles triangle theorems.

▸ Use your smart phone or tablet to scan **the QR Code** and enjoy watching videos.

Unit Objectives :

By the end of this unit, student should be able to :

- recognize the median of a triangle.
- recognize the intersection point of medians of a triangle and the ratio that the point divides each median.
- deduce the relation between the length of the median from the vertex of the right angle in the right-angled triangle and the length of the hypotenuse.
- recognize thirty and sixty triangle.
- recognize the properties of isosceles triangle.
- recognize the properties of equilateral triangle.
- recognize the axis of symmetry of the line segment.
- recognize the axis of symmetry of the isosceles triangle.
- solve miscellaneous problems on the equilateral triangle and the isosceles triangle.
- appreciate the role of geometry in solving of real life problems.

Lesson 1 Medians of triangle

Definition

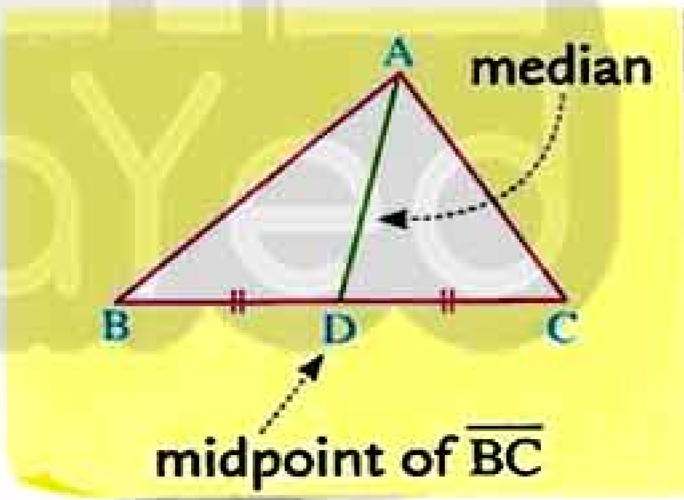
The median of a triangle is the line segment drawn from any vertex of this triangle to the midpoint of the opposite side of this vertex.

For example:

In the opposite figure :

If D is the midpoint of $\overline{BC}$

, then $\overline{AD}$ is a median of Δ ABC



Notice that :

Any triangle has three medians.

Theorem 1

The medians of a triangle are concurrent.

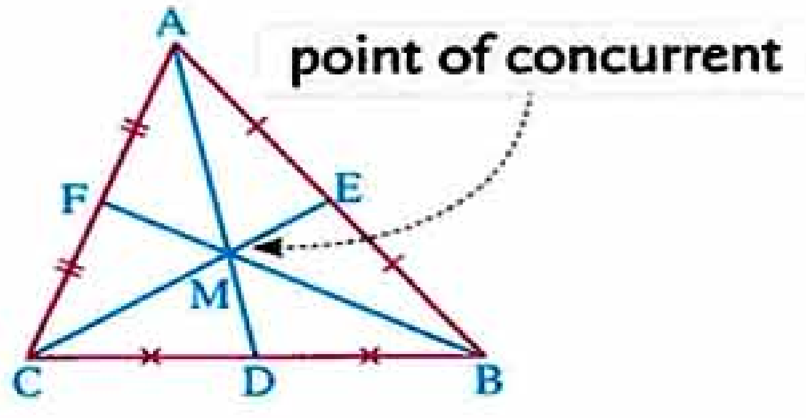
For example:

In the opposite figure :

$\overline{AD}$, $\overline{BF}$ and $\overline{CE}$ are the three medians of Δ ABC ,

and they are concurrent at M

(i.e. $\overline{AD} \cap \overline{BF} \cap \overline{CE} = \{M\}$)



Example 1 **In the opposite figure :**

ABC is a right-angled triangle at B in which :

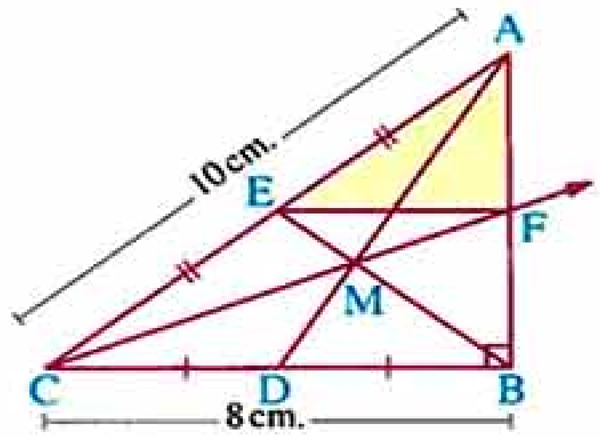
AC = 10 cm. , BC = 8 cm. ,

D and E are the midpoints of $\overline{BC}$ and $\overline{AC}$ respectively

where $\overline{AD} \cap \overline{BE} = \{M\}$

Draw $\overrightarrow{CM}$ to cut $\overline{AB}$ at F

Find the perimeter of Δ AFE



Solution

Given m ($\angle$ ABC) = 90° , AC = 10 cm. , BC = 8 cm. , D is the midpoint of $\overline{BC}$, E is the midpoint of $\overline{AC}$

R.T.F. The perimeter of Δ AFE

Proof In Δ ABC :

$\because$ m ($\angle$ ABC) = 90°

$\therefore (AB)^2 = (AC)^2 - (BC)^2 = 100 - 64 = 36$ $\therefore$ AB = 6 cm.

, $\because$ D is the midpoint of $\overline{BC}$ $\therefore$ $\overline{AD}$ is a median in Δ ABC

, $\because$ E is the midpoint of $\overline{AC}$ $\therefore$ $\overline{BE}$ is a median in Δ ABC

, $\because \overline{AD} \cap \overline{BE} = \{M\}$

$\therefore$ M is the intersection point of the medians of Δ ABC

, $\because M \in \overline{CF}$ $\therefore$ $\overline{CF}$ is a median in Δ ABC

$\therefore$ F is the midpoint of $\overline{AB}$ $\therefore AF = \frac{1}{2} AB = 3$ cm.

, $\because$ E is the midpoint of $\overline{AC}$ $\therefore AE = \frac{1}{2} AC = 5$ cm.

, in Δ ABC :

$\because$ F and E are the midpoints of $\overline{AB}$ and $\overline{AC}$ respectively.

$\therefore FE = \frac{1}{2} BC = 4$ cm.

$\therefore$ The perimeter of Δ AFE = AF + FE + AE

= 3 + 4 + 5 = 12 cm. (The req.)

Unit 4

Theorem 2

The point of concurrence of the medians of the triangle divides each median in the ratio of 1 : 2 from its base.

For example:

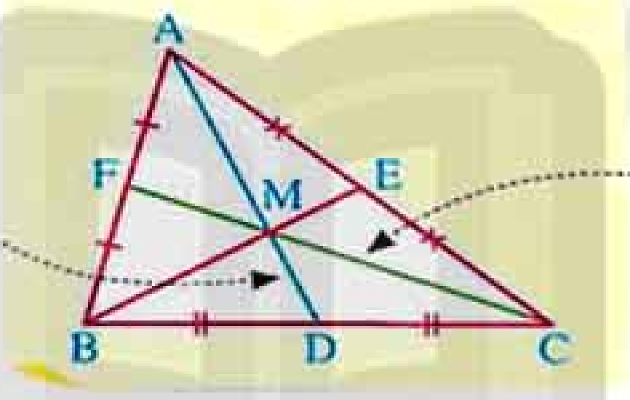
In the following figure :

M is the point of concurrence of the medians of Δ ABC, then :

① $MD = \frac{1}{2} AM$

If AM = 6 cm. ,

then MD = 3 cm.



② CM = 2 FM

If FM = 4 cm. ,

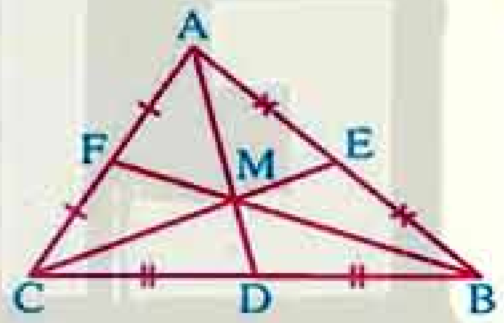
then CM = 8 cm.

Remarks

- The point of concurrence of the medians of the triangle divides each of them in the ratio of 2 : 1 from the vertex.
- **In the opposite figure :**

If ABC is a triangle , M is the point of concurrence of its medians $\overline{AD}$, $\overline{BF}$ and $\overline{CE}$, then:

$MD = \frac{1}{3}$ AD and $AM = \frac{2}{3}$ AD



For example:

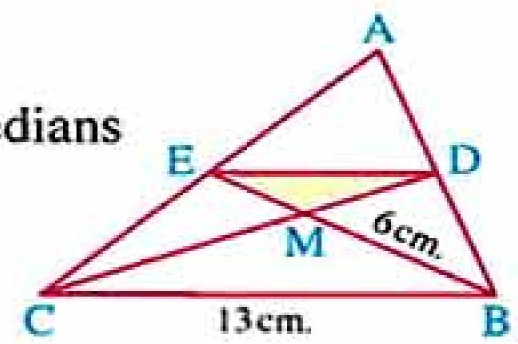
If AD = 9 cm. , then $MD = \frac{1}{3}$ AD = 3 cm. , $AM = \frac{2}{3}$ AD = 6 cm.

Similarly : $MF = \frac{1}{3}$ BF , $BM = \frac{2}{3}$ BF , $ME = \frac{1}{3}$ CE and $CM = \frac{2}{3}$ CE

Example 2 **In the opposite figure :**

ABC is a triangle in which : $\overline{CD}$ and $\overline{BE}$ are two medians intersecting at M , BM = 6 cm. , BC = 13 cm. and DC = 12 cm.

Find the perimeter of Δ DME



Solution

Given: ABC is a triangle in which : $\overline{CD}$ and $\overline{BE}$ are two medians , M is the point of their intersection , BM = 6 cm. , BC = 13 cm. and DC = 12 cm.

R.T.F.: The perimeter of Δ DME

Proof: $\because$ $\overline{CD}$ and $\overline{BE}$ are medians intersecting at the point M

$\therefore$ M is the point of intersection of the medians of Δ ABC

$\therefore$ ME $= \frac{1}{2}$ BM $= \frac{1}{2} \times 6 = 3$ cm.

, DM $= \frac{1}{3}$ DC $= \frac{1}{3} \times 12 = 4$ cm.

$\because$ $\overline{CD}$ and $\overline{BE}$ are two medians in Δ ABC

$\therefore$ D is the midpoint of $\overline{AB}$ and E is the midpoint of $\overline{AC}$

$\therefore$ DE $= \frac{1}{2}$ BC $= \frac{1}{2} \times 13 = 6.5$ cm.

$\therefore$ The perimeter of Δ DME = ME + DM + DE = 3 + 4 + 6.5 = 13.5 cm.

(The req.)

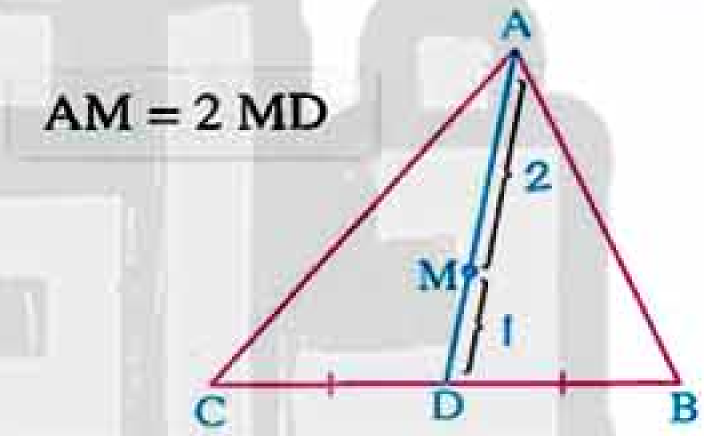
Fact

The point which divides the median in a triangle by the ratio of 1 : 2 from the base is the point of intersection of the medians of this triangle.

In the opposite figure :

If $\overline{AD}$ is a median in Δ ABC and M $\in \overline{AD}$ such that AM = 2 MD ,

then M is the point of intersection of the medians of Δ ABC



Example 3 **In the opposite figure :**

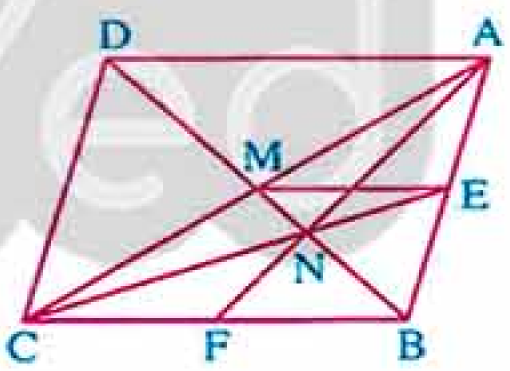
ABCD is a parallelogram ,

M is the point of intersection of its diagonals ,

N $\in \overline{BM}$ where BN = 2 NM

and $\overrightarrow{CN} \cap \overline{AB} = \{E\}$

Prove that : EM $= \frac{1}{2}$ BC



Solution

Given ABCD is a parallelogram , M is the point of intersection of its diagonals , BN = 2 NM , N $\in \overline{BM}$ and $\overrightarrow{CN} \cap \overline{AB} = \{E\}$

R.T.P. EM $= \frac{1}{2}$ BC

Proof $\because$ ABCD is a parallelogram.

$\therefore$ The two diagonals bisect each other.

$\therefore$ M is the midpoint of $\overline{AC}$ $\therefore$ $\overline{BM}$ is a median in Δ ABC

, $\because N \in \overline{BM}$ where $BN = 2\ NM$

$\therefore$ N is the point of intersection of the medians of $\Delta\ ABC$

, $\because \overline{CE}$ passes through the point N $\qquad \therefore \overline{CE}$ is a median in $\Delta\ ABC$

$\therefore$ E is the midpoint of $\overline{AB}$

In $\Delta\ ABC$

$\because$ E is the midpoint of $\overline{AB}$ and M is the midpoint of $\overline{AC}$

$\therefore EM = \frac{1}{2}\ BC$ (Q.E.D.)

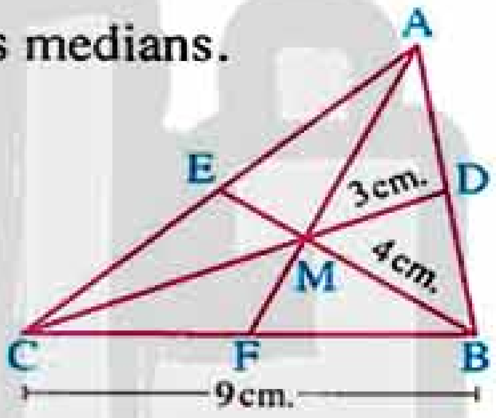
TRY by yourself

In the opposite figure :

ABC is a triangle and M is the point of intersection of its medians.

If MD = 3 cm. , BM = 4 cm. and BC = 9 cm. ,

complete the following :



1 BF = ... cm. 2 MC = ... cm.

3 ME = ... cm.

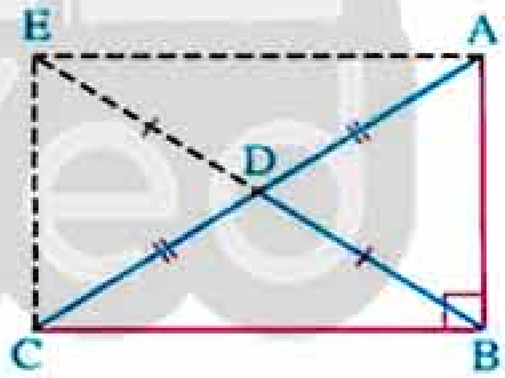
Answers of try by yourself

1 4.5 2 6 3 2

Lesson 2 Medians of triangle "Follow"

Theorem 3

In the right-angled triangle , the length of the median from the vertex of the right angle equals half the length of the hypotenuse.



Given ABC is a triangle in which m ($\angle$ ABC) = 90° ,

$\overline{BD}$ is a median in the triangle ABC

R.T.P. BD = $\frac{1}{2}$ AC

Construction Draw $\overrightarrow{BD}$ and take the point E $\in \overrightarrow{BD}$

such that BD = DE

Proof In the figure ABCE :

$\because$ $\overline{AC}$ and $\overline{BE}$ bisect each other.

$\therefore$ The figure ABCE is a parallelogram.

, $\because$ m ($\angle$ ABC) = 90°

$\therefore$ The figure ABCE is a rectangle.

$\therefore$ BE = AC

, $\because$ BD = $\frac{1}{2}$ BE

$\therefore$ BD = $\frac{1}{2}$ AC (Q.E.D.)

Unit 4

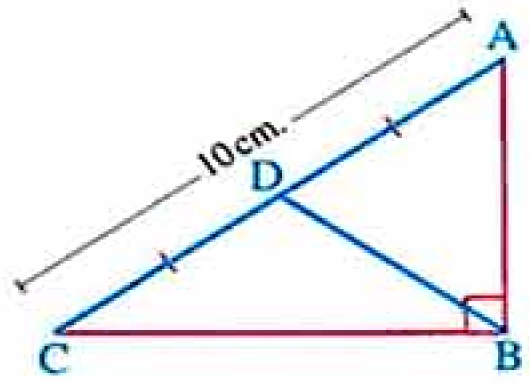
For example:

In the opposite figure :

Δ ABC is a right-angled triangle at B ,

D is the midpoint of $\overline{AC}$ and AC = 10 cm. ,

then DB = 5 cm.



Example 1 **In the opposite figure :**

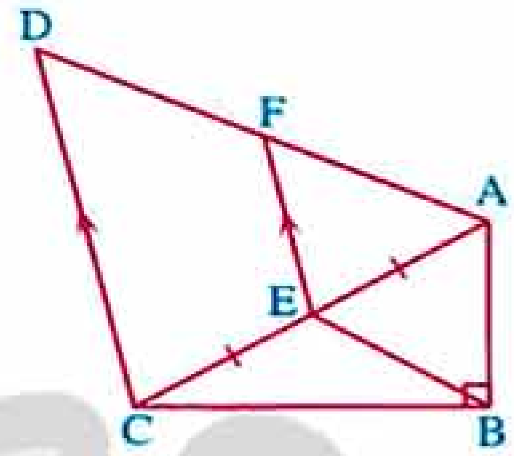
ABCD is a quadrilateral in which

m ($\angle$ ABC) = 90° , AC = CD ,

E is the midpoint of $\overline{AC}$

and F $\in \overline{AD}$ such that $\overline{EF}$ // $\overline{CD}$

Prove that : BE = EF



Solution

Given m ($\angle$ ABC) = 90° , AC = CD ,
E is the midpoint of $\overline{AC}$ and $\overline{EF}$ // $\overline{CD}$

R.T.P. BE = EF

Proof In Δ ABC :

$\because$ m ($\angle$ ABC) = 90° and $\overline{BE}$ is a median

$\therefore$ BE = $\frac{1}{2}$ AC

, $\because$ AC = CD

$\therefore$ BE = $\frac{1}{2}$ CD (1)

In Δ ACD :

$\because$ E is the midpoint of $\overline{AC}$ and $\overline{EF}$ // $\overline{CD}$

$\therefore$ F is the midpoint of $\overline{AD}$

$\therefore$ EF = $\frac{1}{2}$ CD (2)

From (1) and (2) :

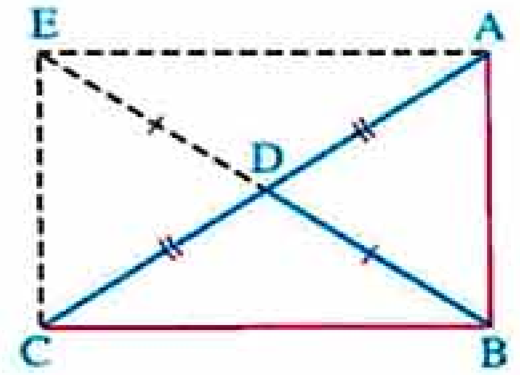
$\therefore$ BE = EF (Q.E.D.)

The converse of theorem 3

If the length of the median drawn from a vertex of a triangle equals half the length of the opposite side to this vertex, then the angle at this vertex is right.

Given ABC is a triangle , $\overline{BD}$ is a median and DA = DB = DC

R.T.P. m ($\angle$ ABC) = 90°



Construction Draw $\overrightarrow{BD}$, then take the point E $\in \overrightarrow{BD}$ such that BD = DE

Proof $\because$ BD = $\frac{1}{2}$ BE = $\frac{1}{2}$ AC

$\therefore$ BE = AC

$\therefore$ In the figure ABCE :

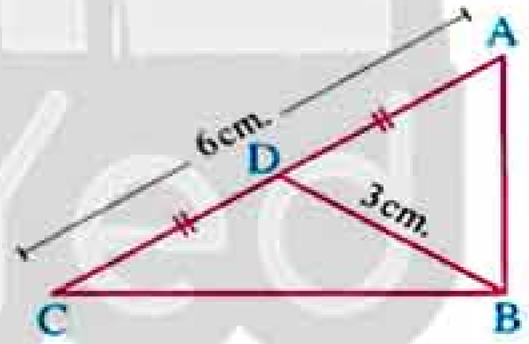
$\overline{AC}$ and $\overline{BE}$ are equal in length and bisect each other.

$\therefore$ The figure ABCE is a rectangle.

$\therefore$ m ($\angle$ ABC) = 90° (Q.E.D.)

For example:

In the opposite figure :



If $\overline{BD}$ is a median in Δ ABC ,

BD = 3 cm. and AC = 6 cm. ,

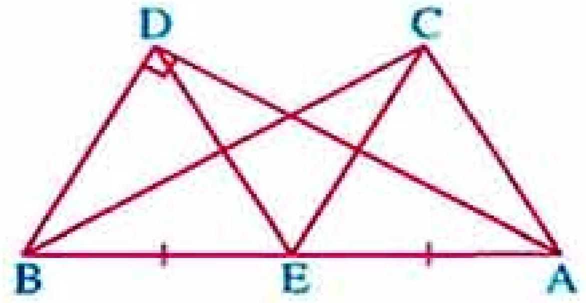
then m ($\angle$ ABC) = 90° "because BD = $\frac{1}{2}$ AC "

Example 2 **In the opposite figure :**

ABD is a right-angled triangle at D ,

E is the midpoint of $\overline{AB}$ and CE = DE

Prove that : m ($\angle$ ACB) = 90°



Solution

Given E is the midpoint of $\overline{AB}$, m ($\angle$ ADB) = 90° , CE = DE

R.T.P. m ($\angle$ ACB) = 90°

Proof

In Δ ADB :

$\because$ m ($\angle$ ADB) = 90° , $\overline{DE}$ is a median

$\therefore$ DE = $\frac{1}{2}$ AB

But CE = DE

$\therefore$ CE = $\frac{1}{2}$ AB

$\therefore$ In Δ ACB :

$\overline{CE}$ is a median with length equals half the length of $\overline{AB}$

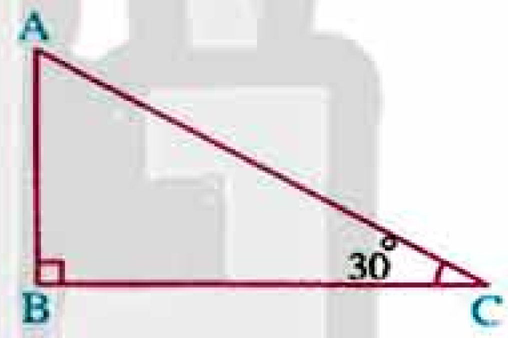
$\therefore$ m ($\angle$ ACB) = 90° (Q.E.D.)

Corollary

The length of the side opposite to the angle of measure 30° in the right-angled triangle equals half the length of the hypotenuse.

i.e.

In the opposite figure :



If Δ ABC is right-angled at B and

m ($\angle$ C) = 30° , then AB = $\frac{1}{2}$ AC

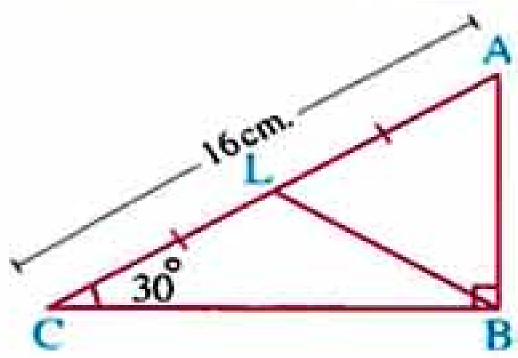
For example:

If AC = 20 cm. , then AB = 10 cm.

Remark

"The right-angled triangle whose measure of one of its angles is 30° , then the measure of the third angle is 60° is called thirty and sixty triangle."

Example 3 **In the opposite figure :**



ABC is a triangle in which m ($\angle$ ABC) = 90° ,

m ($\angle$ C) = 30° , AC = 16 cm. and

L is the midpoint of $\overline{AC}$

Find : 1 The length of each of $\overline{AB}$ and $\overline{BL}$

2 The perimeter of Δ ABL

Solution

Given m ($\angle$ ABC) = 90° , m ($\angle$ C) = 30° ,

AC = 16 cm. and L is the midpoint of $\overline{AC}$

R.T.F. 1 AB , BL 2 The perimeter of Δ ABL

Proof $\because$ $\triangle ABC$ is right-angled at B , $m(\angle C) = 30°$

$\therefore AB = \frac{1}{2} AC = 8$ cm.

, $\because \overline{BL}$ is a median in $\triangle ABC$

$\therefore BL = \frac{1}{2} AC = 8$ cm. (First req.)

$\because AL = \frac{1}{2} AC = 8$ cm.

$\therefore$ The perimeter of $\triangle ABL = 8 + 8 + 8 = 24$ cm. (Second req.)

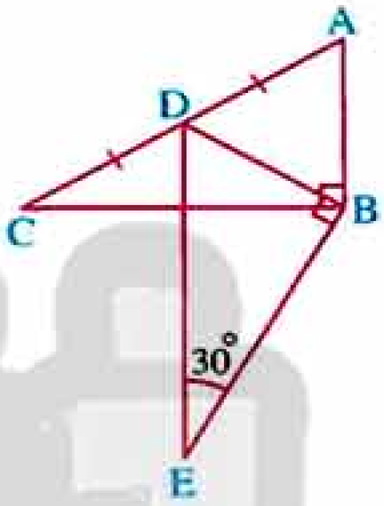
TRY *by yourself*

In the opposite figure :

$m(\angle ABC) = m(\angle DBE) = 90°$,

D is the midpoint of $\overline{AC}$ and $m(\angle E) = 30°$

Prove that : AC = DE



Answers *of try by yourself*

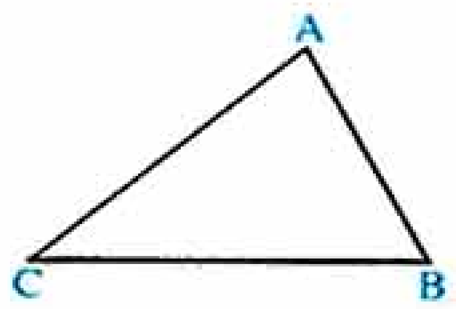
Prove by yourself. (**Hint :** Prove that $BD = \frac{1}{2} AC$ and $BD = \frac{1}{2} DE$)

Lesson 3 The isosceles triangle

Prelude

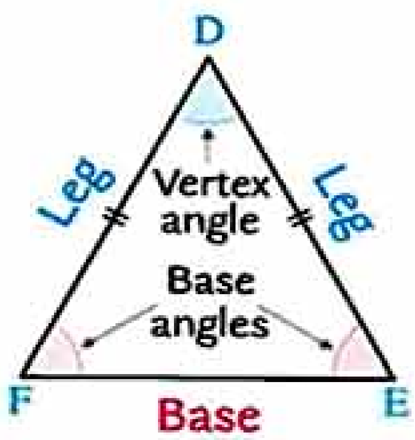
Triangles are classified according to the lengths of their sides into three types which are :

1. Scalene triangle.



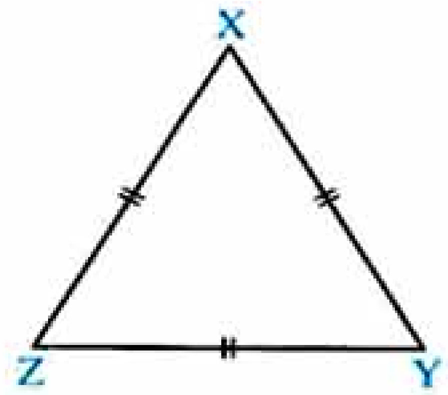
AB ≠ BC ≠ CA

2. Isosceles triangle. (two sides are congruent).



DE = DF

3. Equilateral triangle. (three sides are congruent).



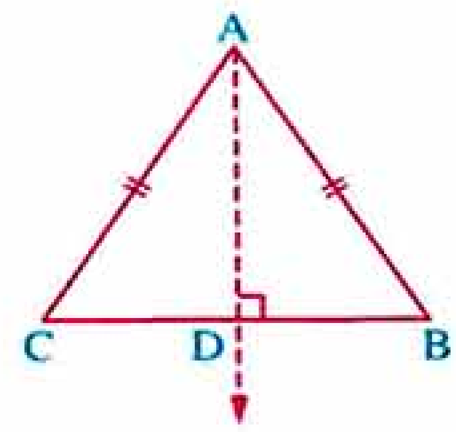
XY = YZ = ZX

- **In the following we will study the relations between the angles in the isosceles triangle and in the equilateral triangle.**

The isosceles triangle theorem

Theorem 1

The base angles of the isosceles triangle are congruent.



Given ABC is a triangle in which $\overline{AB} \equiv \overline{AC}$

R.T.P. $\angle B \equiv \angle C$

Construction Draw $\overrightarrow{AD} \perp \overline{BC}$ where $\overrightarrow{AD} \cap \overline{BC} = \{D\}$

Proof $\because \Delta\Delta$ ADB , ADC in which :

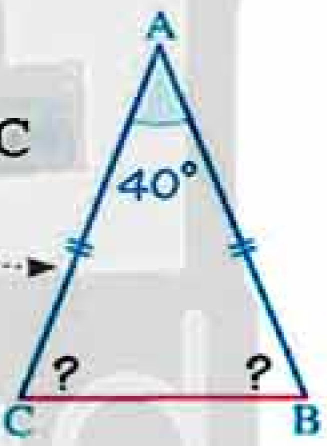
$$\begin{cases} m(\angle ADB) = m(\angle ADC) = 90^\circ & \text{(const.)} \\ \overline{AB} \equiv \overline{AC} & \text{(given)} \\ \overline{AD} \text{ is a common side} \end{cases}$$

$\therefore \Delta ADB \equiv \Delta ADC$,

then we deduce that $\angle B \equiv \angle C$ (Q.E.D)

For example:

In the opposite figure :



If ABC is a triangle in which :

AB = AC , m ($\angle$ A) = 40° ,

then m ($\angle$ B) = m ($\angle$ C) = $\frac{180^\circ - 40^\circ}{2}$ = 70°

Remarks

1. Both of the base angles in the isosceles triangle are acute.
2. The vertex angle in the isosceles triangle may be acute , right or obtuse angle.

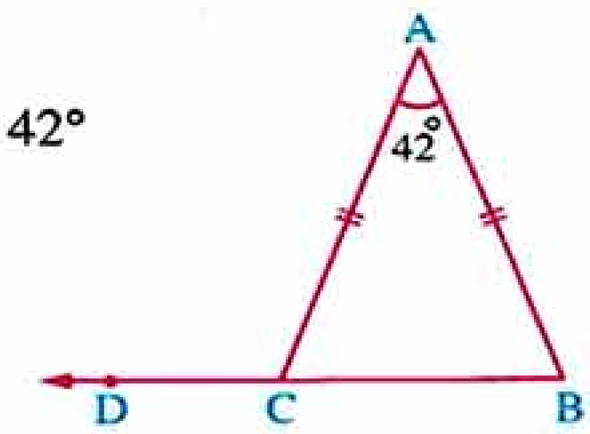
Example 1

In the opposite figure :

ABC is a triangle in which AB = AC , m ($\angle$ A) = 42°

and D $\in \overrightarrow{BC}$

Find : m ($\angle$ ACD)



Solution

Given AB = AC , m ($\angle$ A) = 42° and D $\in \overrightarrow{BC}$

R.T.F. m ($\angle$ ACD)

Proof $\because$ The sum of measures of the interior angles in Δ ABC = 180°

, m ($\angle$ A) = 42°

$\therefore$ m ($\angle$ B) + m ($\angle$ ACB)

= 180° − 42° = 138°

, $\because$ AB = AC (given)

$\therefore$ m ($\angle$ B) = m ($\angle$ ACB) = $\frac{138°}{2}$ = 69°

$\because$ $\angle$ ACD is an exterior angle of Δ ABC

$\therefore$ m ($\angle$ ACD) = m ($\angle$ A) + m ($\angle$ B)

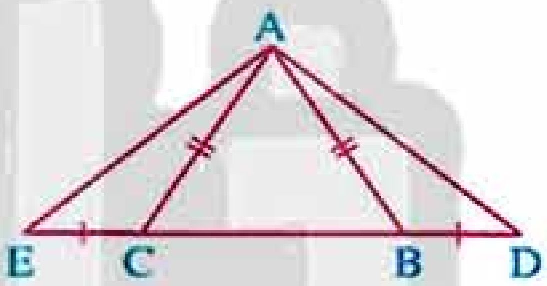
= 42° + 69° = 111° (The req.)

> **Remember that**
> The measure of any exterior angle of a triangle is equal to the sum of measures of the two non-adjacent interior angles.

Example 2 **In the opposite figure :**

B $\in \overline{DE}$, C $\in \overline{DE}$, AB = AC and BD = CE

Prove that : AD = AE



Solution

Given AB = AC and BD = CE

R.T.P. AD = AE

Proof $\because$ AB = AC (given)

$\therefore$ m ($\angle$ ABC) = m ($\angle$ ACB)

$\because$ $\angle$ ABD supplements $\angle$ ABC

, $\angle$ ACE supplements $\angle$ ACB

$\therefore$ m ($\angle$ ABD) = m ($\angle$ ACE)

$\therefore$ In $\Delta\Delta$ ABD , ACE :

$$\begin{cases} AB = AC & \text{(given)} \\ BD = CE & \text{(given)} \\ m(\angle ABD) = m(\angle ACE) & \text{(by proof)} \end{cases}$$

$\therefore$ Δ ABD $\equiv$ Δ ACE , then we deduce that AD = AE (Q.E.D.)

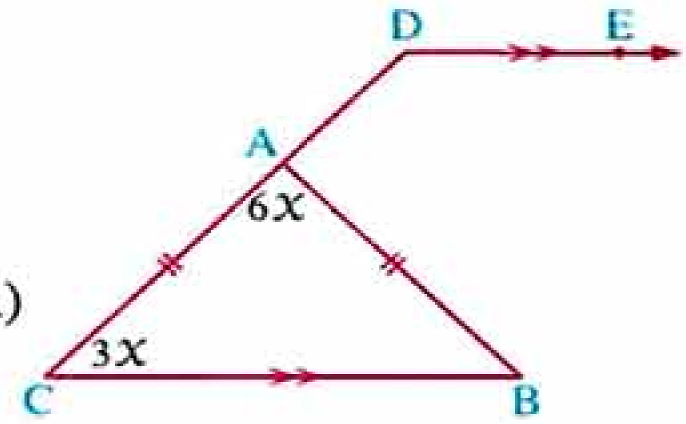
> **Remember that**
> The supplementaries of the equal angles in measures are equal in measures.

Example 3 **In the opposite figure :**

AB = AC , m (∠ BAC) = 6 x ,

m (∠ C) = 3 x and $\overline{BC}$ // $\overrightarrow{DE}$

Find : 1 The value of x 2 m (∠ EDA)



Solution

Given AB = AC , m (∠ BAC) = 6 x , m (∠ C) = 3 x and $\overline{BC}$ // $\overrightarrow{DE}$

R.T.F. 1 The value of x 2 m (∠ EDA)

Proof ∵ AB = AC ∴ m (∠ B) = m (∠ C) = 3 x

, ∵ The sum of measures of the interior angles of the triangle = 180°

∴ 6 x + 3 x + 3 x = 180° ∴ 12 x = 180°

∴ $x = \frac{180°}{12} = 15°$ (First req.)

∴ m (∠ C) = 45° ∵ $\overline{BC}$ // $\overrightarrow{DE}$ and $\overleftrightarrow{CD}$ is a transversal

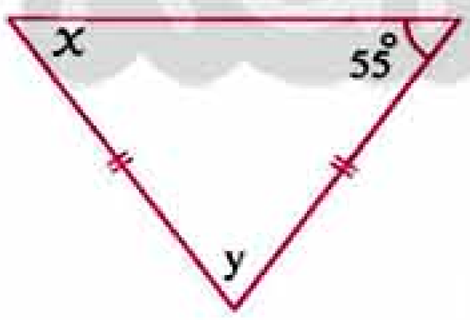
∴ m (∠ EDA) + m (∠ C) = 180° (two interior angles on the same side of the transversal)

∴ m (∠ EDA) = 180° – 45° = 135° (Second req.)

TRY 1 *by yourself*

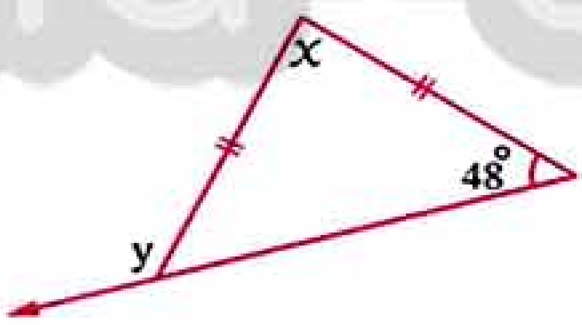
In each of the following figures, find the values of the symbols used as measures for the angles :

1



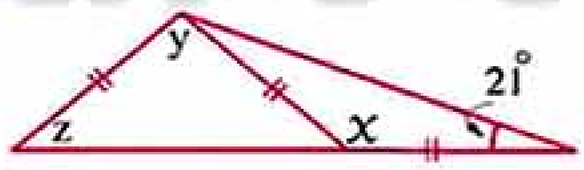
x = ...° , y = ...°

2



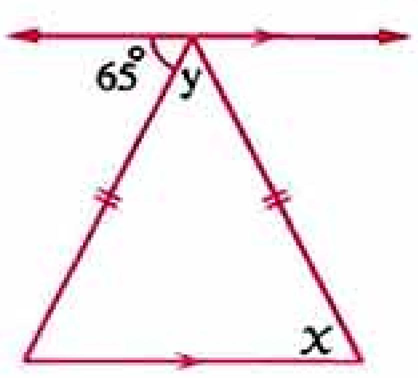
x = ...° , y = ...°

3



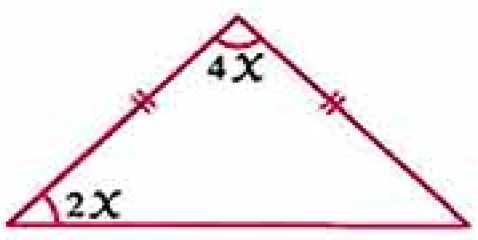
x = ...° , y = ...° , z = ...°

4



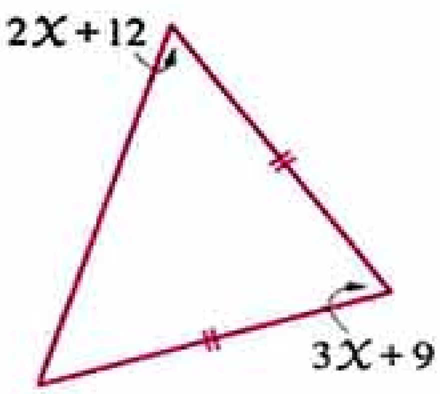
x = ...° , y = ...°

5



x = ...°

6



x = ...°

Unit 4

Corollary

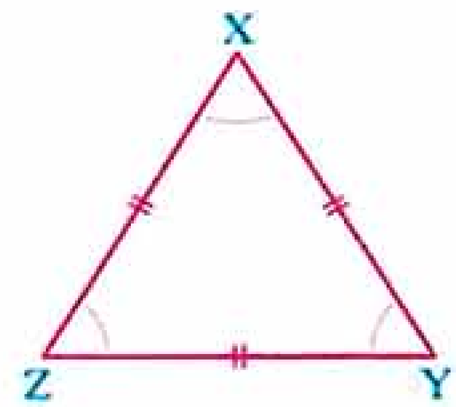
If the triangle is equilateral , then it is equiangular where each angle measure is 60°

For example:

In the opposite figure :

If XYZ is a triangle in which XY = YZ = ZX

, then m (∠ X) = m (∠ Y) = m (∠ Z) = 60°



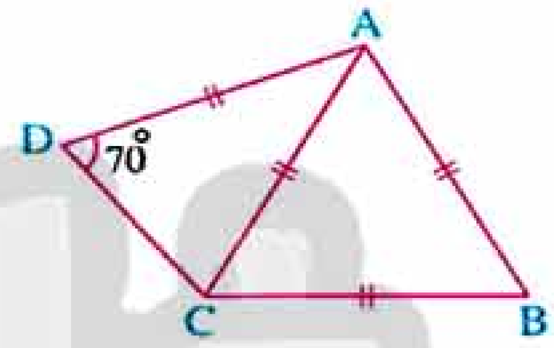
Example 4 **In the opposite figure :**

AB = BC = CA = AD

and m (∠ D) = 70°

Find : 1 m (∠ BCD)

2 m (∠ BAD)



Solution

Given AB = BC = CA = AD and m (∠ D) = 70°

R.T.F. 1 m (∠ BCD)

2 m (∠ BAD)

Proof ∵ Δ ABC is an equilateral triangle.

∴ m (∠ BCA) = 60°

In Δ ACD : ∵ AC = AD

∴ m (∠ ACD) = m (∠ D) = 70°

∴ m (∠ BCD) = m (∠ BCA) + m (∠ ACD)

= 60° + 70° = 130° (First req.)

∵ The sum of measures of the interior angles of the quadrilateral ABCD = 360°

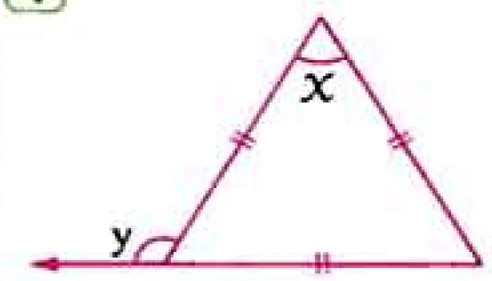
, m (∠ B) = 60°

∴ m (∠ BAD) = 360° – (60° + 130° + 70°) = 100° (Second req.)

TRY 2 by yourself

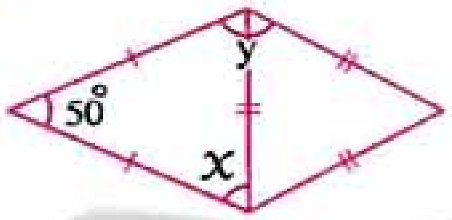
In each of the following figures , find the values of the symbols used as measures for the angles :

1



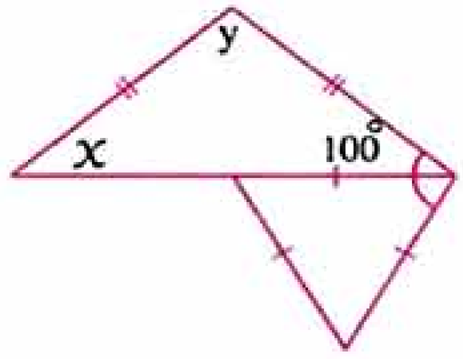
x = ...° , y = ...°

2



x = ...° , y = ...°

3



x = ...° , y = ...°

Answers of try by yourself

1 [1] 55° , 70° [2] 84° , 132° [3] 138° , 96° , 42°
[4] 65° , 50° [5] $22\frac{1}{2}$° [6] 21°

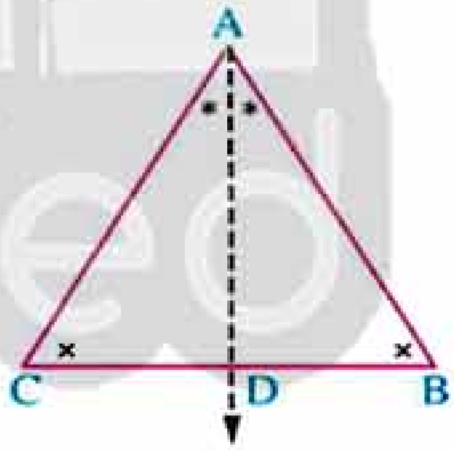
2 [1] 60° , 120° [2] 65° , 125° [3] 40° , 100°

Lesson 4

The converse of the isosceles triangle theorem

Theorem 2

If two angles of a triangle are congruent , then the two sides opposite to these two angles are congruent and the triangle is isosceles.



Given ABC is a triangle in which $\angle B \equiv \angle C$

R.T.P. $\overline{AB} \equiv \overline{AC}$

Construction Bisect $\angle BAC$ by $\overrightarrow{AD}$ to intersect $\overline{BC}$ at D

Proof $\because \angle B \equiv \angle C$

$\therefore m(\angle B) = m(\angle C)$

$\because \overrightarrow{AD}$ bisects $\angle BAC$

$\therefore m(\angle BAD) = m(\angle CAD)$

$\because$ The sum of measures of the interior angles of the triangle = 180°

$\therefore m(\angle ADB) = m(\angle ADC)$

$\therefore$ In $\Delta\Delta$ ABD and ACD :

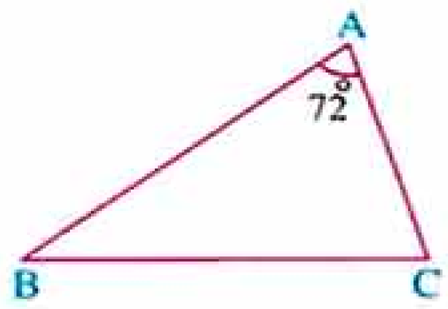
$$\begin{cases} \overline{AD} \text{ is a common side} \\ m(\angle BAD) = m(\angle CAD) \text{ (const.)} \\ m(\angle ADB) = m(\angle ADC) \text{ (by proof)} \end{cases}$$

$\therefore \Delta ABD \equiv \Delta ACD$, then we deduce that

$\overline{AB} \equiv \overline{AC}$, then ΔABC is an isosceles triangle. (Q.E.D.)

Example 1 ABC is a triangle in which m (∠ A) = 2 m (∠ B) = 72°

Prove that : Δ ABC is an isosceles triangle.



Solution

Given m (∠ A) = 2 m (∠ B) = 72°

R.T.P. Δ ABC is an isosceles triangle.

Proof In Δ ABC : ∵ 2 m (∠ B) = 72° ∴ m (∠ B) = $\frac{72°}{2}$ = 36°

∵ m (∠ A) = 72° ∴ m (∠ C) = 180° – (36° + 72°) = 72°

∴ m (∠ A) = m (∠ C) ∴ BC = BA

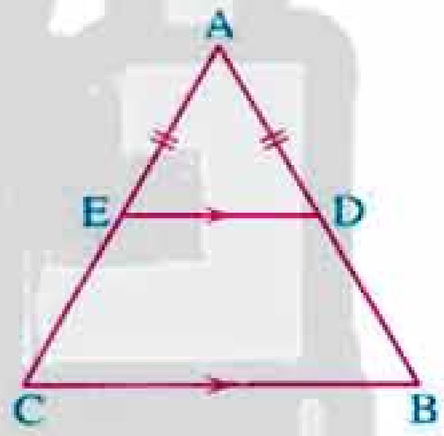
∴ Δ ABC is an isosceles triangle. (Q.E.D.)

Example 2 **In the opposite figure :**

D ∈ $\overline{AB}$ and E ∈ $\overline{AC}$

where AD = AE and $\overline{DE}$ // $\overline{BC}$

Prove that : DB = EC



Solution

Given AD = AE and $\overline{DE}$ // $\overline{BC}$

R.T.P. DB = EC

Proof In Δ ADE : ∵ AD = AE ∴ m (∠ ADE) = m (∠ AED) (1)

∵ $\overline{DE}$ // $\overline{BC}$ and $\overleftrightarrow{AB}$ is a transversal

∴ m (∠ B) = m (∠ ADE) (corresponding angles) (2)

Similarly ∵ $\overline{DE}$ // $\overline{BC}$ and $\overleftrightarrow{AC}$ is a transversal

∴ m (∠ C) = m (∠ AED) (corresponding angles) (3)

From (1) , (2) and (3) : ∴ m (∠ B) = m (∠ C)

∴ AB = AC , ∵ AD = AE

Subtracting ∴ AB – AD = AC – AE ∴ DB = EC (Q.E.D.)

Unit 4

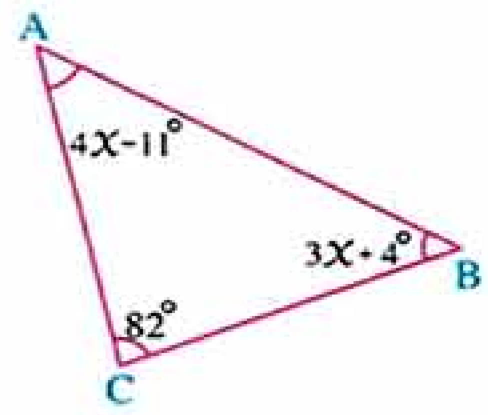
Example 3 **In the opposite figure :**

If m (∠ A) = $4X - 11°$

, m (∠ B) = $3X + 4°$

, m (∠ C) = 82°

, **prove that :** Δ ABC is an isosceles triangle.



Solution

Given m (∠ A) = $4X - 11°$, m (∠ B) = $3X + 4°$, m (∠ C) = 82°

R.T.P. Δ ABC is an isosceles triangle.

Proof ∵ The sum of measures of the interior angles of the triangle = 180°

∴ $4X - 11° + 3X + 4° + 82° = 180°$ ∴ $7X + 75° = 180°$

∴ $7X = 105°$

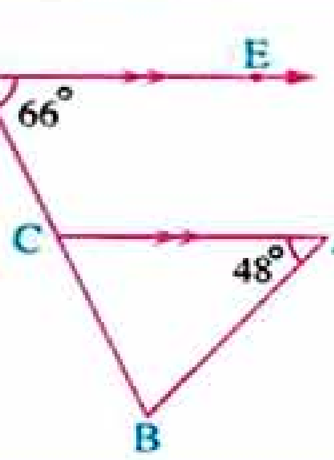
∴ $X = 15°$ ∴ m (∠ A) = 4 × 15° − 11° = 49°

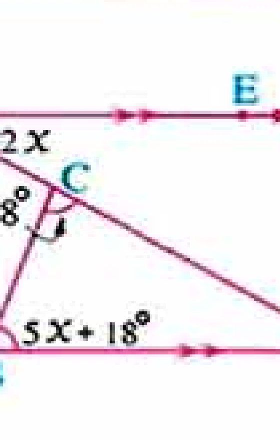
, m (∠ B) = 3 × 15° + 4° = 49° ∴ m (∠ A) = m (∠ B)

∴ BC = AC ∴ Δ ABC is an isosceles triangle. (Q.E.D.)

TRY 1 by yourself

In each of the following figures , write the equal sides in length :

1


2


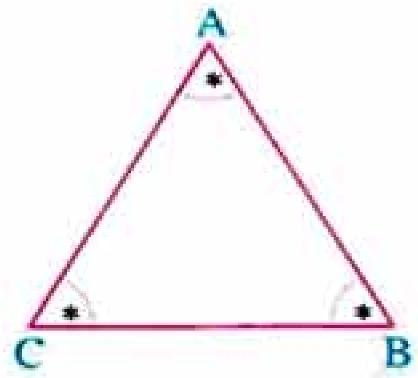
3


4


Corollary

If the angles of a triangle are congruent , then the triangle is equilateral.

For example:



If ABC is a triangle in which :

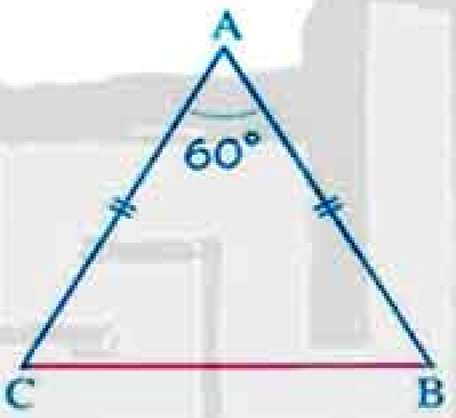
$\angle A \equiv \angle B \equiv \angle C$, then AB = BC = CA

i.e. Δ ABC is an equilateral triangle.

Remark

"The isosceles triangle in which the measure of one of its angles = 60° is an equilateral triangle."

- **In the following figure :**



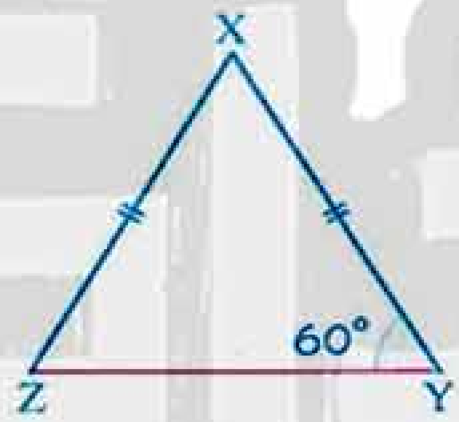
If AB = AC and m ($\angle$ A) = 60°

, then m ($\angle$ B) = m ($\angle$ C) = $\frac{180° - 60°}{2}$

= 60°

$\therefore$ Δ ABC is an equilateral triangle.

- **In the following figure :**



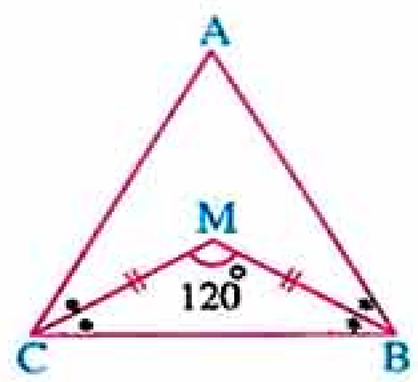
If XY = XZ and m ($\angle$ Y) = 60°

, then m ($\angle$ Z) = 60°

, m ($\angle$ X) = 180° – (60° + 60°) = 60°

$\therefore$ Δ XYZ is an equilateral triangle.

Example 4 **In the opposite figure :**



$\overrightarrow{BM}$ bisects $\angle$ B , $\overrightarrow{CM}$ bisects $\angle$ C ,

MB = MC and m ($\angle$ BMC) = 120°

Prove that : Δ ABC is an equilateral triangle.

Solution

Given $\overrightarrow{BM}$ bisects $\angle$ B , $\overrightarrow{CM}$ bisects $\angle$ C , MB = MC and m ($\angle$ BMC) = 120°

R.T.P. Δ ABC is an equilateral triangle.

Unit 4

Proof

In Δ MBC : $\because$ MB = MC and m ($\angle$ BMC) = 120°

$\therefore$ m ($\angle$ MBC) = m ($\angle$ MCB) = $\frac{180° - 120°}{2}$ = 30°

, $\because$ $\overrightarrow{BM}$ bisects $\angle$ B $\quad\therefore$ m ($\angle$ ABC) = 2 m ($\angle$ MBC) = 60°

, $\because$ $\overrightarrow{CM}$ bisects $\angle$ C $\quad\therefore$ m ($\angle$ ACB) = 2 m ($\angle$ MCB) = 60°

$\therefore$ In Δ ABC : m ($\angle$ BAC) = 180° – (60° + 60°) = 60°

$\therefore$ m ($\angle$ ABC) = m ($\angle$ ACB) = m ($\angle$ BAC) = 60°

$\therefore$ Δ ABC is an equilateral triangle. (Q.E.D.)

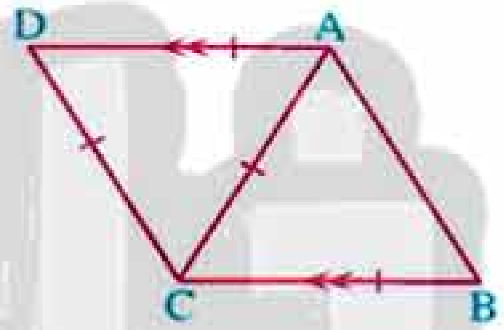
TRY 2 by yourself

In the opposite figure :

ABCD is a quadrilateral in which :

AD = DC = CB = CA , $\overline{AD}$ // $\overline{BC}$

Prove that : Δ ABC is an equilateral triangle.



Answers *of try by yourself*

1 [1] $\overline{XY}$, $\overline{XZ}$ [2] $\overline{AB}$, $\overline{AC}$

[3] $\overline{AB}$, $\overline{AC}$ [4] $\overline{AB}$, $\overline{AC}$

2 Prove by yourself. (**Hint :** Prove that m ($\angle$ ACB) = 60°)

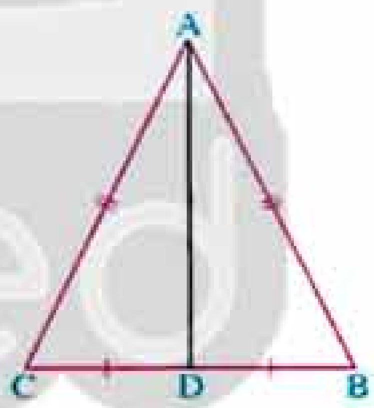
Lesson 5

Corollaries of the isosceles triangle theorems

Corollary 1

The median of an isosceles triangle from the vertex angle bisects it and is perpendicular to the base.

In the opposite figure :



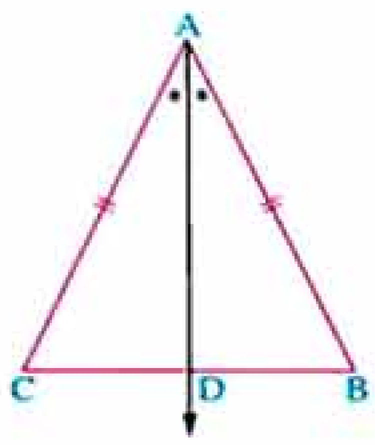
ABC is a triangle in which AB = AC and $\overline{AD}$ is a median , then :

1. $\overrightarrow{AD}$ bisects ∠ BAC i.e. m (∠ BAD) = m (∠ CAD)
2. $\overline{AD} \perp \overline{BC}$

Corollary 2

The bisector of the vertex angle of an isosceles triangle bisects the base and is perpendicular to it.

In the opposite figure :



ABC is a triangle in which AB = AC and $\overrightarrow{AD}$ bisects ∠ BAC , then :

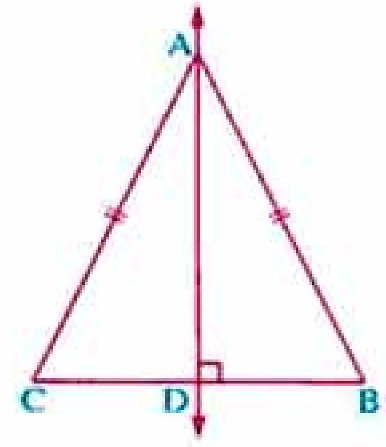
1. D is the midpoint of $\overline{BC}$ i.e. BD = CD
2. $\overline{AD} \perp \overline{BC}$

Corollary 3

The straight line drawn passing through the vertex angle of an isosceles triangle perpendicular to the base bisects each of the base and the vertex angle.

Unit 4

In the opposite figure :



ABC is a triangle in which AB = AC and $\overleftrightarrow{AD} \perp \overline{BC}$, then :

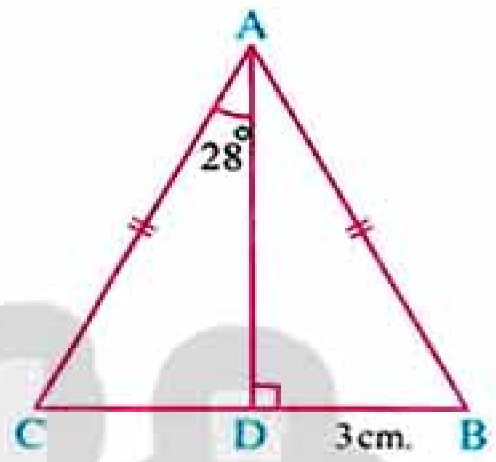
1 D is the midpoint of $\overline{BC}$ i.e. BD = CD

2 m (∠ BAD) = m (∠ CAD)

Notice that :

The previous three corollaries can be proved using the congruence of Δ ABD and Δ ACD

Example 1 **In the opposite figure :**



ABC is an isosceles triangle where AB = AC , D ∈ $\overline{BC}$ such that $\overline{AD} \perp \overline{BC}$, m (∠ CAD) = 28° and BD = 3 cm. **Find :**

1 m (∠ BAC)

2 The length of $\overline{BC}$

Solution

Given AB = AC , m (∠ CAD) = 28° , BD = 3 cm. and $\overline{AD} \perp \overline{BC}$

R.T.F. 1 m (∠ BAC) 2 The length of $\overline{BC}$

Proof In Δ ABC : ∵ AB = AC and $\overline{AD} \perp \overline{BC}$

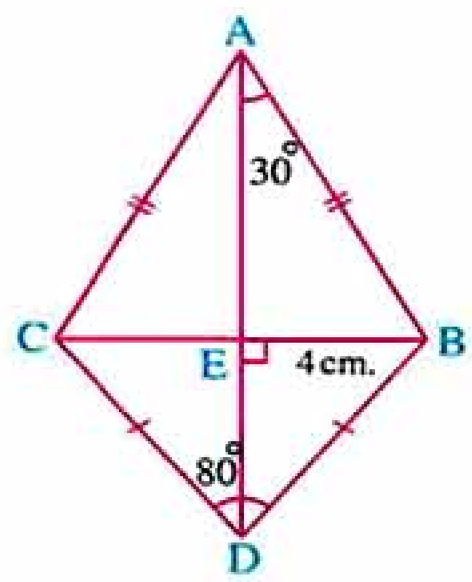
∴ $\overrightarrow{AD}$ bisects each of the vertex angle BAC and the base $\overline{BC}$

∴ m (∠ BAC) = 2 m (∠ CAD) = 2 × 28° = 56° (First req.)

, BC = 2 BD = 2 × 3 = 6 cm. (Second req.)

TRY 1 by yourself

In the opposite figure :



ABDC is a quadrilateral in which :

AB = AC , BD = CD , $\overline{AD} \perp \overline{BC}$,

$\overline{AD} \cap \overline{BC} = \{E\}$, m (∠ BAD) = 30° ,

m (∠ BDC) = 80° and BE = 4 cm.

Complete the following :

1 m (∠ DAC) = ... °

2 m (∠ BDE) = ... °

3 m (∠ ACB) = ...°

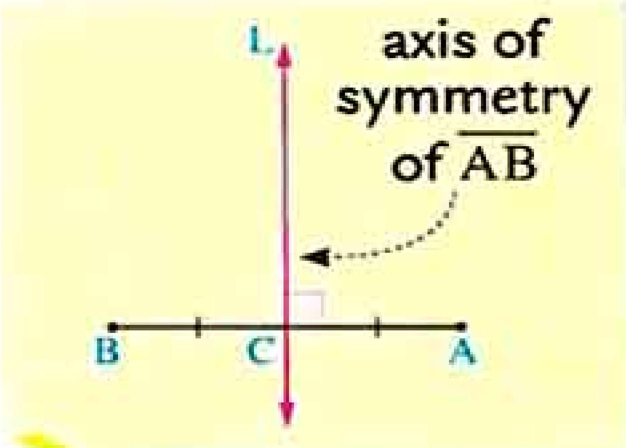
4 EC = ... cm.

5 AC = ... cm.

Axis of symmetry of a line segment

Definition

The straight line perpendicular to a line segment at its middle is called the axis of symmetry for that line segment , in brief it is known as the axis of a line segment.



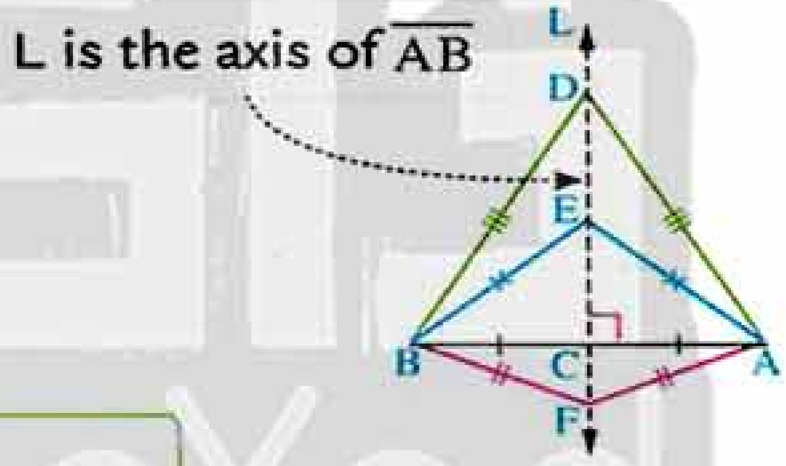
In the opposite figure :

If the straight line L $\perp$ $\overline{AB}$ and C $\in$ the straight line L where C is the midpoint of $\overline{AB}$, then the straight line L is called the axis of $\overline{AB}$

Property

Any point on the axis of symmetry of a line segment is at equal distances from its terminals (end points).

In the opposite figure :



If the straight line L is the axis of $\overline{AB}$,

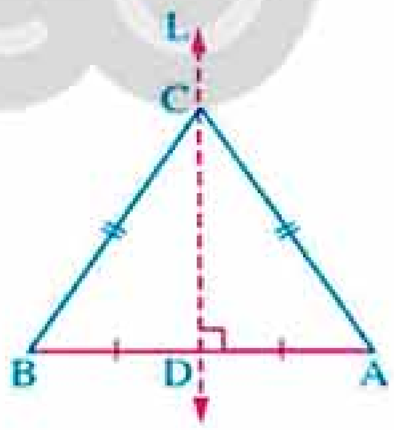
D $\in$ L , E $\in$ L and F $\in$ L , then

DA = DB , EA = EB and FA = FB

The converse of the previous property is true :

i.e. If a point is at equal distances from the two terminals of a line segment , then this point lies on the axis of this line segment.

In the opposite figure :



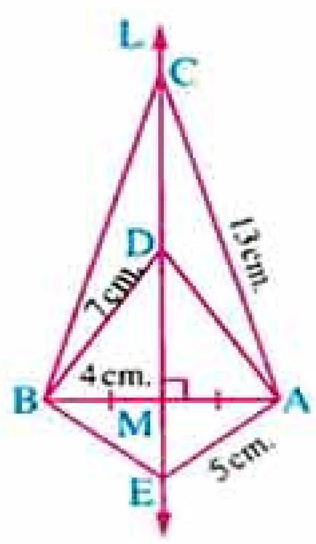
If C is a point such that CA = CB , then the point C lies on the axis of $\overline{AB}$

Example 2 **In the opposite figure :**

The straight line L is the axis of $\overline{AB}$

If the points C , D and E belong to the straight line L , L $\cap$ $\overline{AB}$ = {M} where AC = 13 cm. , DB = 7 cm. , AE = 5 cm. and MB = 4 cm.

Find the length of each of : $\overline{CB}$, $\overline{DA}$, $\overline{EB}$ and $\overline{MA}$



Solution

Given The straight line L is the axis of $\overline{AB}$, C , D and E belong to the straight line L , $L \cap \overline{AB} = \{M\}$

AC = 13 cm. , DB = 7 cm. , AE = 5 cm. and MB = 4 cm.

R.T.F. The lengths of : $\overline{CB}$, $\overline{DA}$, $\overline{EB}$ and $\overline{MA}$

Proof $\because$ C , D and E belong to L (the axis of $\overline{AB}$)

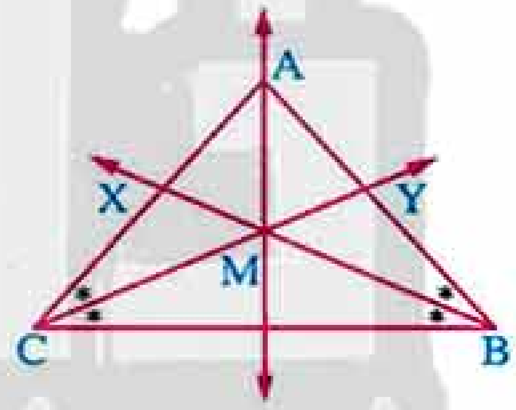
$\therefore$ CB = CA = 13 cm. , DA = DB = 7 cm. ,

EB = EA = 5 cm. , MA = MB = 4 cm. (The req.)

Example 3 Δ ABC is an isosceles triangle where AB = AC , $\overrightarrow{BX}$ bisects $\angle$ ABC and intersects $\overline{AC}$ at X , $\overrightarrow{CY}$ bisects $\angle$ ACB and intersects $\overline{AB}$ at Y

If $\overrightarrow{BX} \cap \overrightarrow{CY} = \{M\}$, **prove that :** $\overleftrightarrow{AM} \perp \overline{BC}$

Solution



Given AB = AC , $\overrightarrow{BX}$ bisects $\angle$ ABC and

$\overrightarrow{CY}$ bisects $\angle$ ACB

R.T.P. $\overleftrightarrow{AM} \perp \overline{BC}$

Proof $\because$ AB = AC $\therefore$ m ($\angle$ ABC) = m ($\angle$ ACB) (1)

, $\because$ $\overrightarrow{BX}$ bisects $\angle$ ABC $\therefore$ m ($\angle$ MBC) = $\frac{1}{2}$ m ($\angle$ ABC) (2)

Similarly

$\because$ $\overrightarrow{CY}$ bisects $\angle$ ACB $\therefore$ m ($\angle$ MCB) = $\frac{1}{2}$ m ($\angle$ ACB) (3)

From (1) , (2) and (3) , we deduce that :

m ($\angle$ MBC) = m ($\angle$ MCB) $\therefore$ MB = MC

i.e. M is at equal distances from B and C

$\therefore$ M $\in$ the axis of $\overline{BC}$ (4)

, $\because$ AB = AC **i.e.** A is at equal distances from B and C

$\therefore$ A $\in$ the axis of $\overline{BC}$ (5)

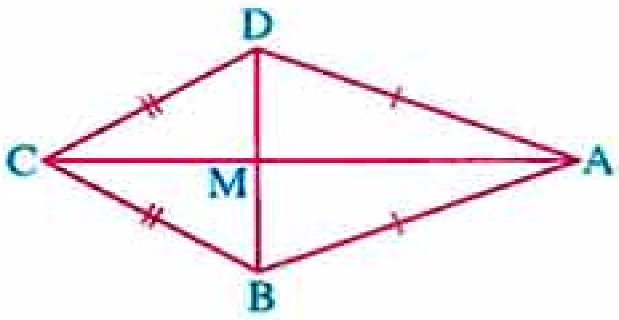
From (4) and (5) : $\therefore$ $\overleftrightarrow{AM}$ is the axis of $\overline{BC}$ $\therefore$ $\overleftrightarrow{AM} \perp \overline{BC}$ (Q.E.D.)

TRY 2 *by yourself*

In the opposite figure :

$\overline{BD} \cap \overline{AC} = \{M\}$, AB = AD and BC = DC

Prove that : M is the midpoint of $\overline{BD}$



Axis of symmetry of the isosceles triangle

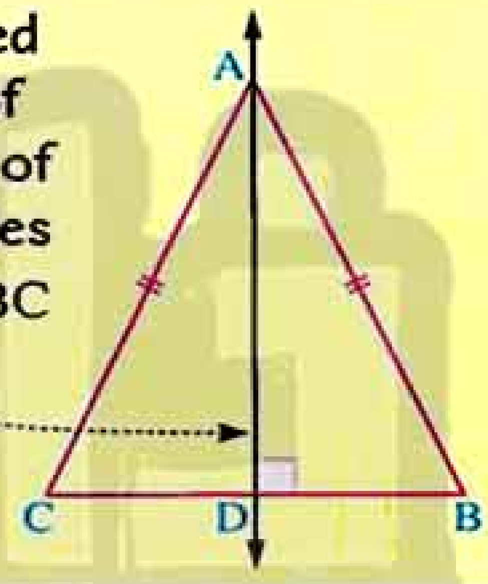
The isosceles triangle has one axis of symmetry.

It is the straight line drawn from the vertex angle perpendicular to its base.

For example:

If ABC is an isosceles triangle where AB = AC and $\overleftrightarrow{AD} \perp \overline{BC}$, then $\overleftrightarrow{AD}$ is called the axis of symmetry of the isosceles triangle ABC

$\overleftrightarrow{AD}$ is called the axis of symmetry of the isosceles triangle ABC

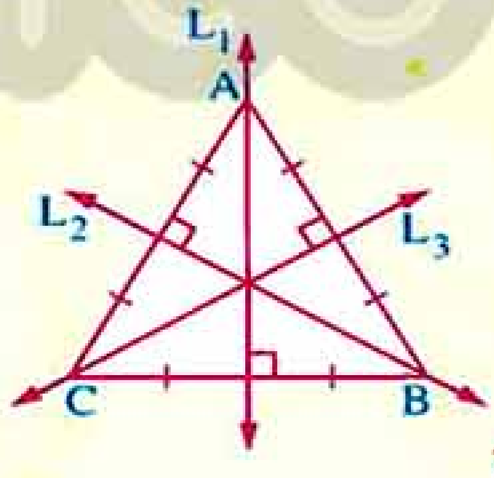


Remarks

1. The equilateral triangle has three axes of symmetry , they are the three perpendiculars drawn from its vertices to the opposite sides.

   In the opposite figure :

   The straight lines L_1 , L_2 and L_3 are the axes of symmetry of the equilateral triangle ABC



2. The scalene triangle has no axes of symmetry.

Answers *of try by yourself*

1. [1] 30° [2] 40° [3] 60° [4] 4 [5] 8
2. Prove by yourself. (**Hint :** Prove that $\overleftrightarrow{AC}$ is the axis of $\overline{BD}$)

UNIT 5

Inequality

Lessons of the unit :

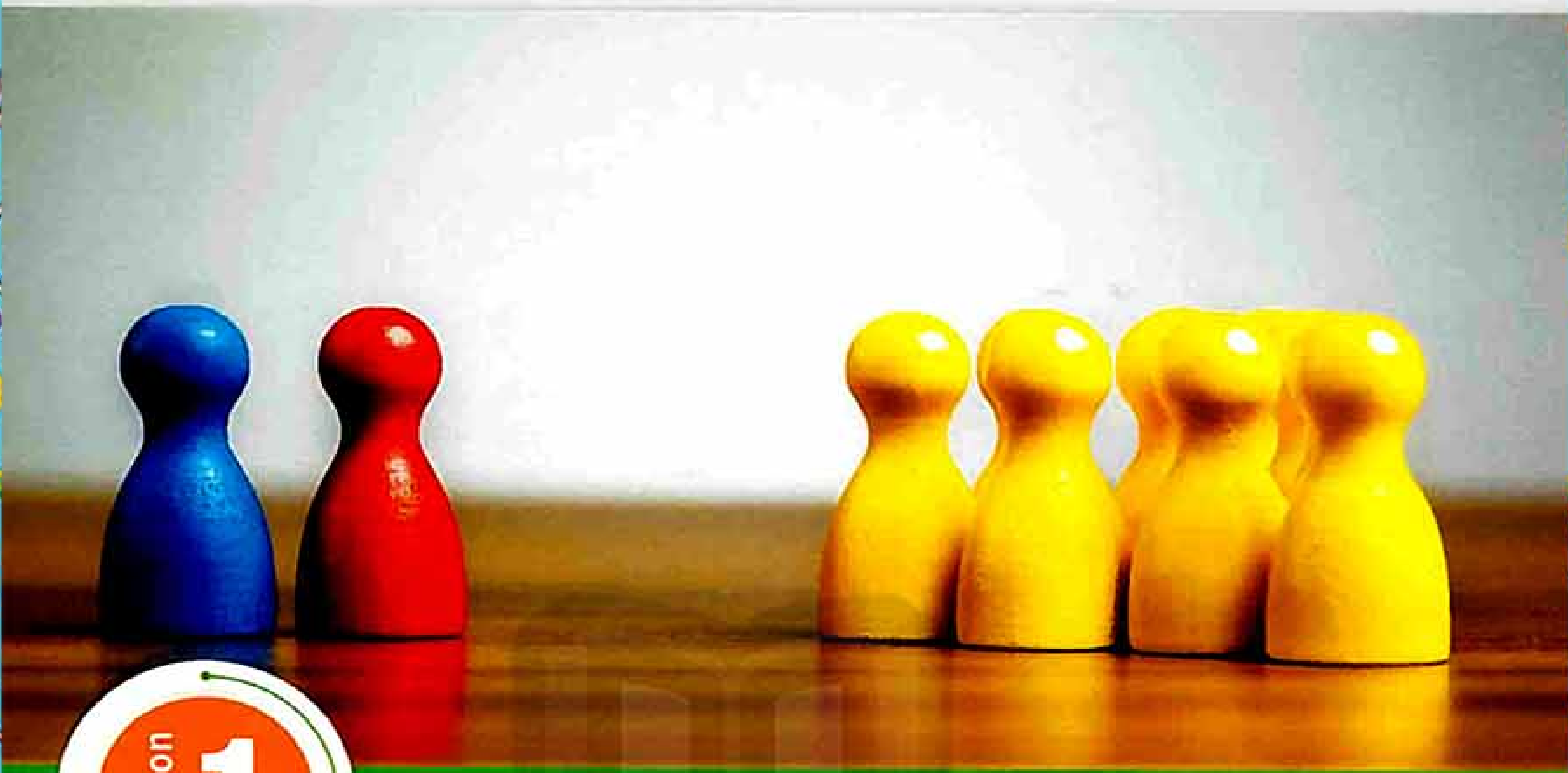
1. Inequality.
2. Comparing the measures of angles in a triangle.
3. Comparing the lengths of sides in a triangle.
4. Triangle inequality.

Use your smart phone or tablet to scan **the QR Code** and enjoy watching videos.

Unit Objectives :

By the end of this unit, student should be able to :

- recognize the concept of inequality.
- recognize the axioms of the inequality relation.
- compare between the measures of angles in the triangle.
- deduce the relation between the measures of two angles in a triangle when the two opposite sides to these angles are not equal in length.
- compare side lengths in the triangle.
- deduce the relation between the lengths of two sides in a triangle when the two opposite angles to these sides are not equal in measure.
- recognize the triangle inequality.
- use the axioms of the inequality relation and the triangle inequality in solving problems in geometry.



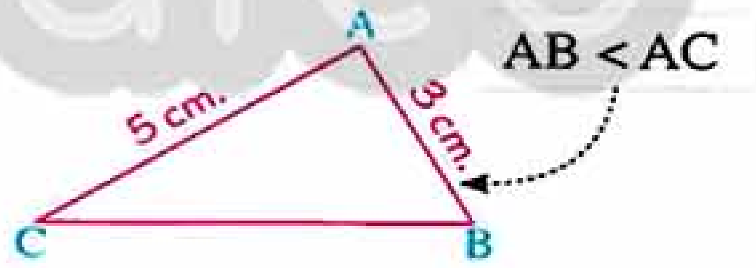
Lesson 1 Inequality

The concept of inequality

- Through our study of the sets of numbers , we had shown the relation of inequality that is used for comparing two different numbers , we expressed that by using one of the two signs $>$ that is read ⟶ « is greater than » or $<$ that is read ⟶ « is smaller than »
- Since the lengths of line segments and measures of angles are numbers , then we can use the relation of inequality to compare between the lengths of two line segments or between the measures of two angles.

For example:

- **In Δ ABC :**

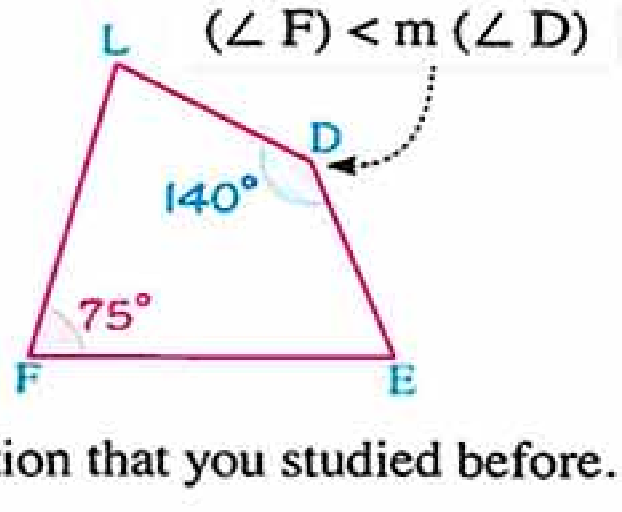


If AC = 5 cm. and AB = 3 cm. , then we deduce that :

The length of $\overline{AC}$ is greater than the length of $\overline{AB}$, then we write AC > AB

or the length of $\overline{AB}$ is smaller than the length of $\overline{AC}$, then we write AB < AC

- **Similarly in the figure DEFL :**



If m ($\angle$ D) = 140° and m ($\angle$ F) = 75° , then we deduce that :

m ($\angle$ D) is greater than m ($\angle$ F) ,

then we write : m ($\angle$ D) > m ($\angle$ F)

or m ($\angle$ F) is smaller than m ($\angle$ D)

, then we write : m ($\angle$ F) < m ($\angle$ D)

In the following , you will be given the axioms of inequality relation that you studied before.

Axioms of inequality relation

For any four numbers a , b , c and d :

1. If $a > b$, then $a + c > b + c$
2. If $a > b$, then $a - c > b - c$
3. If $a > b$, $c > 0$, then $a\,c > b\,c$
4. If $a > b$, $b > c$, then $a > c$
5. If $a > b$, $c > d$, then $a + c > b + d$

Example 1 **In the opposite figure :**

If B and C belong to $\overline{AD}$ such that $AB > CD$

, prove that : $AC > BD$

Solution

Given B and C belong to $\overline{AD}$ and $AB > CD$

R.T.P. $AC > BD$

Proof $\because AB > CD$ (given) and adding BC to both sides

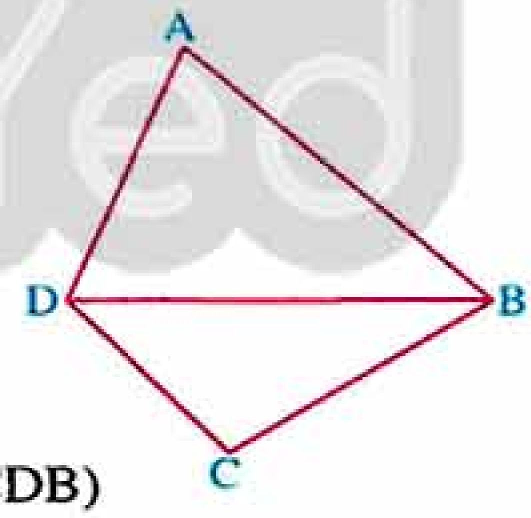
$\therefore AB + BC > CD + BC$

$\therefore AC > BD$ (Q.E.D.)

Example 2 **In the opposite figure :**

If $m(\angle ADB) > m(\angle ABD)$ and $m(\angle CBD) < m(\angle CDB)$

, prove that : $m(\angle ADC) > m(\angle ABC)$



Solution

Given $m(\angle ADB) > m(\angle ABD)$ and $m(\angle CBD) < m(\angle CDB)$

R.T.P. $m(\angle ADC) > m(\angle ABC)$

Proof $\because m(\angle CBD) < m(\angle CDB)$ (given)

$\therefore m(\angle CDB) > m(\angle CBD)$ (1)

, $\because m(\angle ADB) > m(\angle ABD)$ (given) (2)

Adding (1) and (2) :

$\therefore m(\angle CDB) + m(\angle ADB) > m(\angle CBD) + m(\angle ABD)$

$\therefore m(\angle ADC) > m(\angle ABC)$ (Q.E.D.)

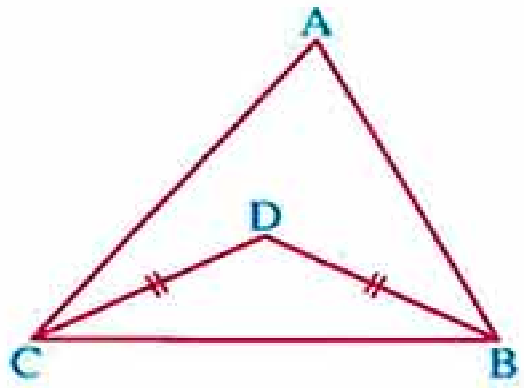
Unit 5

Example 3 **In the opposite figure :**

If m (∠ ABC) > m (∠ ACB)

and BD = DC

, prove that : m (∠ ABD) > m (∠ ACD)



Solution

Given m (∠ ABC) > m (∠ ACB) and BD = DC

R.T.P. m (∠ ABD) > m (∠ ACD)

Proof ∵ DB = DC ∴ m (∠ DBC) = m (∠ DCB) (1)

, ∵ m (∠ ABC) > m (∠ ACB) (given) (2)

Subtracting (1) from (2) :

∴ m (∠ ABC) − m (∠ DBC) > m (∠ ACB) − m (∠ DCB)

∴ m (∠ ABD) > m (∠ ACD) (Q.E.D.)

Remember that

The measure of any exterior angle of a triangle is greater than the measure of any interior angle of the triangle except its adjacent angle.

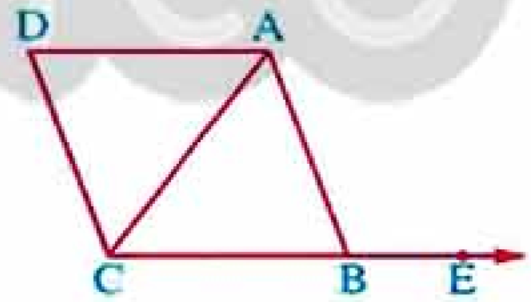
TRY by yourself

In the opposite figure :

ABCD is a parallelogram and E ∈ $\overrightarrow{CB}$

Prove that :

m (∠ ABE) > m (∠ ACD)



Answers *of try by yourself*

Prove by yourself

[**Hint :** m (∠ ABE) > m (∠ BAC) (exterior angle of Δ ABC)

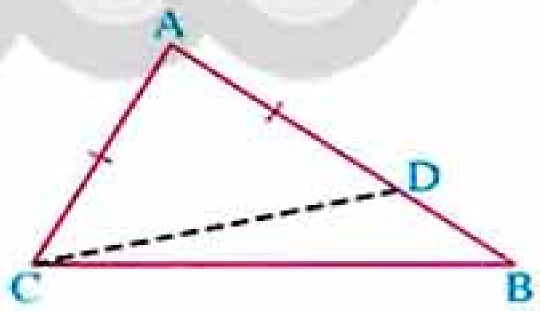
, m (∠BAC) = m (∠ ACD) (alt. angles)]

Lesson 2 Comparing the measures of angles in a triangle

From your study of the previous unit , you learnt that if two sides of a triangle are congruent , then the opposite angles to these sides are equal in measure.
In the following , you shall study the relation between the measures of two angles of a triangle when the two opposite sides to these angles are not equal in length.

Theorem

In a triangle , if two sides have unequal lengths , then the longer is opposite to the angle of the greater measure.



Given ABC is a triangle in which AB > AC

R.T.P. m (∠ ACB) > m (∠ ABC)

Construction Take D $\in \overline{AB}$ such that AD = AC

Proof In Δ ACD :

∵ AD = AC ∴ m (∠ ADC) = m (∠ ACD) (1)

∵ ∠ ADC is an exterior angle of Δ DBC

∴ m (∠ ADC) > m (∠ B) (2)

From (1) and (2) :

∴ m (∠ ACD) > m (∠ B)

, ∵ m (∠ ACB) > m (∠ ACD)

∴ m (∠ ACB) > m (∠ ABC) (Q.E.D.)

Unit 5

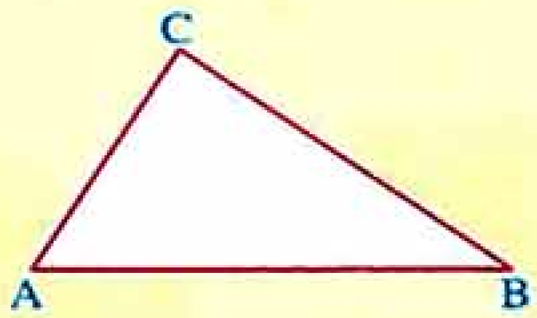
Remark

The greatest angle in measure of the triangle is opposite to the longest side of the triangle and its measure is greater than 60° and the smallest angle in measure of the triangle is opposite to the shortest side of the triangle and its measure is less than 60°

i.e. In ΔABC : If $AB > BC > AC$,

then $m(\angle C) > m(\angle A) > m(\angle B)$

, $m(\angle C) > 60°$ and $m(\angle B) < 60°$



Example 1 ABCD is a quadrilateral in which AB = 5 cm. , BC = 2 cm. , CD = 3 cm. and DA = 4 cm.

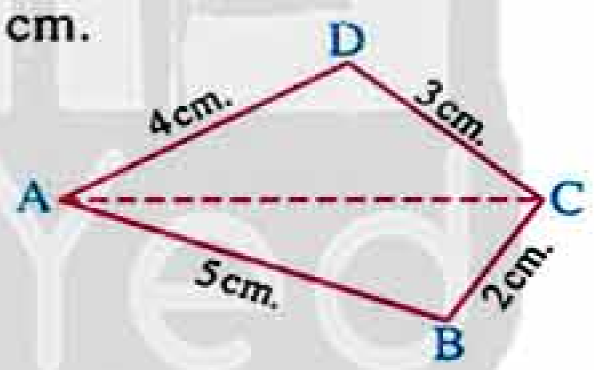
Prove that : $m(\angle DCB) > m(\angle DAB)$

Solution

Given AB = 5 cm. , BC = 2 cm. , CD = 3 cm. and DA = 4 cm.

R.T.P. $m(\angle DCB) > m(\angle DAB)$

Construction Draw $\overline{AC}$



Proof In ΔACD :

$\because AD = 4$ cm. and $CD = 3$ cm.

$\therefore AD > CD$ $\therefore m(\angle ACD) > m(\angle CAD)$ (1)

In ΔABC :

$\because AB = 5$ cm. and $CB = 2$ cm.

$\therefore AB > CB$ $\therefore m(\angle ACB) > m(\angle CAB)$ (2)

Adding (1) and (2) :

$\therefore m(\angle ACD) + m(\angle ACB) > m(\angle CAD) + m(\angle CAB)$

$\therefore m(\angle DCB) > m(\angle DAB)$ (Q.E.D.)

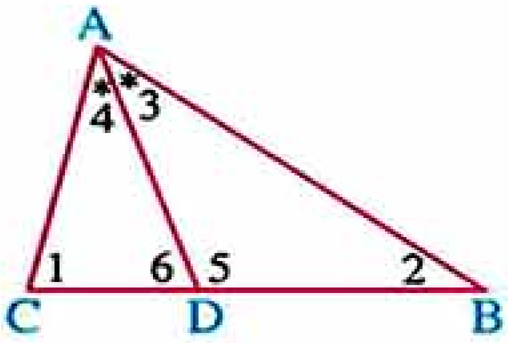
Example 2 ABC is a triangle in which AB > AC and ∠ BAC is bisected by $\overrightarrow{AD}$ which intersects $\overline{BC}$ at D

Prove that : Δ ABD is an obtuse-angled triangle.

Solution

Given ABC is a triangle in which AB > AC and $\overrightarrow{AD}$ bisects ∠ BAC

R.T.P. Δ ABD is an obtuse-angled triangle.



Proof In Δ ABC :

∵ AB > AC ∴ m (∠ 1) > m (∠ 2)

∵ $\overrightarrow{AD}$ bisects ∠ BAC

∴ m (∠ 3) = m (∠ 4)

∴ m (∠ 1) + m (∠ 4) > m (∠ 2) + m (∠ 3)

but m (∠ 1) + m (∠ 4) = m (∠ 5)

because ∠ 5 is an exterior angle of Δ ADC

∴ m (∠ 5) > m (∠ 2) + m (∠ 3)

∴ Δ ABD is an obtuse-angled triangle. (Q.E.D.)

Remember that

If the measure of an angle in a triangle is greater than the sum of measures of the two other angles , then this angle is an obtuse angle.

TRY by yourself

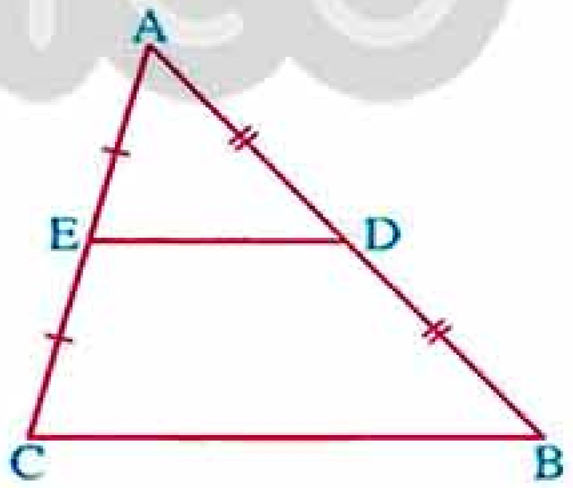
In the opposite figure :

ABC is a triangle in which AB > AC

, D and E are the midpoints

of $\overline{AB}$ and $\overline{AC}$ respectively.

Prove that : m (∠ AED) > m (∠ ADE)



Answers of try by yourself

Prove by yourself (**Hint :** AD > AE because : AB > AC)

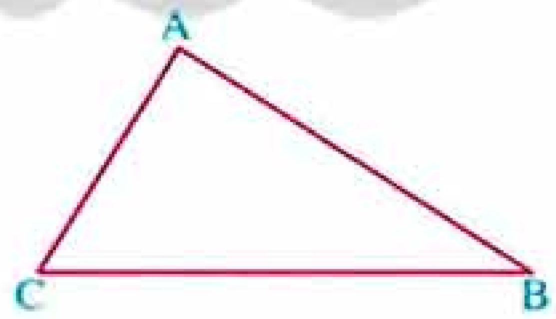
Lesson 3 Comparing the lengths of sides in a triangle

From your previous study , you learnt that : if two angles are equal in measure in a triangle , then the two opposite sides to these angles are equal in length.

In the following , you shall study the relation between the lengths of two sides in a triangle when the two opposite angles are not equal in measure.

Theorem

In a triangle , if two angles are unequal in measure , then the greater angle in measure is opposite to a side greater in length than that opposite to the other angle.



Given ABC is a triangle in which $m(\angle C) > m(\angle B)$

R.T.P. $AB > AC$

Proof $\because$ $\overline{AB}$ and $\overline{AC}$ are two line segments.

$\therefore$ One of the following cases should be verified :

1 $AB > AC$ **2** $AB = AC$ **3** $AB < AC$

Unless $AB > AC$, then either $AB = AC$ **or** $AB < AC$

- If $AB = AC$, then $m(\angle C) = m(\angle B)$ and this contradicts the given where $m(\angle C) > m(\angle B)$
- If $AB < AC$, then $m(\angle C) < m(\angle B)$ according to the previous theorem.

Again this contradicts the given where $m(\angle C) > m(\angle B)$

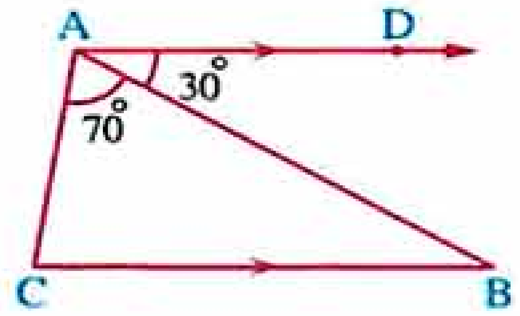
$\therefore$ It should be that $AB > AC$ (Q.E.D.)

Example 1 **In the opposite figure :**

ABC is a triangle in which m ($\angle$ BAC) = 70° ,

$\overrightarrow{AD}$ // $\overline{BC}$ and m ($\angle$ DAB) = 30°

Prove that : AB > AC



Solution

Given $\overrightarrow{AD}$ // $\overline{BC}$, m ($\angle$ BAC) = 70° and m ($\angle$ DAB) = 30°

R.T.P. AB > AC

Proof $\because$ $\overrightarrow{AD}$ // $\overline{BC}$ and $\overleftrightarrow{AB}$ is a transversal to them.

$\therefore$ m ($\angle$ B) = m ($\angle$ DAB) = 30° (alternate angles)

$\therefore$ In Δ ABC : m ($\angle$ C) = 180° – (30° + 70°) = 80°

$\therefore$ m ($\angle$ C) > m ($\angle$ B) $\therefore$ AB > AC (Q.E.D.)

Corollaries

Corollary 1

In the right-angled triangle , the hypotenuse is the longest side.

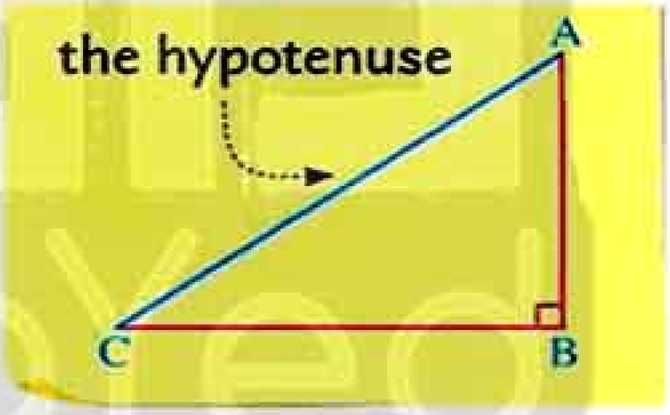
In the opposite figure :

If Δ ABC is right-angled at B , then m ($\angle$ B) > m ($\angle$ A) ,

m ($\angle$ B) > m ($\angle$ C) because $\angle$ B is a right angle and each of

$\angle$ A and $\angle$ C is acute , so we find that :

AC > BC and AC > AB (according to the previous theorem).



Notice that :

In the obtuse-angled triangle, the side opposite to the obtuse angle is the longest side in the triangle.

Corollary 2

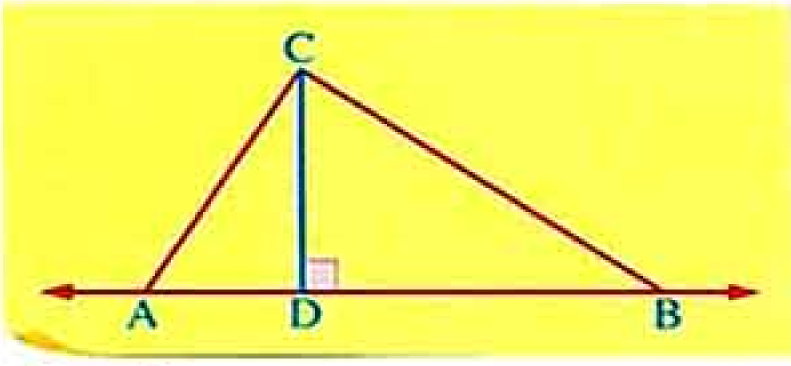
The length of the perpendicular line segment drawn from a point outside a straight line to this line is shorter than any line segment drawn from this point to the given straight line.

In the opposite figure :

If C $\notin$ $\overleftrightarrow{AB}$ and D $\in$ $\overleftrightarrow{AB}$ such that $\overline{CD}$ $\perp$ $\overleftrightarrow{AB}$,

then $\overline{CB}$ is the hypotenuse in Δ CBD

which is right-angled at D ,



$\overline{CA}$ is the hypotenuse in Δ CDA which is right-angled at D and so on ...

According to corollary 1 , we find that CB > CD , CA > CD and so on ...

i.e. CD < CB and CD < CA

Unit 5

Definition

The distance between any point and a given straight line is the length of the perpendicular line segment drawn from this point to the given line.

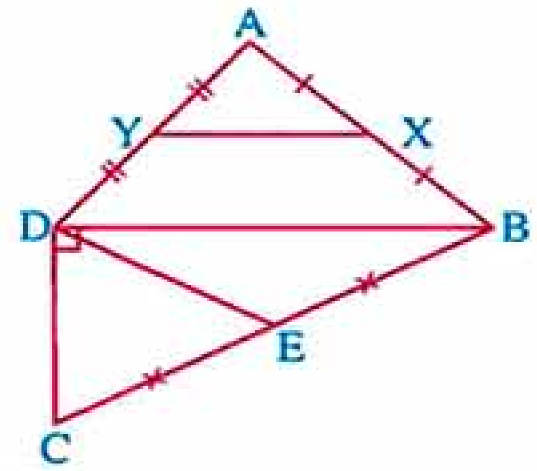
In the previous figure :

The distance between the point C and the straight line $\overleftrightarrow{AB}$ is the length of $\overline{CD}$

Example 2 **In the opposite figure :**

ABCD is a quadrilateral. X , Y and E are the midpoints of $\overline{AB}$, $\overline{AD}$ and $\overline{BC}$ respectively and m ($\angle$ BDC) = 90°

Prove that : DE > XY



Solution

Given X is the midpoint of $\overline{AB}$, Y is the midpoint of $\overline{AD}$, E is the midpoint of $\overline{BC}$ and m ($\angle$ BDC) = 90°

R.T.P. DE > XY

Proof In Δ ABD : $\because$ X is the midpoint of $\overline{AB}$ and Y is the midpoint of $\overline{AD}$

$\therefore XY = \frac{1}{2}$ BD (1)

In Δ DBC : $\because$ m ($\angle$ BDC) = 90° and E is the midpoint of $\overline{BC}$

$\therefore DE = \frac{1}{2}$ BC (2)

, $\because$ $\overline{BC}$ is the hypotenuse of Δ BDC $\therefore$ BC > BD

$\therefore \frac{1}{2} BC > \frac{1}{2} BD$ (3)

From (1) , (2) and (3) : $\therefore$ DE > XY (Q.E.D.)

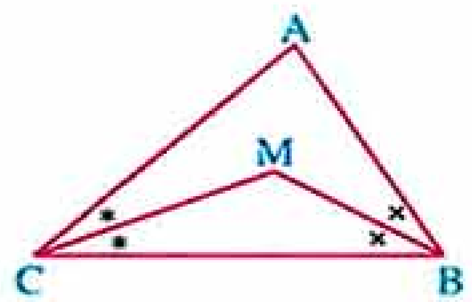
TRY by yourself

In the opposite figure :

ABC is a triangle in which AC > AB ,

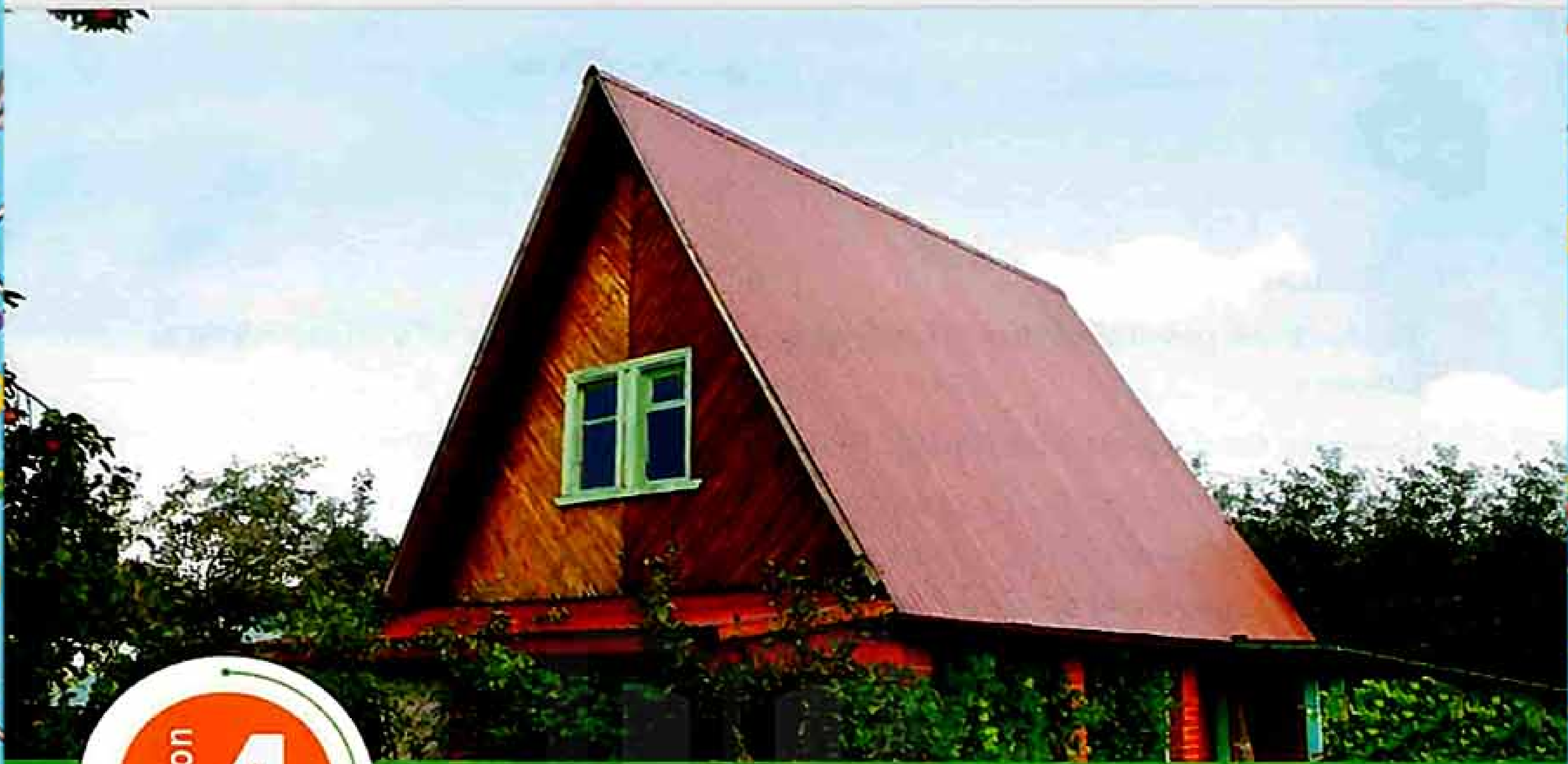
$\overrightarrow{BM}$ bisects $\angle$ ABC and $\overrightarrow{CM}$ bisects $\angle$ ACB

Prove that : MC > MB

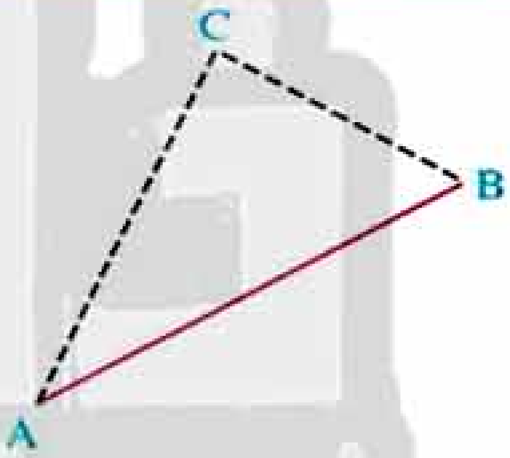


Answers of try by yourself

Prove by yourself [**Hint :** m ($\angle$ ABC) > m ($\angle$ ACB)]



Lesson 4 Triangle inequality



We know that the shortest distance between two points is the length of the line segment joining them.

For example:

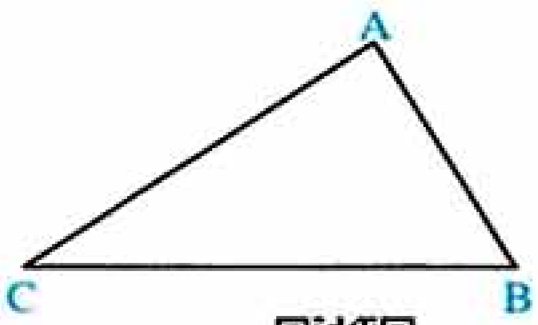
In the opposite figure :

The shortest distance between A and B is the length of $\overline{AB}$

So , for any point $C \notin \overleftrightarrow{AB}$, then $AB < AC + CB$

Generally

In any triangle , the sum of the lengths of any two sides is greater than the length of the third side.



i.e. In any triangle such as ΔABC

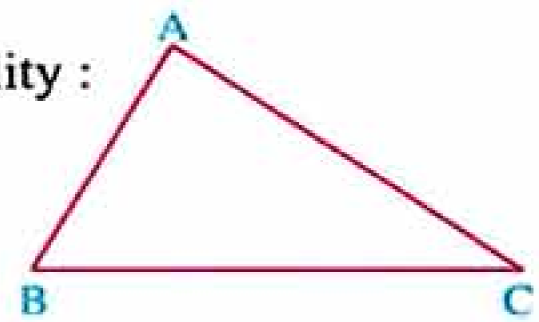
, we get : $AB + BC > AC$

, $BC + CA > AB$

, $CA + AB > CB$

Corollary

The length of any side in a triangle is greater than the difference between the lengths of the other two sides and less than their sum.



And you can prove that as follows :

In the opposite figure ABC is a triangle and from the triangle inequality :

$AC + AB > BC$ (1)

, $\because AB + BC > AC$ i.e. $BC > AC - AB$ (2)

From (1) and (2) , we deduce that : $\boxed{AC - AB < BC < AC + AB}$

Unit 5

Remark

To check the possibility that three lengths can be side lengths of a triangle , do as follows :

Compare the greatest length with the sum of the other two lengths :

- If the greatest length is greater than or equal to the sum of the other two lengths , you deduce that the three given lengths **couldn't be** lengths of the three sides of a triangle. (i.e. no triangle could be drawn with these side lengths).
- If the greatest length is less than the sum of the other two lengths , you deduce that the three given lengths **could be** lengths of the three sides of a triangle. (i.e. a triangle could be drawn with these side lengths).

Example 1 **Is it possible to draw a triangle whose side lengths are as follows (giving reason) :**

1. 5 cm. , 7 cm. , 12 cm.
2. 4 cm. , 6 cm. , 11 cm.
3. 14 cm. , 9 cm. , 7 cm.
4. 8 cm. , 18 cm. , 8 cm.

Solution

1. $\because 5 + 7 = 12$
   $\therefore$ It is not possible to draw a triangle of side lengths 5 cm. , 7 cm. and 12 cm.
2. $\because 4 + 6 < 11$
   $\therefore$ It is not possible to draw a triangle of side lengths 4 cm. , 6 cm. and 11 cm.
3. $\because 9 + 7 > 14$
   $\therefore$ It is possible to draw a triangle of side lengths 14 cm. , 9 cm. and 7 cm.
4. $\because 8 + 8 < 18$
   $\therefore$ It is not possible to draw a triangle of side lengths 8 cm. , 18 cm. and 8 cm.

Example 2 **Find the interval to which the length of the third side of each of the following triangles belongs if the lengths of the other two sides are :**

1. 4 cm. , 3 cm.
2. 4.5 cm. , 7.5 cm.
3. $2\sqrt{5}$ cm. , $2\sqrt{5}$ cm.

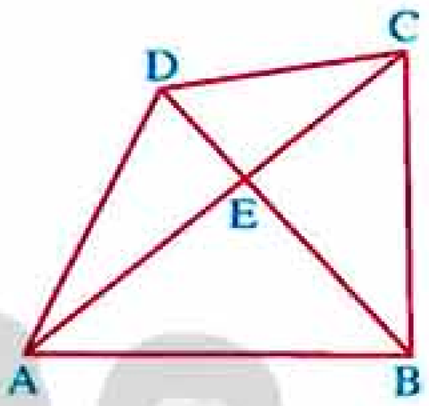
Solution $\because$ The length of any side in a triangle is greater than the difference between the lengths of the other two sides and less than their sum , and let the length of the third side be ℓ cm. , then

1. $4 - 3 < \ell < 4 + 3$ $\quad \therefore 1 < \ell < 7$ $\quad \therefore \ell \in]1 , 7[$
2. $7.5 - 4.5 < \ell < 7.5 + 4.5$ $\quad \therefore 3 < \ell < 12$ $\quad \therefore \ell \in]3 , 12[$
3. $2\sqrt{5} - 2\sqrt{5} < \ell < 2\sqrt{5} + 2\sqrt{5}$ $\quad \therefore 0 < \ell < 4\sqrt{5}$ $\quad \therefore \ell \in]0 , 4\sqrt{5}[$

Example 3 **In the opposite figure :**

ABCD is a quadrilateral whose diagonals intersect at E

Prove that : AC + BD > BC + AD



Solution

Given ABCD is a quadrilateral whose diagonals intersect at E

R.T.P. AC + BD > BC + AD

Proof In Δ EBC : EC + EB > BC (triangle inequality) (1)

In Δ EAD : EA + ED > AD (triangle inequality) (2)

Adding (1) and (2) : $\therefore$ EC + EA + EB + ED > BC + AD

, $\because$ EC + EA = AC , EB + ED = BD $\quad \therefore$ AC + BD > BC + AD (Q.E.D.)

TRY by yourself

1 **Put (✓) in the space in front of each group of the following lengths which can be side lengths of a triangle :**

1. 2 cm. , 3 cm. , 4 cm. ( )
2. 3 cm. , 6 cm. , 2 cm. ( )
3. 10 cm. , 3 cm. , 7 cm. ( )
4. 12 cm. , 5 cm. , 7.5 cm. ( )

2 **Find the interval to which the length of the third side of each of the following ngles belongs if the lengths of the other two sides are :**

1. 6 cm. , 5 cm.
2. 7.5 cm. , 7.5 cm.

Answers of try by yourself

1 (1) , (4)

2 (1)]1 , 11[(2)]0 , 15[

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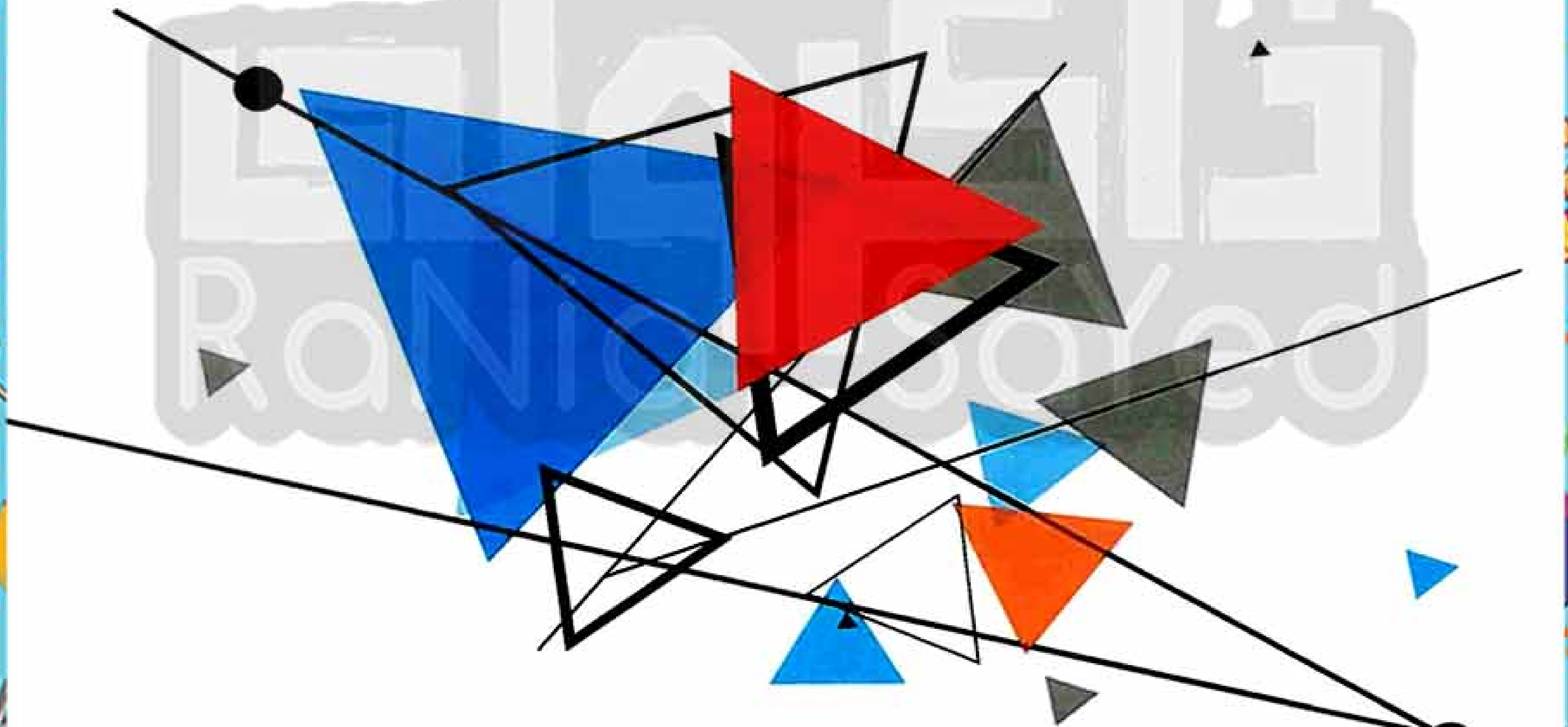
In Mathematics Exercises

For 2nd Prep.

First Term



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A group of supervisors

CONTENTS

First Algebra and Statistics

Second Geometry

First Algebra and Statistics

Revision

From the school book

1 **Complete by writing the following numbers in the form $\frac{a}{b}$ where a and b are two integers and there aren't common factors between them , $b \neq 0$**

1) $0.2 =$ ....
2) $0.3 =$ ....
3) $25\ \% =$ ....
4) $|-0.75| =$ ....
5) $-6 =$ ....
6) $1\frac{1}{4} =$ ....

2 **Choose the correct answer from those given :**

1) $\mathbb{Z}_+ \cup \{0\} =$ ....
(a) $\mathbb{Z}$ (b) $\mathbb{N}$ (c) $\mathbb{C}$ (d) $\mathbb{Q}$

2) $|-2| + |-4| + |6| =$ ....
(a) Zero (b) $|-12|$ (c) -12 (d) 6

3) $\sqrt{a^2} =$ ....
(a) a (b) $-a$ (c) $|a|$ (d) $\pm a$

4) The solution set of the equation : $x + 5 = |-5|$ in $\mathbb{N}$ is ....
(a) $\{0\}$ (b) $\{10\}$ (c) $\{-10\}$ (d) $\varnothing$

5) Which of the following rational numbers lies between $\frac{1}{5}$, $\frac{2}{5}$?
(a) $\frac{2}{10}$ (b) $\frac{1}{10}$ (c) 0.3 (d) -0.3

6) The product of the rational number $\frac{a}{b}$ by its additive inverse equals ....
(a) Zero (b) $-\frac{a}{b}$ (c) $\frac{a^2}{b^2}$ (d) $-\frac{a^2}{b^2}$

7) $3^{10} + 3^{10} + 3^{10} =$ ....
(a) 3^{10} (b) 3^{30} (c) 9^{10} (d) 3^{11}

8) If $a^{-1} = \frac{2}{3}$, then $a =$ ....
(a) $-\frac{2}{3}$ (b) $\frac{3}{2}$ (c) $-\frac{3}{2}$ (d) 1

9) The multiplicative inverse of 5^{-1} is ....
(a) $\frac{1}{5}$ (b) 5 (c) -5 (d) $-\frac{1}{5}$

3 Complete the following :

1. $\sqrt{25+144} =$ ....
2. $\sqrt{0.25} =$ ....
3. $-|-5|-|2| =$ ....
4. $-\sqrt{25}+|-5| =$ ....
5. $\sqrt{0.16}+|-0.6| =$ ....
6. $\dfrac{\sqrt{25-9}}{\sqrt{25}-\sqrt{9}} =$ ....
7. The standard form of the number 0.00015 is ....
8. The standard form of the number 421×10^3 is ....
9. $2^0+2^1+2^2+2^3 =$ ....
10. The sum of the two square roots of the number $2\frac{1}{4}$ equals ....
11. $\left(\frac{2}{3}\right)^2 \times \sqrt{\frac{81}{16}} \times \left(\frac{7}{9}\right)^{\text{zero}} =$ ....

4 Find the value of x which satisfies each of the following equations :

1. $5x+3=20$
2. $7x+11=12$
3. $3x+5=1$
4. $x+3=7$

5 Find the solution set of each of the following equations , where $x \in \mathbb{Q}$:

1. $x^2+12=21$
2. $2x^2-1=-9$
3. $|x|=2$
4. $\sqrt{x^2}=4$

UNIT 1

Real Numbers

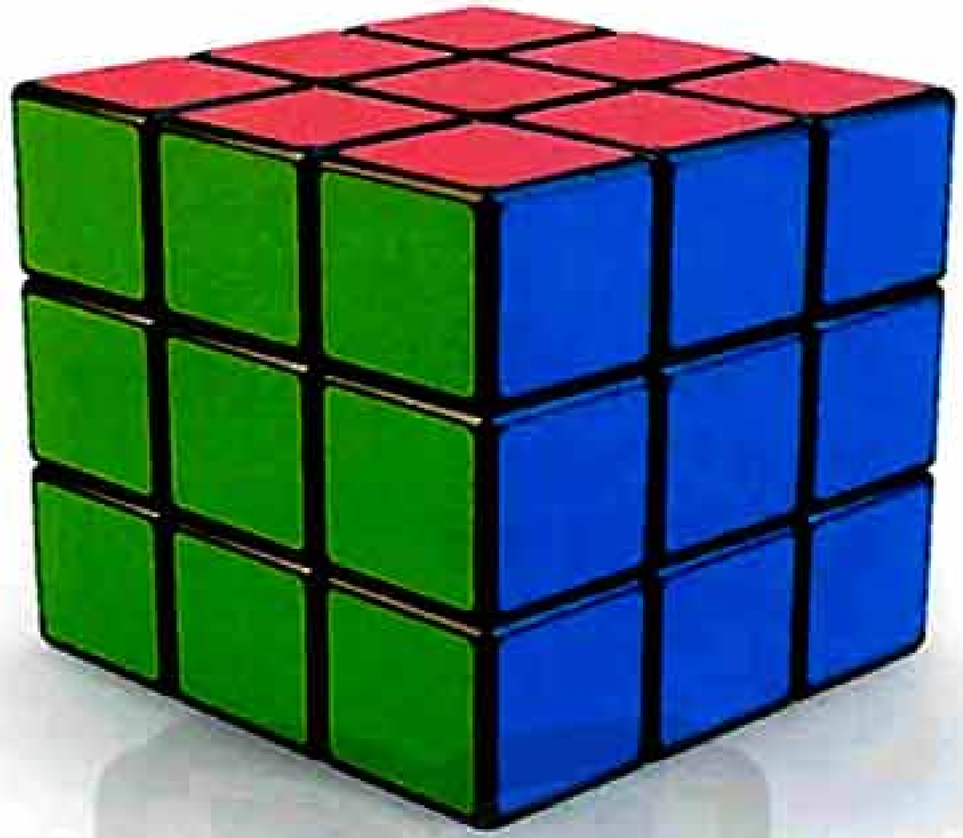
Exercises of the unit :

1. The cube root of a rational number.
2. The set of irrational numbers $\grave{\mathbb{Q}}$
3. The set of real numbers $\mathbb{R}$
   – Ordering numbers in $\mathbb{R}$
4. Intervals.
5. Operations on the real numbers.

✪ Summary of the first part of unit one.
✪ Exams on the first part of unit one.

6. Operations on the square roots.
7. The two conjugate numbers.
8. Operations on the cube roots.
9. Applications on the real numbers.
10. Solving equations and inequalities of the first degree in one variable in $\mathbb{R}$

✪ Summary of the second part of unit one.
✪ Exams on the second part of unit one.



Exercise 1 The cube root of a rational number

From the school book

1 **Complete the following table :**

Number a	8	125	– 27	......	$3\frac{3}{8}$	$-\frac{8}{125}$	......	......
$\sqrt[3]{a}$	......	......	......	– 10	......	......	6	– 4

2 **Find each of the following :**

1. $\sqrt[3]{216}$
2. $\sqrt[3]{-343}$
3. $\sqrt[3]{\frac{64}{125}}$
4. $\sqrt[3]{\frac{-8}{27}}$
5. $\sqrt[3]{0.001}$
6. $\sqrt[3]{-2\frac{10}{27}}$
7. $\sqrt[3]{8x^3}$
8. $\sqrt[3]{-27a^6}$

3 **Complete :**

1. $\sqrt[3]{a^3} =$ ....
2. $\sqrt[3]{\text{..........}} = 4$
3. $\sqrt{16} = \sqrt[3]{\text{..........}}$
4. $|\sqrt[3]{-125}| = \sqrt{\text{..........}}$
5. $\sqrt[3]{8} + \sqrt[3]{-8} =$ ....
6. $\sqrt[3]{27} - \sqrt[3]{64} =$ ....
7. $\sqrt[3]{27} - \sqrt[3]{-27} =$ ....
8. $\sqrt{9} + \sqrt[3]{-8} =$ ....
9. $\sqrt{64} - \sqrt[3]{64} =$ ....
10. $-\sqrt[3]{-1} - \sqrt{1} =$ ....
11. $\frac{-\sqrt[3]{64}}{\sqrt{64}} =$ ....
12. $\sqrt[3]{64 + \text{..........}} = 5$

Exercise 1

4 Choose the correct answer from those given :

1 $\sqrt[3]{(-8)^2} = \dots\dots\dots\dots$

(a) 2 (b) −2 (c) 4 (d) − 4

2 $\sqrt[3]{\left(\frac{1}{8}\right)^2} = \dots\dots\dots\dots$

(a) $\frac{1}{2}$ (b) $\frac{1}{4}$ (c) $\frac{1}{8}$ (d) $\frac{1}{16}$

3 $\sqrt[3]{-64} + \sqrt{16} = \dots\dots\dots\dots$

(a) zero (b) 8 (c) − 8 (d) ± 8

4 $\sqrt{25} - \sqrt[3]{-125} = \dots\dots\dots\dots$

(a) 10 (b) zero (c) 5 (d) ± 5

5 $\sqrt{(-2)^2} + \sqrt[3]{(-2)^3} = \dots\dots\dots\dots$

(a) − 4 (b) 8 (c) 4 (d) zero

6 $\sqrt[3]{3\frac{3}{8}} + \sqrt{0.25} = \dots\dots\dots\dots$

(a) $\frac{3}{2}$ (b) $\frac{1}{2}$ (c) 2 (d) − 2

7 $\sqrt[3]{0.001 \times \frac{1}{8}} = \dots\dots\dots\dots$

(a) $\frac{1}{2}$ (b) 2 (c) $\frac{1}{20}$ (d) 20

8 $\sqrt[3]{1000} \times \sqrt[3]{-0.008} = \dots\dots\dots\dots$

(a) $\frac{1}{2}$ (b) 10 (c) 2 (d) − 2

9 $\sqrt[3]{-27} + \sqrt{12\frac{1}{4}} + \sqrt[3]{0.125} = \dots\dots\dots\dots$

(a) 1 (b) zero (c) −1 (d) $\frac{11}{2}$

10 If $-\sqrt{25} = \sqrt[3]{y}$, then y = …………

(a) 5 (b) −5 (c) 125 (d) −125

Unit 1

11 If $x^3 = 64$, then $\sqrt{x} =$ ....

(a) 4 (b) – 4 (c) 2 (d) – 2

12 $\sqrt[3]{x^6} = \sqrt{\text{..........}}$

(a) x^3 (b) x^2 (c) x (d) x^4

13 If $\frac{x}{3} = \frac{9}{x^2}$, then $x =$ ....

(a) 1 (b) 3 (c) 9 (d) 27

5 Find the value of x in each of the following :

1 $\sqrt[3]{x} = 5$
2 $\sqrt[3]{x} = -\frac{1}{4}$
3 $\sqrt[3]{x} = -\sqrt{4}$
4 $\sqrt[3]{x} - 3 = -1$
5 $x^3 = -8$
6 $x^3 = 64$
7 $x^3 + 5 = 32$
8 $2x^3 = 54$
9 $\frac{1}{5}x^3 = -200$

6 Find the S.S. of each of the following equations in $\mathbb{Q}$:

1 $x^3 + 27 = 0$
2 $8x^3 + 7 = 8$
3 $x^3 + 16 = \frac{3}{8}$
4 $2x^3 - 5 = x^3 + 3$
5 $(x + 3)^3 = 343$
6 $(3x + 1)^3 = -8$
7 $(2x + 1)^3 - 7 = 20$
8 $(5x - 2)^3 + 10 = 18$

7 Find each of the following :

1 $\sqrt[3]{2\frac{1}{4} \div \frac{2}{3}}$
2 $-\sqrt[3]{2^9 \times 3^6}$
3 $\sqrt{\sqrt[3]{729}}$
4 $\sqrt[3]{\sqrt[3]{512}}$
5 $\sqrt{27\sqrt[3]{27}}$

Applications

8 Find the edge length of a cube whose volume is $15\frac{5}{8}$ cm^3. « 2.5 cm. »

9 Find the total area of a cube whose volume is 216 cm^3. « 216 cm^2. »

10 The cube of a number equals 27 Find the square of this number. « 9 »

11 If the half of the cube of a number equals 32 , find this number. « 4 »

12 Find the inner edge length of a cube vessel with capacity of one litre. « 10 cm. »

13 Find the diameter length of a sphere whose volume is $\frac{1372}{81}\pi$ cube unit. « $\frac{14}{3}$ length unit »

14 Find the length of the diameter of a sphere whose volume is 113.04 cm^3. ($\pi = 3.14$) « 6 cm. »

For excellent pupils

15 **Find the S.S. of each of the following equations in $\mathbb{Q}$:**

1. $(x^2 + 6)^3 = 1000$
2. $(x^3 - 14)^2 = 169$
3. $\sqrt[3]{(x-1)^2} = \sqrt[3]{25}$
4. $\sqrt[3]{(x-2)(x^2-4x+4)} = 3$

16 If $\sqrt[3]{\sqrt{x} + 19} = 3$, find the value of $\sqrt[3]{x}$ « 4 »

17 A man was asked about the age of his father and the age of each of his three sons.
His answer was as follows :
My age is half the age of my father. The age of my eldest son is the square root of the age of my father and the age of my middle son is the cube root of the age of my father and the age of my youngest daughter is the quotient of the age of my eldest son by the age of my middle son. Given that the age of my eldest son is twice the age of my middle son.
What is the age of each of his father and his three sons ? « 64 , 8 , 4 , 2 »

Exercise 2

The set of irrational numbers $\bar{\mathbb{Q}}$

From the school book

1 **In each of the following , show which of them is a rational number and which of them is an irrational number :**

1 -5	2 $2\frac{2}{3}$	3 2.06	4 2.3×10^5
5 $-\sqrt{36}$	6 $\sqrt[3]{36}$	7 $\sqrt{7}$	8 zero
9 $\mid -5 \mid$	10 $\sqrt[3]{-\frac{64}{81}}$	11 $\sqrt{\frac{25}{16}}$	12 $\sqrt{\frac{1}{3}}$
13 $\sqrt[3]{3\frac{3}{8}}$	14 $\sqrt[3]{0.343}$	15 $\frac{\pi}{2}$	16 $(-5)^{\text{zero}}$
17 $\frac{\text{zero}}{3}$	18 $\frac{\sqrt{9}}{\sqrt{4}}$	19 $\sqrt{9}+\sqrt{16}$	20 $\sqrt{4}-\sqrt{11}$

2 **Find an approximated value for each of the following numbers :**

1 $\sqrt{11}$ "to the nearest hundredth".

2 $\sqrt[3]{7}$ "to the nearest tenth".

3 $\sqrt[3]{-9}$ "to the nearest tenth".

3 **Find two consecutive integers for each of the following numbers to be included between them :**

1 $\sqrt{5}$ 2 $\sqrt{12}$ 3 $\sqrt[3]{10}$ 4 $\sqrt[3]{-20}$

4 **If x is an integer , find the value of x in each of the following cases :**

1. $x < \sqrt{2} < x + 1$ « 1 »
2. $x < \sqrt{80} < x + 1$ « 8 »
3. $x < \sqrt[3]{5} < x + 1$ « 1 »
4. $x < \sqrt[3]{50} < x + 1$ « 3 »
5. $x < \sqrt[3]{-100} < x + 1$ « – 5 »
6. $x < |-\sqrt{35}| < x + 1$ « 5 »

5 **Find an approximated value for each of the following numbers , then check your answer using the calculator :**

1. $\sqrt{20}$
2. $\sqrt[3]{17}$
3. $\sqrt{5} + 1$
4. $\sqrt[3]{9} - 1$

6 **Choose the correct answer from the given ones :**

1. The irrational number in the following numbers is ....

   (a) $\sqrt{\frac{1}{4}}$ (b) $\sqrt[3]{8}$ (c) $\sqrt{\frac{4}{9}}$ (d) $\sqrt{2}$

2. $\left(\sqrt[3]{-3}\right)^3 =$ ....

   (a) 3 (b) – 3 (c) ± 3 (d) $\sqrt[3]{-9}$

3. The irrational number located between 2 and 3 is ....

   (a) $\sqrt{10}$ (b) $\sqrt{7}$ (c) 2.5 (d) $\sqrt{3}$

4. The irrational number located between –2 and –1 is ....

   (a) – 3 (b) $-1\frac{1}{2}$ (c) $-\sqrt{3}$ (d) $\sqrt{2}$

5. $\sqrt{10} \approx$ ....

   (a) 2.99 (b) 3.71 (c) 3 (d) – 3.2

6. The nearest integer to $\sqrt[3]{25}$ is ....

   (a) 5 (b) 3 (c) 2 (d) 12.5

7. If $n \in \mathbb{Z}_+$, $n < \sqrt{26} < n + 1$, then n = ...

   (a) 25 (b) 5 (c) –5 (d) 24

8. The side length of a square whose area is 6 cm^2 is ....

   (a) a natural number. (b) an integer.

   (c) a rational number. (d) an irrational number.

Unit 1

9 The area of a square whose side length is $\sqrt{3}$ cm. is .... cm^2.

(a) $4\sqrt{3}$ (b) 9 (c) 3 (d) 6

10 The square whose area is 10 cm^2, its side length is .... cm.

(a) 5 (b) -5 (c) $\sqrt{10}$ (d) $-\sqrt{10}$

11 The S.S. of the equation : $\left(x-\sqrt{5}\right)\left(x+\sqrt{3}\right)=0$ in $\mathbb{Q}'$ is ....

(a) $\{\sqrt{5}\}$ (b) $\{-\sqrt{3}\}$ (c) $\{-\sqrt{5},\sqrt{3}\}$ (d) $\{\sqrt{5},-\sqrt{3}\}$

7 Find the value of x in each of the following cases and determine whether $x \in \mathbb{Q}$ or $x \in \mathbb{Q}'$:

1. $5x^2 = 10$ « $\pm\sqrt{2}$ »
2. $4x^2 = 9$ « $\pm\frac{3}{2}$ »
3. $x^3 = 125$ « 5 »
4. $3x^3 = 27$ « $\sqrt[3]{9}$ »
5. $0.1x^2 = 10$ « ± 10 »
6. $0.001x^3 = -8$ « -20 »
7. $(x-1)^2 = 4$ « 3 or -1 »
8. $(x-5)^3 = 1$ « 6 »

8 Find in $\mathbb{Q}'$ the S.S. of each of the following equations :

1. $x^2 = 13$
2. $x^3 = 16$
3. $\frac{2}{5}x^2 = \frac{25}{2}$
4. $\frac{5}{4}x^3 = -2$
5. $125x^3 - 7 = 20$
6. $\frac{1}{4}x^2 + 2 = 66$
7. $(x^3+5)(x^2-3) = \text{zero}$
8. $\left(x+\sqrt{7}\right)\left(x^3-6\right) = \text{zero}$

9 Prove that :

1. $\sqrt{2}$ is included between 1.4 and 1.5
2. $\sqrt{11}$ is included between 3.31 and 3.32
3. $\sqrt[3]{2}$ is included between 1.2 and 1.3
4. $\sqrt[3]{15}$ is included between 2.4 and 2.5
5. $\sqrt[3]{-17}$ is included between -2.6 and -2.5
6. $\sqrt{3}+1$ is included between 2.7 and 2.8

Exercise 2

10 **Determine the point that represents each of the following numbers on the number line :**

1 $\sqrt{3}$ 2 $-\sqrt{11}$ 3 $\sqrt{10}$ 4 $\sqrt{5}+1$ 5 $2-\sqrt{7}$

11 Draw the number line and label point A which represents $\sqrt{2}$

- Label point B which represents $1+\sqrt{2}$
- Label point C which represents $1-\sqrt{2}$

12 Draw the right-angled triangle ABC at B where AB = 1 cm. and BC = 3 cm. , then use the figure to determine the points that represent the following numbers on the number line :

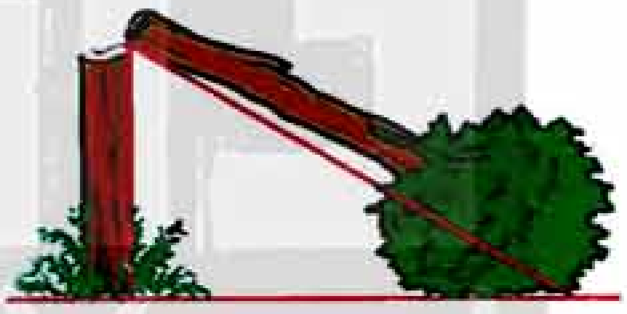
1 $\sqrt{10}$ 2 $-\sqrt{10}$ 3 $2+\sqrt{10}$ 4 $3-\sqrt{10}$

13 Calculate the side length and the diagonal length of a square whose area equals 10 cm^2.

« $\sqrt{10}$ cm. , $\sqrt{20}$ cm. »

Life Application

14 A tree is 3 metres long. Its upper part was broken because of the wind and it made an angle with the surface of the ground. If the length of the left part of the tree is 1 metre , find the distance between the base of the tree and the point of touching of its top with the ground.



« $\sqrt{3}$ metres »

For excellent pupils

15 Without using the calculator , prove that $\sqrt{3}+\sqrt{2}$ is included between 3 and 4

Exercise **3**

The set of real numbers $\mathbb{R}$ and ordering numbers in $\mathbb{R}$

From the school book

1 **Complete the following table by placing (✓) in the suitable place as shown in the first case :**

The number	Natural	Integer	Rational	Irrational	Real
-5	✗	✓	✓	✗	✓
$\sqrt{2}$					
$1\frac{1}{2}$					
$\sqrt[3]{9}$					
$\lvert -2 \rvert$					
$-\sqrt{4}$					
$\frac{5}{2}$					
0.3					
$\sqrt{-1}$					

2 **Complete the following :**

1. $\mathbb{Q} \cap \mathbb{Q}' =$ ....
2. $\mathbb{Q} \cup \mathbb{Q}' =$ ....
3. $\mathbb{R}_+ \cap \mathbb{R}_- =$ ....
4. $\mathbb{R}_+ \cup \mathbb{R}_- =$ ....
5. $\mathbb{R} - \mathbb{Q}' =$ ....
6. $\mathbb{R} - \mathbb{Q} =$ ....

Exercise 3

3 **If $x \in \mathbb{R}$, state whether x is positive or negative or anything else in each of the following cases :**

1 $x > 0$
2 $x < 0$
3 $x > |-4|$
4 $|-5| < x < 7$
5 $-2 < x < 0$
6 $|-1| < x < |-7|$

4 **Put the suitable sign (> , < or =) :**

1 $\sqrt{5}$ 2
2 $\sqrt{7}$ 2.6
3 $\sqrt[3]{24}$ 3
4 $\sqrt[3]{-24}$ .... -2
5 $3-\sqrt{5}$ .... $\sqrt[3]{-1}$
6 $\sqrt[3]{8}$ .... $\sqrt{4}$
7 $1+\sqrt{3}$ .... $\sqrt{5}$
8 $\sqrt[3]{3}-1$ 0.2
9 $\sqrt{2}-1$ .... $1-\sqrt{2}$

5 **Choose the correct answer from those given :**

1 $\mathbb{R}$ = ...
(a) $\mathbb{Q} \cup \grave{\mathbb{Q}}$ (b) $\mathbb{Z}_+ \cup \mathbb{Z}_-$ (c) $\mathbb{R}_+ \cup \mathbb{R}_-$ (d) $\mathbb{N} \cup \mathbb{R}_-$

2 $\{x : x \in \mathbb{R}, x < 0\}$ = ...
(a) $\mathbb{R}_+$ (b) $\mathbb{R}_-$ (c) $\mathbb{R}^*$ (d) $\mathbb{R}$

3 If x is a negative real number , then which of the following numbers is positive ?
(a) x^2 (b) x^3 (c) $2x$ (d) $\frac{x}{2}$

4 If $\frac{1}{a}$ and $\frac{a}{\sqrt{5}}$ are two real numbers included between 0 and 1 , then a = ...
(a) -2 (b) 1 (c) $\sqrt{5}$ (d) 2

5 If $x \in \mathbb{R}_+$, $y \in \mathbb{R}_+$ and $x^2 > y^2$, then ....
(a) $x > y$ (b) $x < y$ (c) $x = y$ (d) $x \le y$

6 $\sqrt{(2-\pi)^2}$ .... $(2-\pi)$ (where π is the ratio between the circumference of the circle and its diameter length)
(a) = (b) < (c) > (d) ≤

7 The S.S. of the equation : $x^2 + 1 = 0$ in $\mathbb{R}$ is ....
(a) $\{-1\}$ (b) $\{1, -1\}$ (c) $\{1\}$ (d) $\varnothing$

6 **Arrange the following numbers ascendingly :**

1 $\sqrt{8}$, $-\sqrt{3}$, $\sqrt{15}$, $\sqrt{5}$, $-\sqrt{7}$ and $-\sqrt{11}$
2 $\sqrt{27}$, $-\sqrt{45}$, $\sqrt{20}$, 0.6 and $\sqrt[3]{-1}$

7 **Arrange the following numbers descendingly :**

1 $\sqrt{62}$, 8 , $-\sqrt{50}$ and $\sqrt{70}$
2 $\sqrt{6}$, 9 , $-\sqrt{10}$, $-\sqrt{7}$, $-\sqrt{50}$ and $\sqrt{101}$

8 Write three positive irrational numbers less than 2

9 Write three negative irrational numbers greater than $-\sqrt{6}$

10 Write four irrational numbers included between 15 and 17

11 Prove that $\sqrt{3}$ is between 1.7 and 1.8 , then represent $\sqrt{3}$, 1.7 and 1.8 on the number line.

12 **Solve the following equations to the nearest hundredth given $x \in \mathbb{R}$:**

1. $x^2 - 6 = 0$
2. $\frac{3}{4} x^2 = 24$
3. $\frac{1}{2} x^2 - 5 = 0$
4. $5 x^3 + 3 = 2$
5. $\frac{3}{4} x^2 + 2 = -11$
6. $\frac{2}{x^3} + 5 = 21 \quad (x \neq 0)$
7. $(x^2 - 9)(x^3 - 5) = 0$
8. $(2 x^3 - 5)(x^2 + 1) = 0$

Geometric Applications

13 Find the side length of a square whose area is 5 cm². Is the side length a rational number ? «$\sqrt{5}$ cm.»

14 Find the edge length of a cube whose volume is 1.728 cm³. Is the edge length a rational number ? «$\frac{6}{5}$ cm.»

15 A cube whose total area is 13.5 cm². Find its edge length. Is the edge length a rational number ? «1.5 cm.»

16 A square is of side length 6 cm. Find its diagonal length. «$\sqrt{72}$ cm.»

17 A square is of area 32 cm². Find its side length and its diagonal length. «$\sqrt{32}$ cm. , 8 cm.»

18 An isosceles right-angled triangle , the length of one side of its right angle = 5 cm. Find the length of its hypotenuse. «$\sqrt{50}$ cm.»

19 A rectangle with dimensions 5 cm. and 7 cm. Find the length of its diagonal. And if its area equals the area of a square , then find the side length of the square and its diagonal length. «$\sqrt{74}$ cm. , $\sqrt{35}$ cm. , $\sqrt{70}$ cm.»

For excellent pupils

20 Without using the calculator , prove that : $\sqrt[3]{3} > \sqrt{2}$

21 Two real numbers , the sum of their squares is 7 and the greater number is 2
Find the other number. «$\sqrt{3}$ or $-\sqrt{3}$»

Exercise 4

Intervals

From the school book

1 Complete the following table :

The interval	Expression by description method	Its representation on the number line
1 [−1 , 2]	$\{x : -1 \leq x \leq 2 , x \in \mathbb{R}\}$	
2 [1 , 3[	....	....
3	$\{x : 0 < x \leq 3 , x \in \mathbb{R}\}$	....
4	....	
5]−∞ , 1]	....	....
6	....	
7	$\{x : x < 4 , x \in \mathbb{R}\}$	....
8 [−2 , ∞[	....	....

2 Choose the correct answer from the given ones :

1. $\mathbb{R} =$ ....

   (a) $\mathbb{R}_+ \cap \mathbb{R}_-$ (b) $\mathbb{R}_+ \cup \mathbb{R}_-$ (c) $]-\infty , \infty[$ (d) $\mathbb{Q} \cap \grave{\mathbb{Q}}$

2. $\mathbb{R}_+ =$ ....

   (a) $]0 , \infty[$ (b) $]-\infty , 0[$ (c) $[0 , \infty[$ (d) $]-\infty , 0]$

Unit 1

3 $\mathbb{R}_- = \dots\dots$

(a) $]0 , \infty[$ (b) $]-\infty , 0[$ (c) $[0 , \infty[$ (d) $]-\infty , 0]$

4 The set of non-negative real numbers = ...

(a) $]0 , \infty[$ (b) $]-\infty , 0[$ (c) $[0 , \infty[$ (d) $]-\infty , 0]$

5 The set of non-positive real numbers = ...

(a) $]0 , \infty[$ (b) $]-\infty , 0[$ (c) $[0 , \infty[$ (d) $]-\infty , 0]$

3 Complete each of the following using one of the symbols $\in$ or $\notin$:

1 $3 \dots\dots [3 , 5]$

2 $-2 \dots\dots]-2 , 1]$

3 $0 \dots\dots [-1 , 4[$

4 $|-3| \dots\dots [2 , \infty[$

5 $\sqrt{9} \dots\dots]-3 , \infty[$

6 $\sqrt[3]{-1} \dots\dots]-\infty , 1[$

7 $1.3 \times 10^{-5} \dots\dots \mathbb{R}_+$

8 $\sqrt{2} \dots\dots [2 , 5]$

9 $5 \dots\dots]\sqrt{5} , \sqrt{23}[$

10 $\sqrt[3]{-125} \dots\dots]-\sqrt{25} , \sqrt{25}]$

4 If $X = [2 , 5[$ and $Y = [-1 , 3[$, find using the number line :

1 $X \cup Y$ 2 $X \cap Y$ 3 $X - Y$

4 $Y - X$ 5 $\grave{X}$ 6 $\grave{Y}$

5 If $X =]-\infty , 3]$ and $Y = [-4 , \infty[$, find using the number line :

1 $X \cup Y$ 2 $X \cap Y$ 3 $X - Y$

4 $Y - X$ 5 $\grave{X}$ 6 $\grave{Y}$

6 If $X = [-1 , 4]$, $Y = [3 , \infty[$ and $Z = \{3 , 4\}$, find the following using the number line :

1 $X \cup Y$ 2 $X \cap Y$ 3 $X - Y$ 4 $X - Z$

5 $Y \cap Z$ 6 $Y - X$ 7 $\grave{X}$ 8 $\grave{Y}$

7 Find using the number line :

1 $[-1 , 4] \cap [2 , 5]$ 2 $[-1 , 4] \cup [2 , 5]$ 3 $]-2 , 3] \cap]0 , 1[$

4 $]-2 , 3] \cup]0 , 1[$ 5 $[2 , 6] - [-1 , 3[$ 6 $[-1 , 3[- [2 , 6]$

7 $[-3 , 0[\cup]0 , 2]$ 8 $[-3 , 0] \cap]0 , 2]$ 9 $[1 , 2] - [-2 , 4]$

10 $[-2 , 4] - [1 , 2]$ 11 $[-1 , 4] \cap [5 , 7[$ 12 $[-1 , 5] -]-1 , 5[$

8 Find using the number line :

1. $[-1, \infty[\cup [-3, 4]$
2. $[2, \infty[\cap]-2, 3[$
3. $]-\infty, 3] \cap [-4, \infty[$
4. $[2, \infty[\cup]-\infty, 3]$
5. $]-\infty, 3] - [-1, \infty[$
6. $]-\infty, -3] - [-3, 1]$
7. $]-\infty, 2] -]-\infty, 0]$
8. $]-\infty, 3[\cup]4, \infty[$

9 Complete the following :

1. $[3, 5] \cup \{3, 5\} =$ ....
2. $]3, 5[\cup \{3, 5\} =$ ....
3. $[3, 5] \cap \{3, 5\} =$ ....
4. $]3, 5[\cap \{3, 5\} =$ ....
5. $[3, 5] - \{3, 5\} =$ ....
6. $]3, 5[- \{3, 5\} =$ ....
7. $\{3, 5\} - [3, 5] =$ ....
8. $\{3, 5\} -]3, 5[=$ ....
9. $]3, 5[\cup \{3\} =$ ....
10. $[3, 5] - \{5\} =$ ....
11. $]2, 5[\cap \{-2, 3, 4\} =$ ....
12. $]-3, 5] \cup \{-2, 3, 4\} =$ ....

10 Complete the following :

1. $]1, 7[\cup]3, 5[=$ ....
2. $]-3, 2] - [0, 2] =$ ....
3. $[3, 4[\cup]3, 4] =$ ....
4. $]2, 5] \cap [2, 5[=$ ....
5. $[3, 5] - [3, 5[=$ ....
6. $[3, 7] - [4, 7] =$ ....
7. $[2, 7] -]2, 7[=$ ....
8. $[-2, 4] \cap [4, 6] =$ ....
9. If $X \cap [2, 7] = [3, 4[$, then $X =$ ....
10. If x is a positive real number , then $x > x^2$ when $x \in]$ .... , ... $[$

11 Choose the correct answer from the given ones :

1. $[-3, 4] - \{-3, 5\} =$ ....
   (a) $]-3, 4[$ (b) $]-3, 4]$ (c) $]-3, 5[$ (d) $[-3, 5[$
2. If $x \in [-3, \infty[$, then ....
   (a) $x < -3$ (b) $x \le -3$ (c) $x > -3$ (d) $x \ge -3$

Unit 1

3 If $X = \{x : x \in \mathbb{R} , 2 < x \le 5\}$, then $[3 , 4]$ X

(a) $\in$ (b) $\notin$ (c) $\subset$ (d) $\not\subset$

4 $\{3\} \cap [3 , 6] =$ ....

(a) $\varnothing$ (b) $\{3\}$ (c) $]3 , 6]$ (d) $\{6\}$

5 $\{8 , 9 , 10\} -]8 , 10[=$ ....

(a) $\varnothing$ (b) $\{8 , 10\}$ (c) $\{9\}$ (d) $\mathbb{N}$

6 The sum of all real numbers in $[-75 , 75]$ is ....

(a) -75 (b) 75 (c) 150 (d) zero

12 Complete the following :

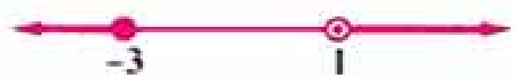
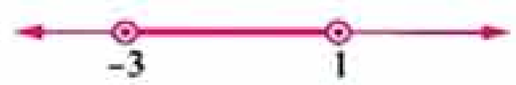
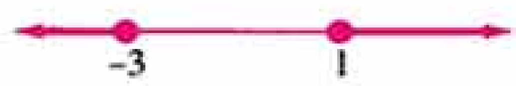
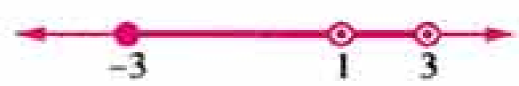
1 $\mathbb{R} \cap [-3 , 3] =$ ....
2 $\mathbb{R} \cup]-1 , 4] =$ ....
3 $\mathbb{R} - [-1 , \infty[=$ ....

4 $\mathbb{R}_- - [-3 , 1] =$ ....
5 $]-2 , 5] - \mathbb{R}_+ =$ ....
6 $[-2 , 2] - \mathbb{R}_- =$ ....

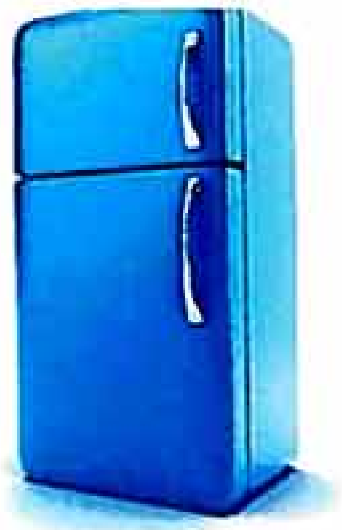
7 $]-3 , 2] \cap \mathbb{Z}_+ =$ ....
8 $\mathbb{N} \cap [-5 , 2[=$ ....
9 $\mathbb{Z} \cap [-1 , 3[=$ ....

10 $\mathbb{R}_+ \cap [0 , 5] =$ ....
11 $\mathbb{R}_- \cap [-3 , 2] =$ ....

13 Choose from column (B) the suitable interval which represents the figure in column (A) :

(A)	(B)
1 	$\mathbb{R} -]-3 , 1]$
2 	$\mathbb{R} - [-3 , 1[$
3 	$\mathbb{R} -]-3 , 1[$
4 	$[-3 , 3[- \{1\}$
5 	$[-3 , 1[$
	$]-3 , 1[$

Exercise 4



Life Application

14 Two kinds of food , the first kind needs to be kept in a temperature between – 3 and 4 degrees , and the other kind needs to be kept in a temperature between 2 and 10 degrees.

What is the temperature needed to keep the two kinds altogether at the same place ?

For excellent pupils

15 **Choose the correct answer from the given ones :**

1 **In the opposite figure :**

If x is a real number , then $x \in$ ....

(a) $\mathbb{R}_-$ (b) $\mathbb{R}_+$ (c) $]-\infty , -1]$ (d) $]-\infty , -1[$

2 If $x \in [-3 , 4]$, then $x^2 \in$ ....

(a) $[9 , 16]$ (b) $[0 , 9]$ (c) $[0 , 16]$ (d) $[-9 , 0]$

3 If $x \in [-5 , 4]$, then $x^2 \in$ ....

(a) $[0 , 16]$ (b) $[16 , 25]$ (c) $[0 , 25]$ (d) $[-5 , 0]$

4 If $x \in [1 , 16]$, then $-\sqrt{x} \in$ ....

(a) $[1 , 4]$ (b) $[-1 , 4]$ (c) $[-4 , -1]$ (d) $[-4 , 0]$

5 If $X \subset \mathbb{R}$, $[2 , 5] - X =]2 , 5[$, then $X =$ ....

(a) $[2 , 5]$ (b) $\{2 , 5\}$ (c) $[2 , 5[$ (d) $]2 , 5]$

6 If $X \subset \mathbb{R}$, $]4 , 7] \cup X = [1 , 7]$, then $X =$ ....

(a) $[1 , 3[$ (b) $[1 , 3]$ (c) $[1 , 4[$ (d) $[1 , 5]$

7 If $M \subset \mathbb{R}$, $M \cap [3 , 8[= [3 , 8[$, then $M =$ ....

(a) $]3 , 8[$ (b) $]3 , 8]$ (c) $[3 , 9]$ (d) $[3 , 7]$

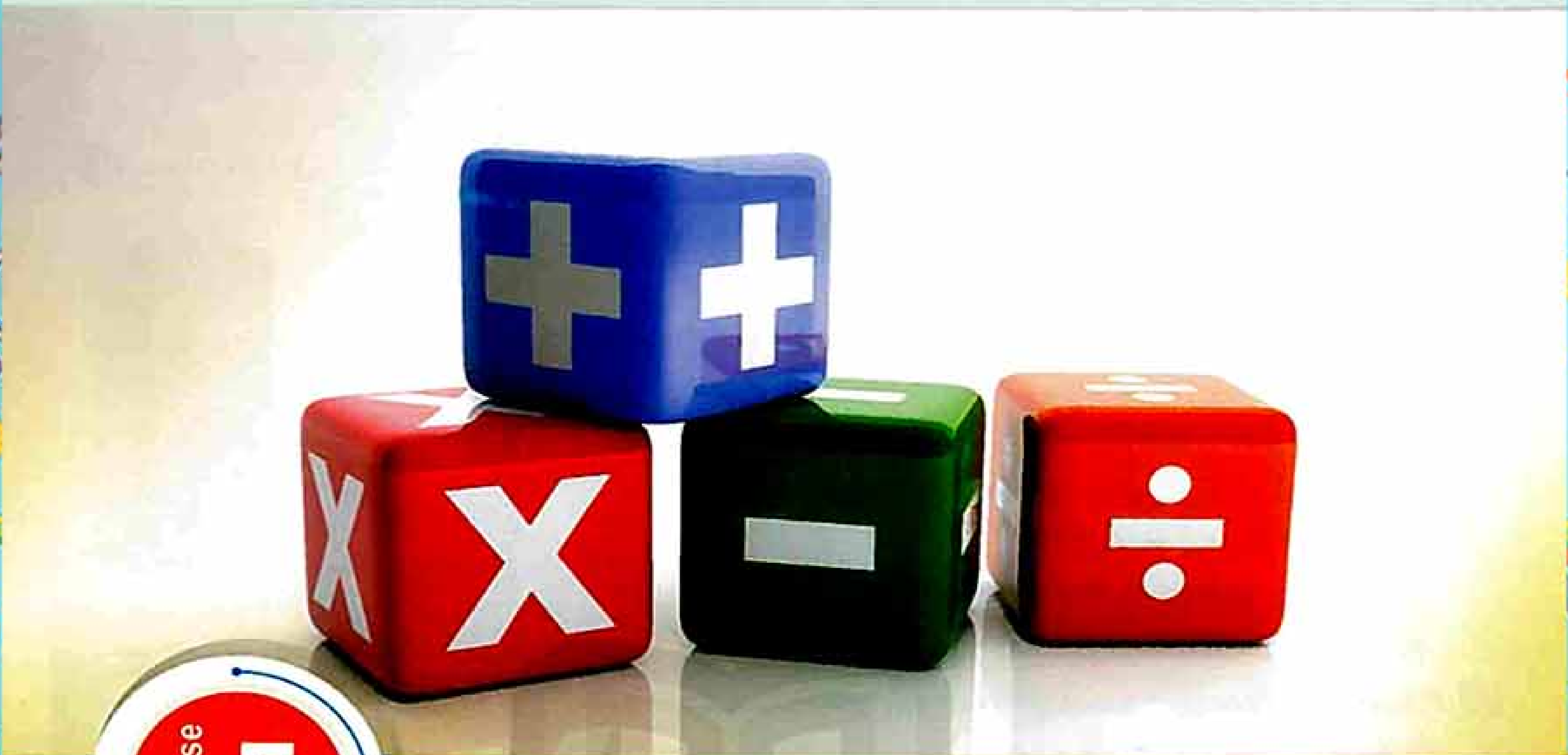
8 If $]-\infty , k[\cap [-2 , 5] = [-2 , 3[$, then $k =$ ....

(a) -2 (b) 5 (c) 3 (d) zero

9 If $[-1 , x] \cap [y , 5] = [2 , 3]$, then $x^y =$ ....

(a) 8 (b) $\frac{1}{5}$ (c) 9 (d) -1

16 If $X \cap Y = [4 , 7]$, $X \cup Y = [3 , 7]$ and $X \subset Y$, **find :** X , Y and $Y - X$



Exercise 5

Operations on the real numbers

From the school book

1 **Find each of the following in the simplest form :**

1. $\sqrt{3} + 2\sqrt{3}$
2. $3\sqrt{2} - 5\sqrt{2}$
3. $2\sqrt{5} - 3\sqrt{5} + \sqrt{5}$
4. $5\sqrt[3]{7} - 8\sqrt[3]{7} + 2\sqrt[3]{7}$
5. $4\sqrt{5} - 2\sqrt{5} + 5\sqrt{5} - \sqrt{5}$
6. $5\sqrt{3} - 7\sqrt{3} + 3\sqrt{3} - \sqrt{3}$

2 **Find each of the following in the simplest form :**

1. $\sqrt{5} - \sqrt{3} + 2\sqrt{5} + \sqrt{3}$
2. $2\sqrt{3} + 5 + \sqrt{3} - 6$
3. $2\sqrt{7} - 3\sqrt{2} + \sqrt{7} + 5\sqrt{7}$
4. $2\sqrt{2} - 3\sqrt[3]{2} + 5\sqrt{2} + \sqrt[3]{2}$
5. $\frac{1}{4}\sqrt{2} + \frac{2}{7}\sqrt{5} + \frac{3}{4}\sqrt{2} - \frac{2}{7}\sqrt{5}$
6. $8\sqrt{\frac{1}{4}} + 2\sqrt[3]{3} - \sqrt[3]{64} - 5\sqrt[3]{3}$

3 **Find the result of each of the following :**

1. $\sqrt{3} \times \sqrt{3}$
2. $-2\sqrt{5} \times 3\sqrt{5}$
3. $2 \times 3\sqrt{2}$
4. $\frac{1}{3}\sqrt{3} \times \sqrt{3}$
5. $(\sqrt[3]{5})^3 \times 3\sqrt{3}$
6. $2\sqrt{3} \times \frac{2\sqrt{7}}{7} \div \frac{20\sqrt{3}}{5\sqrt{7}}$

4 **Find the result of each of the following in the simplest form :**

1. $2(\sqrt{2} + \sqrt{5})$
2. $\sqrt{2}(5 + \sqrt{2})$
3. $\sqrt{7}(\sqrt{7} + 2)$
4. $-\sqrt{3}(-5 - \sqrt{3})$
5. $-2\sqrt{5}(3 - \sqrt{5})$
6. $\sqrt{7}\left(\frac{2}{\sqrt{7}} - \sqrt{7} + 3\right)$
7. $-3(8 + 2\sqrt{3}) + 6\sqrt{3}$
8. $\sqrt{5}(3 - \sqrt{5}) - 2(1 + \sqrt{5})$

Exercise 5

5 **Find the result of each of the following operations :**

1. $(\sqrt{2}+1)(\sqrt{2}-1)$
2. $(4-3\sqrt{2})(4+3\sqrt{2})$
3. $(\sqrt{5}-1)^2$
4. $(2\sqrt{3}+4)^2$
5. $(\sqrt{3}+2)(\sqrt{3}-1)$
6. $(5-\sqrt{3})^2-28$

6 **Make the denominator in each of the following an integer :**

1. $\frac{3}{\sqrt{3}}$
2. $\frac{10}{\sqrt{5}}$
3. $-\frac{6}{\sqrt{3}}$
4. $\frac{8}{\sqrt{6}}$
5. $\frac{2}{3\sqrt{2}}$
6. $\frac{6}{2\sqrt{3}}$
7. $\frac{25}{2\sqrt{10}}$
8. $\frac{\sqrt{2}+3}{\sqrt{2}}$
9. $\frac{\sqrt{5}-15}{2\sqrt{5}}$

7 **Choose the correct answer from those given :**

1. $\sqrt{7}+\sqrt{7}=$ ....
(a) 7 (b) 14 (c) $2\sqrt{7}$ (d) $\sqrt{14}$

2. $\sqrt{3}+(-\sqrt{3})=$ ....
(a) $2\sqrt{3}$ (b) $2\sqrt{6}$ (c) $\sqrt{6}$ (d) zero

3. $2\sqrt{3}+3\sqrt{3}=$ ....
(a) $5\sqrt{6}$ (b) $5\sqrt{3}$ (c) $6\sqrt{3}$ (d) $5\sqrt[3]{3}$

4. $5+7\sqrt{2}-4+\sqrt{2}=$ ....
(a) 15 (b) $1+7\sqrt{2}$ (c) $1+8\sqrt{2}$ (d) $1+6\sqrt{2}$

5. $-2\sqrt{3}\times\sqrt{3}=$ ....
(a) -6 (b) $-2\sqrt{3}$ (c) $2\sqrt{3}$ (d) 6

6. $(2\sqrt[3]{5})^3=$ ....
(a) 10 (b) 20 (c) $4\sqrt[3]{5}$ (d) 40

7. The additive inverse of the number $\frac{6}{\sqrt{2}}$ is ....
(a) $-2\sqrt{3}$ (b) $2\sqrt{3}$ (c) $-3\sqrt{2}$ (d) $3\sqrt{2}$

8. The additive inverse of the number $(\sqrt{2}-\sqrt{5})$ is ....
(a) $\sqrt{2}+\sqrt{5}$ (b) $\sqrt{5}-\sqrt{2}$ (c) $\sqrt{2}-\sqrt{5}$ (d) $-\sqrt{2}-\sqrt{5}$

9 The multiplicative inverse of the number $\sqrt{5}$ is ....

(a) -5 (b) $\frac{-1}{5}$ (c) $\frac{5}{\sqrt{5}}$ (d) $\frac{\sqrt{5}}{5}$

10 The multiplicative inverse of the number $\frac{\sqrt{2}}{6}$ is ....

(a) $\sqrt{3}$ (b) $3\sqrt{2}$ (c) $\sqrt{6}$ (d) $\frac{\sqrt{2}}{2}$

11 $\left(\sqrt{5}+3\sqrt{5}\right) \div \sqrt{5} =$ ....

(a) $3\sqrt{5}$ (b) 3 (c) 5 (d) 4

12 If $x=\sqrt{2}+10$, $y=\sqrt{2}-10$, then $(x+y)^2 =$ ....

(a) 4 (b) 6 (c) 8 (d) $4\sqrt{2}$

8 Complete the following :

1 The multiplicative neutral in $\mathbb{R}$ is .... and the additive neutral in $\mathbb{R}$ is ....

2 The additive inverse of the number $1-\sqrt{2}$ is ....

3 The multiplicative inverse of the number $\frac{2\sqrt{3}}{5}$ is $\frac{\cdots\cdots}{6}$

4 The multiplicative inverse of the number $\frac{3}{\sqrt{3}}$ is $\frac{\cdots\cdots}{\sqrt{3}}$

5 $7+\sqrt{3}=5+(\cdots\cdots+\cdots\cdots)$

6 If $a=\frac{\sqrt{2}}{\sqrt{3}}$, $b=\frac{\sqrt{3}}{\sqrt{2}}$, then $\frac{a}{b}=$ ....

7 $\left(\sqrt{3}-2\right)^2 = 7-$ ....

8 If $\sqrt{x}=\sqrt{2}+1$, then $x=$ ....

9 If $x^2=\left(2\sqrt{3}-\sqrt{7}\right)\left(2\sqrt{3}+\sqrt{7}\right)$, then $x=$ ....

10 If $x^2-y^2=16$, $x-y=\sqrt{2}$, then $x+y=$ ....

11 If the side length of a square is ℓ cm. and its area is 15 cm^2, then the area of the square of side length 2ℓ cm. is ....

12 If $a \in \mathbb{R}$ and $b \in \mathbb{R}$, then $a-b$ means the sum of the number a and .... of the number b

13 If $a \in \mathbb{N}$, $b \in \mathbb{Q}$ and $c \in \mathbb{R}$, then $a+b+c \in$ ....

9 If $x=\sqrt{5}-2$ and $y=\sqrt{5}+2$, find the value of each of the following :

1 $x+y$

2 $x-y$

3 xy

4 x^2-y^2

5 $x^2+2xy+y^2$

6 $x^2-2xy+y^2$

10 If $a - b = 2\sqrt{3}$, find the value of : $a(a-b)^3 + b(b-a)^3$ « 144 »

11 If $x = \sqrt{3+\sqrt{2}}$, find the value of : $x^4 - 2x^2 + 1$ « $6 + 4\sqrt{2}$ »

12 If $x = 3 + \sqrt{5}$ and $y = 1 + \sqrt{8}$, estimate the value of each of the following :

1. x , y
2. $x + y$
3. $x - y$
4. xy

Check the reasonability of each value using your calculator.

13 If $x = \sqrt{15} + 2$ and $y = 4 - \sqrt[3]{25}$, estimate the value of each of the following :

1. x , y
2. $x \times y$
3. $x + y$

Check the reasonability of each value using your calculator.

Geometric Application

14 A rectangle is of dimensions $(6+\sqrt{5})$ cm. and $(6-\sqrt{5})$ cm.

Calculate its perimeter and its area. « 24 cm. , 31 cm.^{2} »

Life Application

15 If the area of each square of Ahmed's chess board is 13 cm.^{2} , find :

1. The side length of the chess board.
2. The diagonal length of the chess board.

« $8\sqrt{13}$ cm. , $\sqrt{1664}$ cm. »

For excellent pupils

16 If the multiplicative inverse of the number $\sqrt{a} - 1$ is $\frac{\sqrt{a}+1}{4}$, find the numerical value of a « 5 »

17 If $x = 2y = 4z = \sqrt{2}$, find the value of : $x^2 + 2y^2 + 4z^2$ « $3\frac{1}{2}$ »

18 If the number y is the additive inverse of x and $\frac{1}{2}(y - x) = 1 - \sqrt{2}$

, prove that : $xy - 2\sqrt{2} = -3$

Summary of the first part of Unit 1

"From lesson 1 to lesson 5"

- The cube root of the number "a" is the number whose cube equals a
  For example : $\sqrt[3]{64} = 4$, $\sqrt[3]{-64} = -4$
- The cube root of the positive number is positive, and the cube root of the negative number is negative.
- $\sqrt[3]{a^3} = a$ For example : $\sqrt[3]{(-5)^3} = -5$
  , $\sqrt[3]{a^n} = a^{\frac{n}{3}}$ where $n \in \mathbb{Z}$
  For example : $\sqrt[3]{a^6} = a^{\frac{6}{3}} = a^2$
- If "a" is a perfect cube number, then the equation : $x^3 = a$ has a unique solution in $\mathbb{R}$
  , which is $\sqrt[3]{a}$
- Each irrational number lies between two rational numbers and can be represented by a point on the number line.
- The set of rational numbers $\mathbb{Q}$ and the set of irrational numbers $\grave{\mathbb{Q}}$ are disjoint sets.
  i.e. $\mathbb{Q} \cap \grave{\mathbb{Q}} = \varnothing$
- $\mathbb{R} = \mathbb{Q} \cup \grave{\mathbb{Q}}$, $\mathbb{R} - \mathbb{Q} = \grave{\mathbb{Q}}$, $\mathbb{R} - \grave{\mathbb{Q}} = \mathbb{Q}$
- $\mathbb{R} = \mathbb{R}_+ \cup \{0\} \cup \mathbb{R}_- =]-\infty , \infty[$, $\mathbb{R}_+ \cap \mathbb{R}_- = \varnothing$
  , $\mathbb{R}_+ =]0 , \infty[$, $\mathbb{R}_- =]-\infty , 0[$
  , the set of non-negative real numbers $= \mathbb{R}_+ \cup \{0\} = [0 , \infty[$
  , the set of non-positive real numbers $= \mathbb{R}_- \cup \{0\} =]-\infty , 0]$
  , $\mathbb{R}^* = \mathbb{R} - \{0\} = \mathbb{R}_+ \cup \mathbb{R}_-$
- It is possible to carry out the operations of intersection , union, difference and complement on the intervals.
- The set of real numbers is closed under addition, subtraction and multiplication operations and is not closed under division operation.
- Each of addition and multiplication operation in $\mathbb{R}$ is commutative and associative, but each of subtraction and division operation in $\mathbb{R}$ is not commutative and associative.
- Zero is the additive neutral in $\mathbb{R}$, and one is the multiplicative neutral in $\mathbb{R}$
- For every real number "a" , there is an additive inverse which is the real number "– a", and for every real number "a" where $a \neq 0$, there is a multiplicative inverse which is the real number $\frac{1}{a}$
- The multiplication in the set of real numbers is distributed on the addition and the subtraction from right and from left.

Exams on the first part of unit one
from lesson (1) to lesson (5)

Model 1

Answer the following questions :

1 Choose the correct answer from those given :

1. $[-2 , 5] - \{-2 , 5\} = \cdots\cdots$
   (a) $\{-2 , 5\}$ (b) $[-2 , 5[$ (c) $]-2 , 5[$ (d) $]-2 , 5]$
2. $\mathbb{Q} \cap \grave{\mathbb{Q}} = \cdots\cdots$
   (a) $\mathbb{Z}$ (b) $\mathbb{R}$ (c) $\grave{\mathbb{Q}}$ (d) $\varnothing$
3. $\sqrt{x^4} = \sqrt[3]{\cdots\cdots}$
   (a) x^6 (b) x^4 (c) x^2 (d) x
4. The irrational number included between 3 and 4 is $\cdots\cdots$
   (a) $\sqrt{7}$ (b) $\sqrt{10}$ (c) $\sqrt[3]{12}$ (d) $3\frac{1}{4}$
5. The multiplicative inverse of the number $\sqrt{3}$ is $\cdots\cdots$
   (a) $\frac{3}{\sqrt{3}}$ (b) -3 (c) 3 (d) $\frac{\sqrt{3}}{3}$
6. If $\frac{x}{4} = \frac{16}{x^2}$, then $x = \cdots\cdots$
   (a) 2 (b) 4 (c) 8 (d) 16

2 Complete the following :

1. $\sqrt[3]{4 + \cdots\cdots} = 3$
2. The square whose side length is $\sqrt{5}$ cm., its area is $\cdots\cdots$ cm^2.
3. $\mathbb{R} - \mathbb{Q} = \cdots\cdots$
4. $[-2 , 7] \cap]-2 , 7[= \cdots\cdots$
5. The additive inverse of the number $5 - \sqrt{3}$ is $\cdots\cdots$

3 [a] If $X = [-2 , 3]$ and $Y = [1 , 5[$, find using the number line each of :

1. $X \cap Y$ 2. $X \cup Y$ 3. $Y - X$

[b] Find the solution set in $\mathbb{R}$ of the equation : $(x^2 - 4)(x^3 - 7) = 0$

Unit 1

4 **[a]** Prove that $\sqrt{12}$ is included between 3.4 and 3.5

[b] A square of side length 5 cm. , find its diagonal length.

5 **[a]** Determine the point which represents the number $\sqrt{5}$ on the number line.

[b] Find the result of each of the following operations :

1 $(\sqrt{3}+1)(\sqrt{3}-1)$ 2 $(\sqrt{7}+2)(\sqrt{7}-1)$

Model 2

Answer the following questions :

1 **Choose the correct answer from those given :**

1 $\mathbb{R} =$ ....

(a) $\mathbb{R}_+ \cup \mathbb{R}_-$ (b) $\mathbb{R}_+ \cap \mathbb{R}_-$ (c) $\mathbb{Q} \cup \acute{\mathbb{Q}}$ (d) $\mathbb{R}^*$

2 The irrational number from the following numbers is ....

(a) $\sqrt{\frac{4}{25}}$ (b) $\sqrt[3]{1}$ (c) $\sqrt{\frac{27}{8}}$ (d) $\sqrt[3]{\frac{1}{64}}$

3 If $-\sqrt{4} = \sqrt[3]{x}$, then $x =$ ....

(a) 8 (b) -8 (c) 4 (d) 16

4 $\sqrt{4} - \sqrt[3]{-8} =$ ....

(a) -2 (b) 4 (c) -4 (d) 8

5 $\{2 , 5 , 7\} -]2 , 7] =$ ....

(a) $\{2\}$ (b) $\{2 , 5\}$ (c) $]2 , 5[$ (d) $[2 , 5]$

6 $\sqrt{4}$ .... $]-2 , \infty[$

(a) $\in$ (b) $\notin$ (c) $\subset$ (d) $\not\subset$

2 **Complete the following :**

1 The additive inverse of the number $\sqrt{7} - \sqrt{2}$ is ....

2 $\mathbb{R}_+ \cup \mathbb{R}_- =$ ....

3 $[-4 , 6[- \mathbb{R}_+ =$ ....

4 The sum of the real numbers in the interval $[-3 , 3[$ is ....

5 The solution set of the equation $x^2 + 25 = 0$ in $\mathbb{R}$ is ....

3 **[a] Find the result of the following in the simplest form :**

$2\sqrt{7}-5\sqrt{2}+\sqrt{7}+5\sqrt{2}$

[b] If $X =]-\infty , 1[$ and $Y = [-2 , 4[$ **, using the number line find in the form of an interval each of the following :**

1. $X \cup Y$
2. $X \cap Y$
3. $\grave{X}$

4 **[a] Find in $\grave{\mathbb{Q}}$ the solution set of each of the following equations :**

1. $\frac{1}{2}x^2 - 3 = 7$
2. $125x^3 - 3 = 5$

[b] Prove that $\sqrt[3]{17}$ is included between 2.57 and 2.58

5 **[a] Simplify to the simplest form :**

1. $(2\sqrt{3}-5)^2$
2. $\sqrt{5}(\sqrt{5}+2)$

[b] Write four irrational numbers included between 11 and 12

Exercise 6 Operations on the square roots

📖 From the school book

1 **Put each of the following in the form $a\sqrt{b}$ where a and b are two integers , b is the least possible value :**

1. $\sqrt{12}$
2. 📖 $\sqrt{28}$
3. 📖 $2\sqrt{72}$
4. $\frac{2}{5}\sqrt{1000}$
5. $2\sqrt{\frac{1}{2}}$
6. $6\sqrt{\frac{2}{3}}$

2 **Simplify each of the following to the simplest form :**

1. 📖 $\sqrt{50}+\sqrt{8}$ « $7\sqrt{2}$ »
2. 📖 $\sqrt{20}-\sqrt{45}$ « $-\sqrt{5}$ »
3. $3\sqrt{2}+\sqrt{8}-\sqrt{18}$ « $2\sqrt{2}$ »
4. $\sqrt{98}-\sqrt{128}-\sqrt{18}+4\sqrt{2}$ « zero »
5. 📖 $2\sqrt{18}+\sqrt{50}+\frac{1}{3}\sqrt{162}$ « $14\sqrt{2}$ »
6. $\sqrt{98}+\sqrt{50}-\frac{1}{2}\sqrt{200}-\sqrt{2}$ « $6\sqrt{2}$ »
7. 📖 $\sqrt{27}+5\sqrt{18}-\sqrt{300}$ « $15\sqrt{2}-7\sqrt{3}$ »

3 **Put each of the following in the simplest form :**

1. $2\sqrt{5}+4\sqrt{20}-5\sqrt{\frac{1}{5}}$ « $9\sqrt{5}$ »
2. $\sqrt{32}-\sqrt{72}+6\sqrt{\frac{1}{2}}$ « $\sqrt{2}$ »
3. 📖 $2\sqrt{5}+6\sqrt{\frac{1}{3}}-\sqrt{12}-5\sqrt{\frac{1}{5}}$ « $\sqrt{5}$ »
4. $\sqrt{3}+\frac{3}{\sqrt{3}}-\sqrt{2}\times\sqrt{6}$ « zero »
5. $\sqrt{18}-\frac{\sqrt{12}}{\sqrt{6}}$ « $2\sqrt{2}$ »
6. $\sqrt{(-5)^2}+\sqrt{18}-\frac{6}{\sqrt{2}}$ « 5 »

4 **Simplify each of the following to the simplest form :**

1. $2\sqrt{3} \times 5\sqrt{2}$ « $10\sqrt{6}$ »
2. $2\sqrt{18} \times 3\sqrt{2}$ « 36 »
3. $\sqrt{5} \times 2\sqrt{10}$ « $10\sqrt{2}$ »
4. $\sqrt{\frac{2}{7}} \times \sqrt{\frac{7}{2}}$ « 1 »
5. $\frac{3\sqrt{15}}{\sqrt{5}}$ « $3\sqrt{3}$ »
6. $12\sqrt{\frac{2}{3}} \times \sqrt{54}$ « 72 »

5 **Simplify each of the following to the simplest form :**

1. $\sqrt{6}\left(\sqrt{3}-\sqrt{2}\right)$
2. $5\sqrt{2}\left(2\sqrt{2}+\sqrt{12}\right)$
3. $\left(3\sqrt{5}-\sqrt{7}\right)\left(3\sqrt{5}+\sqrt{7}\right)$
4. $\left(\sqrt{3}-\sqrt{2}\right)^2$
5. $\left(\sqrt{3}+\sqrt{5}\right)^2-\sqrt{60}$
6. $\sqrt{18}-\frac{12}{\sqrt{6}}+\sqrt{2}\left(2\sqrt{3}-3\right)$

6 **Write each of the following such that the denominator is an integer :**

1. $\frac{\sqrt{3}}{\sqrt{2}}$
2. $\sqrt{\frac{5}{3}}$
3. $\frac{5\sqrt{3}}{\sqrt{5}}$
4. $\frac{4\sqrt{3}-\sqrt{2}}{2\sqrt{3}}$

7 **Choose the correct answer from those given :**

1. $\frac{\sqrt{63}}{\sqrt{7}} =$ ....

   (a) 3 (b) $\sqrt{3}$ (c) 9 (d) ± 3

2. $\sqrt{8}-\sqrt{2} =$ ....

   (a) $\sqrt{6}$ (b) $\sqrt{2}$ (c) 2 (d) 1

3. $\left(\sqrt{8}+\sqrt{2}\right)^2 =$ ....

   (a) $\sqrt{10}$ (b) 10 (c) 18 (d) $\sqrt{18}$

4. $\left(\sqrt{7}-\sqrt{5}\right)\left(\sqrt{7}+\sqrt{5}\right) =$ ....

   (a) 2 (b) 12 (c) $2\sqrt{7}$ (d) $-2\sqrt{5}$

5. $\sqrt{\frac{1}{2}}+\sqrt{\frac{1}{2}} =$ ....

   (a) 1 (b) $\sqrt{\frac{1}{4}}$ (c) $\sqrt{2}$ (d) $\frac{\sqrt{2}}{2}$

6 $\frac{\sqrt{27}}{\sqrt{3}} \div \frac{\sqrt{72}}{\sqrt{2}} = ...$

(a) $\frac{1}{2}$ (b) 2 (c) -2 (d) 4

7 The multiplicative inverse of the number $\sqrt{50}$ is ....

(a) $\frac{\sqrt{2}}{10}$ (b) $\frac{-\sqrt{2}}{10}$ (c) $-5\sqrt{2}$ (d) $5\sqrt{2}$

8 If $x = \frac{\sqrt{6}}{\sqrt{2}}$, then $x^{-1} = ...$

(a) $\sqrt{3}$ (b) $\frac{\sqrt{3}}{2}$ (c) $\frac{\sqrt{3}}{3}$ (d) $2\sqrt{3}$

9 If $x = \sqrt{7} + \sqrt{3}$ and $y = \sqrt{28} + \sqrt{12}$, then $x = ...$

(a) y (b) $\frac{1}{2}$ y (c) 2 y (d) y^2

8 **Complete the following :**

1 $\frac{3\sqrt{2}}{2\sqrt{18}} = ...$ 2 $\sqrt{3} \times \sqrt{6} = 3 \times ....$ 3 $\frac{1}{2}\sqrt{48} = 2 \times ....$

4 If $2\sqrt{27} - 2\sqrt{48} = x\sqrt{3}$, then $x = ...$

5 $\sqrt{5}$, $\sqrt{20}$, $\sqrt{45}$, $\sqrt{80}$, ... (in the same pattern).

6 If $x^2 = \frac{8}{9}$, then x in the simplest form = ...

7 If $x^2 = 5$, then $(x + \sqrt{5})^2 = ...$ or ....

9 **Find the value of each of $x + y$, $x \times y$ in each of the following cases :**

1 $x = 3 + \sqrt{5}$, $y = 1 - \sqrt{5}$ « $4 , -2 - 2\sqrt{5}$ »

2 $x = \sqrt{3} - \sqrt{2}$, $y = \sqrt{3} + \sqrt{2}$ « $2\sqrt{3} , 1$ »

3 $x = 5 - 3\sqrt{2}$, $y = 5 - 3\sqrt{2}$ « $10 - 6\sqrt{2} , 43 - 30\sqrt{2}$ »

10 If $x = \frac{\sqrt{2}}{\sqrt{3}}$ and $y = \frac{\sqrt{3}}{\sqrt{2}}$, find the value of : $6(x + y)$ « $5\sqrt{6}$ »

11 If $x = \frac{10}{\sqrt{5}}$, $y = \sqrt{45} + \sqrt{2}$ and $z = \sqrt{8} + \sqrt{5}$

, find in the simplest form the value of the expression $(x - y + z)^2$ « 2 »

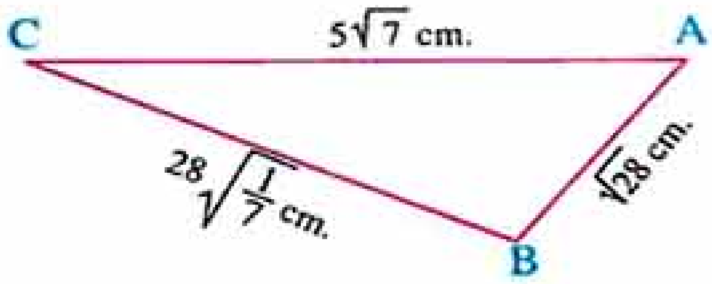
12 If $x = 2\sqrt{5} + \sqrt{2}$, $y = 2\sqrt{5} - \sqrt{2}$

, find the value of the expression : $x^2 + 2xy + y^2$ « 80 »

13 If $x = \sqrt{7} + \frac{1}{2}\sqrt{12}$ and $y = \frac{1}{3}\sqrt{63} - \sqrt{3}$, prove that : $x^2 y^2 = 16$

Geometric Applications

14 **In the opposite figure :**

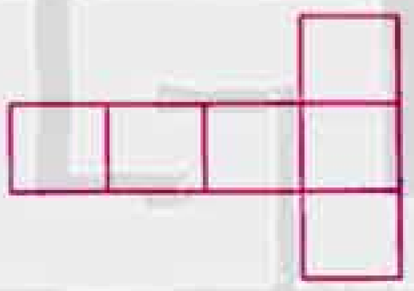


Find the perimeter of Δ ABC in the simplest form.

« $11\sqrt{7}$ cm. »

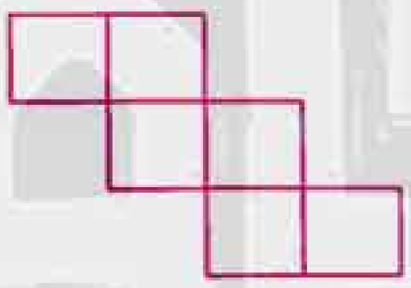
15 Each of the following figures consists of squares equal in area. Find the perimeter of each figure in the simplest form if its area is known :

1



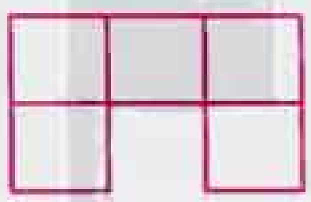
Its area = 300 cm^2.

2



Its area = 72 cm^2.

3



Its area = 40 cm^2.

« $70\sqrt{2}$ cm. , $28\sqrt{3}$ cm. , $24\sqrt{2}$ cm. »

For excellent pupils

16 If $a^x = 6$ and $a^{-y} = \sqrt{3}$, find the value of : a^{x+y} « $2\sqrt{3}$ »

17 **Simplify each of the following to the simplest form :**

1 $\frac{(\sqrt{5})^3 \times (\sqrt{5})^5}{(\sqrt{10})^6}$ « $\frac{5}{8}$ »

2 $\frac{2\sqrt{2} \times (\sqrt{6})^{-3}}{(\sqrt{3})^{-3}}$ « 1 »

18 If $\sqrt{27} + 2\sqrt{\frac{1}{2}} + \sqrt{18} + \sqrt{12} - \sqrt{50} = x\sqrt{2} + y\sqrt{3}$

, find the value of each of x and y where x and y are two rational numbers. « -1 , 5 »

Exercise 7

The two conjugate numbers

From the school book

1 **Write the conjugate number of each of the following numbers :**

1. $\sqrt{5}+\sqrt{3}$
2. $5-2\sqrt{7}$
3. $\sqrt{5}+\frac{2}{\sqrt{2}}$

2 **Make the denominator of each of the following a rational number :**

1. $\frac{5}{\sqrt{7}-\sqrt{2}}$
2. $\frac{\sqrt{3}}{2-\sqrt{3}}$
3. $\frac{\sqrt{7}+3}{\sqrt{7}-3}$

3 If $x=\frac{2}{\sqrt{7}-\sqrt{5}}$ and $y=\sqrt{7}-\sqrt{5}$, find the value of : $(x+y)^2$ « 28 »

4 If $x=\frac{4}{\sqrt{7}-\sqrt{3}}$ and $y=\frac{4}{\sqrt{7}+\sqrt{3}}$, find the value of : $x^2 y^2$ « 16 »

5 If $x=\sqrt{5}+\sqrt{3}$, prove that : $\frac{4}{x}+2x=4\sqrt{5}$

6 If $a=\sqrt{3}+\sqrt{2}$ and $b=\frac{1}{\sqrt{3}+\sqrt{2}}$, find the value of : a^2-b^2 in its simplest form. « $4\sqrt{6}$ »

7 If $x=\sqrt{5}-\sqrt{3}$ and $y=\frac{2}{\sqrt{5}-\sqrt{3}}$, find the value of : $x^2+2xy+y^2$ « 20 »

8 If $x=\sqrt{5}-\sqrt{2}$ and $y=\frac{3}{\sqrt{5}-\sqrt{2}}$, prove that x and y are conjugate numbers , then find the value of : $x^2-2xy+y^2$ « 8 »

9 **If $x = 3 + \sqrt{5}$ and $y = \frac{4}{3 + \sqrt{5}}$, prove that x and y are conjugate numbers , then find :**

1 Their product. 2 $x^2 + y^2$ « 4 , 28 »

10 If $x = \frac{2}{\sqrt{5} - \sqrt{3}}$ and $y = \frac{2}{\sqrt{5} + \sqrt{3}}$, find the value of : $x^2 - xy + y^2$ « 14 »

11 If $x = \sqrt{5} + \sqrt{2}$ and $y = \sqrt{5} - \sqrt{2}$, find the value of : $\frac{x + y}{xy - 1}$ in its simplest form. « $\sqrt{5}$ »

12 If $a = \frac{4}{\sqrt{7} - \sqrt{3}}$ and $b = \frac{4}{\sqrt{7} + \sqrt{3}}$, find the value of : $\frac{a - b}{ab}$ « $\frac{\sqrt{3}}{2}$ »

13 If $x = 2\sqrt{2} - \sqrt{3}$ and $y = \frac{5}{\sqrt{8} - \sqrt{3}}$

, prove that x and y are conjugate numbers and calculate : $\frac{x + y}{xy}$ « $\frac{4\sqrt{2}}{5}$ »

14 If $x = \frac{5\sqrt{2} + 3\sqrt{5}}{\sqrt{5}}$ and $y = \frac{2\sqrt{5} - 3\sqrt{2}}{\sqrt{2}}$

, find the value of each of : 1 $x^2 + y^2$ 2 xy « 38 , 1 »

, then prove that : $x^2 + y^2 = 38\, xy$

15 If $x = \frac{1}{2 + \sqrt{3}}$ and $y = \frac{12}{\sqrt{3}}$, find the value of : $x^2 + y$ « 7 »

16 If $x = \frac{1}{\sqrt{3} - \sqrt{2}}$ and y is the multiplicative inverse of x

, find y , then prove that : $(x + y)^2 = 12$ « $\sqrt{3} - \sqrt{2}$ »

17 If $x = \sqrt{13} + \sqrt{6}$, $xy = 1$, find the value of : $x^2 - 49\, y^2$ « $4\sqrt{78}$ »

18 If $x = \frac{4}{\sqrt{7} - \sqrt{3}}$ and $y^{-1} = \frac{1}{\sqrt{7} - \sqrt{3}}$ (Remember that $y^{-1} = \frac{1}{y}$)

, prove that x and y are conjugate numbers , then find the value of : $x^2 y^2$ « 16 »

Unit 1

19 If $x=\sqrt{7}+\sqrt{5}$ and $y=\frac{2}{x}$, find the value of : $\frac{x+y}{xy}$ in its simplest form. «$\sqrt{7}$»

20 If $x=\frac{\sqrt{6}+\sqrt{5}}{\sqrt{6}-\sqrt{5}}$, prove that : $x+\frac{1}{x}=22$

21 **Complete the following :**

1. $\left(\sqrt{7}+\sqrt{3}\right)\left(\sqrt{7}-\sqrt{3}\right)=$ ....
2. If $x=3+\sqrt{2}$, then its conjugate is .... and the product of multiplying x by its conjugate is ....
3. The conjugate number of the number $\frac{1}{\sqrt{3}-\sqrt{2}}$ is ....
4. The conjugate number of the number $1+\frac{7}{\sqrt{7}}$ in the simplest form is ....
5. The multiplicative inverse for $\left(\sqrt{3}+\sqrt{2}\right)$ in its simplest form is ....
6. If $x=2+\sqrt{5}$ and y is the conjugate number of x , then $(x-y)^2=$ ....
7. If $\frac{x}{5-\sqrt{5}}=5+\sqrt{5}$, then the value of x in its simplest form is ....
8. If $\frac{1}{x}=\sqrt{5}-2$, then the value of x in its simplest form is ....
9. If $x=\sqrt{3}+2$, $y=\sqrt{3}-2$, then $(xy , x+y)=$ ....
10. $\left(\sqrt{2}+\sqrt{3}\right)^{-9}\left(\sqrt{2}-\sqrt{3}\right)^{-9}=$ ....

22 **In each of the following , if a and b are two integers , find the value of each of them :**

1. $\frac{11}{2\sqrt{5}+3}=a\sqrt{5}+b$ «2 , −3»
2. $\frac{3}{2\sqrt{2}-\sqrt{5}}=a\sqrt{2}+b\sqrt{5}$ «2 , 1»
3. $\frac{7}{\sqrt{8}+1}=a+b\sqrt{2}$ «−1 , 2»

23 **Simplify each of the following :**

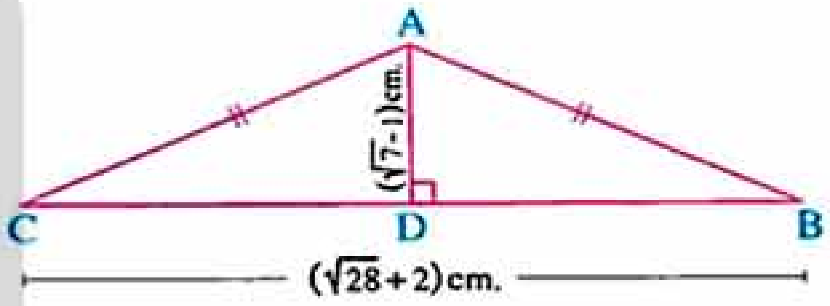
1. $\frac{4}{\sqrt{5}+\sqrt{3}} + \frac{4}{\sqrt{5}-\sqrt{3}}$ « $4\sqrt{5}$ »

2. $\frac{\sqrt{6}-\sqrt{5}}{\sqrt{6}+\sqrt{5}} - \frac{\sqrt{6}+\sqrt{5}}{\sqrt{6}-\sqrt{5}}$ « $-4\sqrt{30}$ »

3. $\sqrt{75} - \sqrt[3]{125} + \frac{10}{\sqrt{3}-1}$ « $10\sqrt{3}$ »

Geometric Application

24 **In the opposite figure :**



Δ ABC is an isosceles triangle in which AB = AC ,

$\overline{AD} \perp \overline{BC}$, BC = $\left(\sqrt{28}+2\right)$ cm. and

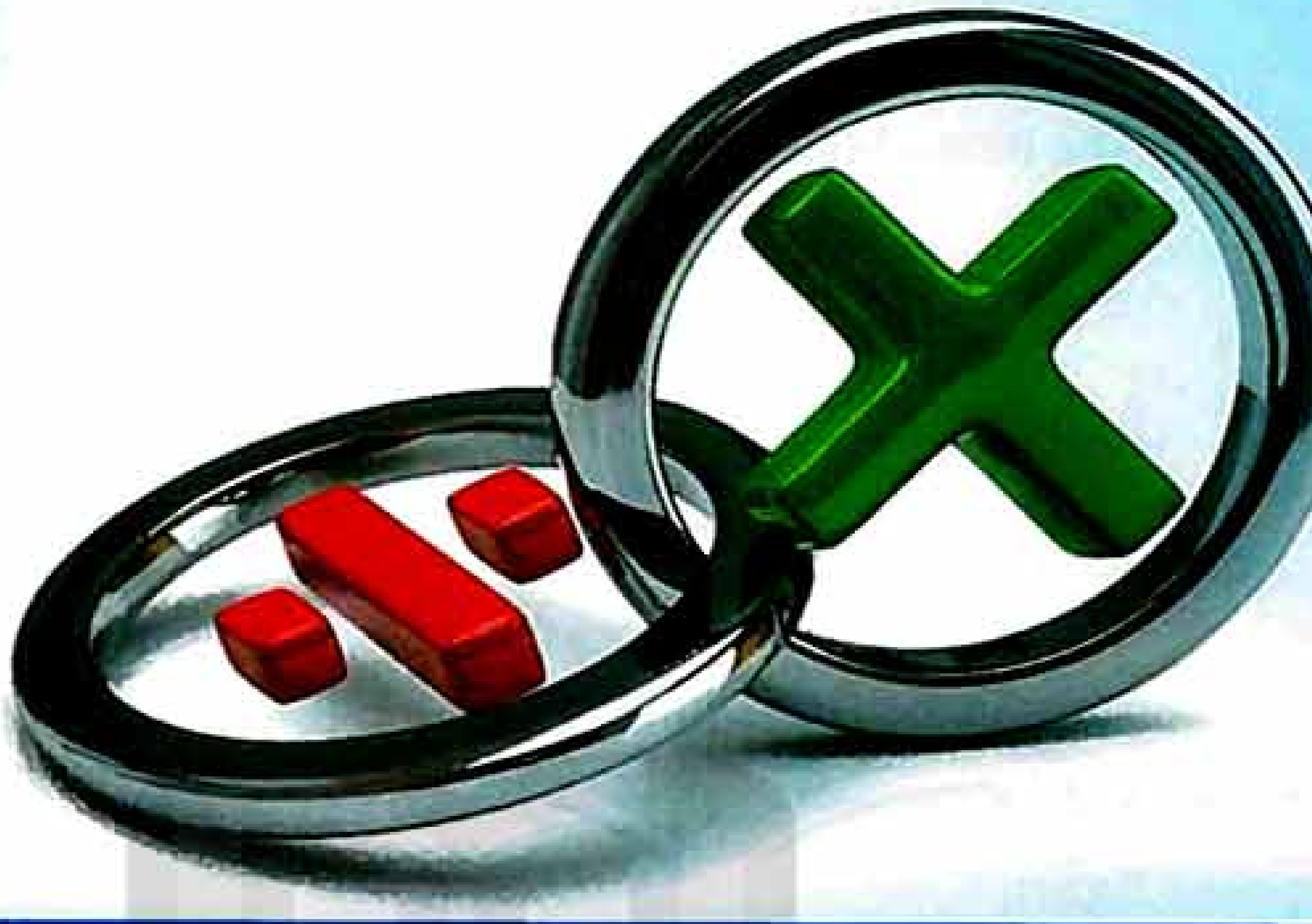
AD = $\left(\sqrt{7}-1\right)$ cm.

Find the area of Δ ABC « 6 cm^2. »

For excellent pupils

25 If $x = \sqrt{5}+1$ and $y = \sqrt{5}-1$, find the value of : $x y^{-1} + y x^{-1}$ « 3 »

26 If $x = \sqrt{7}+\sqrt{6}$ and $y = \sqrt{7}-\sqrt{6}$, find the value of : $\frac{x^8 y^9 - y}{(x+y)^5}$ « zero »



Exercise **8**

Operations on the cube roots

📖 From the school book

1 **Put each of the following in the form $a\sqrt[3]{b}$ where a and b are two integers , b is the least possible positive value :**

1. $\sqrt[3]{16}$
2. $\sqrt[3]{-54}$
3. $2\sqrt[3]{250}$
4. $\frac{2}{3}\sqrt[3]{-135}$
5. $3\sqrt[3]{\frac{1}{3}}$
6. $-10\sqrt[3]{\frac{2}{5}}$

2 **Find the result of each of the following in its simplest form :**

1. $\sqrt[3]{2} \times \sqrt[3]{32}$
2. $\frac{\sqrt[3]{72}}{\sqrt[3]{9}}$
3. $\frac{4\sqrt[3]{-54}}{2\sqrt[3]{-2}}$
4. 📖 $\frac{1}{2}\sqrt[3]{10} \times 6\sqrt[3]{100}$
5. 📖 $\sqrt[3]{\frac{2}{5}} \times \sqrt[3]{\frac{4}{25}}$
6. 📖 $\sqrt[3]{\frac{3}{4}} \div \sqrt[3]{\frac{2}{9}}$

3 **Find the result of each of the following in its simplest form :**

1. $\sqrt[3]{16} - \sqrt[3]{2}$ « $\sqrt[3]{2}$ »
2. 📖 $\sqrt[3]{125} - \sqrt[3]{24}$ « $5 - 2\sqrt[3]{3}$ »
3. $\sqrt[3]{81} + \sqrt[3]{-24}$ « $\sqrt[3]{3}$ »
4. $\sqrt[3]{54} + \sqrt[3]{16} - \sqrt[3]{250}$ « zero »
5. $2\sqrt[3]{54} - 5\sqrt[3]{2} + \sqrt[3]{16}$ « $3\sqrt[3]{2}$ »
6. $\sqrt[3]{16} - \frac{1}{3}\sqrt[3]{54} + \sqrt[3]{-2}$ « zero »
7. $\sqrt[3]{16} + \sqrt[3]{10} \times \sqrt[3]{25}$ « $7\sqrt[3]{2}$ »
8. 📖 $\sqrt[3]{24} - 6\sqrt[3]{13\frac{8}{9}}$ « $-8\sqrt[3]{3}$ »

Exercise 8

4 **Prove that :**

1. $\sqrt[3]{128}+\sqrt[3]{16}-2\sqrt[3]{54}=$ zero
2. $\sqrt[3]{54}\times\sqrt[3]{16}\div\left(\sqrt[3]{4}\times 6\right)=1$

5 **Simplify each of the following to the simplest form :**

1. $\sqrt[3]{81}+\sqrt[3]{-24}-3\sqrt[3]{\frac{1}{9}}$ « zero »
2. $\sqrt[3]{54}+8\sqrt[3]{-\frac{1}{4}}+5\sqrt[3]{16}$ « $9\sqrt[3]{2}$ »
3. $\sqrt[3]{108}-2\sqrt[3]{4}-\sqrt[3]{\frac{1}{2}}$ « $\frac{1}{2}\sqrt[3]{4}$ »
4. $\sqrt[3]{3}-\sqrt[3]{4}\times\sqrt[3]{6}+3\sqrt[3]{\frac{1}{9}}$ « zero »

6 **Simplify each of the following to its simplest form :**

1. $\frac{7}{3}\sqrt{18}+\sqrt[3]{54}-7\sqrt{2}+\sqrt[3]{16}$ « $5\sqrt[3]{2}$ »
2. $\sqrt{27}+\frac{1}{3}\sqrt[3]{27}-9\sqrt{\frac{1}{3}}-1$ « zero »
3. $\sqrt[3]{-16}+\frac{14}{\sqrt{7}}-\sqrt{28}+\sqrt[3]{54}$ « $\sqrt[3]{2}$ »
4. $\sqrt{18}+\sqrt[3]{54}-\frac{\sqrt{216}}{\sqrt{12}}-\sqrt[3]{16}$ « $\sqrt[3]{2}$ »
5. $5\sqrt{2}-\frac{1}{2}\sqrt{200}+\left(\sqrt[3]{5}\times\sqrt[3]{25}\right)$ « 5 »

7 **Simplify the following to its simplest form :**

$2\sqrt[3]{16}\left(3\sqrt[3]{4}+5\sqrt[3]{32}-2\sqrt[3]{\frac{1}{2}}\right)$ « 96 »

8 **Choose the correct answer from those given :**

1. $\sqrt[3]{54}+\sqrt[3]{-2}=$ ....
   (a) $\sqrt[3]{52}$ (b) $\sqrt[3]{2}$ (c) $2\sqrt[3]{2}$ (d) $4\sqrt[3]{2}$
2. $\sqrt[3]{-64}+\sqrt{16}=$ ....
   (a) zero (b) 8 (c) -8 (d) ± 8
3. $\frac{\sqrt[3]{16}}{\sqrt[3]{2}}=$ ....
   (a) 8 (b) -2 (c) 2 (d) $2\sqrt[3]{2}$

Unit 1

4 $\sqrt[3]{2}+\sqrt[3]{2}=$ ....

(a) $\sqrt[3]{2}$ (b) $\sqrt[3]{4}$ (c) $\sqrt[3]{8}$ (d) $\sqrt[3]{16}$

5 $\sqrt[3]{\frac{2}{9}}=$ ....

(a) $\frac{\sqrt[3]{6}}{3}$ (b) $\sqrt[3]{\frac{1}{6}}$ (c) $\sqrt[3]{6}$ (d) $\sqrt[3]{2}$

9 Complete the following :

1 $\sqrt[3]{\frac{2}{3}}\times\sqrt[3]{-12}=$ ....

2 $\sqrt[3]{3}\times\sqrt[3]{9}=\sqrt{\cdots\cdots}$

3 $\sqrt[3]{54}-\sqrt[3]{-16}=\sqrt[3]{\cdots\cdots}$

4 $\frac{1}{2}\sqrt[3]{56}-\sqrt[3]{\frac{7}{27}}=$ ....

5 If $x=2$, $y=\sqrt[3]{-16}$, then $\left(\frac{x}{y}\right)^3=$ ....

6 $\frac{\sqrt[3]{250}-\sqrt[3]{16}}{\sqrt[3]{54}}=$ ....

10 **If $a=\sqrt[3]{5}+1$, $b=\sqrt[3]{5}-1$, find the value of each of the following :**

1 $(a-b)^5$

2 $(a+b)^3$ « 32 , 40 »

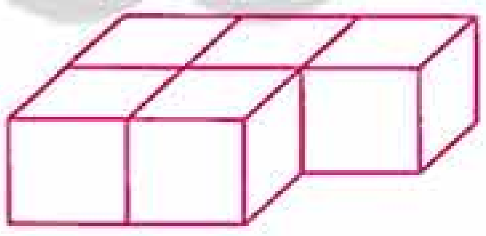
11 If $x=3+\sqrt[3]{6}$, $y=3-\sqrt[3]{6}$, find the value of : $\left(\frac{x-y}{x+y}\right)^3$ « $\frac{2}{9}$ »

12 Find the result of the following in its simplest form :

$\sqrt[3]{32}+4\sqrt[3]{\frac{1}{2}}-(2\sqrt[3]{-2})^2+(\sqrt{2})^{zero}-\left(\frac{2}{\sqrt{2}}\right)^2$ « −1 »

Life Application

13 The opposite figure represents a number of cubic boxes , the volume of each one is 24 dm^3.
Find the area of the ground for putting the boxes.



« $20\sqrt[3]{9}$ dm^2 »

For excellent pupils

14 If $x=\sqrt[3]{2}+1$, $y=\sqrt[3]{2}-1$, prove that : $x^2+y^2=2\sqrt[3]{4}+2$

15 Make the denominator of $\frac{2}{\sqrt[3]{2}}$ a rational number.

Exercise 9

Applications on the real numbers

From the school book

The cube

1 Complete the following :

1. If the edge length of a cube is 5 cm. , then its volume = ... cm^3.
2. The edge length of a cube is 4 cm. , then its total area = ... cm^2.
3. The lateral area of a cube whose edge length is ℓ cm. is .... cm^2.
4. The cube whose volume is ℓ^3 cm^3, its total area = ... cm^2.
5. The cube whose edge length is 2 ℓ cm. , then its volume = ... cm^3.

2 A cube whose lateral area is 36 cm^2. Find :

1. Its total area.
2. Its volume.

« 54 cm^2 , 27 cm^3 »

3 The perimeter of one face of a cube is 12 cm. Find :

1. Its volume.
2. Its lateral area.

« 27 cm^3 , 36 cm^2 »

4 The sum of lengths of all edges of a cube is 60 cm. Find :

1. Its volume.
2. Its total area.

« 125 cm^3 , 150 cm^2 »

5 Choose the correct answer from those given :

1. The volume of a cube is 1 cm^3 , then the sum of its edge lengths = ... cm.

(a) 1 (b) 6 (c) 8 (d) 12

2 The volume of a cube is 64 cm^3, then its lateral area = ... cm^2.

(a) 4 (b) 8 (c) 64 (d) 96

3 If the total area of a cube is 96 cm^2, then the area of one face = ... cm^2.

(a) 16 (b) 64 (c) 24 (d) 48

4 If the area of the six faces of a cube = 54 cm^2, then its volume = ... cm^3.

(a) 54 (b) 44 (c) 72 (d) 27

5 If the volume of a cube = 64 cm^3, then the length of a diagonal of one face = ... cm.

(a) 16 (b) $4\sqrt{2}$ (c) 32 (d) 64

6 The volume of a cube is 5 cm^3. If the edge length became twice the first, then its volume = ... cm^3.

(a) 10 (b) 20 (c) 30 (d) 40

7 The edge length of a cube whose volume is $2\sqrt{2} \text{ cm}^3$ is .... cm.

(a) $\sqrt{2}$ (b) 2 (c) 8 (d) 1.5

The cuboid

6 **The dimensions of the base of a cuboid are 9 cm. and 10 cm. and its height is 5 cm. Find :**

1 Its volume. 2 Its lateral area. 3 Its total area.

« 450 cm^3, 190 cm^2, 370 cm^2 »

7 The dimensions of a cuboid are $\sqrt{2}$ cm., $\sqrt{3}$ cm. and $\sqrt{6}$ cm. Find its volume. « 6 cm^3 »

8 The dimensions of the base of a cuboid are $\sqrt{3}$ cm. and $\left(\sqrt{3}-1\right)$ cm. and its height equals $\left(3+\sqrt{3}\right)$ cm. Calculate its volume. « 6 cm^3 »

9 The lateral area of a cuboid is 480 cm^2 and its base is in the shape of a square whose side length is 10 cm. Calculate its height. « 12 cm. »

10 Find the total area of a cuboid whose volume is 720 cm^3 and its height is 5 cm. with a squared-shape base. « 528 cm^2 »

11 **Which is more in size :**

A cube whose total area is 294 cm^2 or a cuboid with dimensions $7\sqrt{2}$ cm., $5\sqrt{2}$ cm. and 5 cm. ?

12 **In the opposite figure :**

A rectangular piece of cardboard has a length of 25 cm. and a width of 15 cm. A square whose side length = 4 cm. was cut from each of its four corners , then the projected parts were folded to form a basin in the shape of a cuboid.

Find the volume and the total area of that cuboid. « 476 cm^3 , 311 cm^2 »

The circle **Consider $\pi = \frac{22}{7}$ if there are not any other values given.**

13 A circle is of radius length 10.5 cm. Find each of its circumference and its area.

« 66 cm. , 346.5 cm^2 »

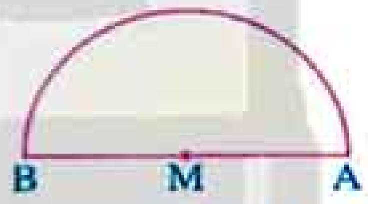
14 The area of a circle is 154 cm^2. Find its circumference and its diameter length.

« 44 cm. , 14 cm. »

15 A circle whose area is 64 π cm^2. Find the length of its radius , then find its circumference approximating it to the nearest integer. (π = 3.14) « 8 cm. , 50 cm. »

16 **In the opposite figure :**

$\overline{AB}$ is a diameter of the semicircle. If the area of this region is 12.32 cm^2, find the perimeter of the figure.

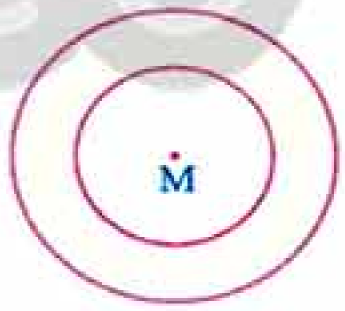


« 14.4 cm. »

17 **In the opposite figure :**

These are two concentric circles at M and their radii lengths are 3 cm. and 5 cm.

Find the area of the shaded part in terms of π



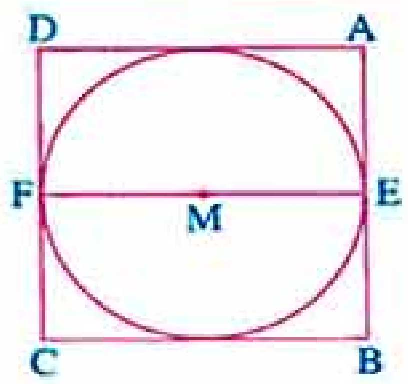
« 16 π cm^2 »

18 **In the opposite figure :**

The circle M is inside the square ABCD

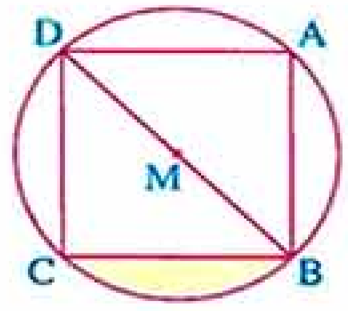
If the area of the shaded part = $10\frac{5}{7}$ cm^2 ,

find the perimeter of this part.



« 35 $\frac{5}{7}$ cm. »

19 **In the opposite figure :**



The square ABCD is inside the circle M

If the radius length of the circle M is 7 cm. ,

find the area of the shaded part and its perimeter. « 14 cm^2 , $(11 + 7\sqrt{2})$ cm. »

The right circular cylinder

Consider $\pi = \frac{22}{7}$ if there are not any other values given.

20 A right circular cylinder , the radius length of its base is 14 cm. and its height is 20 cm. Find the volume and the total area of the cylinder. « 12320 cm^3 , 2992 cm^2 »

21 Find the lateral area for a right circular cylinder of volume 924 cm^3. , and of a height 6 cm. « 264 cm^2 »

22 Find the total area of a right circular cylinder of volume 7536 cm^3 and its height is 24 cm. $(\pi = 3.14)$ « 2135.2 cm^2 »

23 **Which is more in volume :**

A right circular cylinder with base radius length 7 cm. and its height = 10 cm.

or a cube whose edge length is equal to 11 cm. ?

24 **Complete the following :**

1. A right circular cylinder whose base radius length is r cm. and its height = h cm. , then its lateral area = ... cm^2 and its volume = ... cm^3
2. A right circular cylinder with volume $40\pi \text{ cm}^3$ and its height = 10 cm. , then its base radius length = ...
3. A right circular cylinder with volume $500\pi \text{ cm}^3$ and its base radius length = 5 cm. , then its height = ...
4. A right circular cylinder with volume $\pi r^3 \text{ cm}^3$, then its height = ...
5. If the lateral area of a right circular cylinder is $2\pi r^2 \text{ cm}^2$, then its height = ...

25 The circumference of the base of a right circular cylinder is 44 cm. and its height = 25 cm. Find its volume. « 3850 cm^3 »

Exercise 9

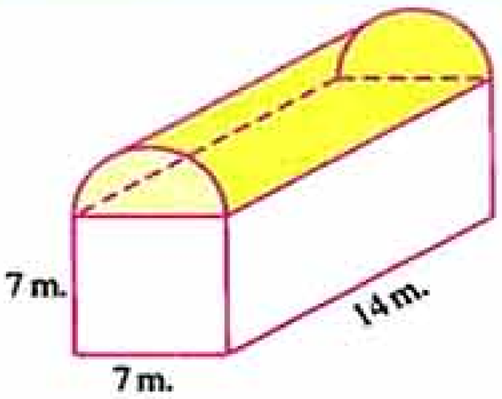
26 The lateral area of a right circular cylinder is 52 cm^2 and the length of the diameter of its base is 8 cm. Find its volume. « 104 cm^3 »

27 Find the height of a right circular cylinder whose height is equal to its base radius length and its volume is 72 π cm^3 « $2\sqrt[3]{9}$ cm. »

28 **In the opposite figure :**

A cuboid-shaped water tank with dimensions 7 m. , 7 m. and 14 m. , and the upper part of it is in the form of half of a right circular cylinder.

Calculate the volume of the tank in m^3



« 955.5 m^3 »

29 A piece of paper has a shape of a rectangle ABCD in which AB = 10 cm. and BC = 44 cm. It was folded to form a right circular cylinder such that $\overline{AB}$ is coincident to $\overline{DC}$

Find the volume of the resulted cylinder. « 1540 cm^3 »

The sphere **Consider $\pi = \frac{22}{7}$ if there are not any other values given.**

30 Find the volume and the surface area of a sphere if the length of its diameter is 4.2 cm. « 38.808 cm^3 , 55.44 cm^2 »

31 The volume of a sphere is 4188 cm^3 Find its radius length. ($\pi = 3.141$) « 10 cm. »

32 The volume of a sphere is 562.5 π cm^3

Find its surface area in terms of π « 225 π cm^2 »

33 **Choose the correct answer from those given :**

1 The volume of the sphere = ...

(a) $4\pi r^2$ (b) $\frac{4}{3}\pi r^3$ (c) $\frac{3}{4}\pi r^3$ (d) $\frac{4}{3}\pi r^2$

2 The sphere whose radius length is $\sqrt[3]{3}$ cm. , its volume = ... cm^3

(a) 4π (b) $4\sqrt{3}\pi$ (c) $\frac{4}{3}\pi$ (d) $\frac{9}{4}\pi$

3 The volume of the sphere whose diameter length is 6 cm. equals .... cm^3

(a) 288 (b) 12π (c) 36π (d) 288π

4 If the volume of a sphere = $\frac{9}{16}\pi$ cm^3 , then its radius length = ... cm.

(a) 3 (b) $\frac{4}{3}$ (c) $\frac{3}{4}$ (d) $\frac{1}{3}$

Unit 1

5 If the surface area of a sphere is 9π cm^2, then its diameter length = ... cm.

(a) 9 (b) 3 (c) 1.5 (d) 6

6 If three quarters of the volume of a sphere equals 8π cm^3, then the length of its radius equals .... cm.

(a) 64 (b) 8 (c) 4 (d) 2

34 Find the radius length of a sphere if its volume equals the volume of a right circular cylinder whose height is 18 cm. and its base radius length is 4 cm. « 6 cm. »

35 Find the volume of a sphere if its radius length equals the radius length of a right circular cylinder with volume 7536 cm^3 and height 24 cm. ($\pi = 3.14$) « $4186\frac{2}{3}$ cm^3 »

36 A lead cuboid is of dimensions 77 cm., 24 cm. and 21 cm. It was melted to make a sphere. Find the radius length of that sphere. « 21 cm. »

37 A metalic sphere, with diameter length 6 cm. has got melt and changed into a right circular cylinder with base radius length 3 cm. Find its height. « 4 cm. »

38 A sphere with volume 36π cm^3 is placed inside a cube. If the sphere touches the six faces of the cube, **find :**

1 The radius length of the sphere.

2 The volume of the cube. « 3 cm., 216 cm^3 »

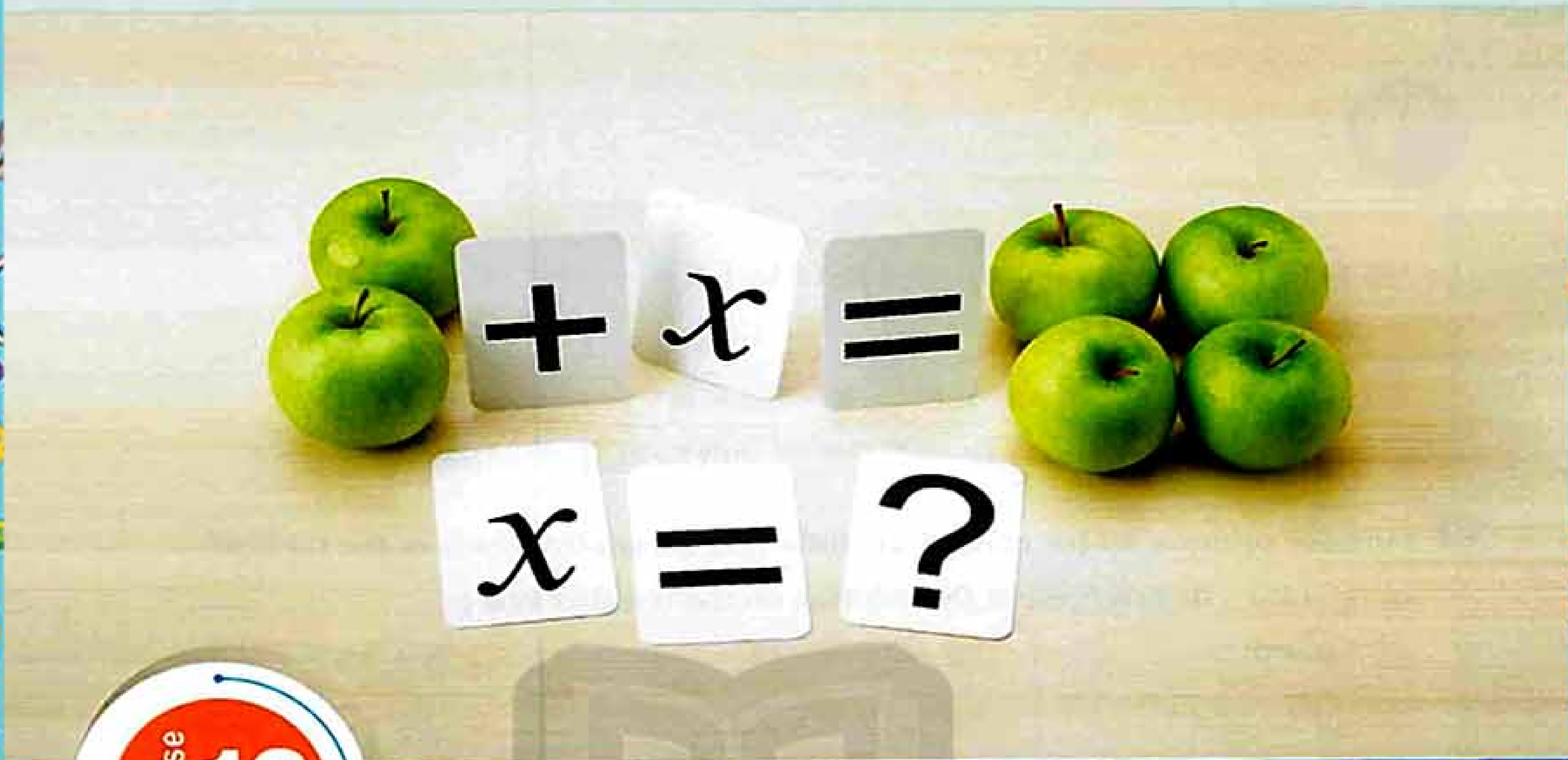
39 A metalic sphere is of radius length 16.8 cm. It is melted and it is converted to 8 small spheres which are equal in volume. Find the radius length of each small sphere. « 8.4 cm. »

40 A right circular cylinder has a height of 20 cm. Find its base radius length if its volume equals $\frac{4}{9}$ of the volume of a sphere with a diameter length of 30 cm. « 10 cm. »

For excellent pupils

41 A cuboid has a square-shaped base whose height = 3 cm. If the sum of lengths of its edges is 52 cm., find its volume. « 75 cm^3 »

42 A hollow metal sphere is with internal radius length 2.1 cm. and external radius length 3.5 cm. Find its mass approximated to the nearest gram taking into consideration that the mass of a cubic centimetre of such a metal is 20 gm. « 2817 gm. »



Exercise 10
Solving equations and inequalities of the first degree in one variable in $\mathbb{R}$

📖 From the school book

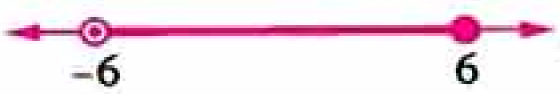
1 **Find the solution set for each of the following equations in $\mathbb{R}$, then represent the solution on the number line :**

1. 📖 $x + 5 = 0$
2. 📖 $5x + 6 = 1$
3. 📖 $2x + 4 = 3$
4. 📖 $2x - 3 = 4$
5. $4x - 1 = |-2|$
6. $\sqrt{5}\,x - 1 = 4$
7. $x - 1 = \sqrt{3}$
8. $2 - \sqrt{6}\,x = |-8|$
9. 📖 $x + 2\sqrt{3} = 3$

2 **Choose the correct answer from those given :**

1. The figure  represents the solution set of the inequality .... in $\mathbb{R}$

   (a) $x > -3$ (b) $x \geq -3$ (c) $x < -3$ (d) $x \leq -3$

2. The figure  represents the solution set of the inequality .... in $\mathbb{R}$

   (a) $-6 < x < 6$ (b) $-6 \leq x < 6$ (c) $-6 < x \leq 6$ (d) $-6 \leq x \leq 6$

3. If $x \in]3 , \infty[$, then ....

   (a) $x < 3$ (b) $x \leq 3$ (c) $x > 3$ (d) $x \geq 3$

4. The S.S. of the inequality : $x > 7$ in $\mathbb{R}$ is ....

   (a) $]-7 , \infty[$ (b) $[7 , \infty[$ (c) $]-\infty , 7[$ (d) $]7 , \infty[$

5 The S.S. of the inequality : $-1 < x \leq 5$ in $\mathbb{R}$ is ………

(a) $]-1 , 5]$ (b) $[-1 , 5]$ (c) $\{-1 , 5\}$ (d) $[-1 , 5[$

6 The S.S. of the inequality : $-x > 3$ in $\mathbb{R}$ is ………

(a) $\{-3\}$ (b) $]3 , \infty[$ (c) $]-\infty , 3[$ (d) $]-\infty , -3[$

3 **Find the solution set for each of the following inequalities in $\mathbb{R}$ in the form of an interval , then represent the solution on the number line :**

1 $2x > 6$
2 $-7x \geq -14$
3 $x + 3 \leq 5$
4 $5 - x > 3$
5 $2x + 5 \geq 3$
6 $1 - 5x < 6$
7 $\frac{1}{2}x + 1 \leq 2$
8 $3 - 2x \leq 7$

4 **Find the solution set for each of the following inequalities in $\mathbb{R}$ in the form of an interval , then represent the solution on the number line :**

1 $3 < x + 2 \leq 6$
2 $-5 < x + 3 < 9$
3 $-3 \leq -x < 3$
4 $1 < 5 - x \leq 3$
5 $\sqrt[3]{-8} \leq x + 1 \leq \sqrt{9}$
6 $5 < 3 - x \leq 3^2$
7 $-8 \leq 3x + 1 \leq 4$
8 $|-3| < 2x - 1 < 5$
9 $-3 < \frac{1}{2}x - 2 \leq$ zero
10 $0 \leq \frac{-2x + 6}{3} < 4$

5 **Find the solution set for each of the following inequalities in $\mathbb{R}$ in the form of an interval , then represent the solution on the number line :**

1 $3x < 2x + 4$
2 $7x - 9 \geq 4x$
3 $5x - 3 < 2x + 9$
4 $7x - 12 \geq 5x - 8$
5 $x - 1 \leq 3 - x$
6 $1 - x \geq -2x - 3$

6 **Find the solution set for each of the following inequalities in $\mathbb{R}$ in the form of an interval , then represent the solution on the number line :**

1 $x + 3 \geq 2x \geq x - 2$
2 $-x < x < 4 - x$
3 $4x \leq 5x + 2 < 4x + 3$
4 $x - 1 < 3x - 1 \leq x + 1$
5 $2 + 2x \leq 3x + 3 < 5 + 2x$
6 $\frac{3x - 4}{6} < x + 1 < \frac{x + 3}{2}$

7 **Complete the following :**

1 If $x - 3 \geq 0$, then x ………
2 If $5x < 15$, then x ………
3 If $1 - x > 4$, then x ………
4 If $-2x \leq 3$, then x ………

5 If $\sqrt{2}\,x \le 4$, then x ....

6 The S.S. of the inequality : $4 < 2x < 8$ in $\mathbb{R}$ is ....

7 The S.S. of the inequality : $-5 \le -x < 2$ in $\mathbb{R}$ is ....

8 The S.S. of the inequality : $2 - x < 0$ in $\mathbb{R}$ is ....

9 If $-3 < x < 3$ where $x \in \mathbb{R}$, then $2x \in]-6 , ...[$

8 **Choose the correct answer from those given :**

1 The S.S. of the inequality : $x + 3 < 3$ in $\mathbb{R}$ is ....

(a) $]-\infty , 0[$ (b) $]-\infty , 0]$ (c) $[0 , \infty[$ (d) $]0 , \infty[$

2 The S.S. of the inequality : $1 > x - 5 > -1$ in $\mathbb{R}$ is ....

(a) $[4 , 6]$ (b) $]4 , 6[$ (c) $]4 , 6]$ (d) $[4 , 6[$

3 If $x > 5$, then $-x$ ....

(a) < -9 (b) ≥ -5 (c) < -5 (d) > -5

4 If $-2 < x < 2$, then $2x + 3$ belongs to ....

(a) $[-1 , 7]$ (b) $]-1 , 5[$ (c) $]-1 , 7[$ (d) $]-4 , 6[$

5 The number 5 belongs to the S.S. of the inequality ....

(a) $x > 5$ (b) $x < 5$ (c) $-x \ge -5$ (d) $-x \ge 5$

Life Application

9 A lift for carrying goods can carry 2200 kg. as a maximum weight. If we have 60 boxes of cans and the weight of one box is 45 kg. , what is the maximum number of boxes can the lift carry in one time without carrying any person ? « 48 boxes »

For excellent pupils

10 Prove that $\sqrt{3}$ belongs to the S.S. of the inequality : $0 < 4 - 2x < 6$ in $\mathbb{R}$

11 If $[4 , 7]$ is the S.S. of the inequality : $a \le x - 3 \le b$, find the value of each of a and b « 1 , 4 »

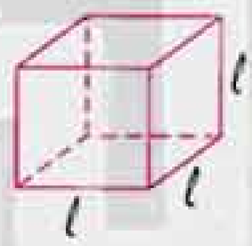
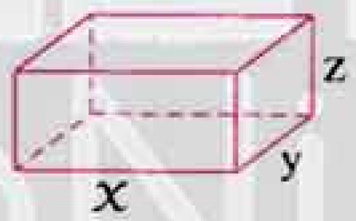
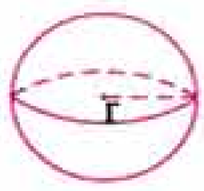
12 If $[m , m + n]$ is the S.S. of the inequality : $\frac{1}{5} \le \frac{2x+1}{5} \le 1$, find the value of n « 2 »

13 If $5 \le \frac{2x}{3} + 1 \le 7$, find the smallest value of the expression : $x - 2$ « 4 »

14 Find in $\mathbb{R}$ the S.S. of the inequality : $\frac{x}{\sqrt{3}-\sqrt{5}} \ge \sqrt{3}+\sqrt{5}$

Summary of the second part of Unit 1
"From lesson 6 to lesson 10"

- If a and b are two non-negative real numbers , then :
  - $\sqrt{a} \times \sqrt{b} = \sqrt{a\,b}$
  - $\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$ (where $b \neq 0$)
- If a and b are two real numbers , then :
  - $\sqrt[3]{a} \times \sqrt[3]{b} = \sqrt[3]{a\,b}$
  - $\frac{\sqrt[3]{a}}{\sqrt[3]{b}} = \sqrt[3]{\frac{a}{b}}$ (where $b \neq 0$)
- If a and b are two positive rational numbers , then each of the two numbers $\left(\sqrt{a} + \sqrt{b}\right)$ and $\left(\sqrt{a} - \sqrt{b}\right)$ is conjugate to the other one and :
  - Their sum $= 2\sqrt{a}$
  - Their product $= a - b$
- If we have a real number whose denominator is written in the form $\left(\sqrt{a} + \sqrt{b}\right)$ or $\left(\sqrt{a} - \sqrt{b}\right)$, we put it in the simplest form by multiplying both the numerator and the denominator by the conjugate of the denominator.
- The following table summarizes the rules of areas and volumes of some solids :

The solid		The lateral area	The total area	The volume
The cube		$4\,l^2$	$6\,l^2$	l^3
The cuboid		$2\,(x + y) \times z$	$2\,(x\,y + y\,z + z\,x)$	$x\,y\,z$
The cylinder		$2\,\pi\,r\,h$	$2\,\pi\,r\,h + 2\,\pi\,r^2 = 2\,\pi\,r\,(h + r)$	$\pi\,r^2\,h$
The sphere		–	$4\,\pi\,r^2$	$\frac{4}{3}\,\pi\,r^3$

Remember that : the circumference of the circle $= 2\,\pi\,r$, the area of the circle $= \pi\,r^2$

- Solving the equation or the inequality is finding the values of the unknown which satisfy this equation or inequality.
- The solution set of the inequality of the first degree in one variable in $\mathbb{R}$ is written in the form of an interval.

Exams on the second part of unit one
from lesson (6) to lesson (10)

Model 1

Answer the following questions :

1 Choose the correct answer from those given :

1. The volume of the sphere of diameter length 3 cm. equals .... cm^3.
   (a) $4.5\,\pi$ (b) $36\,\pi$ (c) $288\,\pi$ (d) 4.5
2. If $x > 3$, then $-x$ ....
   (a) < 3 (b) > -3 (c) < -3 (d) $< \frac{-1}{3}$
3. $\sqrt{20} - \sqrt{5} =$ ....
   (a) $\sqrt{15}$ (b) $\sqrt{5}$ (c) $\sqrt{10}$ (d) 15
4. A cube , its volume is $125\ cm^3$. , then its total area equals .... cm^2.
   (a) 30 (b) 25 (c) 100 (d) 150
5. If $x = \sqrt{7} + \sqrt{3}$ and $y = \sqrt{7} - \sqrt{3}$, then $x\,y =$ ....
   (a) 4 (b) 10 (c) 40 (d) 58
6. $\frac{\sqrt[3]{24}}{\sqrt[3]{3}} =$ ....
   (a) 8 (b) 3 (c) 2 (d) $\sqrt[3]{2}$

2 Complete the following :

1. The multiplicative inverse of the number $\left(\sqrt{3} - \sqrt{2}\right)$ in the simplest form is ....
2. $\sqrt{2} \times \sqrt{12} = 2 \times$ ....
3. $\sqrt[3]{54} - \sqrt[3]{2} =$ .... (in the simplest form)
4. A right circular cylinder , its volume is $500\,\pi\ cm^3$. and the diameter length of its base is 10 cm. , then its height is ....
5. If $1 - x > 5$, then x ....

3 [a] Find in the simplest form the value of the expression :

$$\sqrt{18} + \sqrt[3]{54} - 3\sqrt{2} - \frac{1}{2}\sqrt[3]{16}$$

[b] A sphere , its volume is $36\,\pi\ cm^3$. Calculate its area.

Unit 1

4 **[a] Find in $\mathbb{R}$ the solution set of the inequality :**

$-3 < 2x + 1 < 7$, then represent it on the number line.

[b] A right circular cylinder , its height equals the radius length of its base and its volume is 27π cm^3. Find the radius length of its base.

5 **[a] Simplify to the simplest form :** $\sqrt{32} - \sqrt{72} + 6\sqrt{\frac{1}{2}}$

[b] If $x = \sqrt{5} - \sqrt{2}$ and $y = \frac{3}{\sqrt{5} - \sqrt{2}}$

Prove that : x and y are conjugate , then find : $x^2 + 2xy + y^2$

Model 2

Answer the following questions :

1 **Choose the correct answer from those given :**

1. $\sqrt{3\frac{3}{8}} = \frac{3}{2}\sqrt{\frac{\cdots\cdots}{\cdots\cdots}}$

(a) $\frac{3}{8}$ (b) $\frac{3}{2}$ (c) $\frac{27}{8}$ (d) $\frac{729}{64}$

2. The number $(1 - \sqrt{3})(1 + \sqrt{3})$ is .... number.

(a) a natural (b) a rational (c) an irrational (d) a prime

3. $\sqrt{3} + \sqrt{3} = \cdots\cdots$

(a) 3 (b) $\sqrt{6}$ (c) $2\sqrt{6}$ (d) $2\sqrt{3}$

4. A sphere , its volume is $\frac{4}{3}\pi$ cm^3. , then its diameter length is .... cm.

(a) 0 (b) 1 (c) 2 (d) $\frac{4}{3}$

5. A cube , its volume is $2\sqrt{2}$ cm^3. , then its edge length equals .... cm.

(a) $\sqrt{2}$ (b) 2 (c) 8 (d) 4

6. $\sqrt[3]{2} \times \sqrt[3]{2} = \cdots\cdots$

(a) 2 (b) 4 (c) $\sqrt[3]{4}$ (d) $\sqrt{2}$

2 **Complete the following :**

1. If $x = \frac{1}{\sqrt{8} - \sqrt{5}}$ and $xy = 1$, then y = ...

2 The solution set of the inequality : $4 > -2x$ in $\mathbb{R}$ is ....

3 $\frac{\sqrt[3]{32}}{\sqrt[3]{4}} =$ ....

4 $\left(\sqrt{8}+\sqrt{2}\right)^2 =$ ....

5 A right circular cylinder , its volume is 90π cm^3. , and its height is 10 cm. , then the radius length of its base equals .... cm.

3 **[a] Prove that :** $\sqrt[3]{128}+\sqrt[3]{16}-2\sqrt[3]{54}=0$

[b] If $a=\frac{4}{\sqrt{7}-\sqrt{3}}$ and $b=\sqrt{7}-\sqrt{3}$

, find in the simplest form : $\frac{a-b}{a\,b}$

4 **[a] Simplify :**

$2\sqrt{5}+9\sqrt{\frac{1}{3}}-\sqrt{27}-5\sqrt{\frac{1}{5}}$

[b] A right circular cylinder with volume 36π cm^3. and its height is 4 cm. , and the radius length of its base equals the edge length of a cube. Find the total area of the cube.

5 **[a]** A right circular cylinder , its volume is 231 cm^3. , and its height is 6 cm.

Calculate its lateral area $\left(\pi=\frac{22}{7}\right)$

[b] Find in $\mathbb{R}$ the solution set of the inequality :

$5x-3<2x+9$ and represent it on the number line.

UNIT 2

Relation between Two Variables

Exercises of the unit :

11. Relation between two variables.
12. Slope of straight line.
13. Real life applications on the slope.
- Summary of unit two.
- Unit exams.

Exercise 11 Relation between two variables

From the school book

1 **Complete the following ordered pairs which satisfy the relation : $y = 3x - 1$**

(5 , ...) , (2 , ...) , (0 , ...) , (– 3 , ...)

2 **Show which of the following ordered pairs satisfies the relation : $y - 4x = 7$**

1 (1 , 2)   2 (3 , – 5)   3 (– 1 , 3)

3 **Find four ordered pairs satisfying each of the following relations :**

1 $2x - y = 5$ 2 $y = \frac{1}{2}x + 5$

3 $y = 2$ 4 $2x = 5$

4 **Using the linear relations , complete the following tables :**

1 $4x - y = -1$

x	0	1	2	3
y	....	....	....	....

2 $y = 5x + 15$

x	– 4	– 3	– 2
y	....	....	....

3 $a - b = 4$

a	1	....	....
b	....	0	– 1

4 $a - 3b = 5$

a	2	....	– 1
b	....	0	....

Unit 2

5 **If $y - 2x = 1$, find :**

1. y at $x = 3$
2. y at $x = -5$
3. x at $y = 1$
4. x at $y = -1$

6 If (3 , 6) satisfies the relation : $y = kx$, find the value of k « 2 »

7 If (3 , 1) satisfies the relation : $y - 3x = a$, find the value of a « – 8 »

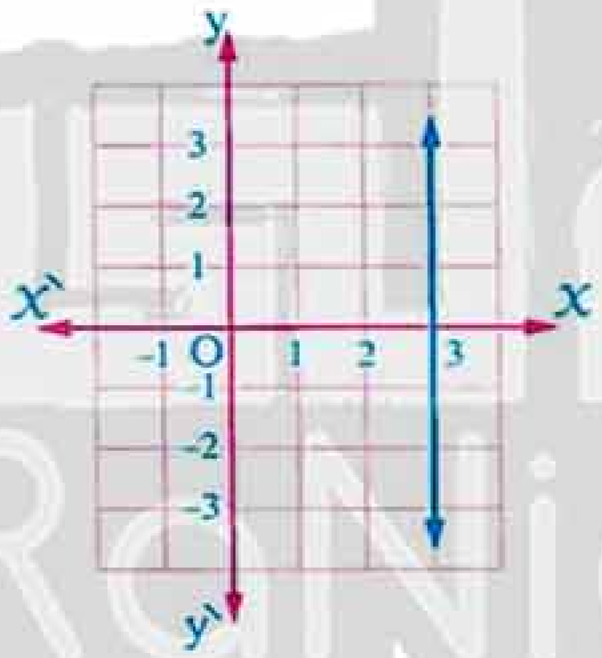
8 Find the value of b , where (– 3 , 2) satisfies the relation : $3x + by = 1$ « 5 »

9 If (3 , a) satisfies the relation : $y - 2x = 4$, find the value of a « 10 »

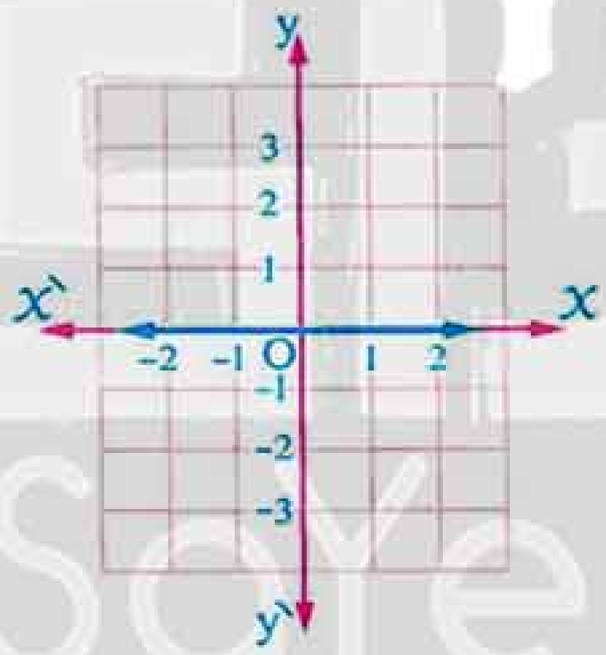
10 Find the value of k , where (k , 2 k) satisfies the relation : $x + y = 15$ « 5 »

11 **Find the relation that is represented by the line in each figure below :**

1.



2.



12 **Represent graphically each of the following relations :**

1. $x + y = 2$
2. $x - y = 3$
3. $x + 2y = 3$
4. $y - 3x = 1$
5. $y = -2x$
6. $y - 2x = -1$
7. $2x = 5$
8. $y + 1 = 0$

13 Graph the relation : $2x + 3y = 6$ If the straight line representing this relation intersects the x-axis at the point A and the y-axis at the point B , find the area of the triangle OAB where O is the origin point. « 3 square units »

14 If the straight line which represents the relation : $2x - y = a$ intersects the x-axis at the point (3 , b) , find a and b «6 ,0»

15 **Choose the correct answer from those given :**

1. If (2 , – 5) satisfies the relation : $3x - y + c = 0$, then c = ...
   (a) 1 (b) – 1 (c) 11 (d) – 11
2. Which of the following ordered pairs satisfies the relation : $2x + y = 5$?
   (a) (– 1 , 3) (b) (1 , 3) (c) (3 , 1) (d) (2 , 2)
3. (3 , 2) does not satisfy the relation ....
   (a) $y + x = 5$ (b) $3y - x = 3$ (c) $y + x = 7$ (d) $x - y = 1$
4. The point (3 , 5) lies on the straight line which represents the relation ....
   (a) $y = 3x - 5$ (b) $2x - y = 1$ (c) $3x + y = 1$ (d) $y = 3x - 1$
5. If (– 1 , 5) satisfies the relation : $3x + ky = 7$, then k = ...
   (a) 2 (b) – 2 (c) 1 (d) 10
6. The opposite table represents the relation between x and y , which of the following expresses this relation ?

x	1	2	3	4
y	– 2	– 5	– 8	– 11

   (a) $x + y = -1$ (b) $x - y = 3$
   (c) $3x + y = 1$ (d) $y = -x - 3$
7. The opposite table shows the relation between x and y , which is ....

x	1	2	3	4	5
y	1	3	5	7	9

   (a) $y = x + 4$ (b) $y = x + 1$
   (c) $y = 2x - 1$ (d) $y = 3x - 2$
8. The relation which expresses the two ordered pairs (2 , 1) and (4 , 3) together is ....
   (a) $y = \frac{1}{2}x$ (b) $y = 2x - 5$ (c) $y = x - 1$ (d) $y = 3x + 3$
9. The relation : $3x + 8y = 24$ is represented by a straight line intersecting the y-axis at the point ....
   (a) (0 , 8) (b) (8 , 0) (c) (0 , 3) (d) (3 , 0)

10 The relation $2x + 7y = 14$ is represented by a straight line intersecting the x-axis at the point ....

(a) (2 , 0) (b) (0 , 2) (c) (7 , 0) (d) (0 , 7)

11 The image of the point (5 , 3) by reflection in the straight line $x = 1$ is ....

(a) (5 , – 3) (b) (– 5 , 3) (c) (– 3 , 3) (d) (6 , 3)

12 The image of the point (4 , – 2) by reflection in the straight line y = 1 is ....

(a) (4 , 2) (b) (– 4 , – 2) (c) (4 , – 1) (d) (4 , 4)

16 Two even natural numbers , twice the first plus the second equals 12
Find the different possibilities of the two numbers.

Geometric Application

17 The perimeter of a rectangle is 14 cm. What are the different possibilities of the length and the width given that each of them belongs to $\mathbb{Z}_+$?

Life Applications

18 Essam has 10 bills of L.E. 5 and other bills of L.E. 20
He bought some goods from a shopping centre for L.E. 65
Determine the different possibilities to pay this amount of money. Find the relation and graph it.

19 The selling price of a computer table is L.E. 100 and its chair is L.E. 50 If the store sells in one week with L.E. 500 , what are the represented expectations to the number of sold computer tables and chairs ?
Represent the relation graphically.

For excellent pupils

20 The perimeter of an isosceles triangle is 19 cm. What are the different possible lengths of its sides given that its sides lengths $\in \mathbb{Z}_+$?

Notice that : The sum of the lengths of any two sides of the triangle is greater than the length of the third side.

Exercise **12**

Slope of straight line

From the school book

1 **Classify the slope of the straight line in each of the following figures showing whether it is (positive – negative – zero – undefined) :**

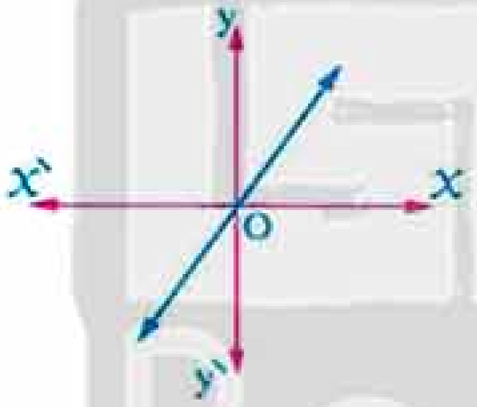


Fig. (1)

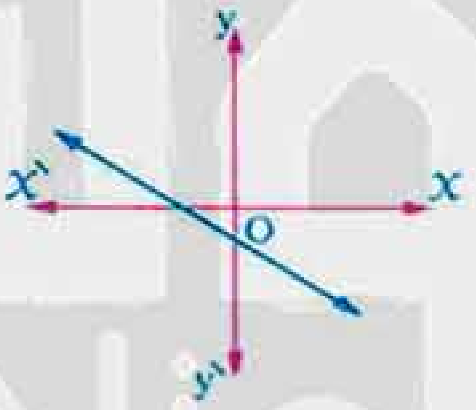


Fig. (2)

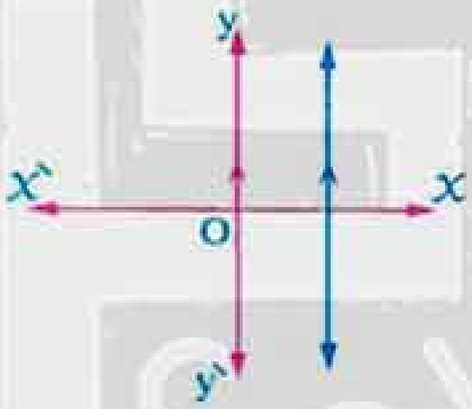


Fig. (3)

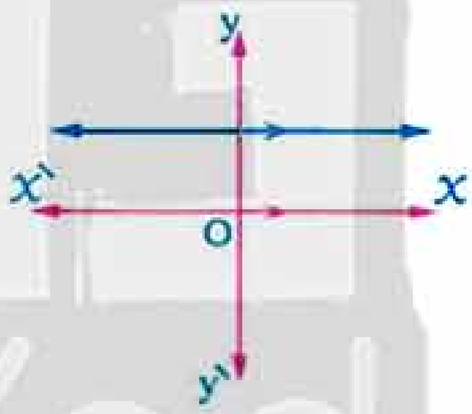
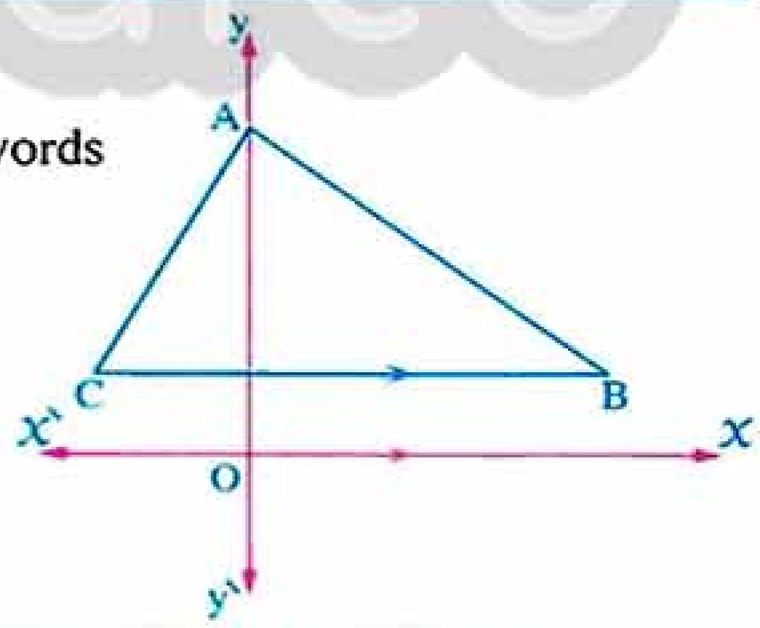


Fig. (4)

2 **In the opposite figure :**

ABC is a triangle. Complete by using one of the following words (positive , negative , zero , undefined)

1. The slope of $\overleftrightarrow{AB}$ is ....
2. The slope of $\overleftrightarrow{BC}$ is ....
3. The slope of $\overleftrightarrow{AO}$ is ....
4. The slope of $\overleftrightarrow{AC}$ is ....



3 **Complete the following :**

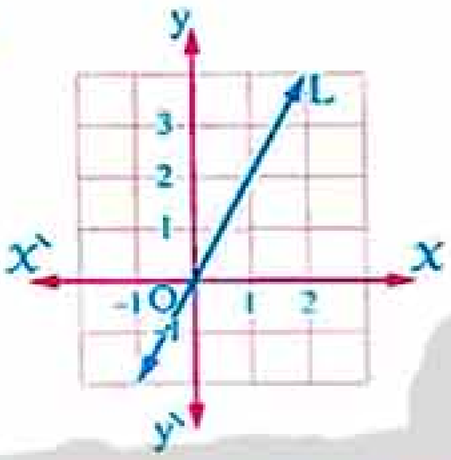
1. The slope of any horizontal straight line equals ....
2. The slope of any straight line parallel to y-axis is ....
3. The straight line whose slope = zero is parallel to ....
4. If A , B and C are collinear , then the slope of $\overleftrightarrow{AB}$ = the slope of ....

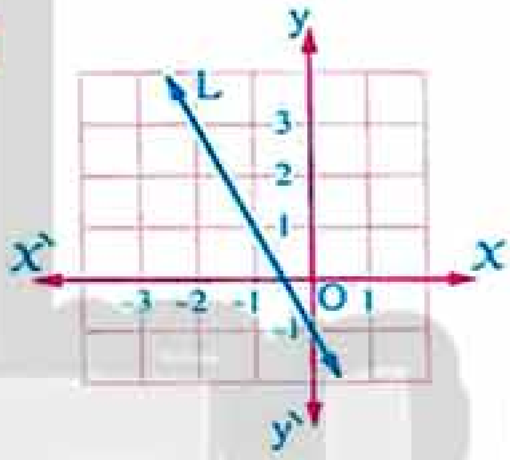
Unit 2

4 **Find the slope of the straight line passing through the two points in each of the following :**

1. A (1 , 3) , B (3 , 4)
2. A (1 , 2) , B (5 , 0)
3. A (3 , 2) , B (6 , 5)
4. A (2 , – 1) , B (4 , – 1)
5. A (1 , 3) , B (2 , 3)
6. A (5 , 2) , B (5 , 4)
7. A (3 , – 1) , B (3 , 2)
8. A (3 , – 2) , B (4 , 1)
9. A (– 1 , 3) , B (2 , 1)
10. N (4 , – 2) , K (– 1 , – 7)
11. E (– 3 , – 1) , O (0 , 0)
12. A (– 6 , – 9) , B (– 1 , – 1)

5 **Find the slope of the straight line L in each of the following graphs :**

1


2


6 **In the opposite figure :**

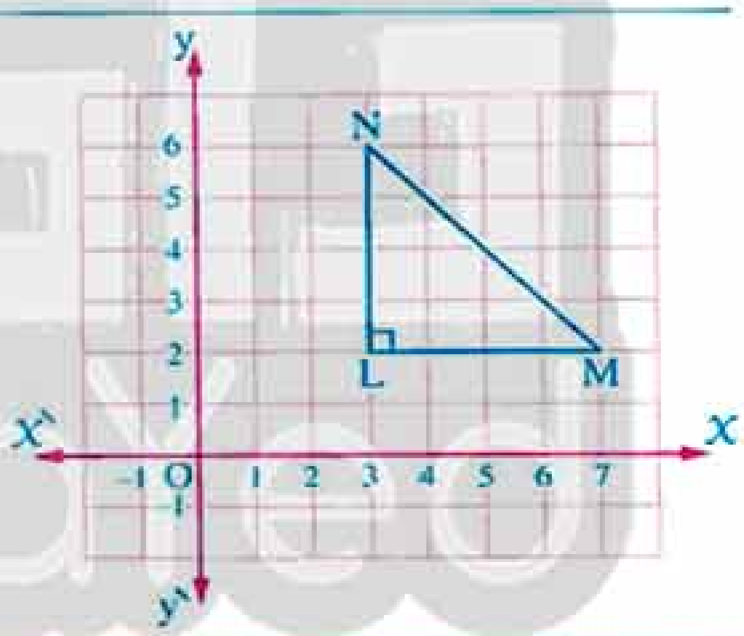
LMN is a right-angled triangle at L

, where m (∠ M) = 45°

Given that L (3 , 2) and M (7 , 2)

, find the coordinates of N

and calculate the slope of $\overleftrightarrow{MN}$



7 If A (2 , – 1) , B (10 , 3) and C (2 , 3) , find the slope of each of $\overleftrightarrow{AB}$, $\overleftrightarrow{BC}$ and $\overleftrightarrow{CA}$

Draw the triangle ABC on a square grid , then mention the type of the triangle according to the measures of its angles.

8 If the slope of the straight line which passes through the two points (1 , 3) and (3 , k) equals 3 , find the value of k « 9 »

9 If the slope of the straight line which passes through the two points (3 , c) and (5 , – 2) equals – 3 , find the value of c « 4 »

10 If A (– 1 , 4) and B (x , 2) and the slope of $\overleftrightarrow{AB}$ equals – 2

, find the value of x « zero »

11 If the straight line which passes through the two points $(-2, y)$ and $(3, -1)$ has a slope -0.6, find the value of y «2»

12 Find the value of k such that the straight line passing through the two points $(3, 4)$ and $(2, k)$ is parallel to x-axis. «4»

13 Find the value of x such that the straight line which passes through the two points $(2x, 3)$ and $(6, 7)$ is parallel to y-axis. «3»

14 Find the value of y such that the straight line passing through the two points $(3, 6)$ and $(-2, 3y)$ is perpendicular to y-axis. «2»

15 Are the points $(-5, 11)$, $(0, 8)$ and $(5, 5)$ collinear ?

16 Find the slope of each of $\overleftrightarrow{AB}$, $\overleftrightarrow{BC}$ and $\overleftrightarrow{AC}$, where $A(2, 1)$, $B(3, 2)$ and $C(4, 5)$ and represent each line graphically. What do you observe ?

17 **In each of the following, prove that the points A, B and C are collinear :**

1. $A(1, 1)$, $B(2, 2)$, $C(-3, -3)$
2. $A(4, -3)$, $B(-6, 7)$, $C(5, -4)$
3. $A(-2, 12)$, $B(2, 4)$, $C(6, -4)$

18 **In each of the following, prove that the points A, B and C are not collinear :**

1. $A(2, 1)$, $B(3, 0)$, $C(5, -1)$
2. $A(-1, 2)$, $B(3, 1)$, $C(7, 2)$
3. $A(0, -3)$, $B(2, 2)$, $C(-3, -3)$

19 Find the slope of the line $\overleftrightarrow{AB}$, where $A(-1, 3)$ and $B(2, 5)$
Is the point $C(8, 1) \in \overleftrightarrow{AB}$? «$\frac{2}{3}$»

20 Find the value of y such that the points $(4, 1)$, $(-2, 7)$ and $(3, y)$ are collinear. «2»

For excellent pupils

21 If the straight line which passes through the points $(3, -1)$, $(x, 1)$ and $(9, y)$ has a slope $= \frac{2}{3}$, find the value of each of x and y «6, 3»

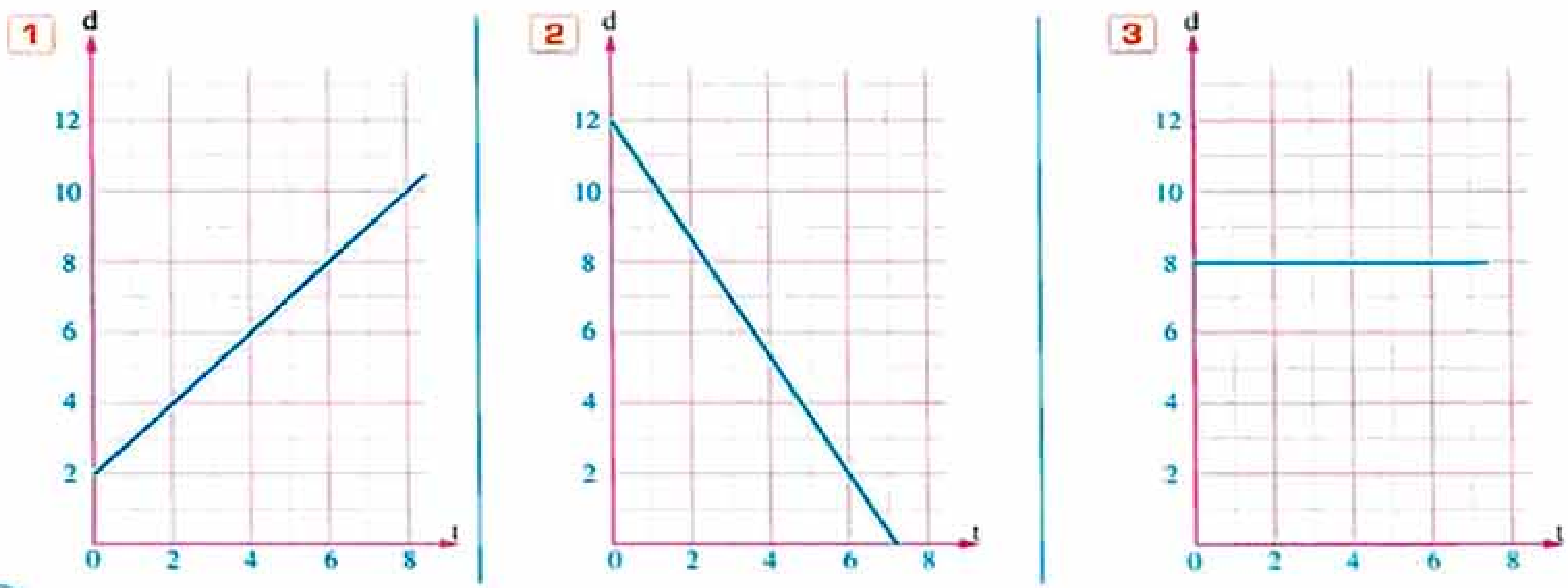
Exercise 13 Real life applications on the slope

From the school book

1 A car moves with uniform velocity such that it covers 180 km. per 3 hours. If the car moves for 5 hours , what is the covered distance ? « 300 km. »

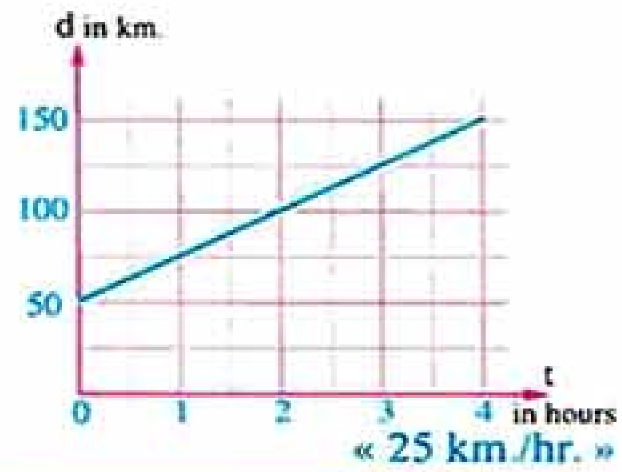
2 An irrigation machine consumes 2.47 litres of diesel to work for 3 hours. If the machine works for 10 hours , how many litres of diesel will the machine consume ? « $8\frac{7}{30}$ litres »

3 The following diagrams show the relation between the covered distance (in m.) and the elapsed time (in sec.) of an object. Determine the position of the object at the starting motion and its position after 6 seconds (when t = 6 sec.) Find the slope of the line in each case , and state what it represents.



Exercise 13

4 The opposite graph represents the motion of a car moving with uniform velocity.
Determine the velocity of the car.

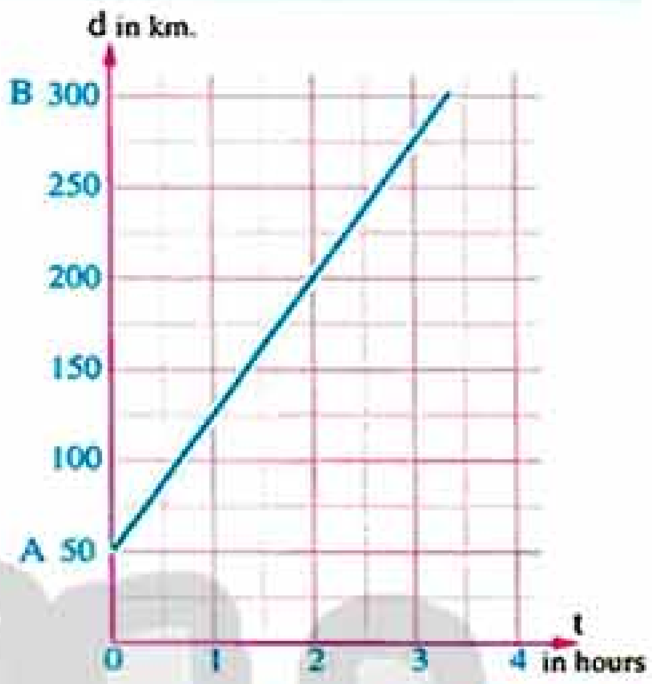


« 25 km./hr. »

5 Bassim drove his car from the city A to the city B
The opposite graph shows the relation between the distance d in km. and the time t in hours.

Answer the following :

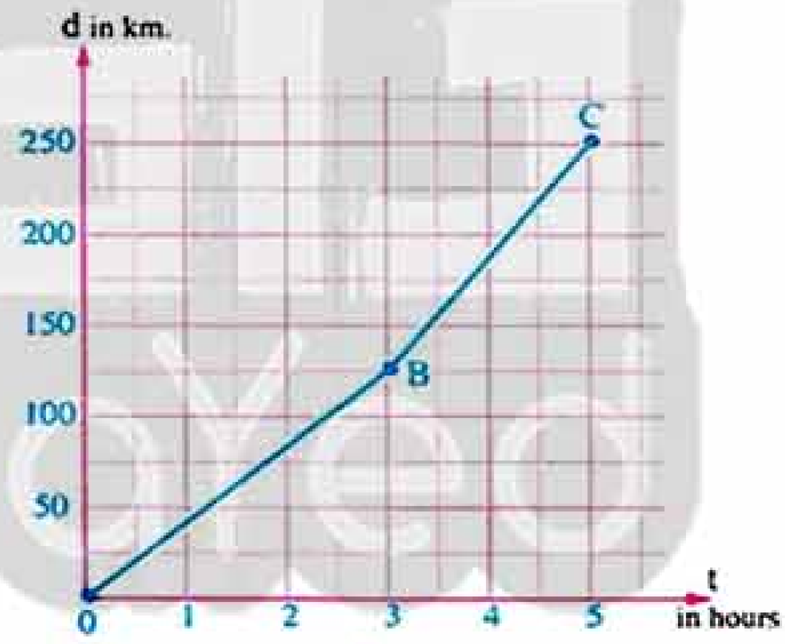
1. What is the uniform velocity of the car of Bassim ?
2. Find the distance between the car and the point 0 after three hours from the moment of beginning.



« 75 km./hr. , 275 km. »

6 **The opposite graph represents the motion of a car :**

1. Find the velocity of the car within the first three hours from the beginning , then find the velocity within the next two hours.
2. Find the average velocity of the car within the total time.

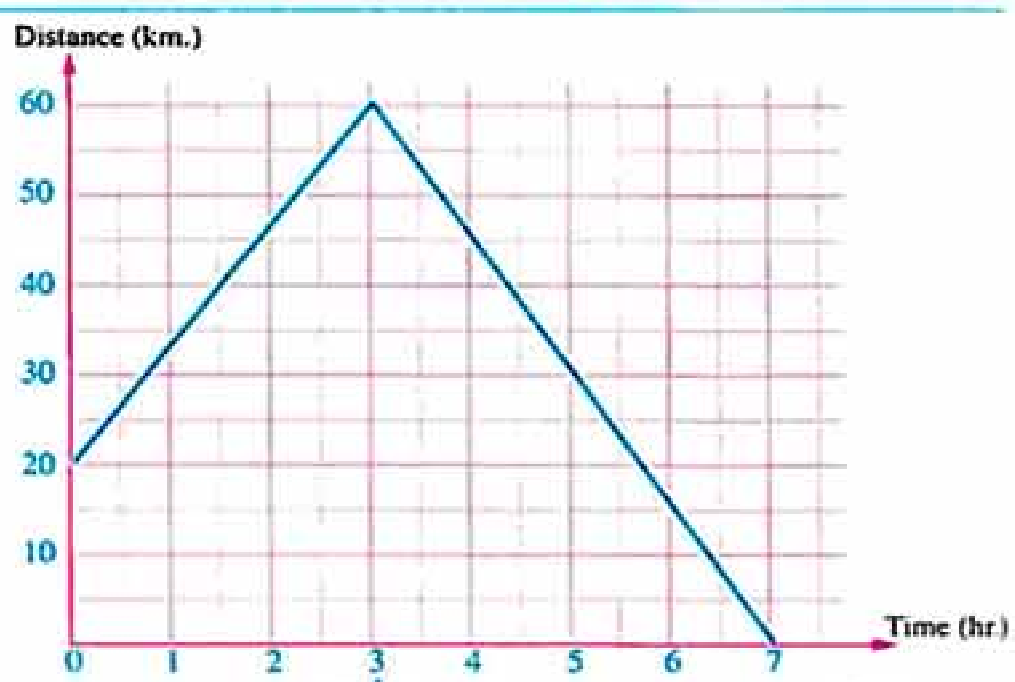


« $41\frac{2}{3}$ km./hr. , $62\frac{1}{2}$ km./hr. , 50 km./hr. »

7 **The opposite figure represents the motion of a bicycle measured from a constant point.**
Find the regular velocity of the bicycle during :

1. The first three hours.
2. The next four hours.

Find the total distance covered by the bicycle.



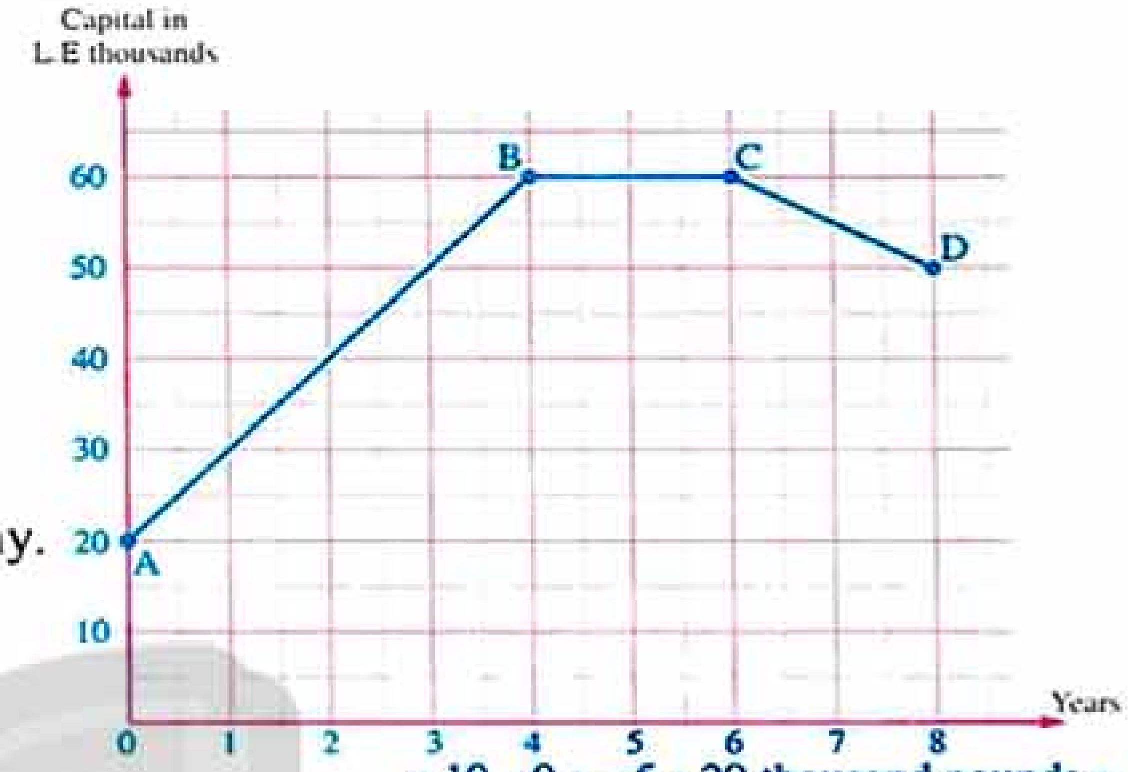
« $13\frac{1}{3}$ km./hr. , 15 km./hr. , 100 km. »

8 **The opposite figure shows the capital change of a company during 8 years :**

1. Find the slope of each of $\overleftrightarrow{AB}$, $\overleftrightarrow{BC}$ and $\overleftrightarrow{CD}$

   What is the meaning of each ?

2. Find the starting capital of the company.

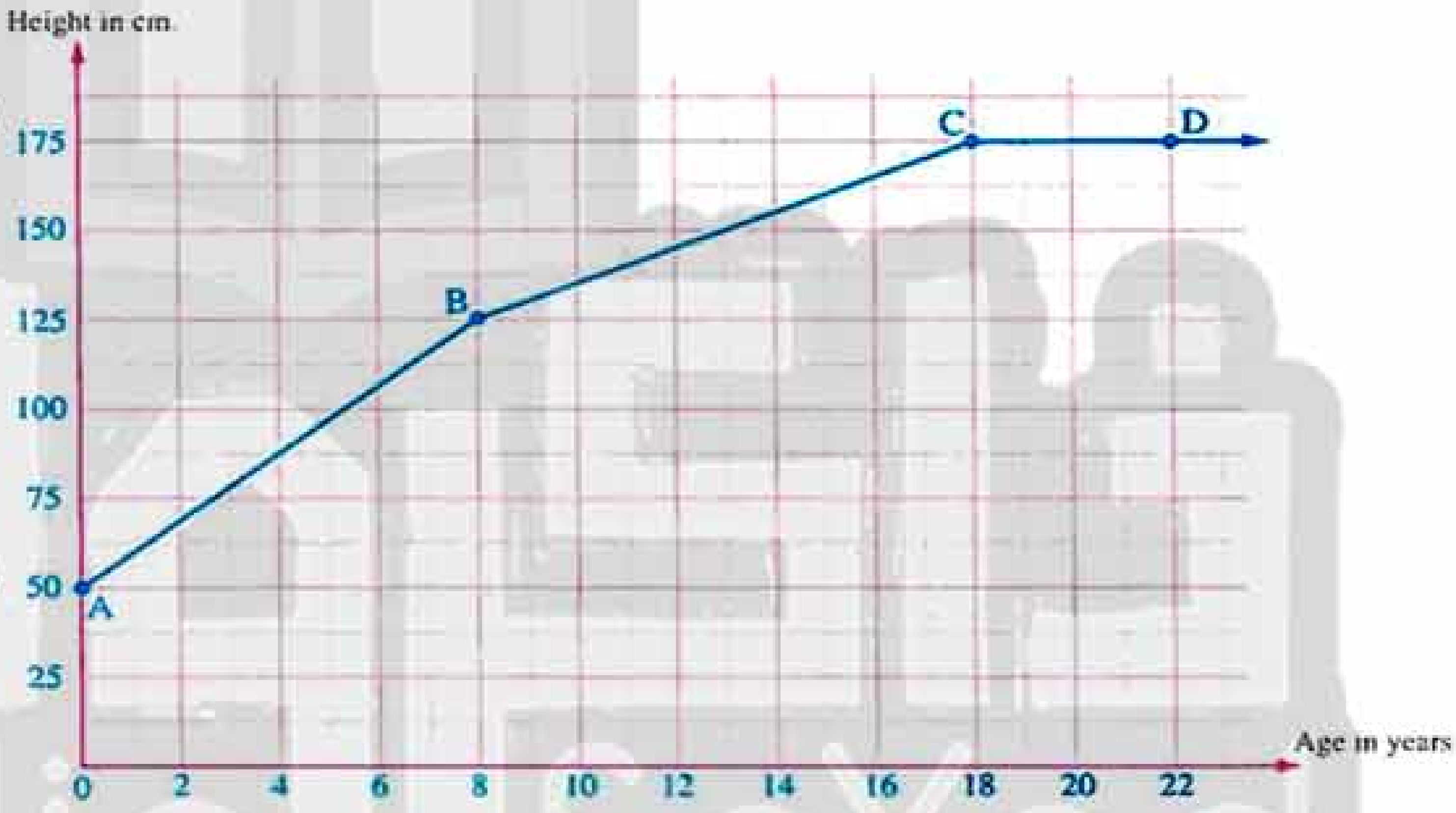


« 10 , 0 , – 5 , 20 thousand pounds »

9 **The opposite figure shows the relation between the height of a person (in cm.) and his age (in years) :**

1. Find the slope of each of $\overleftrightarrow{AB}$, $\overleftrightarrow{BC}$ and $\overleftrightarrow{CD}$

   What is the meaning of each ?

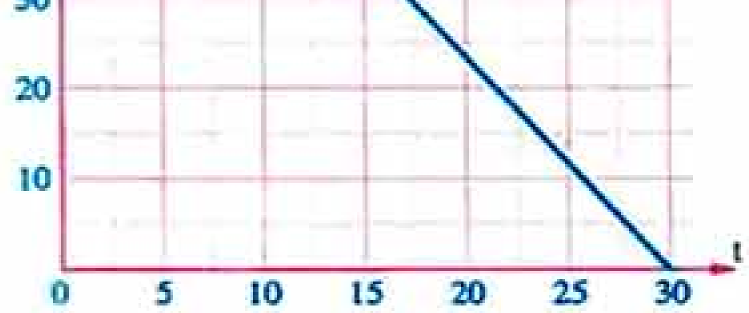


2. Calculate the difference between the height of this person when he was 8 years old and his height when he was 30 years old.

« $9\frac{3}{8}$, 5 , 0 , 50 cm. »

10 **Magdi filled the tank of his car by fuel. The opposite figure represents the relation between the time (t) in hours and the amount of remained fuel in the tank (y) in litres :**

1. What is the greatest capacity of the tank ?
2. When will the tank become empty ?
3. What is the amount of remained fuel after 15 hours ?
4. What is the range of consumption of fuel in each hour ?

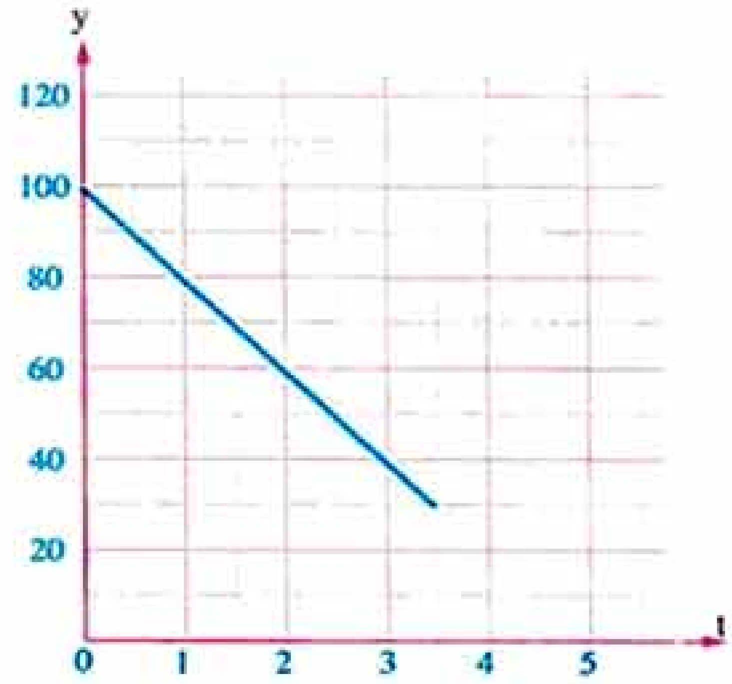


« 70 L , 30 hr. , 35 L , $2\frac{1}{3}$ L /hr. »

Exercise 13

11 **A person read a book.**
The opposite graph shows the relation between the time (t) in hours and the number of remained pages (y) :

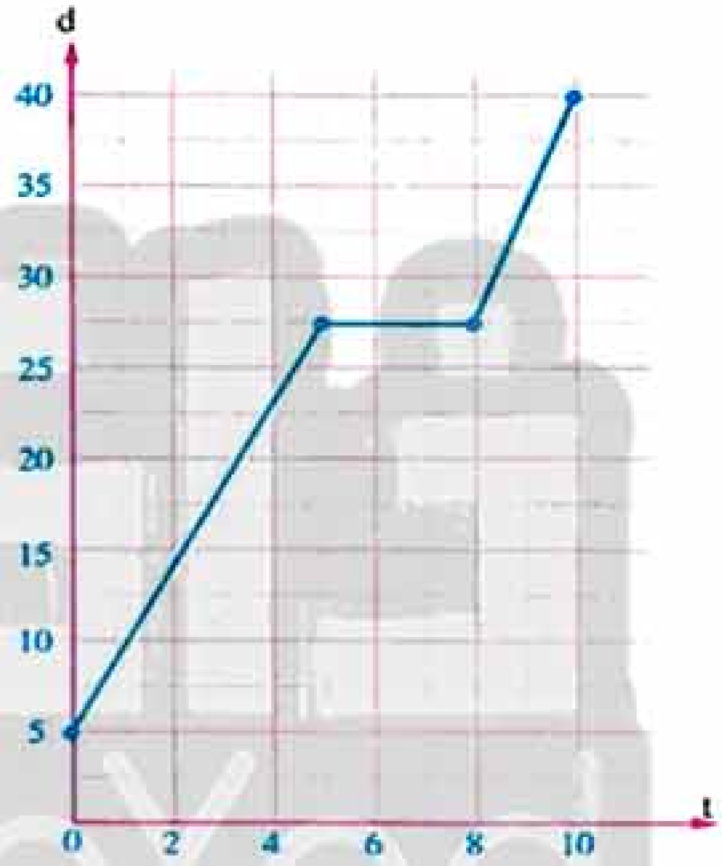
1. How many pages are remained in the beginning ?
2. Find the rate of reading pages per hour.
3. When does this person finish reading this book ?



« 100 pages , 20 pages/hr. , after 5 hours »

12 **A farmer wanted to complete digging a well in his farm. He rented a digging machine. The opposite graph shows the depth of the well (d) in metres after time (t) in hours , find :**

1. The depth of the well before beginning digging.
2. The depth of the well after finishing digging.
3. The total time which the machine took in digging the well.
4. The average of depth of the well which the machine digs within the first five hours.
5. The average of the depth of the well within the last two hours of digging.



« 5 m. , 40 m. , 10 hr. , 4.5 m./hr. , 6.25 m./hr. »

13 **The opposite graph shows the relation between the distance in km. and the time (t) in hours for a bicycle which moved between two towns A and B going and returning back.**
Answer the following :

1. What is the uniform velocity during the going trip ?
2. What is the average velocity during returning back ?
3. What is the meaning of the horizontal line segment in the graph ?

« 20 km./hr. , 12 km./hr. »

Unit 2

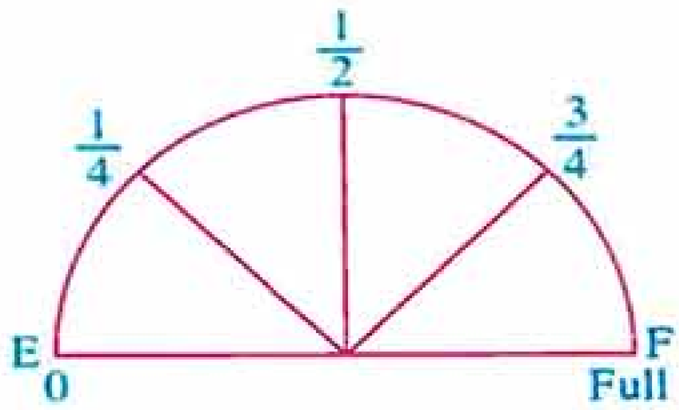
14 Hazem filled up the 40 L tank of his car.

After covering a distance of 120 km. , the fuel gauge shows that the rest of fuel is $\frac{3}{4}$ of the tank.

Draw a diagram to show the relation between the amount of fuel in the tank and the covered distance

(This relation is linear).

Calculate the covered distance until the tank totally gets empty.



« 480 km. »

15 The opposite diagram shows the relation between the covered distance (in km.) and the elapsed time (in hr.) for two trains A and B between two railway stations.

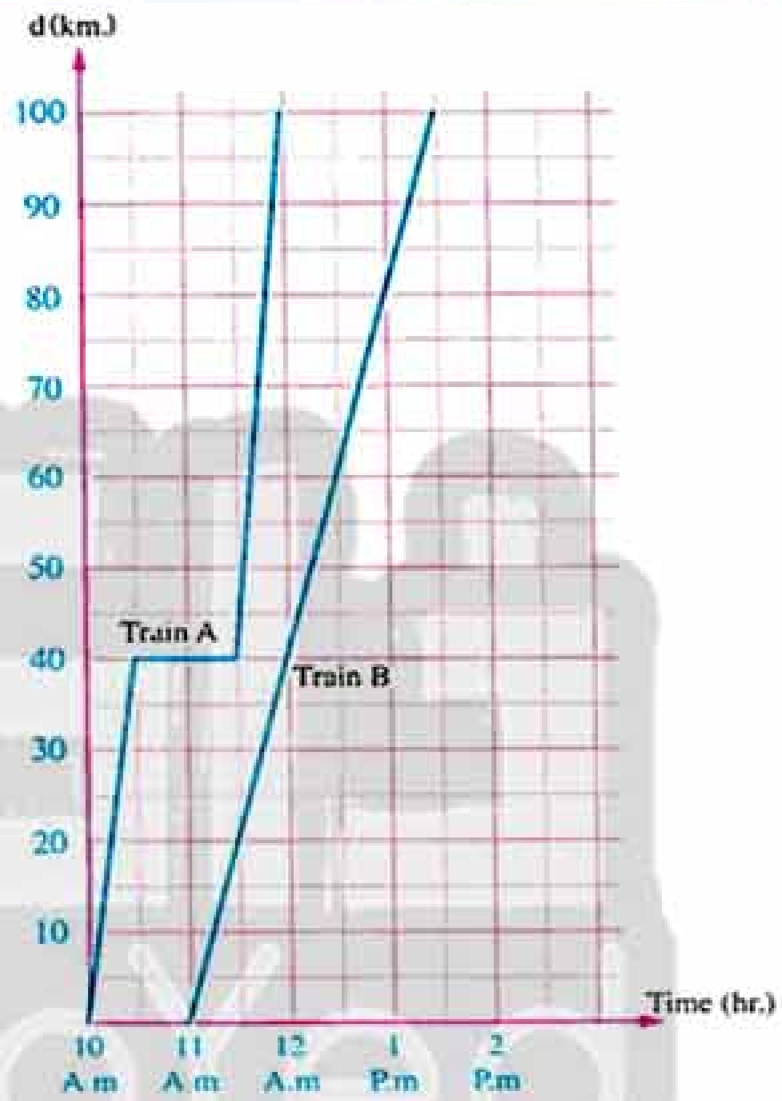
Use the diagram to find :

1 The distance between the two railway stations.

2 The elapsed time of each train.

3 The average speed of each train.

4 The meaning of the horizontal segment in the diagram of train A



« 100 km. , 2 hr. , 2.5 hr. , 50 km./hr. , 40 km./hr. »

16 **The opposite graph shows the race of 100 metres between a rabbit and a tortoise.**

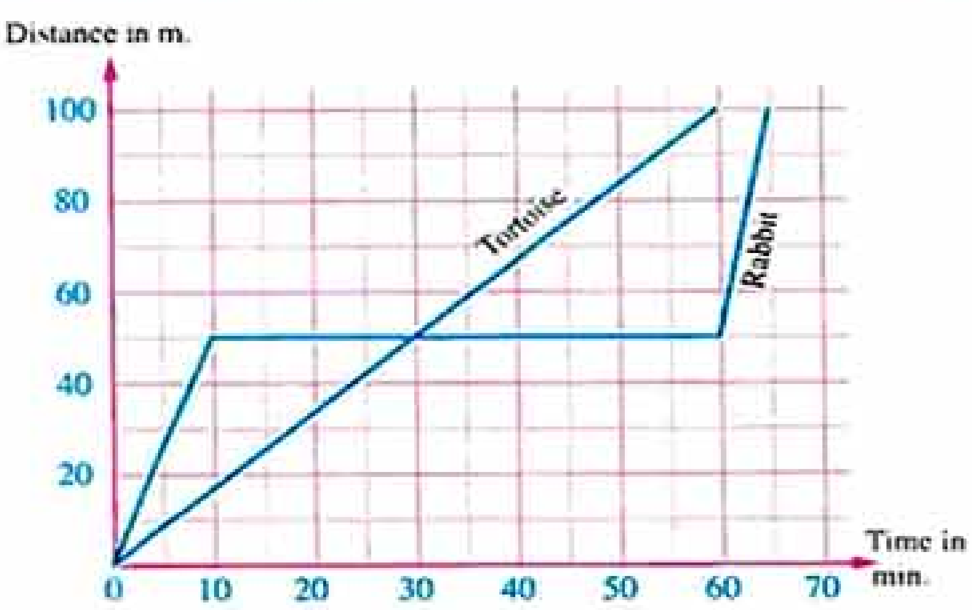
Answer the following :

1 Which of them is the winner ?

2 What is the velocity of the tortoise ?

3 What is the average velocity of the rabbit ?

4 What is the meaning of the horizontal line segment in the graph ?



« tortoise , $1\frac{2}{3}$ m./min. , $1\frac{7}{13}$ m./min. »

For excellent pupils

17 **During the motion of a bicycle with a uniform velocity in a straight line , the distances between the bicycle and a fixed point have been registered after periods measured in hours from the moment of beginning the motion in the following table :**

The distance between the bicycle and the fixed point	125	150	175	200
The passed time in hours	2	4	6	8

Graph the relation between the distance between the bicycle and the fixed point and the passed time. **From the graph , find :**

1. The velocity of the bicycle in km./hr.
2. The distance between the bicycle and the fixed point after 300 minutes.
3. The time at which the bicycle is at a distance = 187.5 km. from the fixed point.
4. The distance between the starting point of the bicycle and the fixed point.

« 12.5 km./hr. , 162.5 km. , 7 hr. , 100 km. »

Summary of Unit 2

- The linear relation is a relation of the first degree between two variables x and y , it is in the form : $a x + b y = c$ where a , b and c are real numbers , a and b are not both equal to zero , and there is an infinite number of ordered pairs which satisfy this relation , and it is represented graphically by a straight line.
- To graph a linear relation , you need to graph at least two ordered pairs satisfying this relation , you can add a third ordered pair to check that the three points lie on the same straight line which is the graphic representation of the relation.
- The relation : $y = 0$ is represented by x-axis.
- The relation : $x = 0$ is represented by y-axis.
- The linear relation $a x + b y = 0$ is represented graphically by a straight line passing through the origin point.
- The slope of the straight line $= \frac{\text{the change in y-coordinates}}{\text{the change in } x\text{-coordinates}} = \frac{\text{the vertical change}}{\text{the horizontal change}}$

  i.e. $S = \frac{y_2 - y_1}{x_2 - x_1}$ where $x_1 \neq x_2$
- The slope of the straight line parallel to x-axis equals zero
- The slope of the straight line parallel to y-axis is undefined.

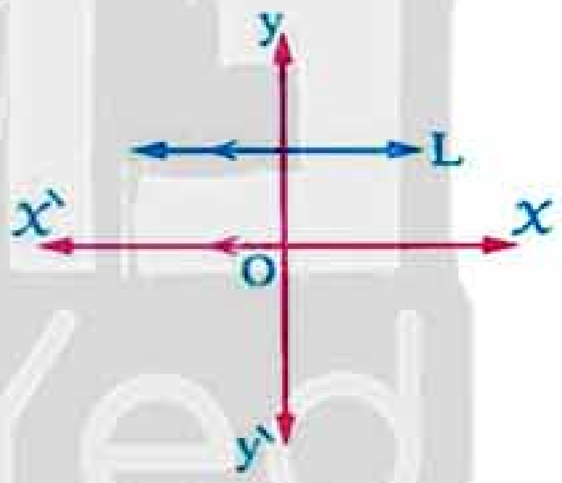
Exams on Unit Two

Model 1

Answer the following questions :

1 Choose the correct answer from those given :

1. Which of the following ordered pairs satisfies the relation : $2x + y = 5$?
   (a) $(-3, -1)$ (b) $(3, 1)$ (c) $(1, 3)$ (d) $(2, 2)$
2. If $(2k, k)$ satisfies the relation : $y + 2x = 5$, then $k = $ ....
   (a) 1 (b) 2 (c) 3 (d) 4
3. The slope of the straight line passing through the two points $(2, 3)$, $(-5, 3)$ is ....
   (a) 2 (b) 1 (c) 0 (d) undefined.
4. The relation : $x - 3 = 0$ is represented by a straight line of slope ....
   (a) 0 (b) undefined (c) 5 (d) -5
5. **In the opposite figure :**
   The slope of the straight line L is ....
   (a) positive. (b) negative.
   (c) 0 (d) undefined.



6. $(3, 1)$ does not satisfy the relation ....
   (a) $y + x = 4$ (b) $2x - y = 5$
   (c) $3y + x = 4$ (d) $4y + 2x = 10$

2 Complete the following :

1. The relation : $3x + 4y = 12$ is represented by a straight line intersecting the x-axis at the point ....
2. If the slope of the straight line passing through the two points $(3, y)$, $(5, -2)$ is -3, then $y = $ ....
3. If $(-1, 5)$ satisfies the relation : $3x + ky = 7$, then $k = $ ....
4. The slope of the straight line that is parallel to the y-axis is ....
5. If the straight line : $ax + by + c = 0$ passes through the origin point, then $c = $ ....

3 **[a]** Represent graphically the relation : $2x + y = 4$

[b] Prove that the points A $(4, 3)$, B $(1, 1)$ and C $(-5, -3)$ are collinear.

Unit 2

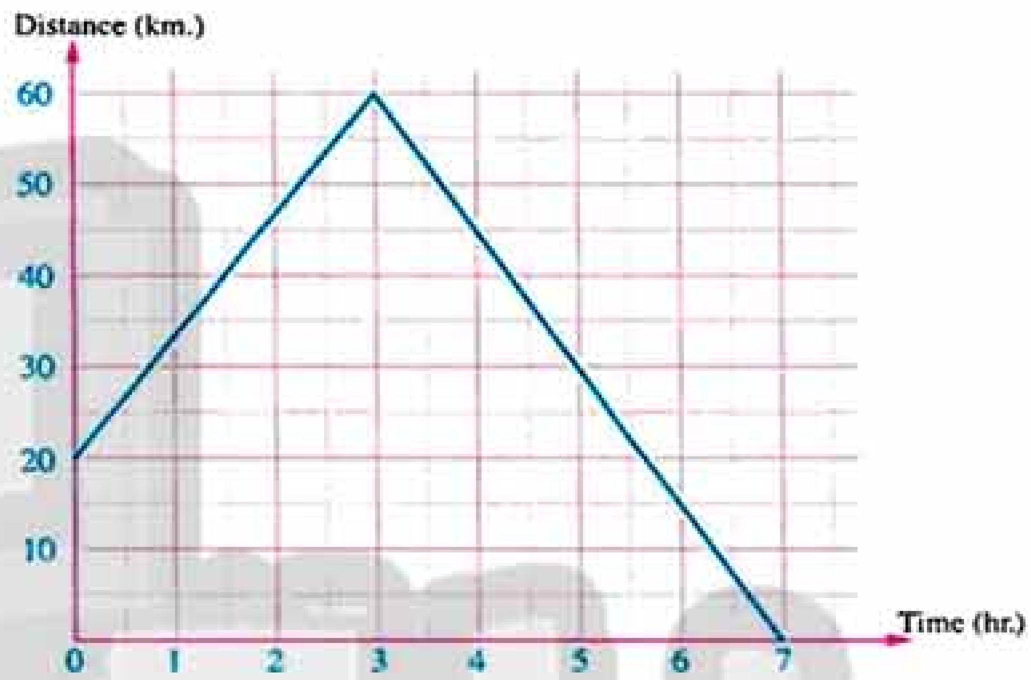
4 [a] Represent graphically the straight line that represents the relation : $2y - 3x = -6$ and if the straight line intersects the x-axis at the point A and intersects the y-axis at the point B , find the area of Δ OAB where O is the origin point.

[b] Find the value of y such that the straight line passing through the two points $(4, -1)$, $(-2, 2y)$ is perpendicular to the y-axis.

5 The opposite figure represents the movement of a bicycle from a fixed point.

Find :

1. The velocity of the bicycle during the first three hours.
2. The velocity of the bicycle during the next four hours.
3. The total distance.

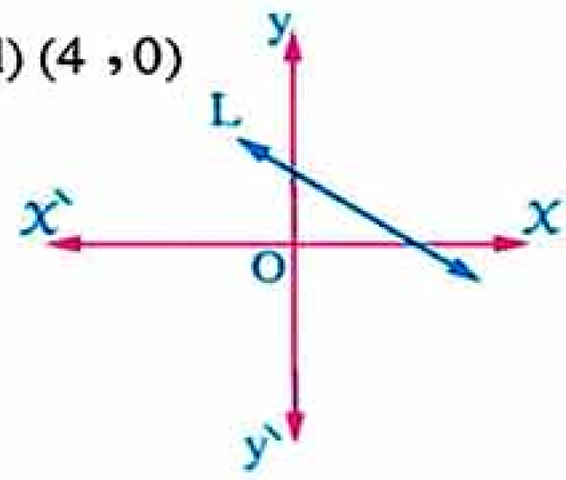


Model 2

Answer the following questions :

1 **Choose the correct answer from those given :**

1. The ordered pair which does not satisfy the relation : $y = x + 1$ is ....
   (a) $(0, 1)$ (b) $(2, 3)$ (c) $(1, 2)$ (d) $(2, 5)$
2. If $(5, 2m)$ satisfies the relation : $y = 3x - 1$, then m = ...
   (a) 2 (b) 7 (c) 10 (d) 14
3. If the slope of the straight line representing the relation $x + my = 5$ is undefined , then m = ...
   (a) 1 (b) -1 (c) 5 (d) zero.
4. The relation : $2x + 3y = 12$ is represented by a straight line intersecting the y-axis at the point ....
   (a) $(6, 0)$ (b) $(0, 6)$ (c) $(0, 4)$ (d) $(4, 0)$
5. **In the opposite figure :**
   The slope of the straight line L is ....
   (a) positive. (b) negative.
   (c) zero. (d) undefined.



6 The slope of the straight line $\overleftrightarrow{yy'}$ is ....

(a) zero. (b) undefined. (c) 1 (d) – 1

2 Complete the following :

1 The slope of the straight line parallel to X-axis is ....

2 If (2 , –1) satisfies the relation : $2X + 3y + c = 0$, then c = ...

3 The straight line which represents the relation :
$y = 2X + 5$ intersects X-axis at the point ....

4 The relation : $X - 5 = 0$ is represented by a straight line whose slope is ....

5 If the slope of $\overleftrightarrow{AB}$ = the slope of $\overleftrightarrow{BC}$, then A , B and C are ....

3

[a] Represent graphically the relation : $y - 2X + 1 = 0$

[b] If the straight line which represents the relation : $X - 2y = a$ intersects y-axis at the point (b , 3) , then find the value of each of a and b

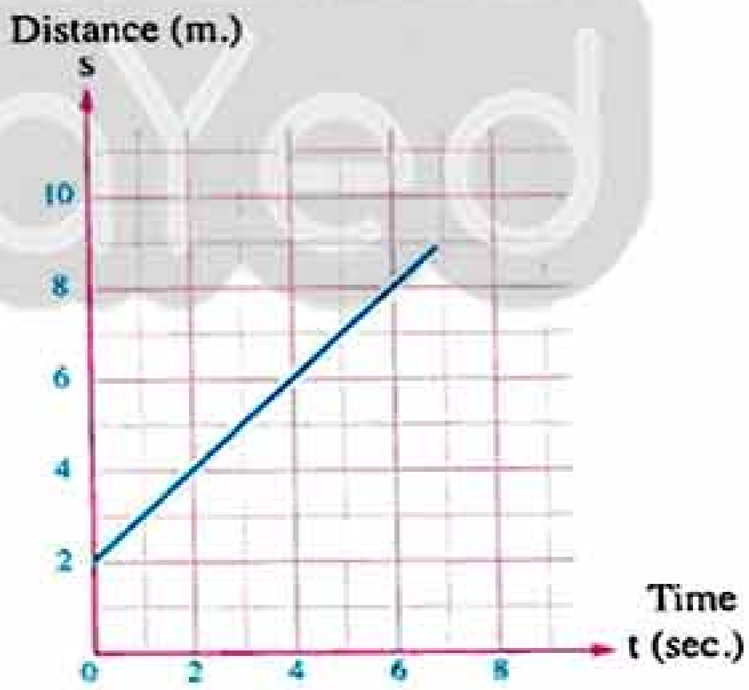
4

[a] If the slope of the straight line which passes through the two points (3 , a) and (5 , 4) equals 3 , find the value of a

[b] Prove that the points A (2 , – 3) , B (4 , – 5) and C (0 , – 1) are collinear.

5

The opposite graph represents the relation between the distance (s) in metres which a particle away from the observer and the elapsed time(t) in seconds.



First : Find the distance between the particle and the observer :

1 at beginning the motion.

2 after t = 6 sec.

Second : Find the slope of the straight line which represents the relation.

UNIT 3 Statistics

Exercises of the unit :

14. Collecting and organizing data.
15. The ascending and descending cumulative frequency tables and their graphical representation.
16. Mean.
17. Median.
18. Mode.

- Summary of unit three.
- Unit exams.

Exercise 14

Collecting and organizing data

From the school book

1 The following are the weights of 40 pupils of one class of the second year preparatory in kg. The required is forming the frequency table with sets.
Use the subsets (25– , 30– , 35– , ...) :

36	30	42	37	25	34	35	28	30	28
29	36	38	32	44	39	34	36	35	30
30	35	30	38	27	41	33	39	31	36
36	33	37	31	43	35	40	31	39	45

2 The following are the weekly wages of 40 workers in a factory in L.E. :

47	71	36	94	54	64	87	89	62	57
51	61	44	52	70	66	56	32	69	36
79	48	77	90	65	99	96	67	60	55
95	75	81	84	78	38	49	94	48	59

Required : Form a frequency table with sets (use the subsets : 30– , 40– , 50– , ... , 90–)
What is the set with the highest frequency? What is the set with the lowest frequency ?

3 The following are the scores of 30 students in a monthly math exam :

25	35	40	20	30	37	40	33	22	38
35	36	28	37	39	28	32	26	29	37
23	34	35	36	29	38	40	35	37	31

1 From a frequency table with sets for these scores.
2 Find the total number of excellent students. The excellence rate is 36 marks or more.
« 12 students »

Unit 3

4 **The following are the marks of the students in a class in the second year preparatory in algebra exam. Given that their number is 40 students and the full mark is 20 marks :**

7	11	7	13	14	3	18	13	10	14
16	8	15	12	5	15	11	12	6	11
8	9	15	8	15	14	7	10	14	19
10	7	2	10	12	4	11	17	13	15

The required is forming a frequency table with sets for the marks of students in algebra using the subsets 0– , 4– , 8– , ... and so on , then find the percentage of the number of students who obtained 12 marks at least. « 47.5% »

5 **The following are the heights of 50 persons in centimetres :**

155	183	163	181	186	144	199	150	182	166
197	126	188	158	153	130	163	166	154	173
137	163	146	198	164	156	173	177	157	118
138	187	178	173	184	143	147	142	176	160
170	194	154	167	149	112	196	128	126	156

Using the previous data :

1. Find the least height in these data and the greatest height and the range in which these two heights lie. « 112 cm. , 199 cm. , 87 cm. »
2. Form a frequency table using sets of length 10 centimetres for each.

6 **In a military camp , the heights of 55 soldiers were measured in centimetres , their measures were as follows :**

169	194	200	185	165	188	166	186	181	176	173
177	179	188	170	193	180	173	173	184	192	167
182	168	186	189	171	179	172	175	175	181	166
185	177	175	165	190	172	177	178	184	166	174
178	177	172	174	175	179	195	176	189	187	189

Form a frequency table using the sets (165– , 170– , 175– , ·····)

From the table , find :

1. The number of soldiers whose heights are less than 185 cm. « 39 soldiers »
2. The number of soldiers whose heights are 180 cm. at least. « 22 soldiers »

Exercise 15 The ascending and descending cumulative frequency tables and their graphical representation

From the school book

First Problems on the ascending cumulative frequency curve

1 **The following table shows the frequency distribution of the scores of 50 students in an experimental math exam :**

Sets	2–	6–	10–	14–	18–	22–	26–	Total
Frequency	3	5	9	10	12	7	4	50

Graph the ascending cumulative frequency curve.

2 **The following frequency table represents the marks of 60 pupils in math :**

Sets	10–	20–	30–	40–	50–	Total
Frequency	9	11	13	17	10	60

Graph the ascending cumulative frequency curve and if the success mark is 30 marks , find the number of failed pupils. « 20 pupils »

3 **The following table shows the frequency distribution of 100 factories according to the number of weekly work hours :**

Sets of hours	50–	60–	70–	80–	90–	100–	Total
Number of factories	5	16	30	22	15	12	100

1 Graph the ascending cumulative frequency curve of this distribution.

2 From the graph , find the number of factories which work less than 75 hours in the week. « 37 factories »

3 Find the percentage of the number of factories which work less than 75 hours in the week. « 37 % »

Second Problems on the descending cumulative frequency curve

4 **The following table shows the frequency distribution of the daily wages of some workers :**

Sets	5–	10–	15–	20–	25–	30–	Total
Frequency	10	14	24	30	12	10	100

Graph the descending cumulative frequency curve.

5 **A class has 50 pupils , the following table shows the distribution of studying hours among them every day :**

Sets	1–	2–	3–	4–	5–	6–	7–	Total
Freq.	2	3	5	12	15	7	6	50

1 Graph the descending cumulative frequency curve of this distribution.

2 From the graph , find the number of pupils who study 6 hours or more daily.

« 13 pupils »

3 Find the percentage of the number of pupils who study 6 hours or more daily. « 26 % »

6 **The following table shows the frequency distribution of a group of 60 persons according to their weights in kg. :**

Sets of weights in kg.	55 –	60 –	65 –	70 –	75 –	80 –	85 –	Total
No. of persons	8	12	18	....	7	3	2	60

Complete the table , then graph the descending cumulative frequency curve of this distribution and from the graph , find the number of persons who weigh 68 kg. or more for each.

« 28 persons »

Third Problems on the two curves together

7 **Graph the ascending and descending curves for the following frequency distribution :**

Sets	8–	12–	16–	20–	24–	28–	32–	36–	40–	Total
Freq.	4	7	12	18	20	19	11	6	3	100

8 **The following table shows the frequency distribution of the scores of 1000 students in a final year exam :**

Percentage	20 –	30 –	40 –	50 –	60 –	70 –	80 –	90 –	Total
Number of students	30	70	160	260	150	130	110	90	1000

1. Graph the ascending and descending cumulative frequency curves.
2. Find the number of students whose scores are less than 75% « 740 students »
3. Find the number of students whose scores are 85% or more. « 140 students »

9 **The following are the scores of 100 students in an experimental math exam :**

Sets	0 –	10 –	20 –	30 –	40 –	50 –	Total
Frequency	8	14	15	28	23	12	100

1. Form both the ascending and descending cumulative frequency tables.
2. Graph both the ascending and descending cumulative frequency curves on the same graph paper.
3. From the graph , find the number of students who got less than 40 marks and those who got 40 marks or more. « 65 students , 35 students »
4. Find the percentage of the number of students who succeeded given that the success mark is 20 marks. « 78 % »
5. Find the percentage of the number of students who got 45 marks or more. « 23 % »

For excellent pupils

10 **A factory has 120 workers whose experiences are from 5 years to 35 years.**

The opposite table shows the descending cumulative frequency distribution for those workers according to the years of experience :

Lower boundaries of sets	Descending cumulative frequency
5 and more	120
10 and more	113
15 and more	93
20 and more	64
25 and more	27
30 and more	12
35 and more	0

1. Deduce from the table the frequency table.
2. Form the ascending cumulative frequency table.
3. Graph the ascending cumulative frequency curve.
4. From the graph , deduce the number of workers whose experience years are less than 17.5 years. « 40 workers »

Exercise 16 Mean

From the school book

1 Complete the following :

1. The mean of a set of values = $\dfrac{\cdots\cdots\cdots\cdots}{\cdots\cdots\cdots\cdots}$

2. The centre of the set = $\dfrac{\cdots\cdots + \cdots\cdots}{2}$

3. The arithmetic mean of the values : 5 , 12 , 17 , 6 is ....

4. If the lower limit of a set is 8 and the upper limit of the same set is 14 , then its centre is ....

5. If the lower limit of a set is 4 and its centre is 9 , then its upper limit is ....

6. If the mean of a frequency distribution is 39.4 and the total of frequencies is 100 , then the total of the products of frequencies of the sets by their centres is ....

2 Choose the correct answer from those given :

1. The mean of the values : 2 – a , 4 , 1 , 5 , 3 + a is ....
(a) 1 (b) 2 (c) 3 (d) 15

2. If the mean of marks of 5 pupils is 20 , then the sum of their marks is .... marks.
(a) 4 (b) 15 (c) 25 (d) 100

3. The centre of the first set of the sets : 7 – , 13 – , 19 – , 25 – is ....
(a) 6 (b) 7 (c) 10 (d) 13

4. If the upper limit of a set is 14 and its centre is 10 , then its lower limit is ....
(a) 5 (b) 6 (c) 20 (d) 24

5. If the beginning of a set is 5 and its centre is 7.5 , then the length of the set is ....
(a) 5 (b) 7.5 (c) 10 (d) 12.5

3 **Find the mean of the following frequency distribution :**

Sets	5 –	15 –	25 –	35 –	Total
Frequency	6	8	4	2	20

« 21 »

4 **The following table shows the frequency distribution of marks of 10 students in mathematics :**

Sets	10 –	20 –	30 –	40 –	50 –	Total
Frequency	1	2	4	2	1	10

1 Calculate the mean of marks of students.

2 If the mark of success is 30 , calculate the number of failed students.

« 35 marks , 3 students »

5 **The following table shows the frequency distribution of weekly wages of 100 workers in one factory :**

Sets	16 –	20 –	24 –	28 –	32 –	36 –	Total
Frequency	10	15	22	25	20	8	100

Calculate the mean. « 28.16 »

6 **The following table shows the frequency distribution of extra wages of 30 workers :**

Sets	15 –	25 –	35 –	45 –	55 –	65 –	75 –	Total
Freq.	2	3	5	8	6	4	2	30

Find the arithmetic mean. « 51 »

7 **The following table shows the frequency distribution of the heights of 120 students in centimetres :**

Height (in cm.)	140 –	144 –	148 –	152 –	156 –	160 –	Total
Frequency	12	20	38	22	17	11	120

Find the mean. « 151.5 cm. »

8 **The following table shows the frequency distribution of number of daily studying hours of 50 pupils in a class :**

Number of hours	1 –	2 –	3 –	4 –	5 –	6 –	7 –	Total
Number of pupils	2	3	5	12	15	7	6	50

1. Calculate the mean of the number of hours of study per day.
2. Find the number of pupils who study less than 4 hours daily. « 5.1 hours , 10 pupils »

9 **The following table shows the distribution of marks of 40 students in one exam :**

Sets	5 –	15 –	......	35 –	45 –	Total
Number of students	3	......	12	10	5	40

1. Complete the table.
2. Calculate the mean.
3. Find the number of students whose marks are not less than 35 marks.

« 31 marks , 15 students »

10 **The following table shows the frequency distribution of the weights of 30 children in kg. :**

Weight (kg.)	6 –	10 –	14 –	18 –	22 –	26 –	30 –	Total
Frequency	2	3	.....	8	6	4	2	30

Complete the table , then find the mean of this distribution. « 20.4 kg. »

11 **Using the following set frequency table (given that the sets are equal in range) :**

Sets	10 –	20 –	x –	40 –	50 –	60 –	Total
Frequency	10	17	20	32	k + 2	4	100

Find :

1. The value of each of x and k
2. The mean of this distribution. « $x = 30$, k = 15 , 39.1 »

12 **The following table shows the frequency distribution of weights of 50 pupils in kg. in one school :**

Weight in kg.	30 –	35 –	40 –	45 –	50 –	55 –	Total
Number of pupils	7	3 k	4 k	10	8	4	50

1 Calculate the value of k

2 Find the mean of this distribution. « 3 , 44 kg. »

13 **The following table shows the frequency distribution of 50 workers days-off :**

Sets	2 –	6 –	10 –	14 –	18 –	22 –	26 –	Total
Frequency	4	5	8	k – 2	7	5	1	50

Find :

1 The value of k

2 The mean. « 22 , 15.2 days »

14 If the mean of the scores of a student during the first 5 months is 23.8 , what is the score of the 6^{th} month if the mean of his scores is 24 marks ? « 25 marks »

15 If the mean of marks of Magdi in 4 exams is 16 marks , what is the mark which he should obtain in the fifth exam so that his mean in the five exams will be 18 marks ?

« 26 marks »

For excellent pupils

16 **The opposite table is for finding the mean of marks of m pupils in one exam :**

1 Deduce the value of each of : a , b , c , d , e , f , x , y , z and m

2 Find the mean of these marks.

Sets	Centres of sets	Frequency	Centres of sets × frequency
0 –	a	5	10
4 –	6	b	90
d –	c	30	300
12 –	e	z	y
16 –	f	10	x
Total		m	1140

Exercise 17

Median

From the school book

1 Complete the following :

1. The median of the values : 9 , 4 , 8 , 1 and 3 is ....
2. The median of the values : 3 , 7 , 2 , 9 , 5 and 11 is ....
3. The order of the median of the values : 7 , 6 , 5 , 8 and 4 is ....
4. If the order of the median of a set of values is the fourth , then the number of these values equals ....
5. If the median of the values : k + 1 , k + 2 , k + 5 , k + 4 and k + 3 where k is a positive integer is 13 , then k = ...
6. The point of intersection of the ascending and descending cumulative frequency curves determines .... on the set-axis.

2 Using the ascending cumulative frequency curve , find the median of the following frequency distribution :

Sets	0–	2–	4–	6–	Total
Frequency	1	2	2	5	10

« 6 »

3 **The following table shows the frequency distribution of 40 persons according to the percentage of intelligence of each of them :**

Sets of intelligence percentage	40 –	50 –	60 –	70 –	80 –	90 –	Total
Number of persons	1	3	8	14	10	4	40

Using the ascending cumulative frequency curve , find the madian of percentage of intelligence. « Approximately 75 % »

4 **The following table shows the frequency distribution of 100 factories according to the number of weekly working hours :**

Sets of hours	50 –	60 –	70 –	80 –	90 –	100 –	Total
Number of factories	5	8	12	28	33	14	100

Find using the descending cumulative frequency curve the median number of hours of work of these factories. « 89.5 hours »

5 **The following table shows the frequency distribution of 50 workers' wages in pounds :**

Sets of wages	300 –	400 –	500 –	600 –	700 –	Total
Number of workers	8	12	18	7	5	50

Graph the descending cumulative frequency curve , then find the median. « 520 pounds »

6 **The following table shows the frequency distribution of marks of 60 students in mathematics exam :**

Sets of marks	5 –	10 –	15 –	20 –	25 –	30 –	35 –	Total
Number of students	2	5	14	20	13	5	1	60

Find the median mark. « 22 marks »

7 **The following table shows the frequency distribution of weights of 20 children in kg. :**

Sets	5 –	15 –	25 –	35 –	45 –	Total
Frequency	3	4	7	4	2	20

Find the median weight in kg. using the ascending and descending cumulative frequency curves of this distribution. « 29 kg. »

8 **The following table shows the distribution of the students of a secondary school in a governorate according to their ages in years :**

Sets of ages in years	14 –	15 –	16 –	17 –	18 –	19 –	Total
Frequency	90	130	110	80	70	20	500

Graph the ascending and descending cumulative frequency curves of this distribution , then find the median age.

« 16.3 years »

9 **The following table shows the frequency distribution of the marks of 90 students in a monthly exam :**

Sets of marks	10 –	14 –	18 –	22 –	26 –	30 –	34 –	Total
Number of students	8	10	24	21	12	9	6	90

Find the median mark using the ascending and descending cumulative frequency curves.

« 22.5 marks »

10 **The following table shows the frequency distribution for the scores of 50 students in an examination :**

Sets	2 –	6 –	10 –	14 –	18 –	22 –	26 –	Total
Frequency	3	5	9	10	12	7	4	50

Find : 1 The mean of the student's score.

2 The median.

« 16.8 , 17.6 »

11 **From the following frequency table with equal sets in range :**

Sets	10 –	20 –	X –	40 –	50 –	60 –	Total
Frequency	10	17	20	32	k + 2	4	100

1 Find the value of each of X and k

« $X = 30$, k = 15 »

2 Graph the ascending and descending cumulative curves on one figure, then calculate the median.

« 41 »

Exercise **18**

Mode

From the school book

1 **Complete the following :**

1. The mode of a set of values is ....
2. The mode of the values : 5 , 3 , 8 , 5 , 9 is ....
3. The mode of the values : 8 , 7 , 8 , 7 , 6 , 5 , 8 is ....
4. If the mode of the values : 4 , a , 5 , 3 is 3 , then a = ...
5. If the mode of the values : 12 , 7 , $x + 1$, 7 , 12 is 7 , then x = ...
6. If the mode of the values : 4 , 11 , 8 , 2^x is 4 , then x = ...

2 **A factory has 600 workers. A sample of 120 workers is taken such that it represents the all groups very well. It is found that the distribution of their ages in years is as the following table :**

Age	25–	30–	35–	40–	45–	50–	Total
Number of workers	12	17	18	40	25	8	120

Draw the histogram , then deduce the mode age. « 43 years »

3 **The following table shows the frequency distribution of marks of 100 pupils in an exam :**

Sets of marks	10–	14–	18–	22–	26–	30–	34–	Total
Number of pupils	2	10	15	40	25	6	2	100

Find the mode mark using the histogram of this distribution. « 24.5 marks »

4 The following is the frequency distribution of 100 workers in one of the factories according to their daily wages :

Sets of wages in pounds	10 –	15 –	20 –	25 –	30 –	35 –	40 –	Total
Number of workers	6	12	16	24	20	14	8	100

Draw the histogram of this frequency distribution , then deduce the mode wage of the worker.

« 28.5 pounds »

5 Find the mode of the following frequency distribution for the scores of 40 students in an examination :

Sets of marks	30 –	40 –	50 –	60 –	70 –	80 –	Total
Frequency	3	4	12	8	7	6	40

« 57 »

6 The following is the frequency distribution of ages of 45 persons :

Sets of ages in years	12 –	14 –	16 –	18 –	20 –	22 –	24 –	Total
Number of persons	5	7	8	12	6	4	3	45

Find the mode age.

« 18.8 years »

7 The following table shows the frequency distribution of the heights of 200 students :

Height in cm.	110 –	115 –	120 –	125 –	130 –	135 –	140 –	Total
Number of students	10	12	28	35	60	40	15	200

Graph the frequency histogram , then find the mode height.

« 132.75 cm. »

8 The following table shows the frequency distribution of 102 cows according to the weekly amount of milk in galoons :

Sets of milk in galoons	14 –	16 –	18 –	20 –	22 –	24 –	Total
Number of cows	8	16	28	20	18	12	102

Use the histogram of this distribution to find the mode of the weekly amount of milk.

« 19.2 galoons »

9 **The following table shows the frequency distribution of marks of 100 pupils in mathematics at the end of the year :**

Marks	15 –	20 –	25 –	30 –	35 –	40 –	45 –	50 –	55 –	Total
Number of pupils	4	6	8	12	16	20	22	7	5	100

Graph the histogram of that distribution , then find the mode mark. « 45.5 marks »

10 **The following table shows the frequency distribution of the weights of 100 children in kg. :**

Weight in kg.	10 –	14 –	18 –	22 –	26 –	30 –	Total
Frequency	5	15	30	24	17	9	100

Find the mode weight. « 20.8 kg. »

11 **The following table shows the frequency distribution of the weights of 50 students in kg. :**

Weight in kg.	30 –	35 –	40 –	45 –	50 –	55 –	Total
Number of students	k + 4	3 k	4 k	3 k + 1	3 k – 1	k + 1	50

1 Find the value of k « 3 »

2 Graph the frequency histogram , then find the mode. « 43 kg. »

12 **The following table shows the frequency distribution with equal range sets for the weekly wages of 100 workers in a factory :**

Sets of wages in L.E.	70 –	80 –	90 –	100 –	x –	120 –	130 –
Number of workers	10	13	k – 4	20	16	14	11

Find : 1 The value of each of x and k « $x = 110$, k = 20 »

2 The mode of wages in L.E. « 105 pounds »

13 **The following is the frequency distribution of 100 workers of building according to the number of weekly working hours :**

Sets of working hours	35 –	45 –	55 –	65 –	75 –	85 –	Total
Number of workers	15	30	23	20	8	4	100

The required is finding :

1 The mean. « 58.8 hours »

2 The median. « 57.5 hours »

3 The mode. « 52 hours »

14 The following is the frequency distribution of the weekly bonus of 100 workers in a factory :

Bonus in L.E.	20 –	30 –	40 –	50 –	60 –	70 –
No. of workers	10	k	22	26	20	8

1 Calculate the value of k « 14 »

2 Find the mean of this distribution. « 50.6 pounds »

3 Find the mode value of the weekly bonus using the histogram. « 54 pounds »

15 The following table shows the frequency distribution for the weights of 50 students in kg. at a school :

Weight in kg.	30 –	35 –	40 –	45 –	50 –	55 –	Total
Number of students	7	3 k	4 k	10	8	4	50

1 Find the value of k « 3 »

2 Calculate the mean. « 44 kg. »

3 Draw the ascending cumulative frequency curve.

4 Draw the histogram and find the mode of weights. « 43 kg. »

5 Find the median. « 43.5 kg. »

Summary of Unit 3

- You can represent the frequency table with sets by the ascending or the descending cumulative frequency curves.
- The range is the difference between the greatest value and the smallest value.
- The mean of a set of values $= \dfrac{\text{The sum of values}}{\text{Number of values}}$
- The mean of frequency distribution with sets $= \dfrac{\text{The sum of } (\mathcal{X} \times f)}{\text{The sum of } f}$

  where f is frequency and $\mathcal{X}$ is the centre of the set and equals $\dfrac{\text{its lower limit + its upper limit}}{2}$
- The median is the middle value in a set of values after arranging it ascendingly or descendingly such that the number of values which are less than it is equal to the number of values which are greater than it ,

  if the values number is odd , then the median is the value lying in the middle exactly ,
  if the values number is even , then the median $= \dfrac{\text{The sum of the two values lying in the middle}}{2}$
- The intersection point of the ascending and the descending cumulative frequency curves determines the median on the sets axis.
- The mode of a set of values is the most common value in the set , or it is the value which is repeated more than any other values.

Exams on Unit Three

Model 1

Answer the following questions :

1 Choose the correct answer from those given :

1. The median of the values : 15 , 22 , 9 , 11 , 33 is ....
   (a) 9 (b) 15 (c) 18 (d) 90
2. The arithmetic mean of the values : 19 , 32 , 27 , 6 , 6 is ....
   (a) 90 (b) 32 (c) 18 (d) 6
3. It the mode of the values : 4 , 5 , a , 3 is 3 , then a = ...
   (a) 3 (b) 4 (c) 5 (d) 6
4. If the median of the values : k + 1 , k + 2 , k + 5 , k + 4 , k + 3 is 13 , then k = ...
   (a) 2 (b) 5 (c) 10 (d) 13
5. If the arithmetic mean of the marks of five pupils is 30 , then the sum of their marks equals .... marks.
   (a) 15 (b) 6 (c) 100 (d) 150

2 Complete the following :

1. If the order of the median of a set of values is the fifth , then the number of values equals ....
2. If the mode of the values : 15 , 9 , $x + 6$, 9 , 15 is 9 , then x = ...
3. The point of intersection of the ascending and descending cumulative curves determines .... on the horizontal axis.
4. If the arithmetic mean of the values : 1 , 6 , 4 , 4 , 5 k is 7 , then k = ...
5. The centre of the set whose lower boundary is 2 and its upper boundary is 6 , is ....

3 **The following table shows the frequency distribution of marks of 10 students in a methematics exam :**

Sets	10 –	20 –	30 –	40 –	50 –	Total
Frequency	1	2	4	2	1	10

1. Find the arithmetic mean of marks.
2. If the mark of success is 30 , find the number of failure students.

4 **Find using the following frequency distribution :**

Sets	0 –	2 –	4 –	6 –	k –	Total
Frequency	m	5	8	7	6	30

1. The values of k and m.
2. The median using the ascending cumulative frequency curve.

5 **Find the mode of the following frequency distribution of marks of 40 students in an exam :**

Sets of marks	30 –	40 –	50 –	60 –	70 –	80 –	Total
Frequency	4	8	12	7	5	4	40

Model 2

Answer the following questions :

1 **Choose the correct answer from those given :**

1. The order of the median of the values : 4 , 5 , 6 , 7 and 8 is the ....
   (a) third. (b) fourth. (c) fifth. (d) sixth.
2. If the arithmetic mean of the values : 18 , 23 , 29 , 2 k – 1 and k is 18 , then k = ...
   (a) 1 (b) 7 (c) 29 (d) 90
3. The mode of the values : 14 , 11 , 10 , 11 , 14 , 15 and 11 is ....
   (a) 14 (b) 10 (c) 11 (d) 15
4. The arithmetic mean of the values : 3 – a , 5 , 1 , 4 and 2 + a equals ....
   (a) 1 (b) 2 (c) 3 (d) 15
5. If the centre of a set is 10 and its lower boundary is 4 , then its upper boundary is ....
   (a) 10 (b) 4 (c) 7 (d) 16

Unit 3

2 **Complete the following :**

1 The point of intersection of the ascending and the descending cummulative frequency curves determines .... on the vertical axis.

2 The most common value of a set of values is called ....

3 If the arithmetic mean of a frequency distribution is 35.7 and the total of frequencies is 200 , then the total of the products of frequencies of each set by its centre is ....

4 If the order of the median of a set of values is the ninth , then the number of these values is ....

5 If the mode of the values : 13 , 7 , $x + 2$, 7 and 13 is 7 , then $x =$ ....

3 **Find the arithmetic mean of the following frequency distribution :**

Sets	5 –	15 –	25 –	35 –	45 –	Total
Frequency	3	10	12	10	5	40

4 **Find using the following frequency distribution :**

Sets	0 –	2 –	k –	6 –	8 –	Total
Frequency	3	4	7	m + 2	1	20

1 The values of k and m

2 The median using the descending cummulative frequency curve of this distribution.

5 **600 workers at a factory , a sample of 120 workers is chosen such that it represents the society completely to be found that their ages are distributed as the following table :**

Sets of ages	25 –	30 –	35 –	40 –	45 –	50 –	Total
Number of workers	12	16	18	40	25	9	120

Graph the histogram , then find the mode age.

TIMSS Problems

Accumulative basic skills

1 Complete the following :

1. A turtle covers 80 metres per hour , then it covers 8 metres in .... minutes.
2. The sum of the real numbers in the interval $[-12 , 12]$ equals ....
3. If $\bigcirc + \square = 20$, $\bigcirc + \bigcirc + \square = 35$, then $\bigcirc =$ ....
4. In three games of bowling , Sara gained 139 , 143 , 144 points , then the number of points she needs in the 4^{th} game so that the mean of points is 145 , is ....
5. Two boxes of apples , the sum of their weights is 54 kg. The first has 12 kg. more than the second , then the number of kilograms in the second box is .... kg.
6. $300 \div 200 = 1 \div$ ....
7. $(301 + 302 + 303 + \cdots + 325) - (1 + 2 + 3 + \cdots + 25) =$ ....
8. If four times a number is 48 , then $\frac{1}{3}$ this number is ....
9. Gamal has 3 sisters and 5 brothers , his sister Sara has x sisters and y brothers , then x y = ...
10. If a + b + c = 26 , a + b = 15 , b + c = 20 , then b = ...
11. Three girls can perform a work in 36 hours , then the needed hours for four girls to perform the same work is .... hours.
12. If

$$\begin{array}{cccc} & \square & \square & \bigcirc \\ & \square & \square & \triangle \\ + & \square & \triangle & \square \\ \hline 2 & 0 & 1 & 6 \end{array}$$

, then $\square =$ .... , $\triangle =$ .... , $\bigcirc =$ ....

Basic skills

2 Choose the correct answer from the given ones :

1. The number 3.015 lies on the number line between ....
   (a) $\frac{5}{2}$, 3 (b) $\frac{7}{2}$, $\frac{11}{3}$ (c) 3 , $\frac{16}{5}$ (d) 3.12 , 3.15

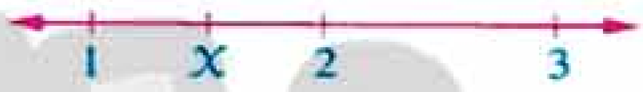
2. Which of the following numbers lies between 0.07 , 0.08 ?
   (a) 0.00075 (b) 0.0075 (c) 0.075 (d) – 0.75

3. Which of the following is different in value ?
   (a) $1 \div 9 + 9 - 1$ (b) $1 + 9 \div 9 - 1$ (c) $1 - 9 + 9 \times 1$ (d) $1 \times 9 - 9 + 1$

4. If x is a negative number , which of the following is a positive number ?
   (a) x^2 (b) x^3 (c) $2x$ (d) $\frac{x}{2}$

5. The greatest number of the following is ....
   (a) – 1.25 (b) – 0.125 (c) – 0.0125 (d) – 0.00125

6. The best estimation to the number opposite to x is ....



   (a) 1.1 (b) 1.2 (c) 1.5 (d) 1.7

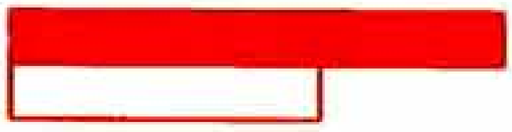
7. If 10% of x equals y , then x = ...
   (a) 0.1 y (b) y (c) 9 y (d) 10 y

8. If $x = (-2)^4$, $y = -2^4$, then ....
   (a) $x = y$ (b) $x > y$ (c) $x < y$ (d) $x \leq y$

9. $\sqrt{\sqrt{81 \times 81 \times 81 \times 81}}$ = ...
   (a) 3 (b) 9 (c) 27 (d) 81

10. For any number k , then $k + k + (k \times k \times k)$ can be written as ....
   (a) $2k^2 + 3k$ (b) $5k$ (c) k^5 (d) $2k + k^3$

11. A machine produces two kinds of rods , one is red and of length (10 ± 0.5) cm. and the other is white and of length (6 ± 0.5) cm.



   If we put two rods as shown in the opposite figure , then the smallest difference between their lengths may be ....
   (a) 4 cm. (b) 5 cm. (c) 3 cm. (d) 8.5 cm.

12. All numbers divisible by 4 and 15 are divisible by ....
   (a) 6 (b) 8 (c) 24 (d) 45

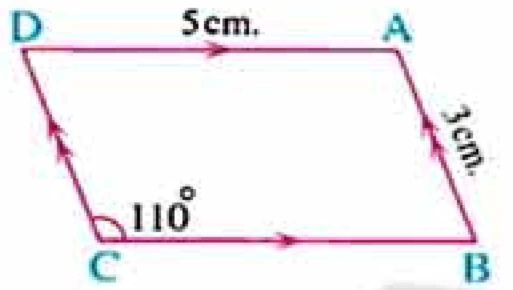
Second

Geometry

Revision

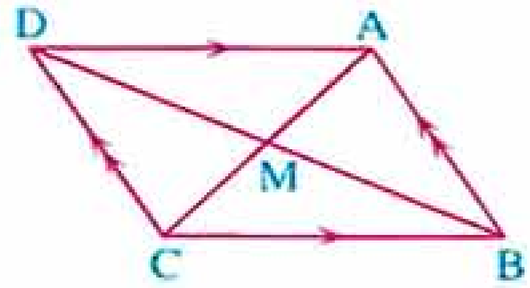
1 Complete the following using the given data of each figure :

1



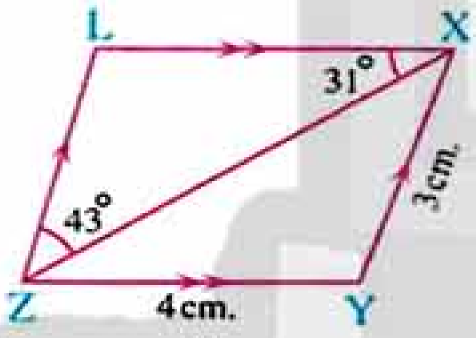
BC = ... cm. , CD = ... cm. ,
m (∠ A) = ...° and m (∠ D) = ...°

2 AC = 8 cm. and BM = 7 cm.



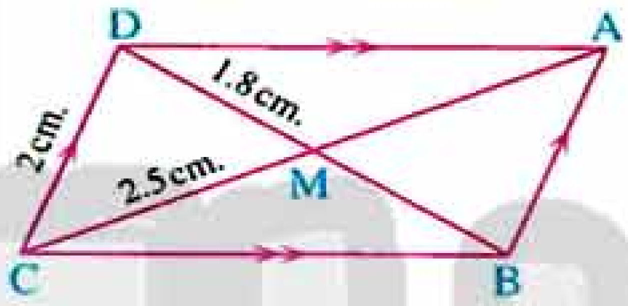
AM = ... cm. and
BD = ... cm.

3



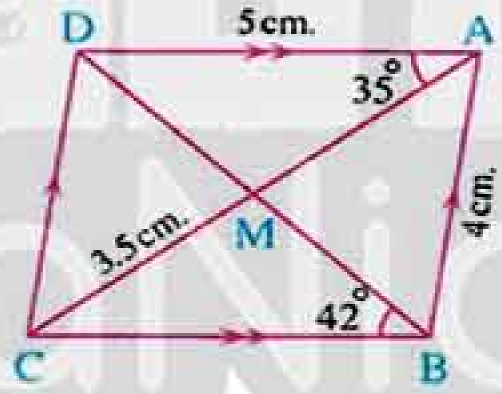
m (∠ Y) = ...°
The perimeter of ▱ XYZL = ... cm.

4



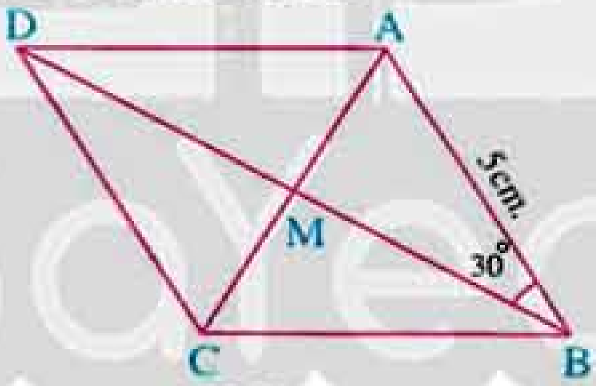
The perimeter of Δ ABM = ... cm.

5



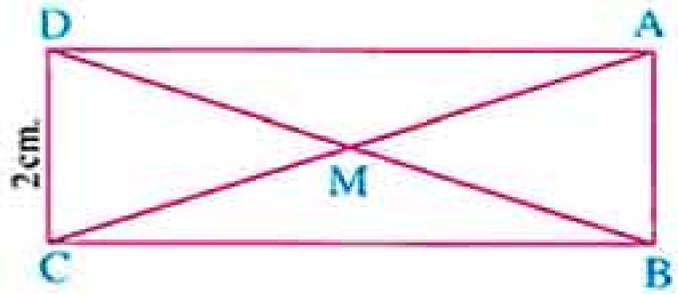
The perimeter of Δ ABC = ... cm. ,
m (∠ AMB) = ...°

6 ABCD is a rhombus



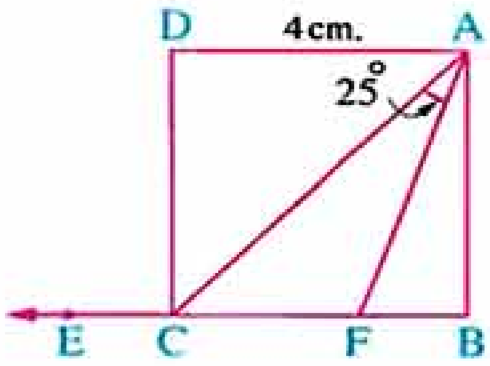
AD = ... cm. ,
m (∠ BAM) = ...°

7 ABCD is a rectangle and AC = 6 cm.



AB = ... cm. , DM = ... cm. ,
The perimeter of Δ ABM = ... cm.

8 ABCD is a square , $E \in \overrightarrow{BC}$

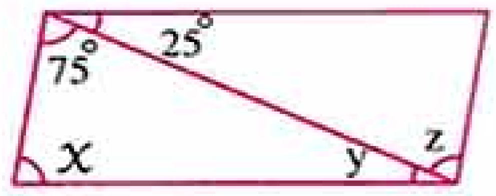


The perimeter of the square ABCD = ... cm. ,
m (∠ ACE) = ...° ,
m (∠ AFC) = ...°

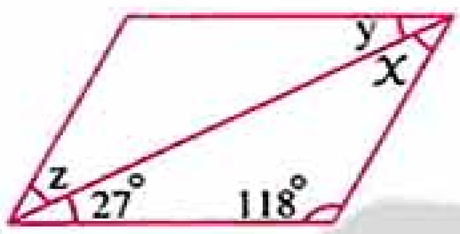
2 **Find the values of x, y and z in each of the following parallelograms :**

1

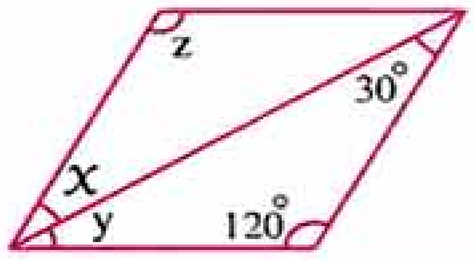
2



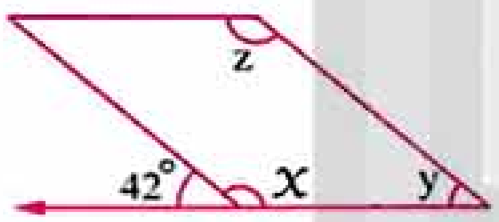
3



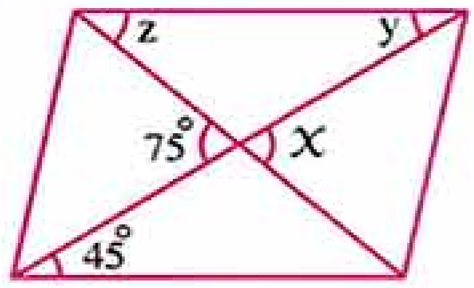
4



5



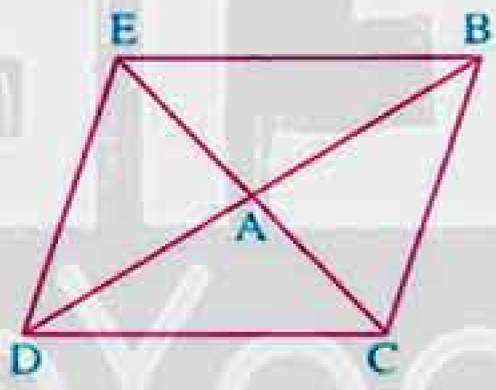
6



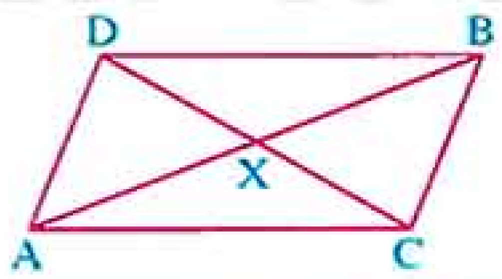
3 **Find the length of $\overline{AB}$ in each of the following parallelograms :**

1

2 BD = 15 cm.



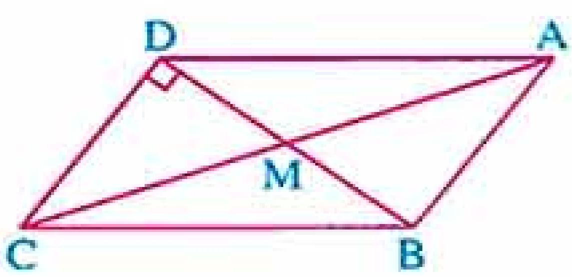
3 AX = 7 cm.



4 AB = $\frac{1}{3}$ AD

5 BC = $\frac{1}{2}$ AB

6 BC = 15 cm.
and BM = 6 cm.



4 **Find the values of x , y and z in each of the following figures :**

Rectangle

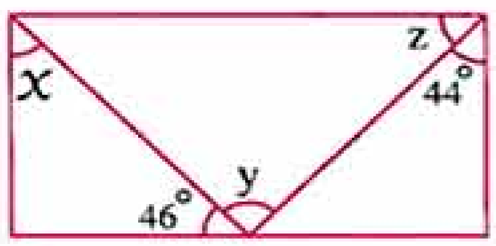


Fig. (1)

Square

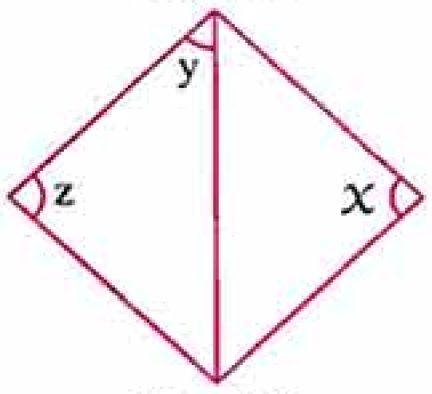


Fig. (2)

Rhombus

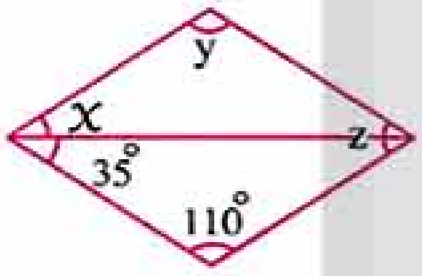


Fig. (3)

Rectangle

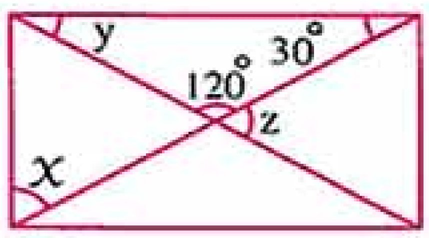
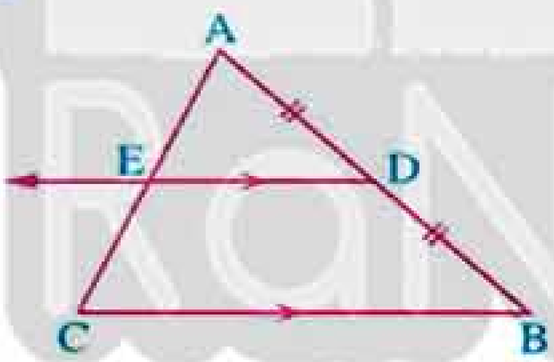


Fig. (4)

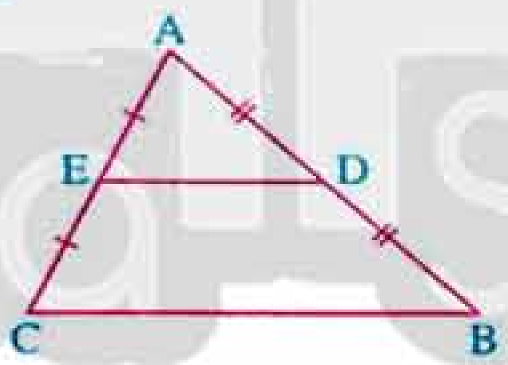
5 **Complete the following :**

1



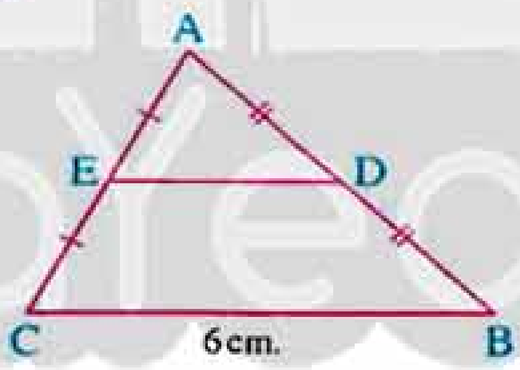
If D is the midpoint of $\overline{AB}$, $\overrightarrow{DE}$ // $\overline{BC}$, then .... is the midpoint of ....

2



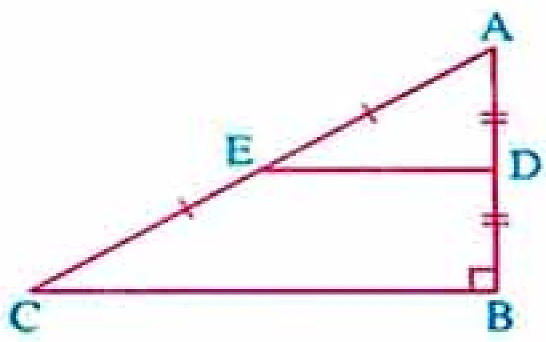
If D and E are the midpoints of $\overline{AB}$ and $\overline{AC}$ respectively , then .... // ....

3



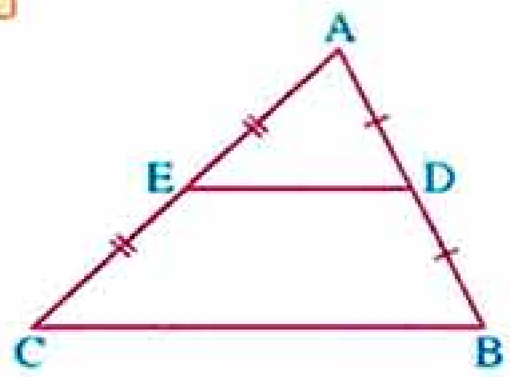
If D and E are the midpoints of $\overline{AB}$ and $\overline{AC}$ respectively , BC = 6 cm. , then DE = ... cm.

4



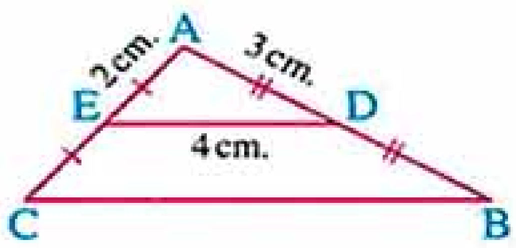
If m (∠ B) = 90° ,
D and E are the midpoints of $\overline{AB}$ and $\overline{AC}$ respectively ,
then m (∠ ADE) = ...°

5



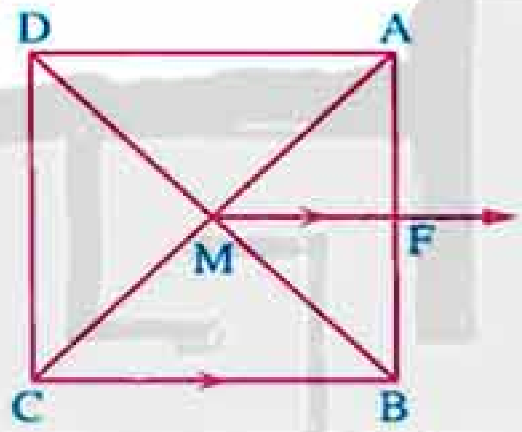
If D and E are the midpoints of $\overline{AB}$ and $\overline{AC}$ respectively , and the perimeter of Δ ABC = 24 cm. , then the perimeter of Δ ADE = ... cm.

6



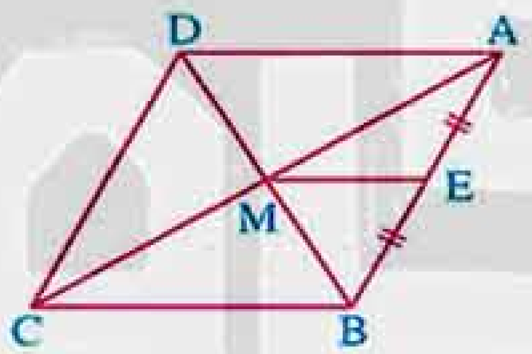
If D and E are the midpoints of $\overline{AB}$ and $\overline{AC}$ respectively , AD = 3 cm. , AE = 2 cm. and DE = 4 cm. , then the perimeter of the shape DBCE = ... cm.

7



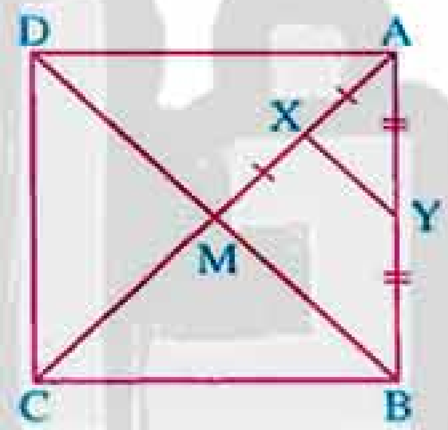
If the perimeter of the square ABCD = 20 cm. ,
$\overrightarrow{MF}$ // $\overrightarrow{CB}$,
then AF = ... cm.

8



If ABCD is a rhombus its perimeter is 24 cm. ,
E is the midpoint of $\overline{AB}$,
then ME = ... cm.

9



If ABCD is a square ,
X and Y are the midpoints of $\overline{AM}$ and $\overline{AB}$ respectively ,
AC = 12 cm. ,
then XY = ... cm. ,
m (∠ AYX) = ... °

UNIT 4

Medians of Triangle – Isosceles Triangle

Exercises of the unit :

1. Medians of triangle.
2. Medians of triangle "Follow".
3. The isosceles triangle.
4. The converse of the isosceles triangle theorem.
5. Corollaries of the isosceles triangle theorems.

- Summary of unit four.
- Unit exams.

Exercise 1

Medians of triangle

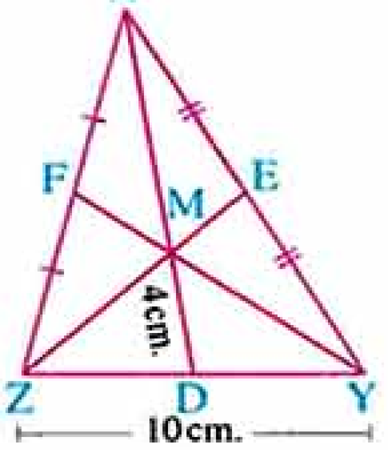
From the school book

1 Complete the following :

1. In Δ ABC , if D is the midpoint of $\overline{BC}$, then $\overline{AD}$ is called ....
2. The number of medians of the triangle is ....
3. The medians of the triangle intersect at ....
4. The point of concurrence of the medians of the triangle divides each median in the ratio .... : .... from its base.
5. The point of concurrence of the medians of the triangle divides each median in the ratio .... : .... from the vertex.
6. The point of intersection of the medians of the triangle divides each of them in the ratio 2 : .... from the base.

2 Using data given for each of the following figures , find the required below each figure :

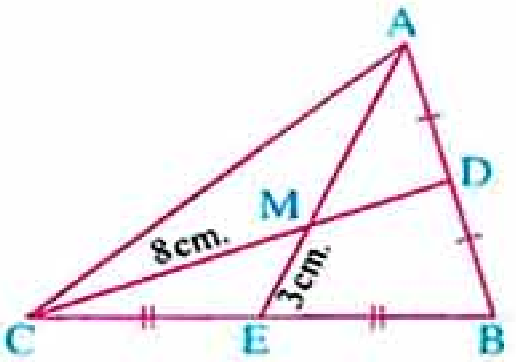
1



XM = ... cm. and

YD = ... cm.

2



MA = ... cm. ,

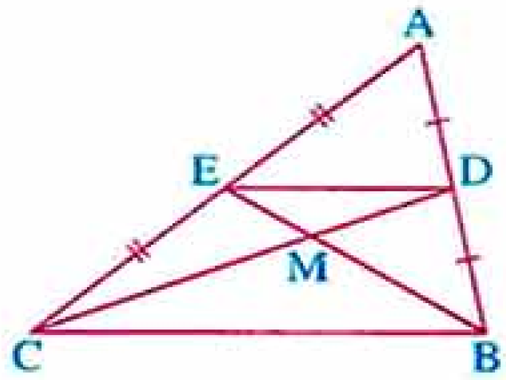
MD = ... cm. ,

ME = ... AE

and MC = ... CD

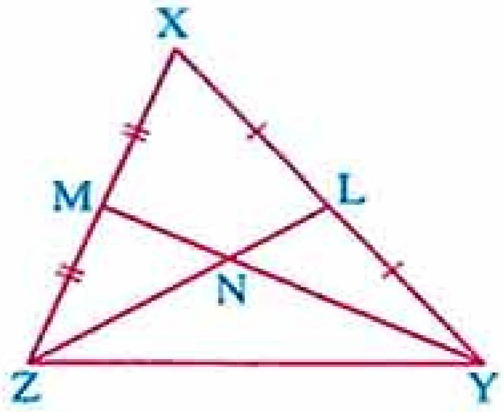
Unit 4

3



If BC = 12 cm. , BE = 9 cm. and MC = 8 cm. , then DE = ... cm. , ME = ... cm. and MD = ... cm.

4



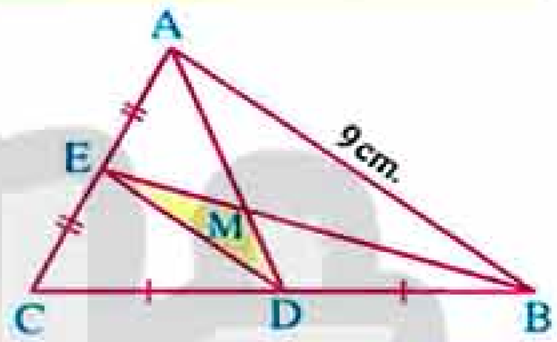
If LZ = 15 cm. , YM = 18 cm. and XY = 20 cm. , then NL = ... cm. , NY = ... cm. and the perimeter of Δ NLY = ... cm.

3 **In the opposite figure :**

ABC is a triangle in which D is the midpoint of $\overline{BC}$, E is the midpoint of $\overline{AC}$ and $\overline{AD} \cap \overline{BE} = \{M\}$

If AD = 6 cm. and AB = BE = 9 cm. ,

calculate : The perimeter of Δ MDE

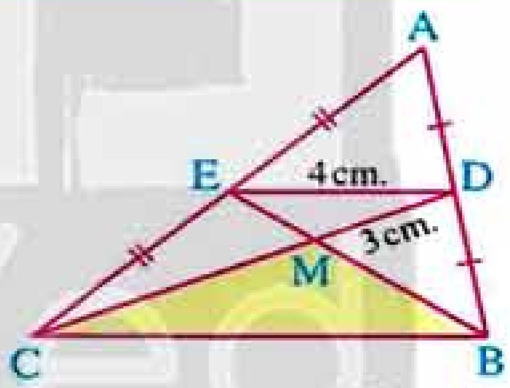


« 9.5 cm. »

4 **In the opposite figure :**

If D is the midpoint of $\overline{AB}$, E is the midpoint of $\overline{AC}$ and $\overline{BE} \cap \overline{DC} = \{M\}$, DE = 4 cm. , DM = 3 cm. and BE = 6 cm. ,

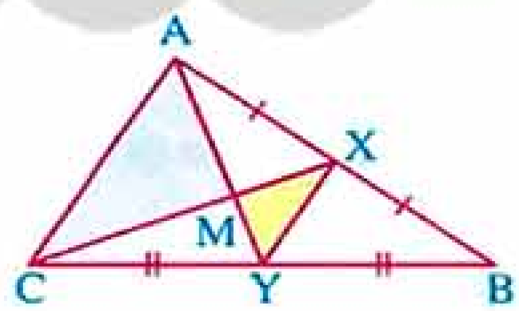
find : The perimeter of Δ BMC



« 18 cm. »

5 **In the opposite figure :**

ABC is a triangle , X is the midpoint of $\overline{AB}$, Y is the midpoint of $\overline{BC}$, XY = 5 cm. and $\overline{XC} \cap \overline{AY} = \{M\}$ where CM = 8 cm. , YM = 3 cm. **Find :**



1 The perimeter of Δ MXY

2 The perimeter of Δ MAC

« 12 cm. , 24 cm. »

6 In Δ ABC , BC = 8 cm. , F and E are the midpoints of $\overline{AB}$ and $\overline{AC}$ respectively and $\overline{BE} \cap \overline{CF} = \{M\}$ If BM = 4 cm. and CM = 6 cm. , **find :** The perimeter of Δ MFE « 9 cm. »

7 In Δ ABC , F and E are the midpoints of $\overline{AB}$ and $\overline{AC}$ respectively and $\overline{BE} \cap \overline{CF} = \{M\}$ Draw $\overrightarrow{AM}$ such that $\overrightarrow{AM} \cap \overline{BC} = \{D\}$ If BC = 10 cm. and AD = 12 cm. ,

find the length of each of : $\overline{BD}$ and $\overline{AM}$ « 5 cm. , 8 cm. »

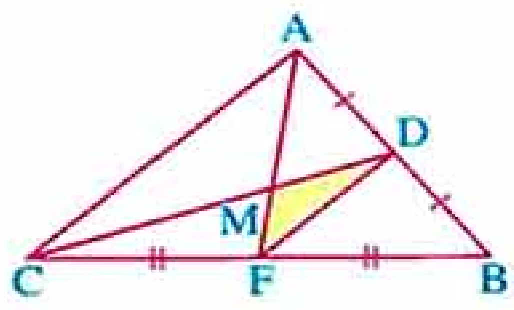
8 **In the opposite figure :**

$\overline{AF}$ and $\overline{CD}$ are two medians in Δ ABC ,

$\overline{AF} \cap \overline{CD} = \{M\}$

If the perimeter of Δ AMC = 36 cm. ,

find : The perimeter of Δ MFD



« 18 cm. »

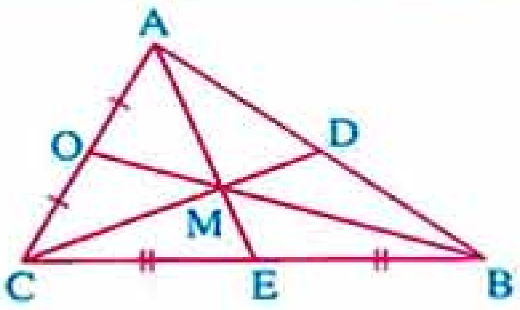
9 **In the opposite figure :**

ABC is a triangle in which E is the midpoint of $\overline{BC}$,

O is the midpoint of $\overline{AC}$, $\overline{AE} \cap \overline{BO} = \{M\}$

If AM + BM + CM = 18 cm.

, **find :** AE + BO + CD



« 27 cm. »

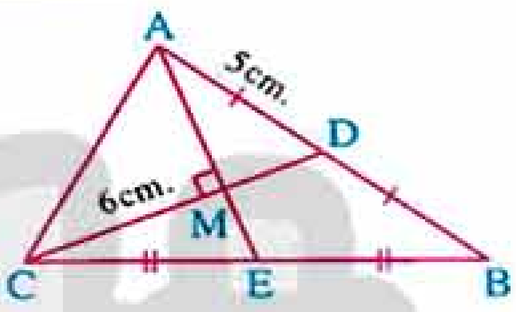
10 **In the opposite figure :**

M is the point of concurrence of the medians

of Δ ABC , $\overline{AM} \perp \overline{CD}$

, MC = 6 cm. , AD = 5 cm.

Find : The length of $\overline{ME}$



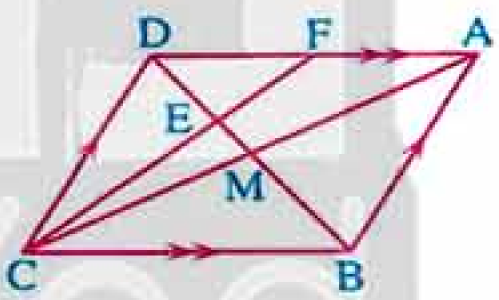
« 2 cm. »

11 **In the opposite figure :**

ABCD is a parallelogram, its diagonals intersect at M ,

E $\in \overline{DM}$ where DE = 2 EM , draw $\overrightarrow{CE}$ to cut $\overline{AD}$ at F

Prove that : AF = FD



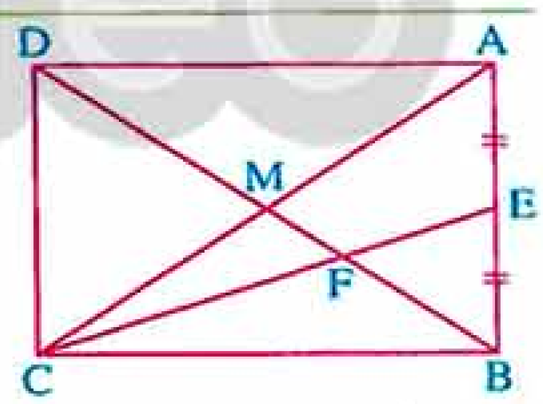
12 **In the opposite figure :**

ABCD is a rectangle , its diagonals intersect at M ,

E is the midpoint of $\overline{AB}$, $\overline{CE} \cap \overline{BD} = \{F\}$

1 **Prove that :** F is the intersection point of the medians of the triangle ABC

2 If BF = 4 cm. , **find :** The length of $\overline{AM}$



« 6 cm. »

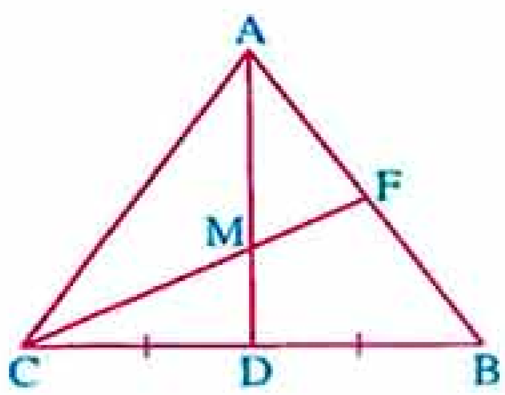
13 **In the opposite figure :**

ABC is a triangle in which D is the midpoint of $\overline{BC}$,

AB = AC , M $\in \overline{AD}$ where AM = $\frac{2}{3}$ AD and

$\overrightarrow{CM} \cap \overline{AB} = \{F\}$

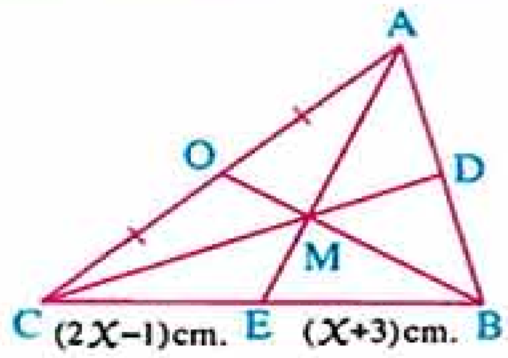
Prove that : BF = $\frac{1}{2}$ AC



Unit 4

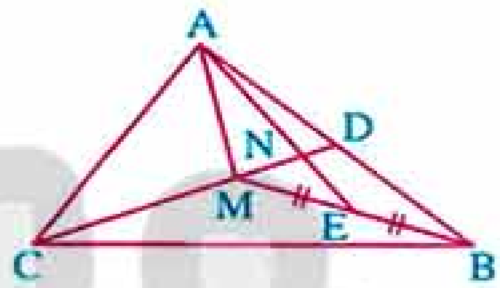
14 ABC is a triangle where point D is the midpoint of $\overline{BC}$ and point $M \in \overline{AD}$, $AM = 2\ MD$
Draw $\overrightarrow{CM}$ to intersect $\overline{AB}$ at point E If $EC = 12$ cm., **then find :** The length of $\overline{EM}$ « 4 cm. »

15 **In the opposite figure :**



ABC is a triangle in which O is the midpoint of $\overline{AC}$,
$BO = 3\ MO$, $E \in \overline{BC}$ where :
$BE = (x + 3)$ cm., $EC = (2x - 1)$ cm.
, $D \in \overline{AB}$, $\overline{BO} \cap \overline{CD} \cap \overline{AE} = \{M\}$
Find : The length of $\overline{BC}$ « 14 cm. »

16 **In the opposite figure :**

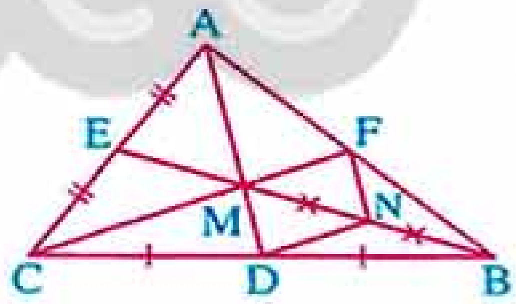


$M \in \overline{CD}$, M is the point of concurrence of the medians of Δ ABC, $N \in \overline{DM}$ where $ND = (x - 1)$ cm.
, $MN = (x + 3)$ cm., $\overrightarrow{AN}$ is drawn to intersect $\overline{BM}$ at E which is the midpoint of $\overline{BM}$
Find : The length of $\overline{MC}$ « 24 cm. »

17 ABCD is a parallelogram whose diagonals intersect at M, E is the midpoint of $\overline{BC}$, $\overline{DE}$ intersects $\overline{AC}$ at F

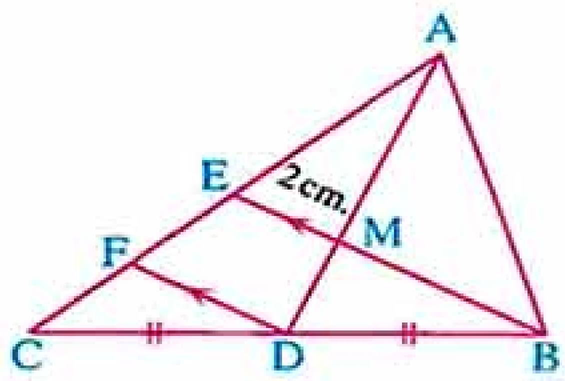
Prove that : **1** $\overrightarrow{BF}$ bisects $\overline{CD}$ **2** $CF = \frac{1}{3}\ AC$

18 **In the opposite figure :**



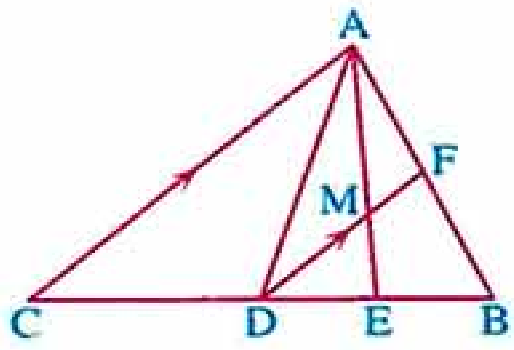
$\overline{AD}$ and $\overline{BE}$ are medians in the triangle ABC intersecting at M,
$\overrightarrow{CM} \cap \overline{AB} = \{F\}$, if N is the midpoint of $\overline{MB}$
, **prove that :** The figure FNDM is a parallelogram.

19 **In the opposite figure :**



ABC is a triangle in which D is the midpoint of $\overline{BC}$
, $M \in \overline{AD}$ where $AM = 2\ MD$
, $\overrightarrow{BM} \cap \overline{AC} = \{E\}$
, $ME = 2$ cm., draw $\overrightarrow{DF} \parallel \overline{BE}$ and cut $\overline{AC}$ at F
Find : The length of $\overline{DF}$ « 3 cm. »

20 **In the opposite figure :**



ABC is a triangle in which D is the midpoint of $\overline{BC}$ and E is the midpoint of $\overline{BD}$, Draw $\overrightarrow{DF}$ // $\overline{AC}$ and cut $\overline{AE}$ at M and $\overline{AB}$ at F

Prove that : $DM = \frac{1}{3} AC$

21 ABC is a triangle , D is the midpoint of $\overline{AB}$ and E is the midpoint of $\overline{AC}$

If $\overline{CD} \cap \overline{BE} = \{M\}$ Draw $\overrightarrow{AM}$ to intersect $\overline{BC}$ at F

Prove that : The figure DBFE is a parallelogram.

For excellent pupils

22 ABCD is a parallelogram , its diagonals intersect at M and $X \in \overrightarrow{AB}$ where $AB = BX$

Draw $\overline{DX}$ to intersect $\overline{BC}$ at Y and $\overline{AC}$ at Z

Prove that : $\overrightarrow{BZ}$ intersects $\overline{DC}$ at the midpoint of $\overline{DC}$

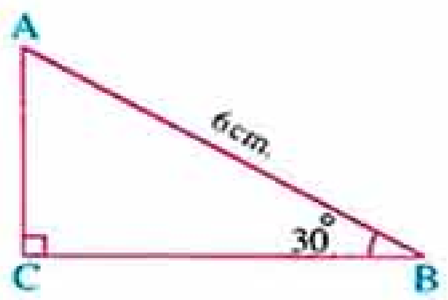
Exercise 2 Medians of triangle "Follow"

From the school book

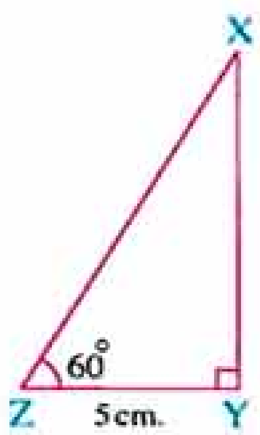
1 Complete the following :

1. The number of medians in the right-angled triangle is ....
2. The length of the median from the vertex of the right angle in the right-angled triangle equals ....
3. If the length of the median drawn from a vertex of a triangle equals half the length of the opposite side to this vertex , then the angle at this vertex is ....
4. The length of the side opposite to the angle of measure 30° in the right-angled triangle equals ....
5. The length of the hypotenuse in thirty and sixty triangle equals .... the length of the side opposite to the angle whose measure is 30°

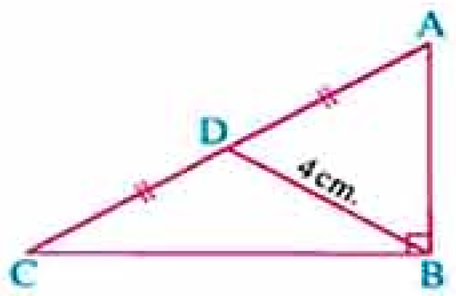
2 Using data given for each of the following figures , find the required below each figure :

1.


AC = ... cm.

2.


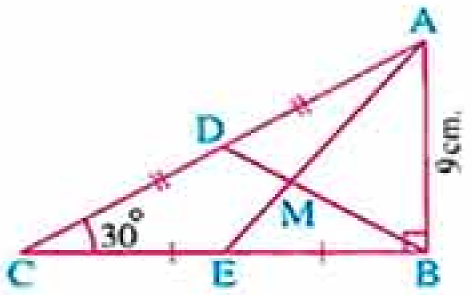
XZ = ... cm.

3.


AC = ... cm.

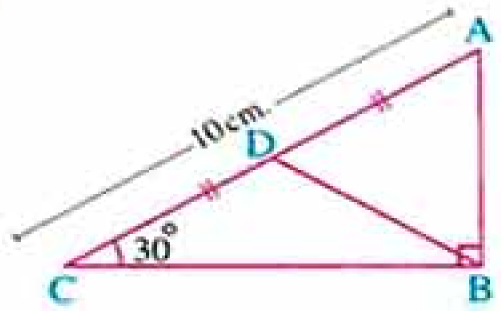
Exercise 2

4



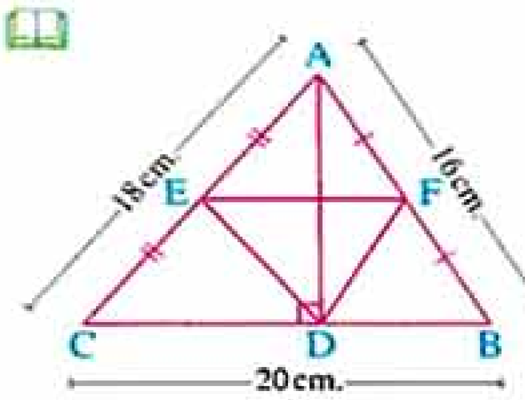
AC = ... cm. ,
BD = ... cm. ,
MD = ... BD
and MD = ... cm.

5



BD = ... cm. ,
AB = ... cm.
and the perimeter of
Δ ABD = ... cm.

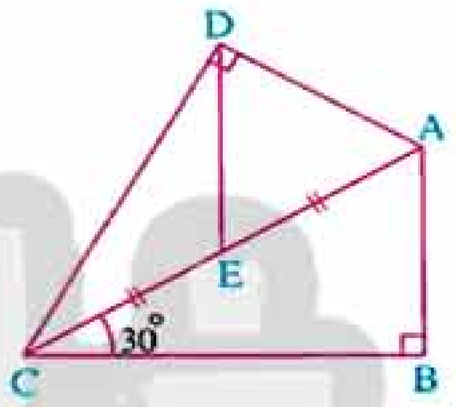
6



DF = ... cm. ,
DE = ... cm. ,
FE = ... cm.
and the perimeter of
Δ DEF = ... cm.

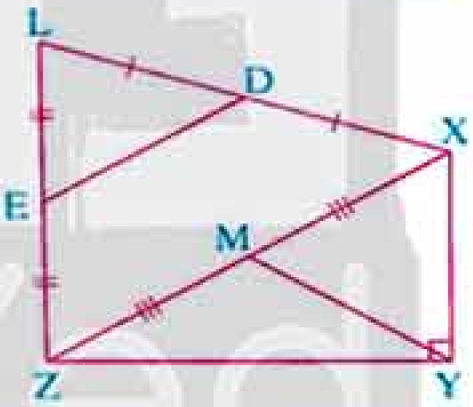
3 **In the opposite figure :**

m ($\angle$ ABC) = m ($\angle$ ADC) = 90° ,
m ($\angle$ ACB) = 30° and
E is the midpoint of $\overline{AC}$
Prove that : AB = DE



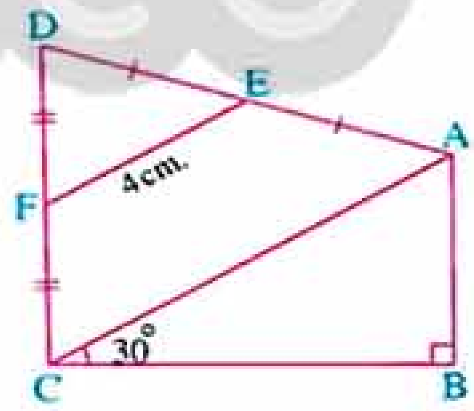
4 **In the opposite figure :**

m ($\angle$ XYZ) = 90° , D is the midpoint of $\overline{XL}$,
E is the midpoint of $\overline{ZL}$ and
M is the midpoint of $\overline{XZ}$
Prove that : DE = YM



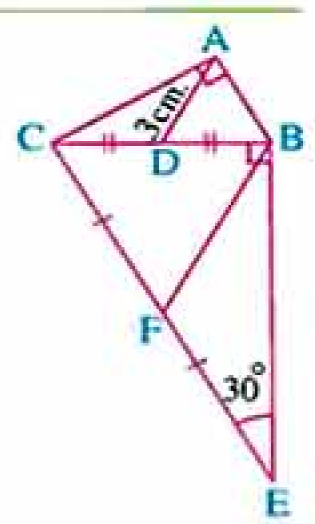
5 **In the opposite figure :**

ABCD is a quadrilateral in which m ($\angle$ B) = 90° ,
E is the midpoint of $\overline{AD}$, F is the midpoint of $\overline{CD}$,
m ($\angle$ ACB) = 30° and EF = 4 cm.
Find by proof : The length of $\overline{AB}$ « 4 cm. »



6 **In the opposite figure :**

m ($\angle$ BAC) = m ($\angle$ CBE) = 90°
, m ($\angle$ BEC) = 30°
, D and F are the midpoints
of $\overline{BC}$ and $\overline{CE}$ respectively and AD = 3 cm.
Find : The length of $\overline{BF}$ « 6 cm. »



Unit 4

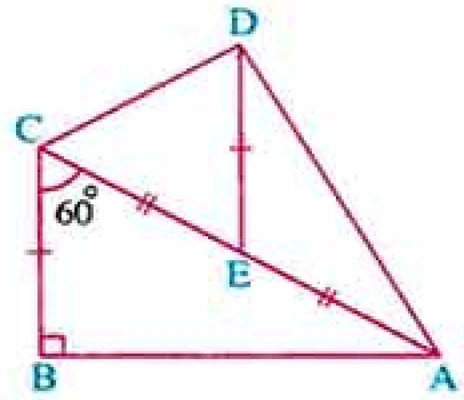
7 **In the opposite figure :**

ABC is a right-angled triangle at B , m (∠ ACB) = 60° ,

E is the midpoint of $\overline{AC}$ and

DE = BC

Prove that : m (∠ ADC) = 90°



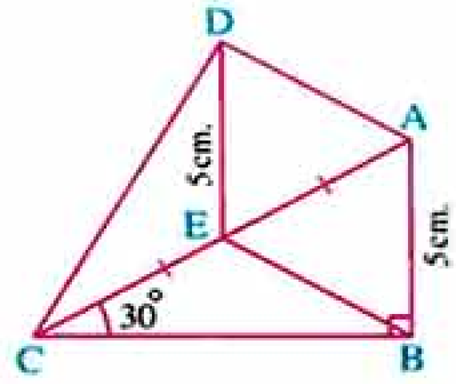
8 **In the opposite figure :**

ABC is a right-angled triangle at B ,

m (∠ ACB) = 30° , AB = 5 cm. and

E is the midpoint of $\overline{AC}$ If DE = 5 cm. ,

prove that : m (∠ ADC) = 90°



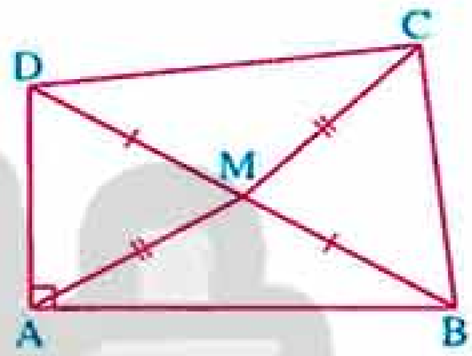
9 **In the opposite figure :**

ABCD is a quadrilateral in which m (∠ BAD) = 90° ,

M is the midpoint of $\overline{BD}$ and CM = AM

Prove that :

m (∠ BCD) = 90°



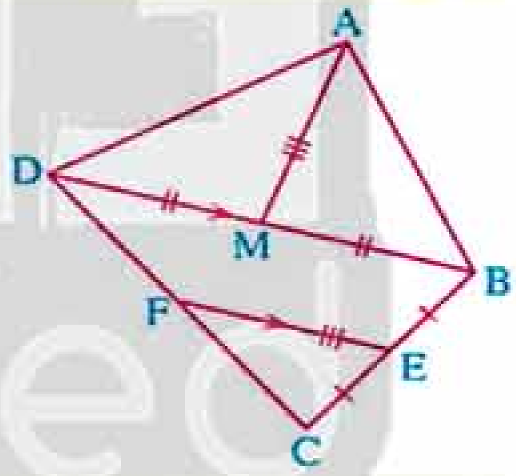
10 **In the opposite figure :**

ABD is a triangle , M is the midpoint of $\overline{BD}$,

E is the midpoint of $\overline{BC}$,

F ∈ $\overline{CD}$, $\overline{EF}$ // $\overline{BD}$ and AM = EF

Prove that : m (∠ BAD) = 90°



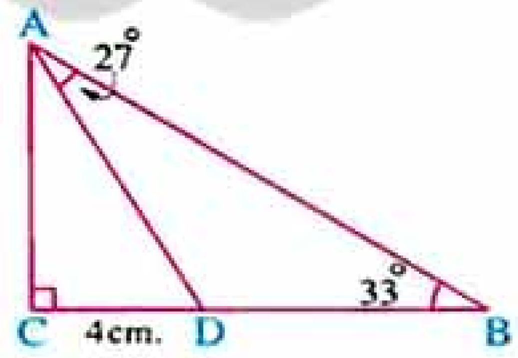
11 **In the opposite figure :**

ABC is a triangle in which m (∠ B) = 33°

, m (∠ C) = 90° , D ∈ $\overline{BC}$ where CD = 4 cm.

, m (∠ BAD) = 27°

Find : The length of $\overline{AD}$ « 8 cm. »



12 **In the opposite figure :**

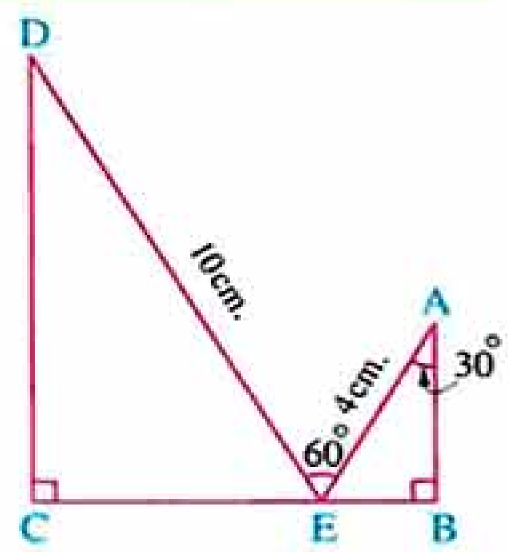
ABE is a right-angled triangle at B

, DCE is a right-angled triangle at C

where E ∈ $\overline{BC}$, AE = 4 cm.

, DE = 10 cm. , m (∠ A) = 30° , m (∠ AED) = 60°

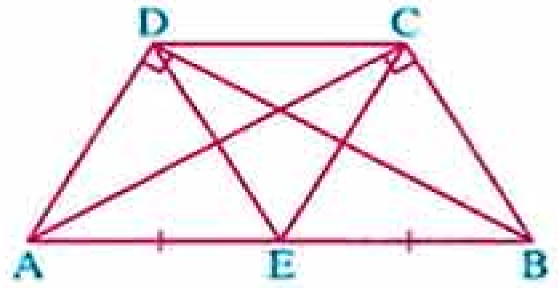
Find : The length of $\overline{BC}$ « 7 cm. »



13 **In the opposite figure :**

ADB is a right-angled triangle at D ,
ACB is a right-angled triangle at C and E is the midpoint of $\overline{AB}$

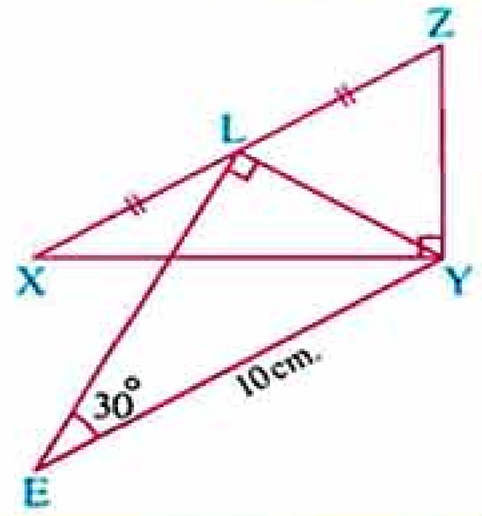
Prove that : Δ CED is an isosceles triangle.



14 **In the opposite figure :**

m (∠ YLE) = 90° , m (∠ E) = 30° , YE = 10 cm. ,
m (∠ XYZ) = 90° and
L is the midpoint of $\overline{XZ}$

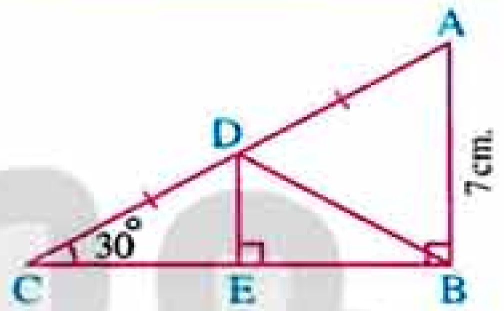
Find by proof : The length of $\overline{XZ}$ « 10 cm. »



15 **In the opposite figure :**

ABC is a right-angled triangle at B , D is the midpoint of $\overline{AC}$, $\overline{DE} \perp \overline{BC}$, AB = 7 cm. and m (∠ C) = 30°

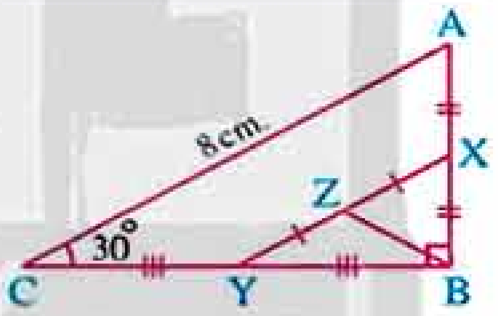
Find the length of each of : $\overline{BD}$ and $\overline{DE}$



« 7 cm. , 3.5 cm. »

16 **In the opposite figure :**

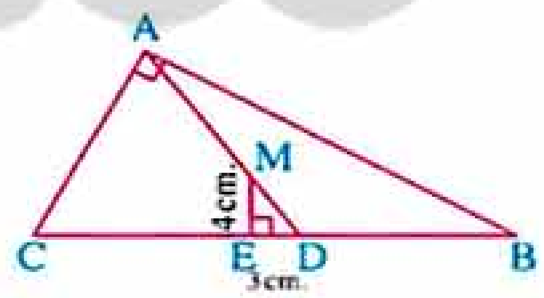
ABC is a triangle in which m (∠ ABC) = 90° , m (∠ C) = 30° ,
X , Y and Z are the midpoints of $\overline{AB}$, $\overline{BC}$ and $\overline{XY}$ respectively and AC = 8 cm.



Find the length of each of : $\overline{AB}$, $\overline{XY}$ and $\overline{BZ}$ « 4 cm. , 4 cm. , 2 cm. »

17 **In the opposite figure :**

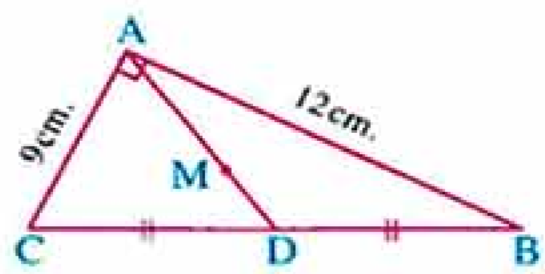
ABC is a right-angled triangle at A
, M is the point of concurrence of its medians
, E ∈ $\overline{DC}$ where $\overline{ME} \perp \overline{DC}$, DE = 3 cm.
and ME = 4 cm.



Find : The length of $\overline{BC}$ « 30 cm. »

18 **In the opposite figure :**

m (∠ BAC) = 90° , AB = 12 cm. , AC = 9 cm.
$\overline{AD}$ is a median of Δ ABC and M is the point of concurrence of the medians of Δ ABC



Find : The length of $\overline{AM}$ « 5 cm. »

Unit 4

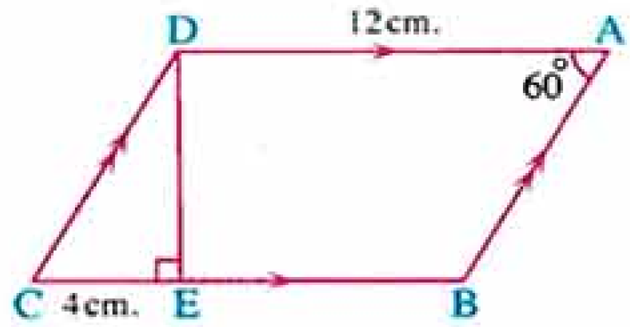
19 **In the opposite figure :**

ABCD is a parallelogram in which

m (∠ A) = 60° , $\overline{DE} \perp \overline{BC}$

, AD = 12 cm. and EC = 4 cm.

Find : The perimeter of the parallelogram ABCD



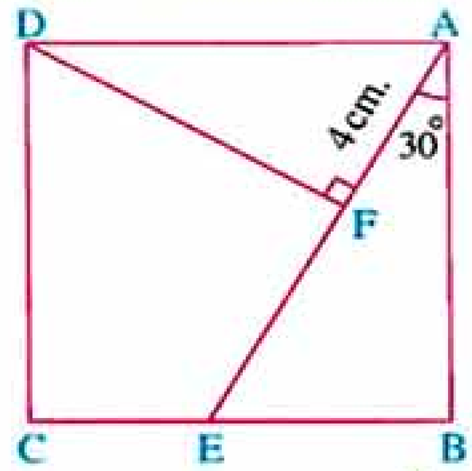
« 40 cm. »

20 **In the opposite figure :**

ABCD is a square , E ∈ $\overline{BC}$ where m (∠ BAE) = 30° and

$\overline{DF} \perp \overline{AE}$ If AF = 4 cm. ,

calculate : The area of the square ABCD



« 64 cm^2 »

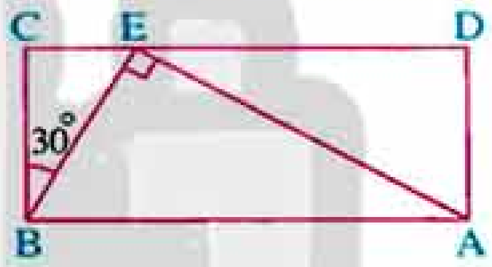
21 **In the opposite figure :**

ABCD is a rectangle , E ∈ $\overline{DC}$

where m (∠ CBE) = 30°

and m (∠ AEB) = 90°

Prove that : CE = $\frac{1}{4}$ AB



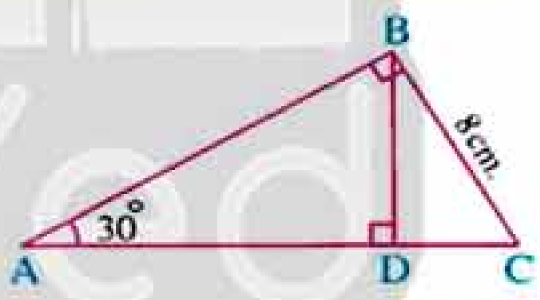
22 **In the opposite figure :**

ABC is a right-angled triangle at B ,

m (∠ A) = 30° ,

D ∈ $\overline{AC}$ such that $\overline{BD} \perp \overline{AC}$

If BC = 8 cm. , **find :** The length of $\overline{AD}$



« 12 cm. »

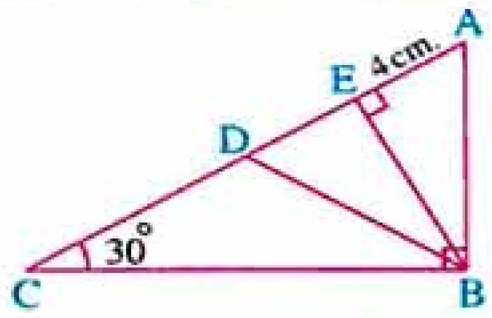
23 **In the opposite figure :**

ABC is a triangle in which m (∠ ABC) = 90° ,

m (∠ C) = 30° , $\overline{BE} \perp \overline{AC}$, $\overline{BD}$ is a median

in Δ ABC and AE = 4 cm.

Calculate : The perimeter of Δ ABD



« 24 cm. »

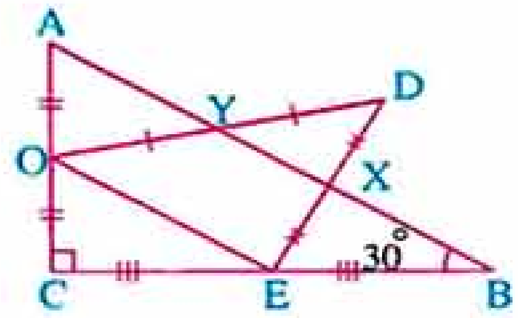
24 **In the opposite figure :**

ABC is a right-angled triangle at C in which m (∠ B) = 30°

, E , O , X , Y are the midpoints of $\overline{BC}$, $\overline{AC}$

, $\overline{DE}$, $\overline{DO}$ respectively

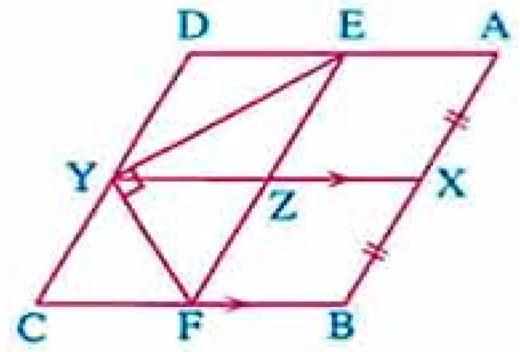
Prove that : XY = $\frac{1}{2}$ AC



25 **In the opposite figure :**

ABCD is a parallelogram in which X is the midpoint of $\overline{AB}$, draw $\overrightarrow{XY} \parallel \overline{BC}$ and cut $\overline{DC}$ at Y , if $E \in \overline{AD}$, $F \in \overline{BC}$ where m ($\angle$ EYF) = 90° and $\overline{EF} \cap \overline{XY} = \{Z\}$

prove that : $YZ = \frac{1}{2}$ EF



26 ABC is a triangle in which AB = AC and $\overrightarrow{AD}$ is drawn to be perpendicular to $\overline{BC}$ where $\overrightarrow{AD} \cap \overline{BC} = \{D\}$ If E and F are the two midpoints of $\overline{AB}$ and $\overline{AC}$ respectively,

prove that : DE + DF = AB

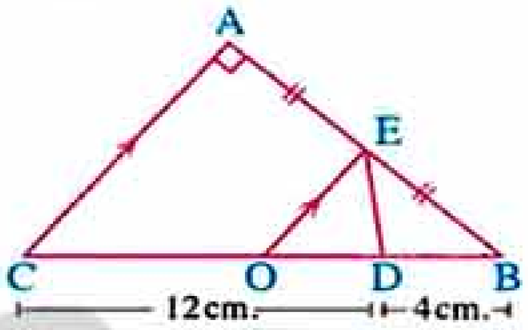
27 **In the opposite figure :**

ABC is a right-angled triangle at A

, E is the midpoint of $\overline{AB}$, $O \in \overline{BC}$

where $\overline{EO} \parallel \overline{AC}$, $D \in \overline{BO}$ where BD = 4 cm. , DC = 12 cm.

Find : The length of $\overline{DE}$

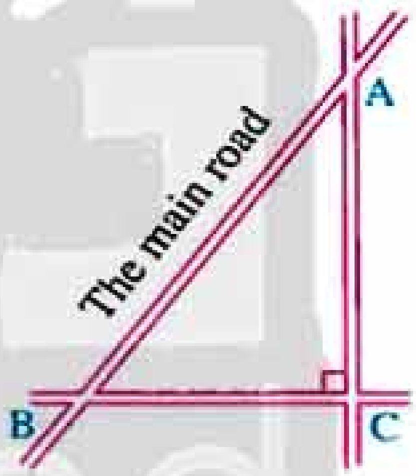


« 4 cm. »

Life Application

28 The opposite figure is a sketch for three towns A , B and C such that the distance between the towns A and C is 40 km. and the distance between the towns B and C is 30 km.

If we want to build a service station lying on the main road at the half-way between the towns A and B , also we want to build a road linking this station to the town C , then how long will this road be ?



« 25 km. »

For excellent pupils

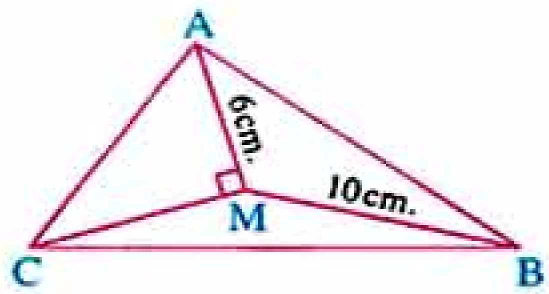
29 **In the opposite figure :**

M is the point of concurrence of the medians of Δ ABC

, AM = 6 cm. , BM = 10 cm.

, m ($\angle$ AMC) = 90°

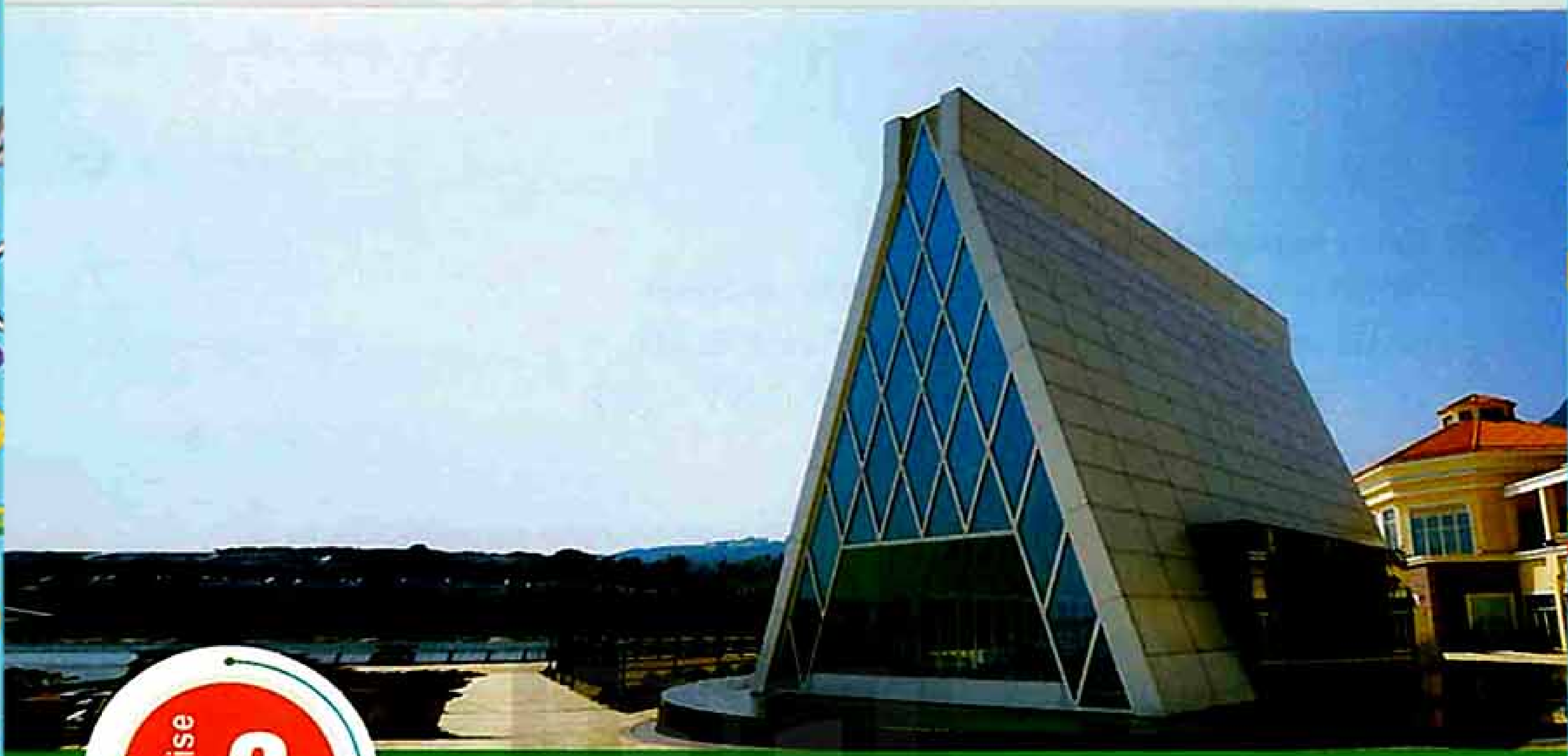
Find by proof : 1 The length of $\overline{AC}$ 2 The length of $\overline{MC}$



« 10 cm. , 8 cm. »

30 ABCD is a parallelogram , X is an interior point in it such that $\overrightarrow{DX}$ bisects $\angle$ ADC , $\overrightarrow{CX}$ bisects $\angle$ DCB , if the point Y is the midpoint of $\overline{DC}$

, **prove that :** XY = YC

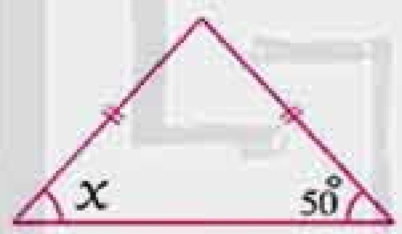


Exercise 3

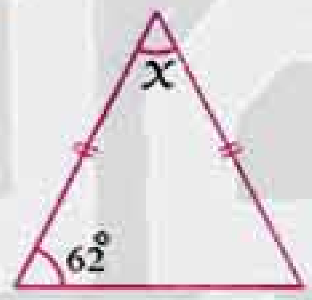
The isosceles triangle

From the school book

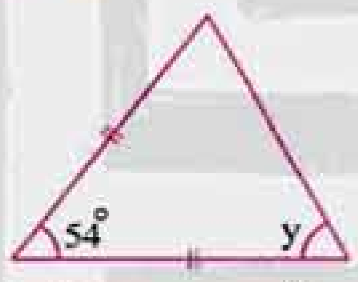
1 **In each of the following , find the value of the symbol used for the measure of the angle :**

1


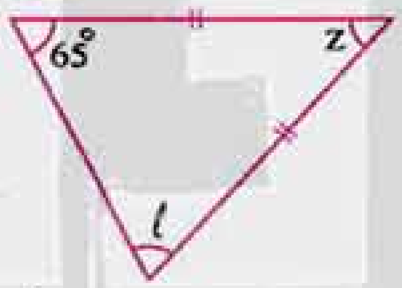
x = ...°

2


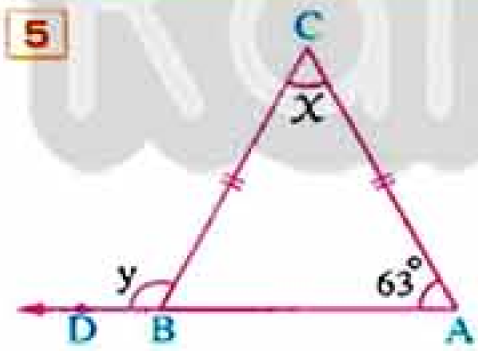
x = ...°

3


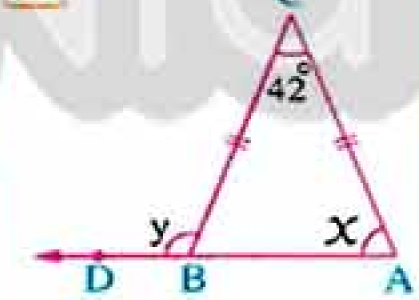
y = ...°

4


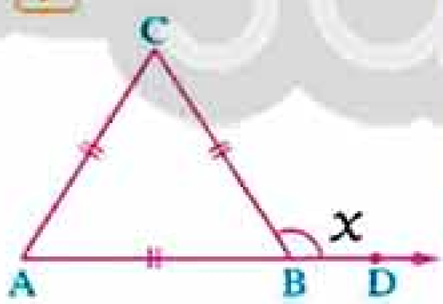
ℓ = ...° ,
z = ...°

5


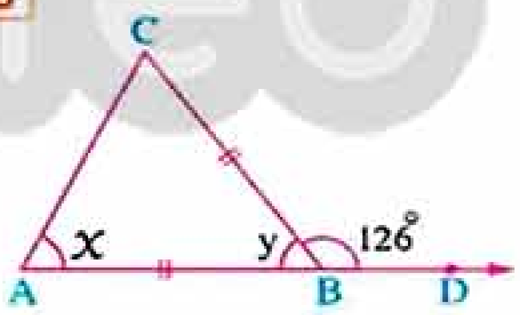
x = ...° ,
y = ...°

6


x = ...° ,
y = ...°

7


x = ...°

8


x = ...° ,
y = ...°

2 **Complete the following :**

1 The base angles of the isosceles triangle are ....

2 The measure of each angle in the equilateral triangle equals ....°

3 In Δ DEF , if DE = DF , then m (∠ E) = m (∠ ....)

4 In the isosceles triangle , if the measure of one of the two base angles is 65° , then the measure of its vertex angle equals ....°

5 In the isosceles triangle , if the measure of the vertex angle equals 40° , then the measure of one of the two base angles equals ....°

6 In Δ ABC , if AB = AC and m ($\angle$ A) = 80° , then m ($\angle$ B) = m ($\angle$ ....) = ...°

3 Choose the correct answer from those given :

1 In Δ XYZ , if XY = YZ = XZ , then m ($\angle$ X) = ...

(a) 30° (b) 60° (c) 90° (d) 180°

2 The measure of the exterior angle of the equilateral triangle equals ....

(a) 60° (b) 90° (c) 120° (d) 180°

3 If Δ ABC is right-angled at A and AB = AC , then m ($\angle$ B) = ...

(a) 30° (b) 45° (c) 60° (d) 90°

4 If the measure of one of the two base angles in the isosceles triangle is 30° , then the triangle is ....

(a) obtuse-angled. (b) acute-angled.

(c) right-angled. (d) equilateral triangle.

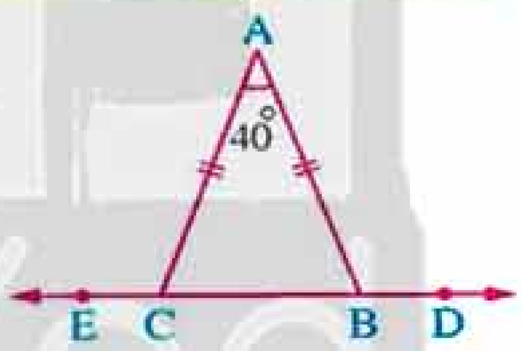
5 In Δ XYZ , if XY = XZ , then the exterior angle at the vertex Z is ....

(a) acute. (b) obtuse. (c) right. (d) reflex.

4 In the opposite figure :

ABC is an isosceles triangle in which AB = AC ,

m ($\angle$ A) = 40° and D $\in \overrightarrow{CB}$, E $\in \overrightarrow{BC}$



1 **Find :** m ($\angle$ ABC)

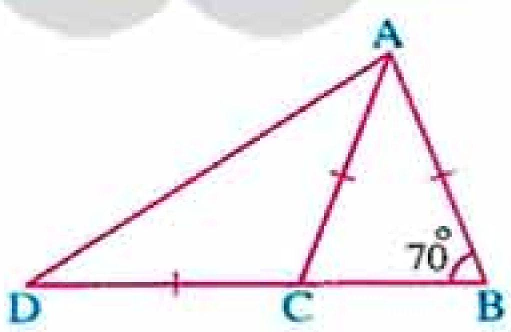
2 **Prove that :** $\angle$ ABD $\equiv$ $\angle$ ACE

« 70° »

5 In the opposite figure :

AB = AC = CD

and m ($\angle$ B) = 70°



Find by proof :

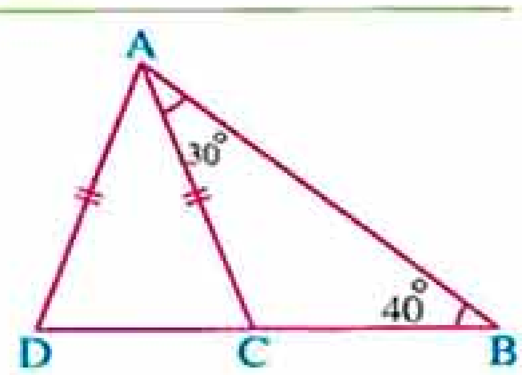
m ($\angle$ BAD)

« 75° »

6 In the opposite figure :

m ($\angle$ B) = 40° , m ($\angle$ BAC) = 30°

and AC = AD



Find by proof :

1 m ($\angle$ D) 2 m ($\angle$ CAD)

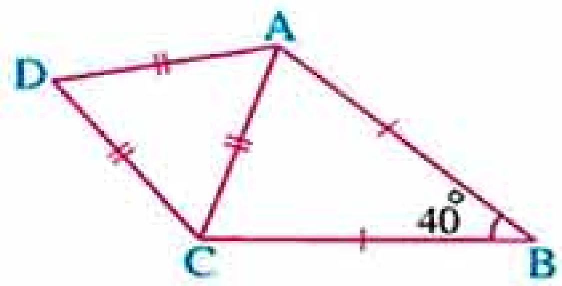
« 70° , 40° »

7 **In the opposite figure :**

AD = DC = AC , AB = BC

and m (∠ ABC) = 40°

Find : m (∠ BAD)



« 130° »

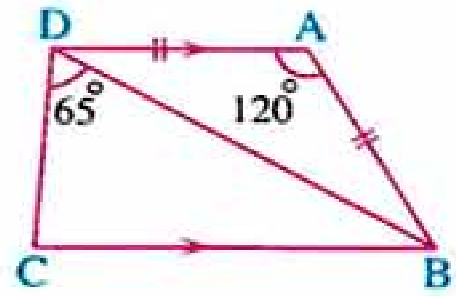
8 **In the opposite figure :**

AB = AD , $\overline{AD}$ // $\overline{BC}$,

m (∠ BAD) = 120° and m (∠ BDC) = 65°

Find :

1 m (∠ ADB) 2 m (∠ C)



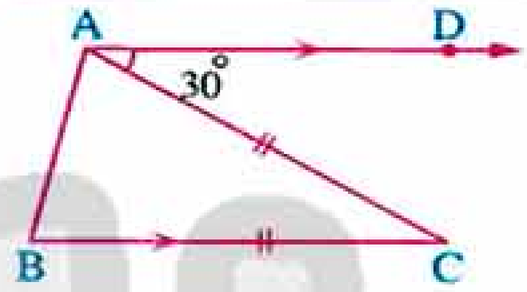
« 30° , 85°»

9 **In the opposite figure :**

ABC is a triangle in which AC = BC ,

$\overrightarrow{AD}$ // $\overline{BC}$ and m (∠ DAC) = 30°

Find : The measures of the angles of Δ ABC



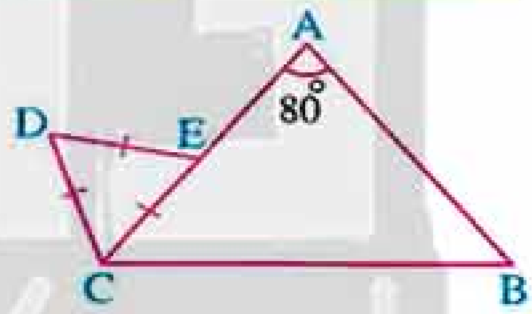
« 30° , 75° , 75° »

10 **In the opposite figure :**

AB = AC , m (∠ BAC) = 80°

and CE = ED = CD

Find by proof : m (∠ BCD)



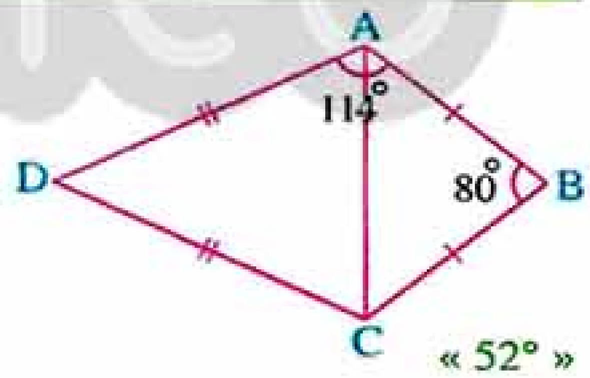
« 110° »

11 **In the opposite figure :**

AB = BC , AD = CD , m (∠ BAD) = 114°

and m (∠ B) = 80°

Find : m (∠ ADC)



« 52° »

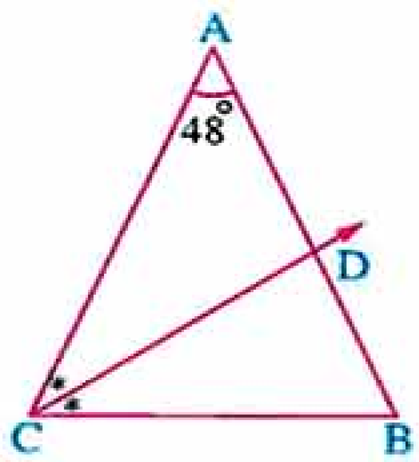
12 **In the opposite figure :**

AB = AC , m (∠ BAC) = 48° , $\overrightarrow{CD}$ bisects ∠ BCA

and intersects $\overline{AB}$ at D

Find :

1 m (∠ B) 2 m (∠ BCD)

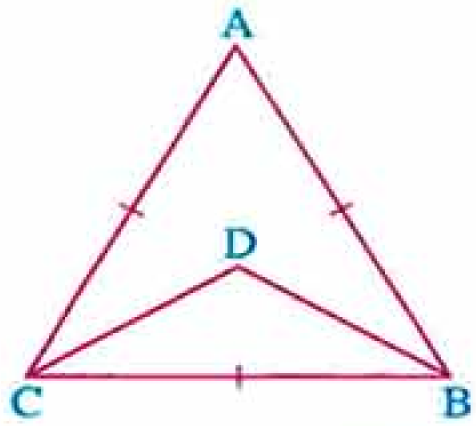


« 66° , 33° »

13 **In the opposite figure :**

ABC is an equilateral triangle and the two bisectors of $\angle$ B and $\angle$ C intersect together at D

Find : m ($\angle$ BDC)

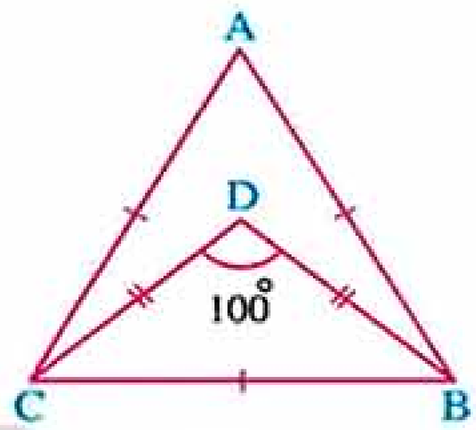


« 120° »

14 **In the opposite figure :**

ABC is an equilateral triangle , DB = DC and m ($\angle$ BDC) = 100°

Find by proof : m ($\angle$ ABD)



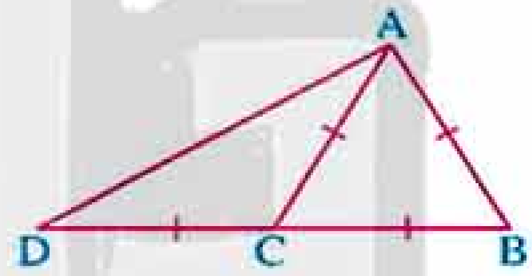
« 20° »

15 **In the opposite figure :**

ABC is an equilateral triangle.

D $\in$ $\overrightarrow{BC}$ such that BC = CD

Prove that : $\overline{BA} \perp \overline{AD}$

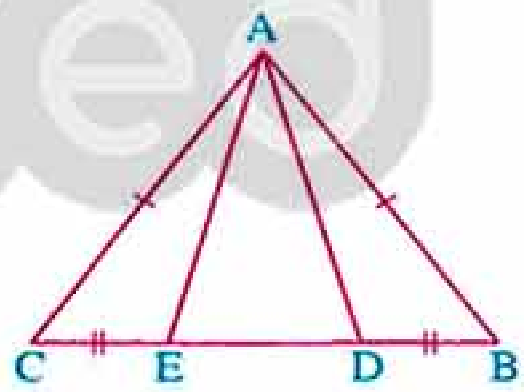


16 **In the opposite figure :**

ABC is an isosceles triangle in which AB = AC , D $\in$ $\overline{BC}$ and E $\in$ $\overline{BC}$, such that BD = EC

Prove that :

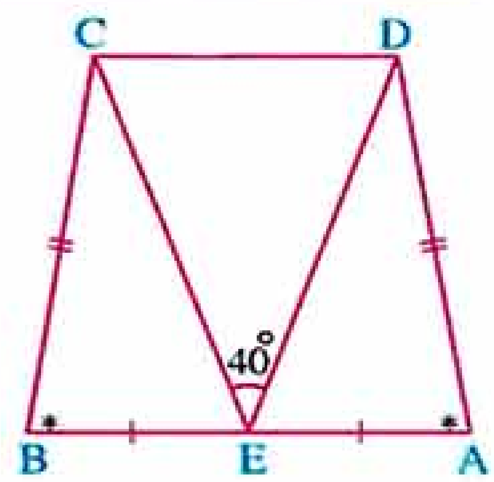
1. Δ ADE is an isosceles triangle.
2. $\angle$ AED $\equiv$ $\angle$ ADE



17 **In the opposite figure :**

E is the midpoint of $\overline{AB}$, AD = BC , m ($\angle$ A) = m ($\angle$ B) and m ($\angle$ DEC) = 40°

Find : m ($\angle$ EDC)



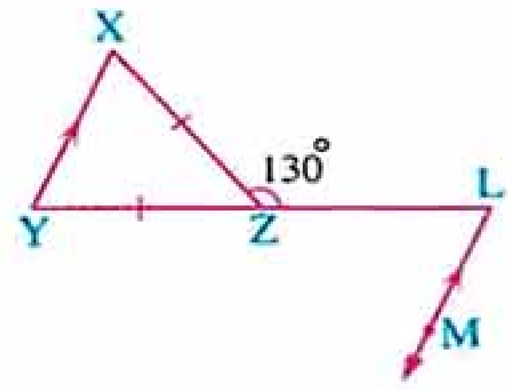
« 70° »

Unit 4

18 **In the opposite figure :**

$Z \in \overline{LY}$, $XZ = YZ$, $m(\angle LZX) = 130°$ and $\overleftrightarrow{LM} \parallel \overline{XY}$

Find : $m(\angle MLY)$



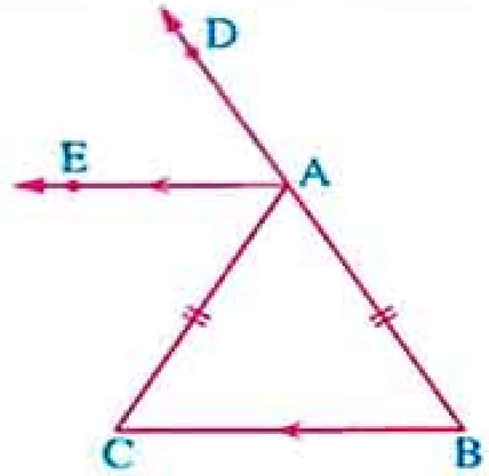
« 65° »

19 **In the opposite figure :**

$A \in \overrightarrow{BD}$, $AB = AC$ and $\overrightarrow{AE} \parallel \overline{BC}$

Prove that :

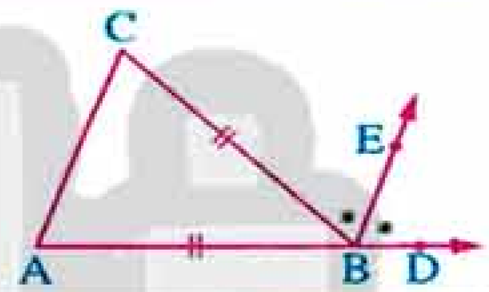
$\overrightarrow{AE}$ bisects $\angle DAC$



20 **In the opposite figure :**

$AB = BC$ and $\overrightarrow{BE}$ bisects $\angle CBD$

Prove that : $\overrightarrow{BE} \parallel \overline{AC}$

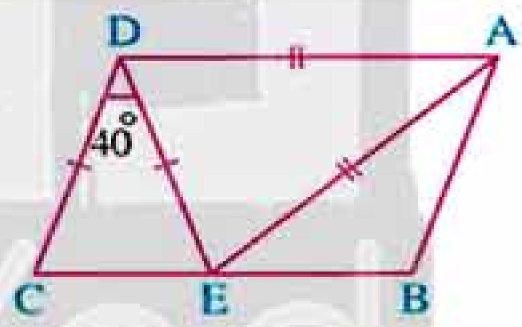


21 **In the opposite figure :**

ABCD is a parallelogram , $E \in \overline{BC}$, where $AE = AD$, $DE = DC$ and $m(\angle EDC) = 40°$

Find : 1 $m(\angle AED)$

2 $m(\angle BAE)$



« 70° , 30° »

22 **In the opposite figure :**

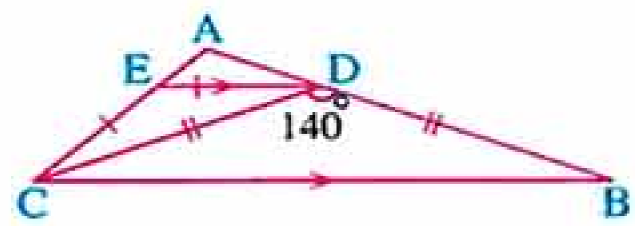
ABC is a triangle in which

$D \in \overline{AB}$, $E \in \overline{AC}$

where $\overline{DE} \parallel \overline{BC}$, $DE = EC$

, $DB = DC$ and $m(\angle BDC) = 140°$

Find : $m(\angle A)$



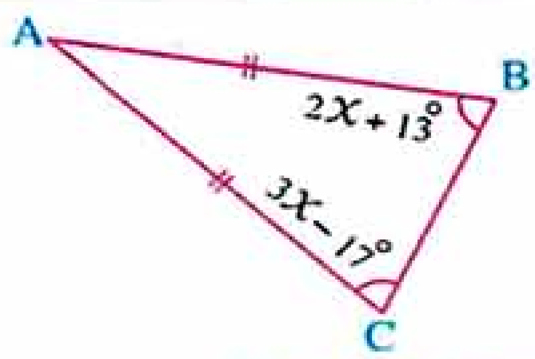
« 120° »

23 **In the opposite figure :**

$AB = AC$, $m(\angle B) = 2x + 13°$

and $m(\angle C) = 3x - 17°$

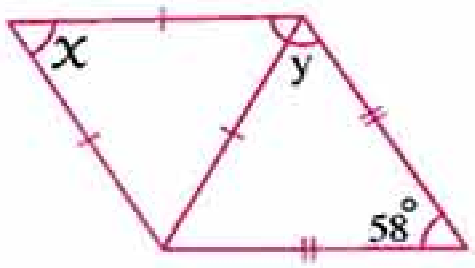
Find : The measures of the angles of ΔABC



« 73° , 73° , 34° »

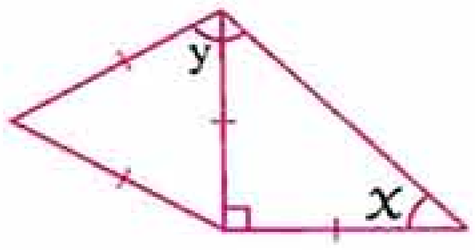
24 In each of the following figures , find the value of the symbol used for the measure of the angle :

1



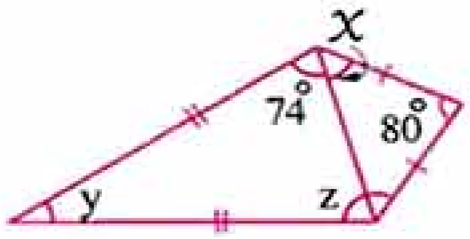
$x =$ ....° ,
y = ...°

2



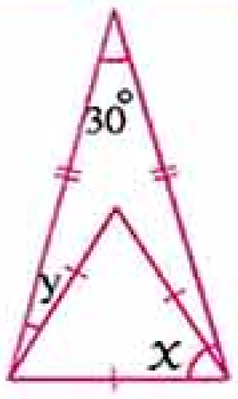
$x =$ ....° ,
y = ...°

3



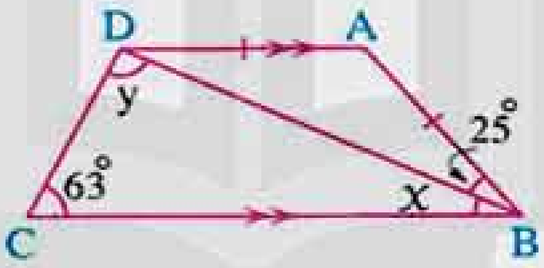
$x =$ ....° , y = ...° ,
z = ...°

4



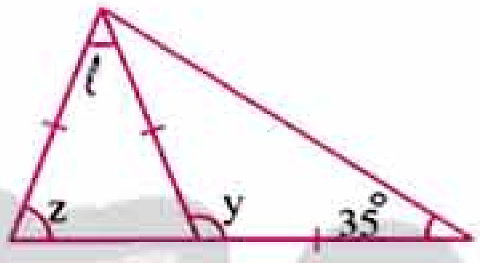
$x =$ ....° ,
y = ...°

5



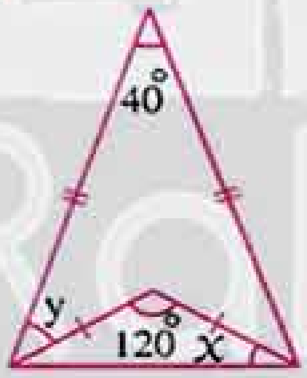
$x =$ ....° ,
y = ...°

6



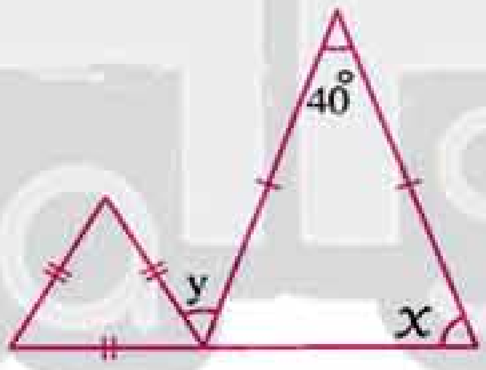
y = ...° , ℓ = ...° ,
z = ...°

7



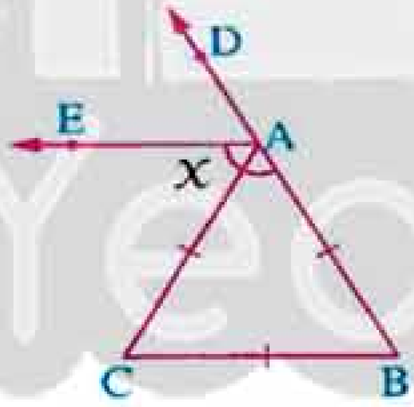
$x =$ ....° ,
y = ...°

8



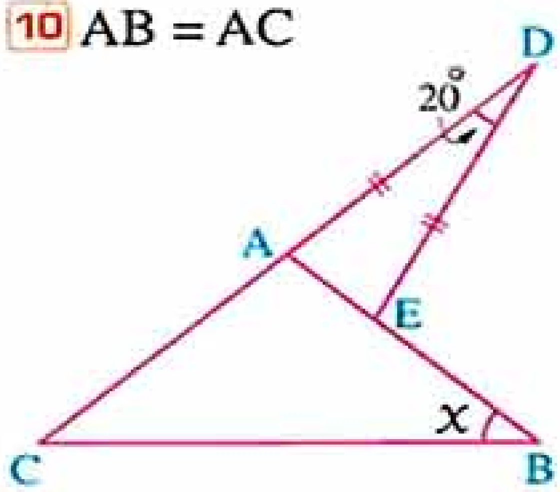
$x =$ ....° ,
y = ...°

9 $\overrightarrow{AE}$ bisects ∠ CAD



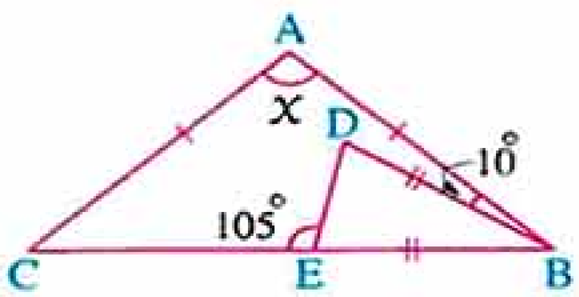
$x =$ ....°

10 AB = AC



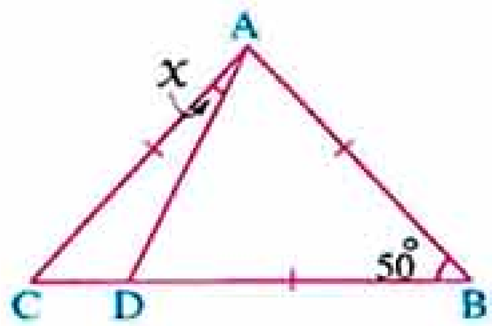
$x =$ ....°

11



$x =$ ....°

12

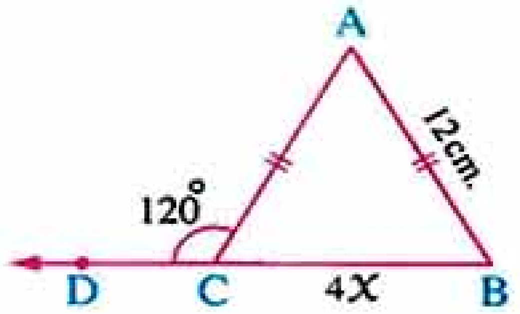


$x =$ ....°

Unit 4

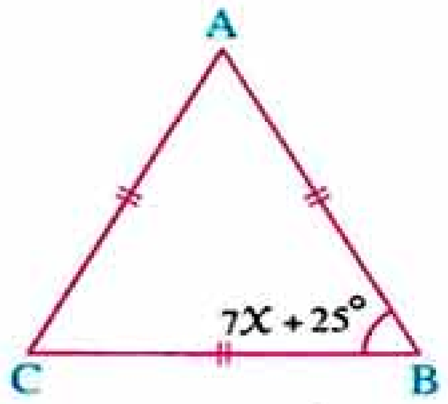
25 **Find the value of x in each of the following figures :**

1



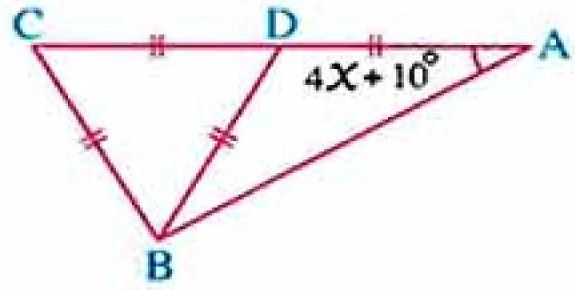
x = ... cm.

2



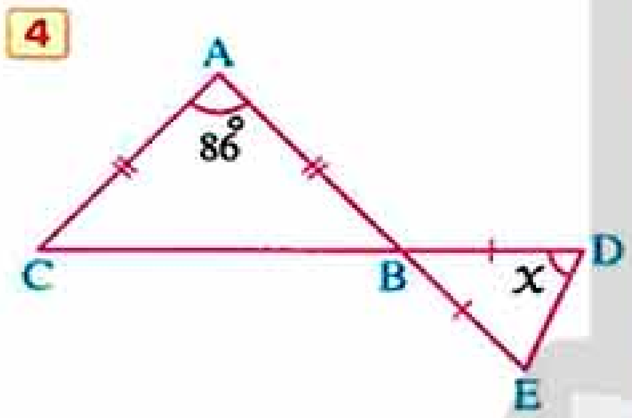
x = ...°

3



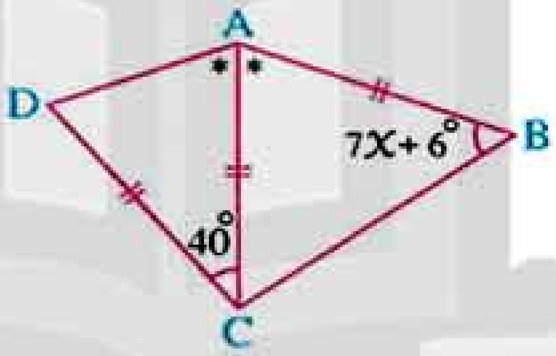
x = ...°

4



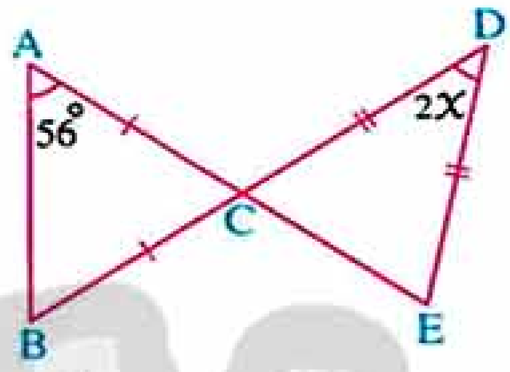
x = ...°

5



x = ...°

6



x = ...°

26 **In the opposite figure :**

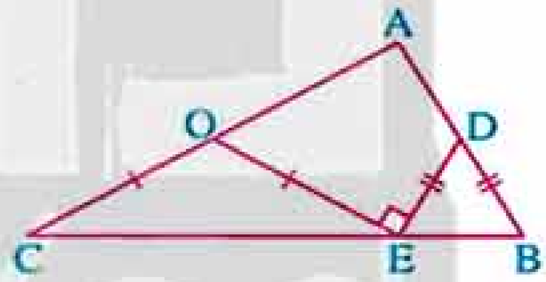
ABC is a triangle in which

$D \in \overline{AB}$, $E \in \overline{BC}$, $O \in \overline{AC}$

where m (∠ DEO) = 90° , DB = DE

and OE = OC

Find : m (∠ A)



« 90° »

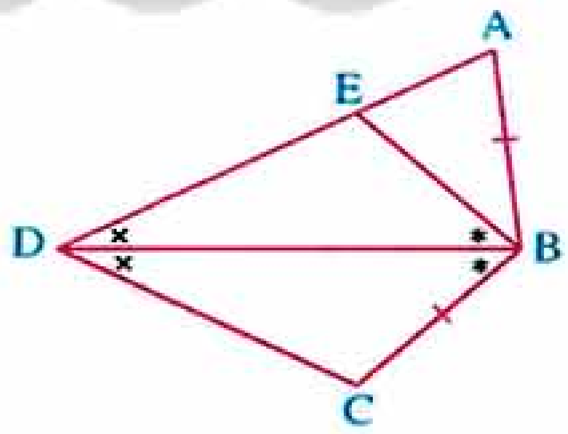
27 **In the opposite figure :**

BA = BC , $E \in \overline{AD}$

and $\overleftrightarrow{BD}$ bisects each

of ∠ CBE and ∠ CDE

Prove that : m (∠ A) + m (∠ C) = 180°

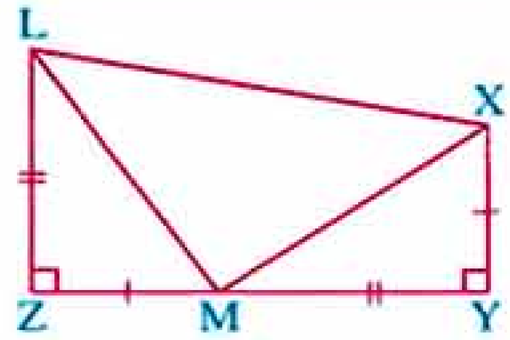


28 **In the opposite figure :**

m (∠ Y) = m (∠ Z) = 90°

, XY = MZ and YM = ZL

Find by proof : m (∠ MXL)



« 45° »

29 **In the opposite figure :**

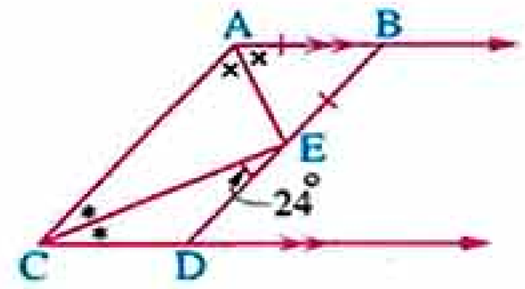
$\overrightarrow{AB} // \overrightarrow{CD}$, $E \in \overline{BD}$

where $\overrightarrow{AE}$ bisects $\angle BAC$

, $\overrightarrow{CE}$ bisects $\angle DCA$

, BA = BE and m ($\angle$ DEC) = 24°

Find : m ($\angle$ ABE)



« 48° »

Life Application

30 **In the opposite figure :**

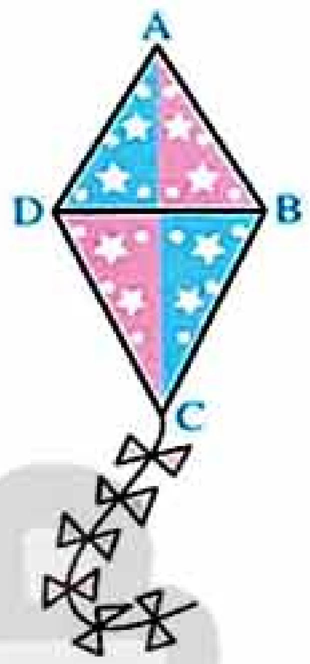
ABCD is a kite that consists of two triangles

, Δ ABD is an equilateral triangle and

Δ BCD is an isosceles triangle where

CB = CD , if m ($\angle$ C) = 50°

Find : m ($\angle$ ABC)



« 125° »

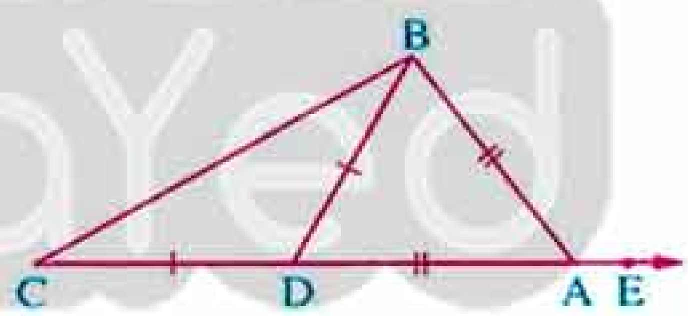
For excellent pupils

31 **In the opposite figure :**

ABC is a triangle , $D \in \overline{AC}$ such that BD = DC

AD = AB and $E \in \overrightarrow{CA}$

Prove that : m ($\angle$ BAE) = 4 m ($\angle$ BCD)

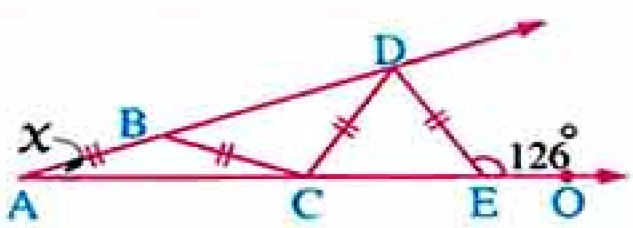


32 **In the opposite figure :**

m ($\angle$ A) = x° , AB = BC = CD = DE

and m ($\angle$ DEO) = 126°

Find : The value of x

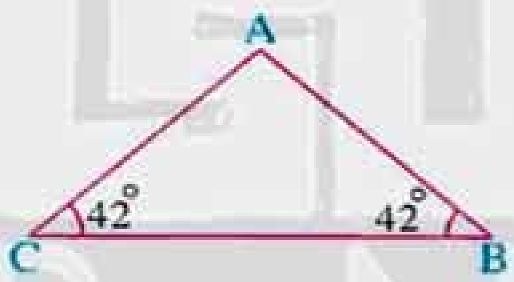


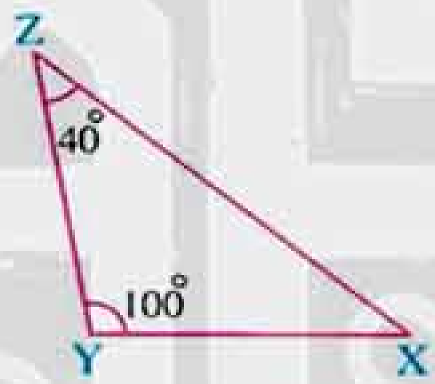
« 18° »

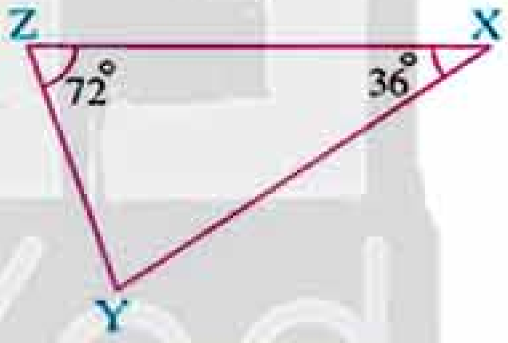
Exercise 4 The converse of the isosceles triangle theorem

From the school book

1 In each of the following figures , write the equal sides in length :

1


2


3


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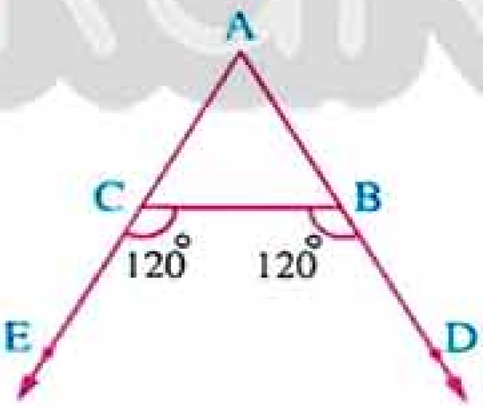
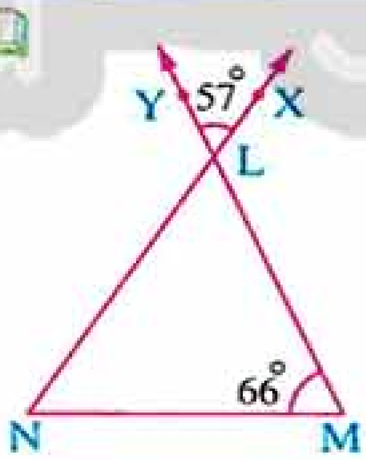
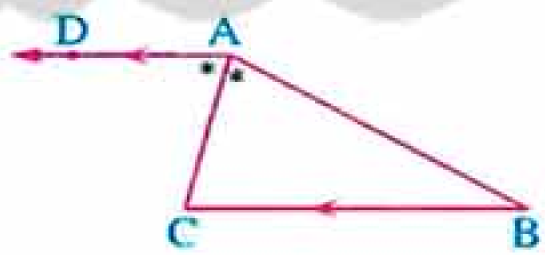


5


6


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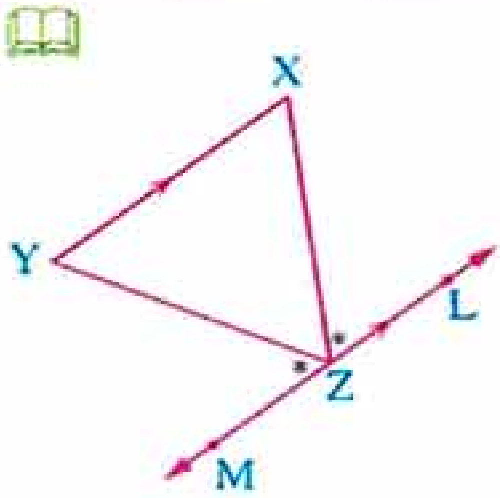

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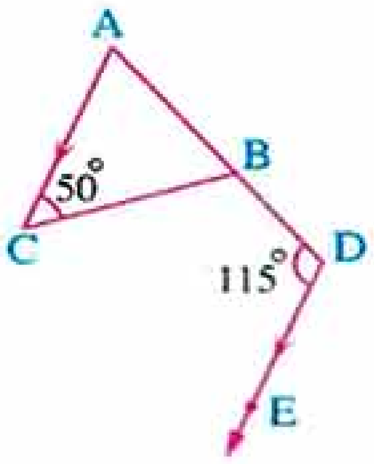

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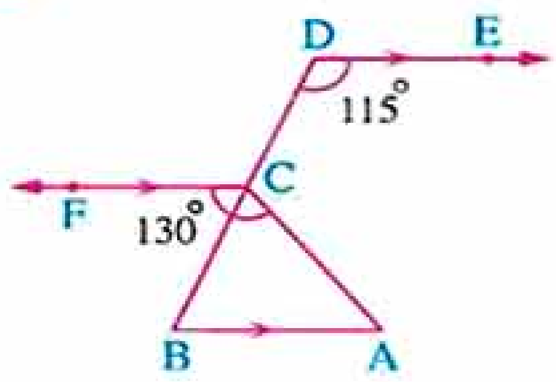


Exercise 4

2 **Complete the following :**

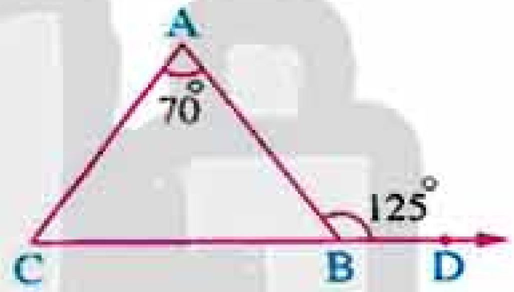
1. If two angles in the triangle are congruent , then the two sides opposite to these two angles are .... and the triangle is ....
2. If the three angles in the triangle are congruent , then the triangle is ....
3. In Δ ABC , if m ($\angle$ A) = 50° and m ($\angle$ B) = 80° , then the triangle is ....
4. If the measure of one angle in the right-angled triangle is 45° , then the triangle is ....
5. If the measure of one angle of an isosceles triangle is 60° , then the triangle is ....
6. ABC is a triangle in which AB = AC and m ($\angle$ A) = 60°
   If its perimeter = 18 cm. , then BC = ... cm.

3 **In the opposite figure :**

D $\in \overrightarrow{CB}$, m ($\angle$ ABD) = 125°

and m ($\angle$ A) = 70°

Prove that : Δ ABC is an isosceles triangle.

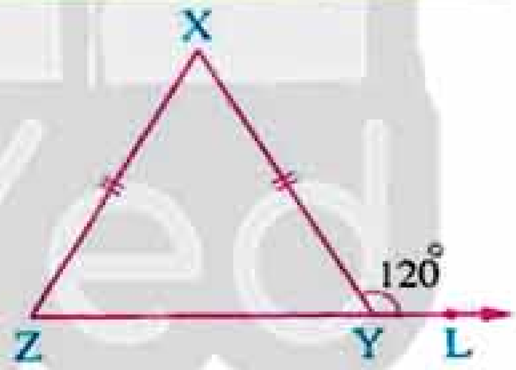


4 **In the opposite figure :**

XY = XZ , m ($\angle$ XYL) = 120°

and L $\in \overrightarrow{ZY}$

Prove that : Δ XYZ is an equilateral triangle.

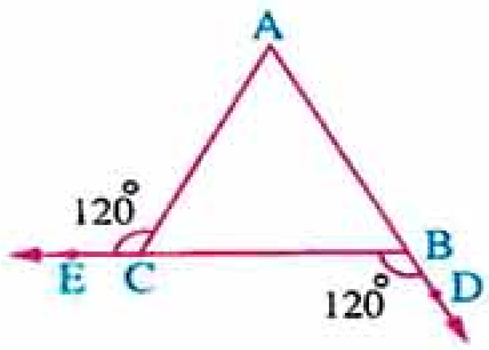


5 **In the opposite figure :**

D $\in \overrightarrow{AB}$, E $\in \overrightarrow{BC}$ and

m ($\angle$ CBD) = m ($\angle$ ACE) = 120°

Prove that : Δ ABC is an equilateral triangle.

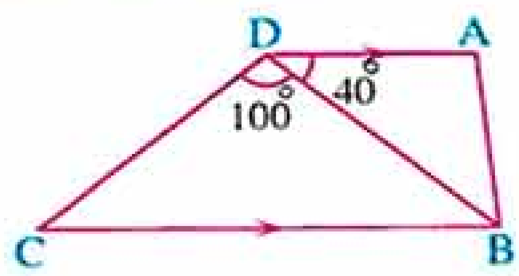


6 **In the opposite figure :**

$\overline{AD}$ // $\overline{BC}$, m ($\angle$ ADB) = 40°

and m ($\angle$ BDC) = 100°

Prove that : Δ DBC is an isosceles triangle.



Unit 4

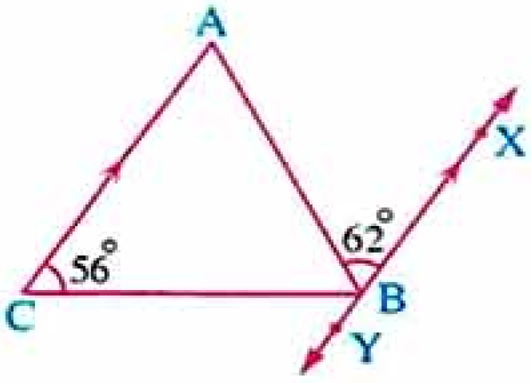
7 **In the opposite figure :**

$B \in \overleftrightarrow{XY}$, $\overleftrightarrow{XY} \parallel \overline{AC}$

, m (∠ ABX) = 62° and

m (∠ C) = 56°

Prove that : AC = BC



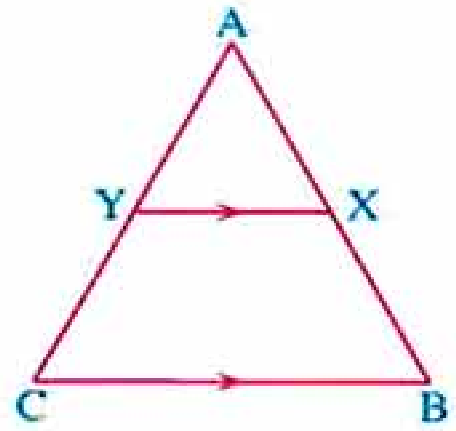
8 **In the opposite figure :**

ABC is a triangle in which AB = AC , $X \in \overline{AB}$,

$Y \in \overline{AC}$ and $\overleftrightarrow{XY} \parallel \overline{BC}$

Prove that : 1 Δ AXY is an isosceles triangle.

2 XB = YC



9 ABC is a triangle in which $D \in \overline{AB}$ and $E \in \overline{BC}$ such that BD = BE

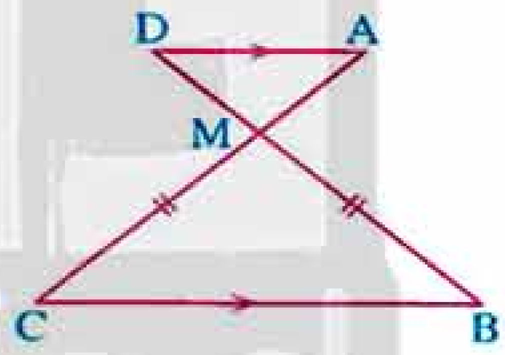
So if $\overline{DE} \parallel \overline{AC}$, **prove that :** AB = BC

10 **In the opposite figure :**

$\overline{AC} \cap \overline{BD} = \{M\}$,

MB = MC and $\overline{AD} \parallel \overline{BC}$

Prove that : MA = MD

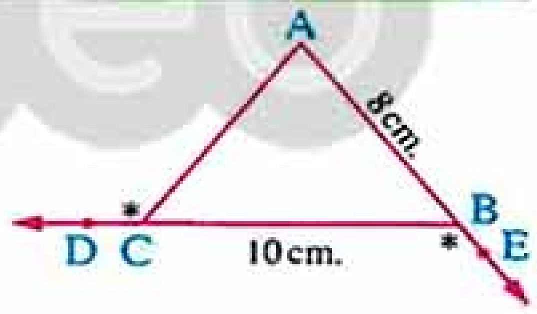


11 **In the opposite figure :**

$B \in \overrightarrow{AE}$, $C \in \overrightarrow{BD}$, AB = 8 cm. ,

BC = 10 cm. and m (∠ EBC) = m (∠ ACD)

Find : The perimeter of Δ ABC « 26 cm. »



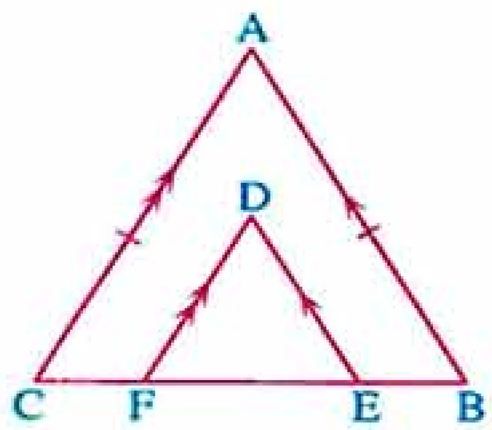
12 **In the opposite figure :**

AB = AC , $\overline{DE} \parallel \overline{AB}$ and $\overline{DF} \parallel \overline{AC}$

Prove that :

1 DE = DF

2 m (∠ BAC) = m (∠ EDF)



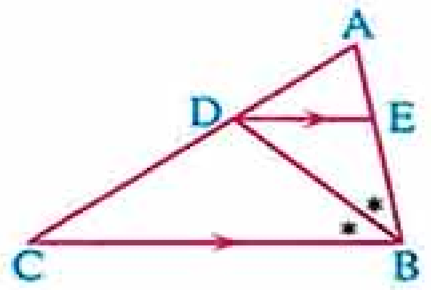
Exercise 4

13 **In the opposite figure :**

ABC is a triangle

, $\overrightarrow{BD}$ bisects $\angle$ ABC and $\overline{ED}$ // $\overline{BC}$ where E $\in$ $\overline{AB}$

Prove that : Δ EBD is an isosceles triangle.

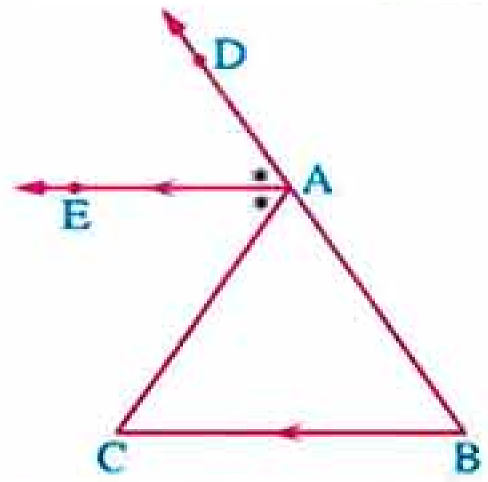


14 **In the opposite figure :**

A $\in$ $\overrightarrow{BD}$, $\overrightarrow{AE}$ // $\overline{BC}$

and $\overrightarrow{AE}$ bisects $\angle$ CAD

Prove that : AB = AC

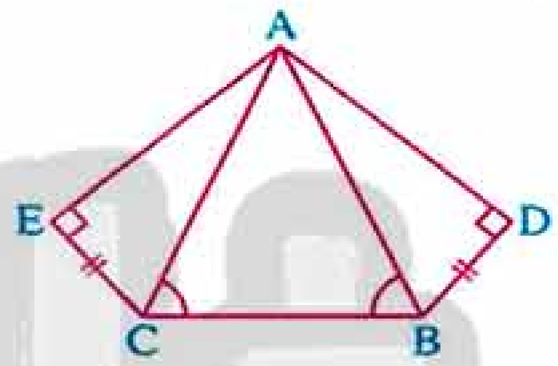


15 **In the opposite figure :**

BD = CE , m ($\angle$ ABC) = m ($\angle$ ACB)

and m ($\angle$ D) = m ($\angle$ E) = 90°

Prove that : m ($\angle$ DAB) = m ($\angle$ CAE)



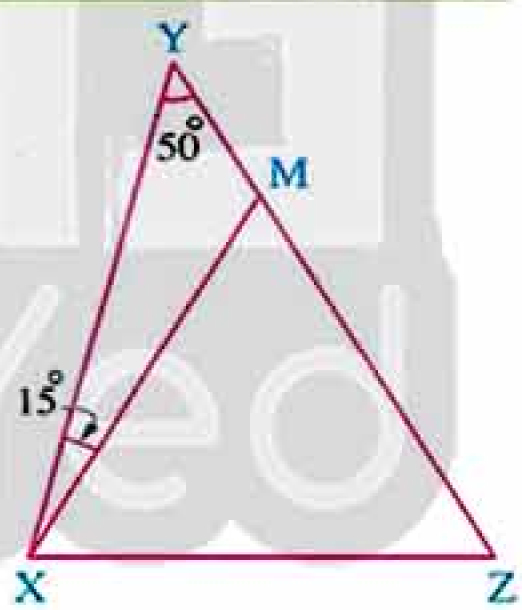
16 **In the opposite figure :**

YZX is a triangle in which YZ = YX

, m ($\angle$ Y) = 50°

and m ($\angle$ YXM) = 15°

Prove that : Δ MZX is an isosceles triangle.

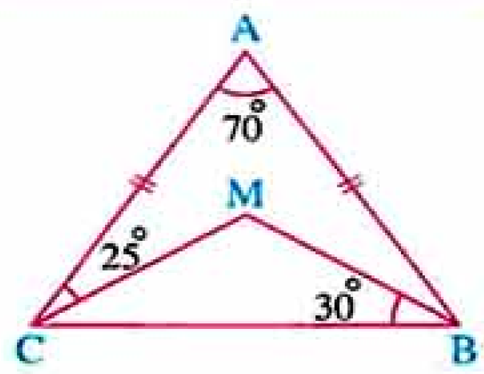


17 **In the opposite figure :**

ABC is a triangle in which AB = AC , m ($\angle$ A) = 70°

, m ($\angle$ MCA) = 25° and m ($\angle$ MBC) = 30°

Prove that : Δ MBC is an isosceles triangle.

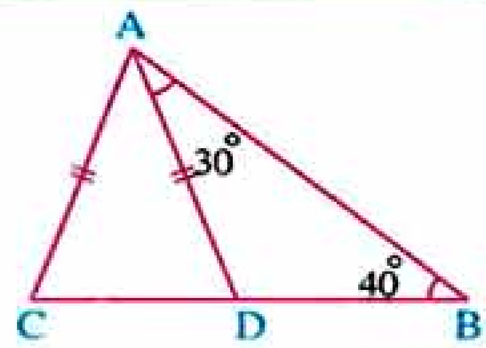


18 **In the opposite figure :**

AD = AC , m ($\angle$ B) = 40°

and m ($\angle$ BAD) = 30°

Prove that : AB = CB



Unit 4

19 ABC is a triangle in which AB = AC , $\overrightarrow{BD}$ bisects ∠ ABC and $\overrightarrow{CD}$ bisects ∠ ACB

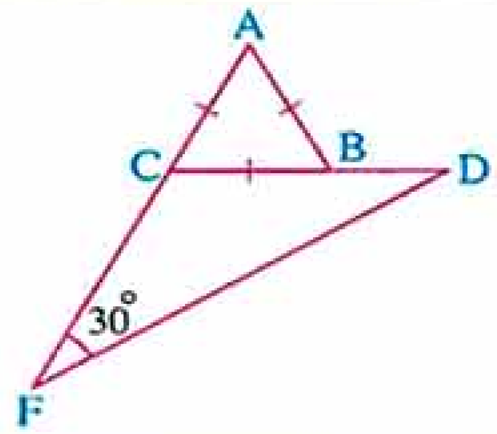
Prove that : Δ DBC is an isosceles triangle.

20 **In the opposite figure :**

ABC is an equilateral triangle , F ∈ $\overrightarrow{AC}$,

D ∈ $\overrightarrow{CB}$ and m (∠ DFC) = 30°

Prove that : Δ DCF is an isosceles triangle.



21 **In the opposite figure :**

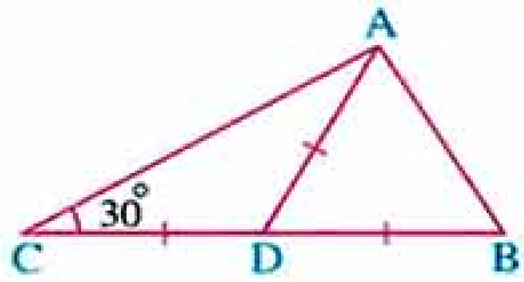
D ∈ $\overline{BC}$ such that DA = DB = DC

and m (∠ C) = 30°

Prove that :

1 Δ ABD is an equilateral triangle.

2 Δ ABC is a right-angled triangle.



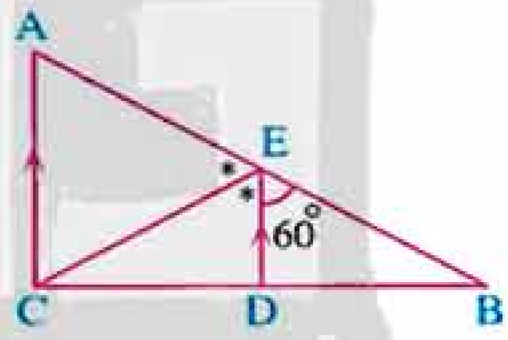
22 **In the opposite figure :**

ABC is a triangle in which E ∈ $\overline{AB}$,

$\overline{ED}$ // $\overline{AC}$, m (∠ BED) = 60°

and $\overrightarrow{EC}$ bisects ∠ AED

Prove that : Δ AEC is an equilateral triangle.



23 **In the opposite figure :**

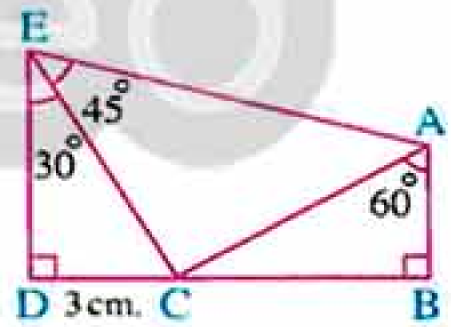
C ∈ $\overline{BD}$, m (∠ B) = m (∠ D) = 90° ,

m (∠ CED) = 30°

, m (∠ AEC) = 45° , m (∠ BAC) = 60°

and CD = 3 cm.

Find : The length of $\overline{AC}$



« 6 cm. »

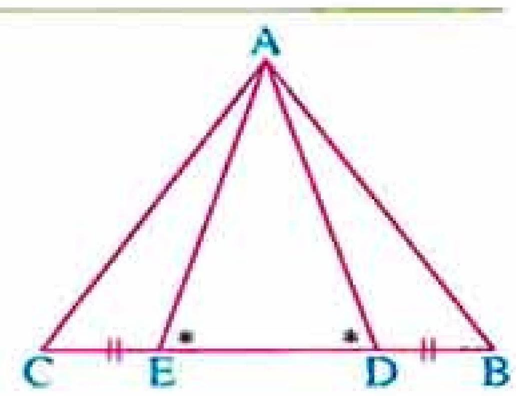
24 **In the opposite figure :**

∠ ADE ≡ ∠ AED

, B , D , E , C are collinear

and BD = CE

Prove that : Δ ABC is an isosceles triangle.

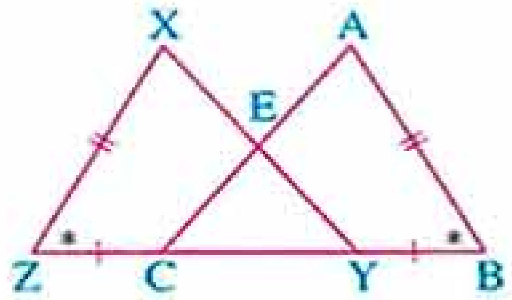


25 **In the opposite figure :**

$Y \in \overline{BZ}$, $C \in \overline{BZ}$, $AB = XZ$,

$BY = CZ$, $\overline{XY} \cap \overline{AC} = \{E\}$ and $m(\angle B) = m(\angle Z)$

Prove that : Δ EYC is an isosceles triangle.

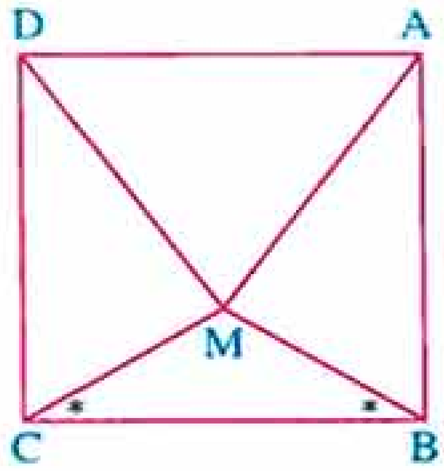


26 **In the opposite figure :**

ABCD is a square.

M is a point inside it such that : $m(\angle MBC) = m(\angle MCB)$

Prove that : Δ AMD is an isosceles triangle.



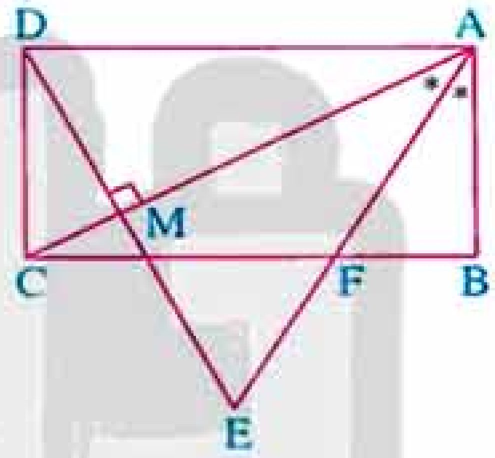
27 **In the opposite figure :**

ABCD is a rectangle in which

$\overline{AC}$ is a diagonal , $\overrightarrow{AE}$ bisects $\angle$ BAC

and $\overline{DE} \perp \overline{AC}$ where $\overrightarrow{AE} \cap \overline{DE} = \{E\}$, $\overline{AC} \cap \overline{DE} = \{M\}$

Prove that : DA = DE



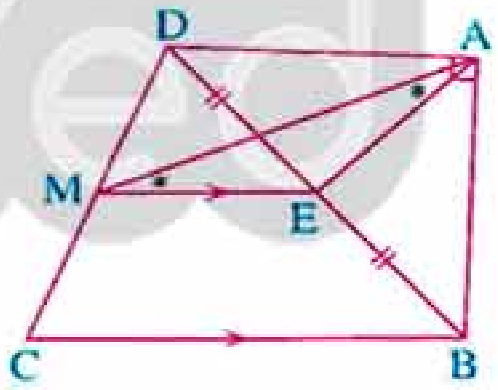
28 **In the opposite figure :**

ABCD is a quadrilateral in which

$m(\angle BAD) = 90°$, E is the midpoint of $\overline{BD}$ and $M \in \overline{DC}$

such that $\overline{EM} \parallel \overline{BC}$ and $m(\angle EAM) = m(\angle EMA)$

Prove that : BD = BC

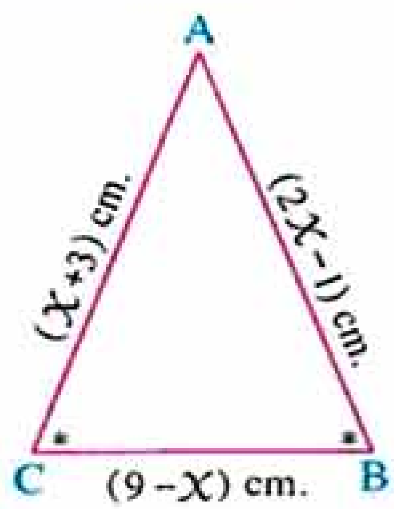


29 **In the opposite figure :**

ABC is a triangle in which :

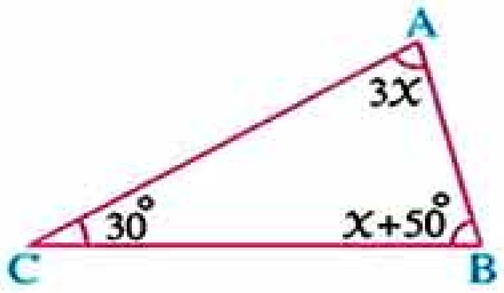
$m(\angle B) = m(\angle C)$

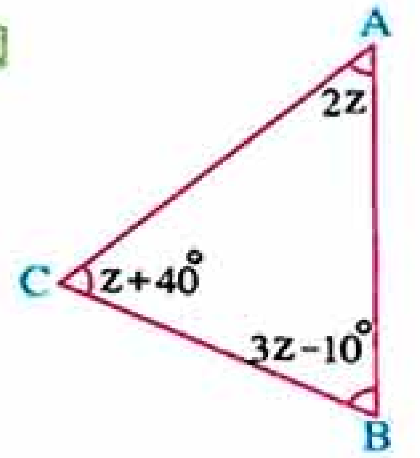
Find : The perimeter of the triangle.

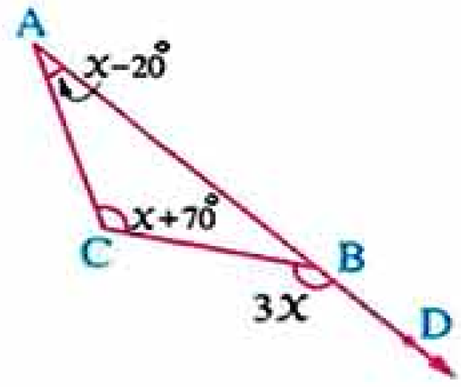


« 19 cm. »

30 **In each of the following figures , write the equal sides in length showing the steps of solution :**

1 

2 

3 

31 **Choose the correct answer from those given :**

1 In Δ ABC , if CA = CB and m ($\angle$ C) = m ($\angle$ A) , then m ($\angle$ B) = ...

(a) 30° (b) 60° (c) 90° (d) 120°

2 If the sum of measures of two congruent angles in a triangle = $\frac{2}{3}$ the sum of measures of its angles , then the triangle is ....

(a) right-angled. (b) isosceles. (c) equilateral. (d) scalene.

3 ABC is a triangle in which m ($\angle$ A) = 30° and m ($\angle$ B) : m ($\angle$ C) = 1 : 4 , then Δ ABC is ....

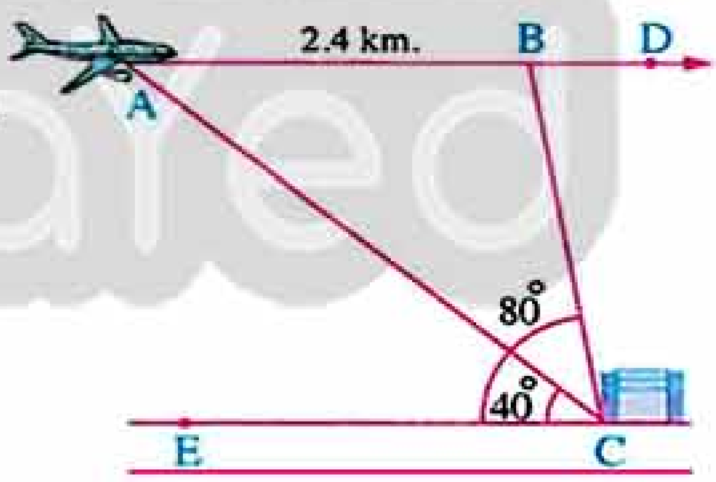
(a) right-angled. (b) isosceles. (c) equilateral. (d) scalene.

Life Application

32 **In the opposite figure :**

A plane is flying in a direction which is parallel to the earth , from the point A to the point D , the airport controller observed it and m ($\angle$ ACE) = 40° when it was on the point A and when the plane travelled 2.4 km. to the point B , m ($\angle$ BCE) = 80°

Find : The length of $\overline{BC}$



« 2.4 km. »

For excellent pupils

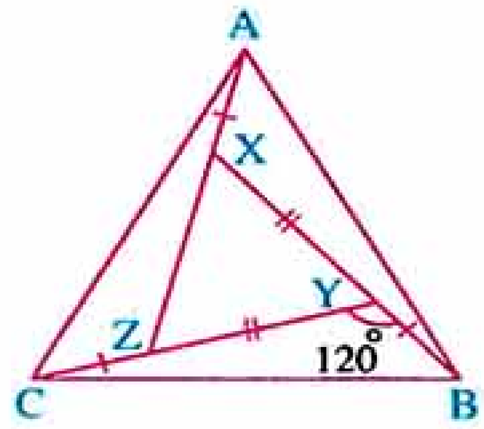
33 **In the opposite figure :**

Y $\in$ $\overline{BX}$, Z $\in$ $\overline{YC}$, X $\in$ $\overline{AZ}$,

AX = BY = CZ , XY = YZ

and m ($\angle$ BYC) = 120°

Prove that : The triangle ABC is an equilateral triangle.



Exercise 5

Corollaries of the isosceles triangle theorems

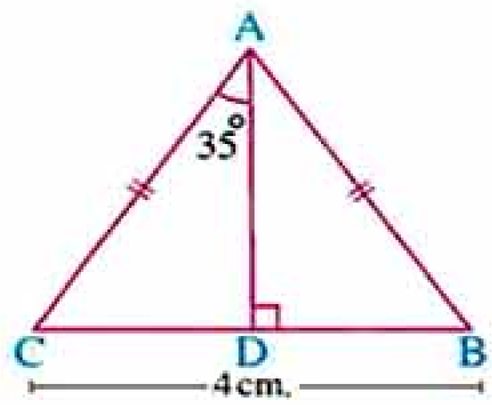
From the school book

1 Complete the following :

1. The straight line drawn from the vertex of the isosceles triangle perpendicular to the base is called ....
2. The number of axes of symmetry in the equilateral triangle equals ....
3. The number of axes of symmetry in the isosceles triangle equals ....
4. The number of axes of symmetry in the scalene triangle equals ....
5. The median of the isosceles triangle drawn from the vertex angle ....
6. The bisector of the vertex angle of the isosceles triangle ....
7. The straight line passing through the vertex angle of the isosceles triangle perpendicular to its base ....
8. The axis of the line segment is ....
9. Any point belonging to the axis of a line segment is .... from its two terminals.
10. If C belongs to the axis of symmetry of $\overline{AB}$, then .... = ...
11. In ΔABC , if $m(\angle A) = m(\angle B) = 60°$, then the number of axes of symmetry of ΔABC is ....
12. In ΔABC , if $m(\angle A) = m(\angle B) \neq 60°$, then the number of axes of symmetry of ΔABC is ....
13. In ΔABC , if $AB = AC$, $m(\angle A) = 60°$, then the number of axes of symmetry of ΔABC is ....
14. If the measure of one of the angles of a right-angled triangle is 45° , then the number of axes of symmetry of it is ....
15. If ΔABC has one axis of symmetry and $m(\angle ABC) = 120°$, then $m(\angle A) =$ ....°

Unit 4

2 **In the opposite figure :**

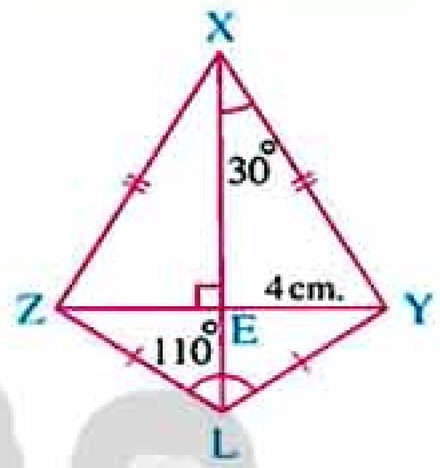


If AB = AC , $\overline{AD} \perp \overline{BC}$, BC = 4 cm. and

m (∠ DAC) = 35° , **complete the following :**

1. m (∠ BAD) = ...°
2. m (∠ BAC) = ...°
3. m (∠ B) = ...°
4. BD = ... cm.
5. The axis of symmetry of Δ ABC is ....

3 **In the opposite figure :**



XYLZ is a quadrilateral in which

XY = XZ , LY = LZ , $\overline{XL} \perp \overline{YZ}$,

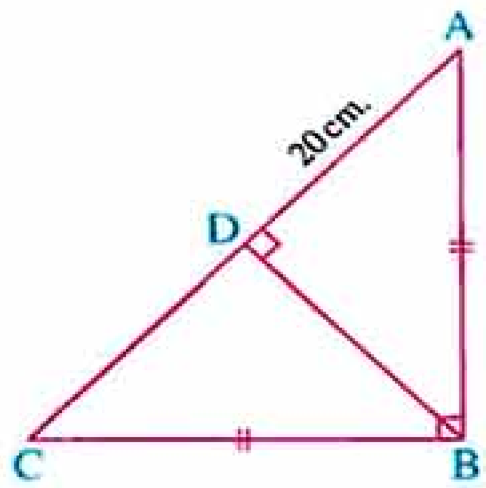
$\overline{XL} \cap \overline{YZ} = \{E\}$, m (∠ YXL) = 30° ,

m (∠ YLZ) = 110° and YE = 4 cm.

Complete the following :

1. m (∠ LXZ) = ...°
2. m (∠ YLE) = ...°
3. m (∠ XZY) = ...°
4. EZ = ... cm.
5. XZ = ... cm.
6. XE = ... cm.
7. The number of axes of symmetry of Δ XYZ is ....
8. The number of axes of symmetry of Δ YLZ is ....
9. The area of Δ XYZ = ... cm^2

4 **In the opposite figure :**



ABC is a right-angled triangle at B and it is also an isosceles triangle , $\overline{BD} \perp \overline{AC}$ and AD = 20 cm. Find the length of $\overline{AC}$ and m (∠ DBC) , then deduce that Δ BDC is an isosceles triangle.

« 40 cm. , 45° »

Exercise 5

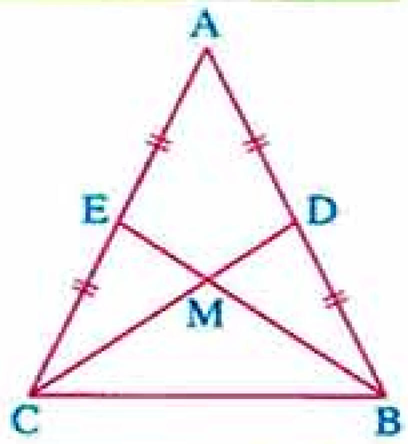
5 ABC is an isosceles triangle in which AB = AC , BC = 6 cm. , $\overline{AD} \perp \overline{BC}$ cutting it at D and m (∠ BAD) = 25° **Find :** The length of $\overline{BD}$ and m (∠ B) « 3 cm. , 65° »

6 **In the opposite figure :**

AB = AC , D and E are the midpoints of $\overline{AB}$ and $\overline{AC}$ respectively and $\overline{BE} \cap \overline{CD} = \{M\}$

Prove that :

1 $\overrightarrow{AM} \perp \overline{BC}$ 2 $\overrightarrow{AM}$ bisects ∠ BAC



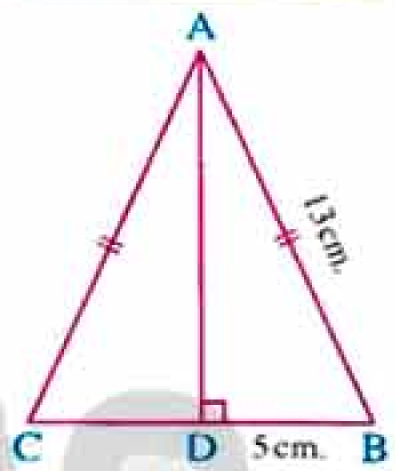
7 **In the opposite figure :**

In Δ ABC , AB = AC , $\overline{AD} \perp \overline{BC}$, AB = 13 cm. and BD = 5 cm.

Find :

1 The length of $\overline{BC}$

2 The area of Δ ABC



« 10 cm. , 60 cm^2. »

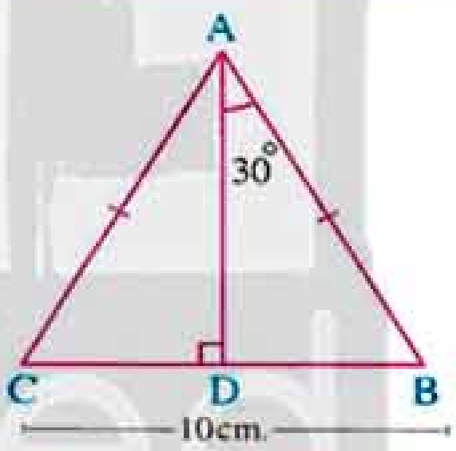
8 **In the opposite figure :**

AB = AC , BC = 10 cm. , m (∠ BAD) = 30° and $\overline{AD} \perp \overline{BC}$

1 Find the length of each of : $\overline{BD}$ and $\overline{AD}$

2 How many axes of symmetry are there at Δ ABC ?

3 Find the area of Δ ABC



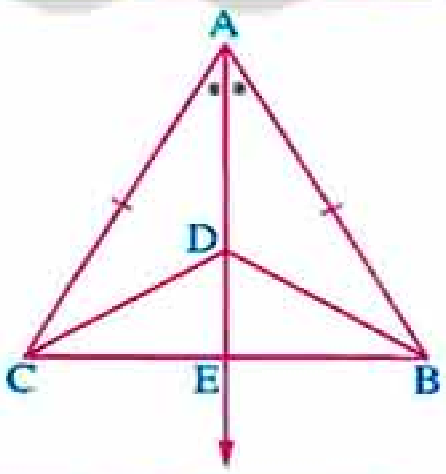
« 5 cm. , $5\sqrt{3}$ cm. , $25\sqrt{3}$ cm^2. »

9 **In the opposite figure :**

ABC is a triangle in which AB = AC , $\overrightarrow{AE}$ bisects ∠ BAC , $\overline{AE} \cap \overline{BC} = \{E\}$ and $D \in \overline{AE}$

Prove that :

1 BE = $\frac{1}{2}$ BC 2 BD = CD



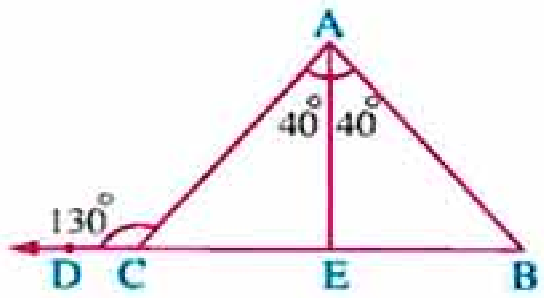
10 **In the opposite figure :**

$C \in \overrightarrow{BD}$, m (∠ ACD) = 130°

and m (∠ BAE) = m (∠ CAE) = 40°

Prove that :

1 $\overline{AE} \perp \overline{BC}$ 2 E is the midpoint of $\overline{BC}$



Unit 4

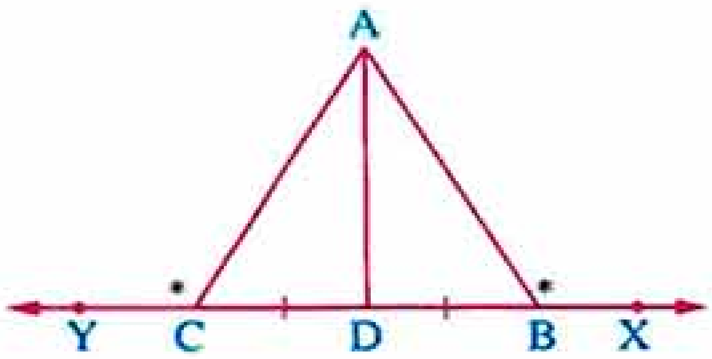
11 **In the opposite figure :**

X , B , C , D and Y are collinear points ,

$\overline{AD}$ is a median of Δ ABC and

m ($\angle$ ABX) = m ($\angle$ ACY)

Prove that : $\overline{AD} \perp \overline{BC}$



12 **In the opposite figure :**

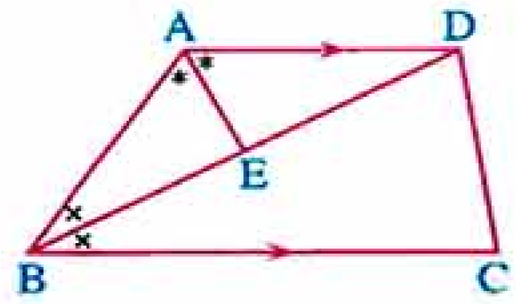
ABCD is a quadrilateral in which

$\overline{AD}$ // $\overline{BC}$, $\overrightarrow{BD}$ bisects $\angle$ ABC and

$\overrightarrow{AE}$ bisects $\angle$ BAD

Prove that :

1 AB = AD 2 $\overline{AE} \perp \overline{BD}$ 3 BE = ED



13 **In the opposite figure :**

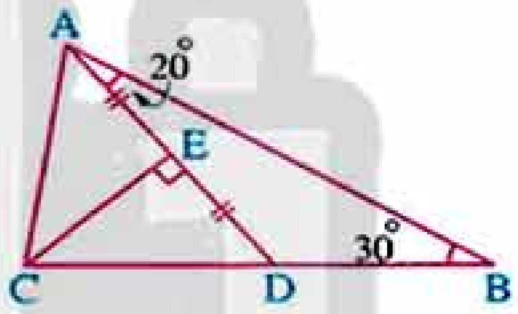
ABC is a triangle in which

m ($\angle$ B) = 30° , D $\in \overline{BC}$

where m ($\angle$ BAD) = 20°

, E is the midpoint of $\overline{AD}$ and $\overline{CE} \perp \overline{AD}$

Find : m ($\angle$ ACE) « 40° »



14 **In the opposite figure :**

ABC is a triangle in which

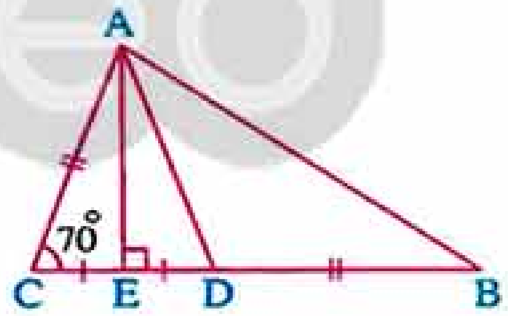
m ($\angle$ C) = 70° , D $\in \overline{BC}$

where BD = AC

, E is the midpoint of $\overline{DC}$

and AE $\perp \overline{DC}$

Find : m ($\angle$ B) « 35° »

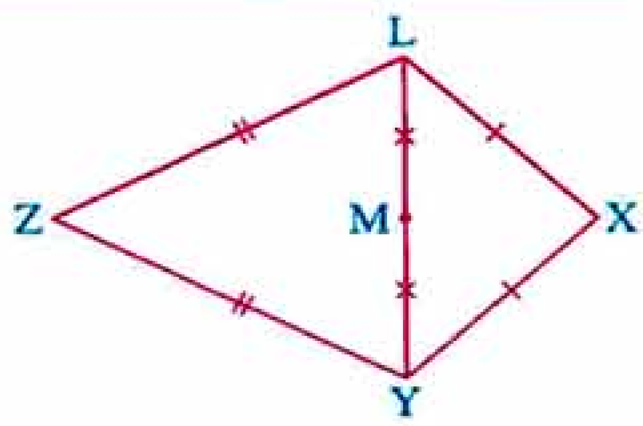


15 **In the opposite figure :**

XY = XL , ZY = ZL and LM = YM

Prove that :

X , M and Z are on the same straight line.

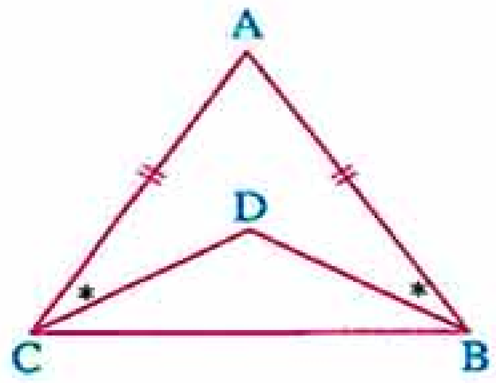


16 **In the opposite figure :**

ABC is a triangle , D is a point inside it such that

m (∠ ABD) = m (∠ ACD) and AB = AC

Prove that : $\overleftrightarrow{AD}$ is the axis of symmetry of $\overline{BC}$



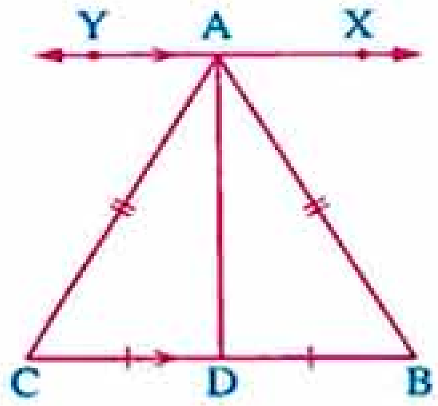
17 **In the opposite figure :**

ABC is a triangle in which AB = AC ,

D is the midpoint of $\overline{BC}$ and $\overleftrightarrow{XY}$ passes through the vertex A

such that $\overleftrightarrow{XY}$ // $\overline{BC}$

Prove that : $\overline{AD} \perp \overleftrightarrow{XY}$

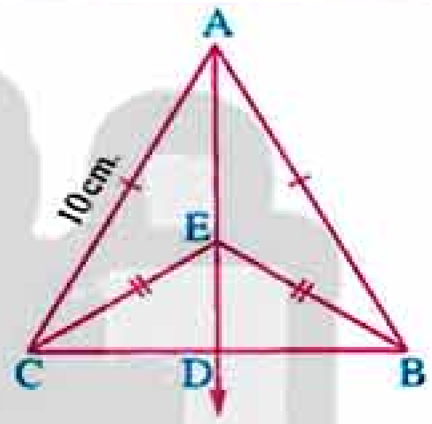


18 **In the opposite figure :**

AB = AC = 10 cm. , EB = EC and $\overrightarrow{AE} \cap \overline{BC} = \{D\}$

Prove that : BD = DC , and if BC = 6 cm.

Find the length of each of : $\overline{CD}$ and $\overline{AD}$



« 3 cm. , $\sqrt{91}$ cm. »

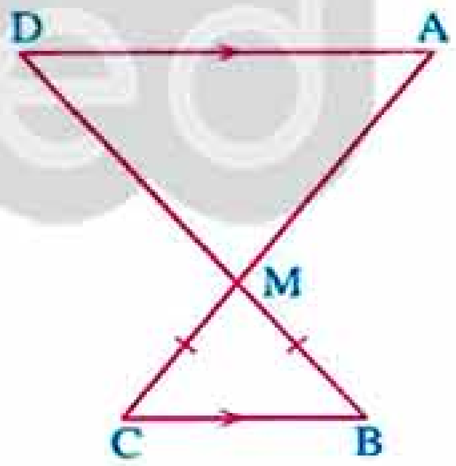
19 **In the opposite figure :**

$\overline{AC} \cap \overline{BD} = \{M\}$, $\overline{AD}$ // $\overline{BC}$ and MB = MC

Prove that :

1 Δ AMD is an isosceles triangle.

2 The axis of symmetry of Δ AMD is the same of Δ BMC



20 **In the opposite figure :**

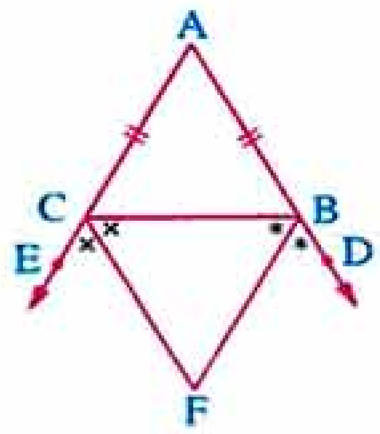
AB = AC , D ∈ $\overrightarrow{AB}$, E ∈ $\overrightarrow{AC}$,

$\overrightarrow{BF}$ bisects ∠ DBC and $\overrightarrow{CF}$ bisects ∠ BCE

Prove that :

1 Δ BFC is an isosceles triangle.

2 $\overleftrightarrow{AF}$ is the axis of symmetry of $\overline{BC}$



Unit 4

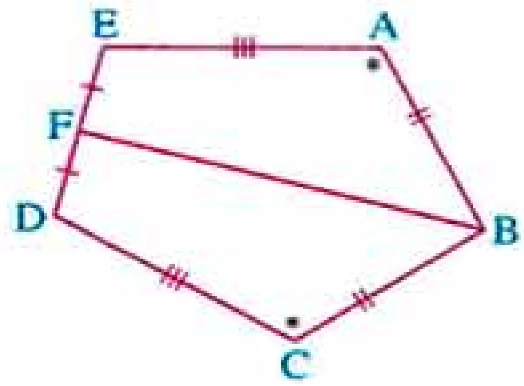
21 **In the opposite figure :**

$AB = BC$, $AE = CD$,

$m(\angle BAE) = m(\angle BCD)$

and F is the midpoint of $\overline{DE}$

Prove that : $\overline{BF} \perp \overline{DE}$



22 **In the opposite figure :**

ABCD is a quadrilateral in which

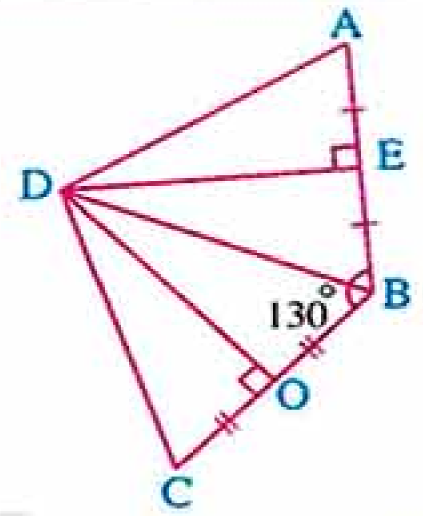
$m(\angle ABC) = 130°$

, E is the midpoint of $\overline{AB}$

, O is the midpoint of $\overline{BC}$

, $\overline{DE} \perp \overline{AB}$ and $\overline{DO} \perp \overline{BC}$

Find : $m(\angle ADC)$



« 100° »

23 **Choose the correct answer from those given :**

1. If ABCD is a quadrilateral in which $AB = AD$ and $BC = DC$, then $\overleftrightarrow{AC}$ is .... $\overline{BD}$

(a) parallel to (b) equal to

(c) the axis of symmetry of (d) congruent to

2. The triangle whose sides lengths are 2 cm. , $(X + 3)$ cm. and 5 cm. becomes an isosceles triangle when $X =$ .... cm.

(a) 1 (b) 2 (c) 3 (d) 4

3. If the length of any side in a triangle = $\frac{1}{3}$ of the perimeter of the triangle , then the number of axes of symmetry of the triangle equals ....

(a) 1 (b) 2 (c) 3 (d) zero

4. If $\overleftrightarrow{XY}$ is the axis of symmetry of $\overline{AB}$, then ....

(a) $AX = BY$ (b) $AX = BX$ (c) $BY = XY$ (d) $AY = BX$

5. In the rhombus ABCD , the axis of symmetry of $\overline{AC}$ is ....

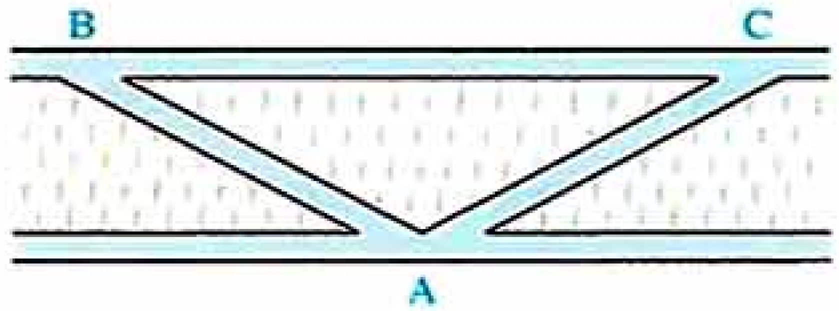
(a) $\overleftrightarrow{BD}$ (b) $\overleftrightarrow{AB}$ (c) $\overleftrightarrow{AD}$ (d) $\overleftrightarrow{CD}$

6. In the square ABCD , $\overleftrightarrow{BD}$ is the axis of symmetry of ....

(a) $\overline{AB}$ (b) $\overline{AC}$ (c) $\overline{AD}$ (d) $\overline{CD}$

Life Application

24 The opposite figure represents a geometrical sketch for building two roads $\overline{AB}$ and $\overline{AC}$, each of them is of length 1 km. , on a cultivated area of width $\frac{1}{2}$ km. in a park for bikes to move.



Find : 1 The measures of angles of Δ ABC « 30° , 120° , 30° »

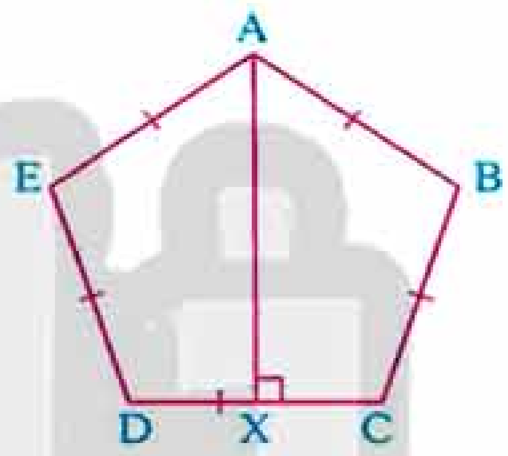
2 The distance the cyclist moves if he starts at the point B and moves around Δ ABC , then he returned to the point B again to the nearest km. « 4 km. »

For excellent pupils

25 **In the opposite figure :**

ABCDE is a regular pentagon and $\overline{AX} \perp \overline{CD}$

Find : m ($\angle$ DAX)



« 18° »

Summary of Unit 4

- The median of a triangle is the line segment drawn from any vertex of this triangle to the midpoint of the opposite side of this vertex.
- The medians of a triangle are concurrent.
- The point of concurrence of the medians of the triangle divides each median in the ratio of 1 : 2 from its base or in the ratio of 2 : 1 from the vertex.
- The point which divides the median in a triangle in the ratio of 1 : 2 from the base is the point of intersection of the medians of this triangle.
- In the right-angled triangle , the length of the median from the vertex of the right angle equals half the length of the hypotenuse.
- If the length of the median drawn from a vertex of a triangle equals half the length of the opposite side to this vertex , then the angle at this vertex is right.
- The length of the side opposite to the angle of measure 30° in the right-angled triangle equals half the length of the hypotenuse.
- The base angles of the isosceles triangle are congruent. (i.e. equal in measure)
- If two angles of a triangle are congruent , then the two sides opposite to these two angles are congruent and the triangle is isosceles.
- If the triangle is equilateral , then it is equiangular where each angle measure is 60°
- If the angles of a triangle are congruent , then the triangle is equilateral.
- The isosceles triangle in which the measure of one of its angles = 60° is an equilateral triangle.
- The median of an isosceles triangle from the vertex angle bisects it and is perpendicular to the base.
- The bisector of the vertex angle of an isosceles triangle bisects the base and is perpendicular to it.

- The straight line drawn passing through the vertex angle of an isosceles triangle perpendicular to the base bisects each of the base and the vertex angle.
- The axis of symmetry of a line segment is the straight line perpendicular to it from its midpoint.
- Any point on the axis of symmetry of a line segment is at equal distances from its terminals (end points).
- If a point is at equal distances from the two terminals of a line segment, then this point lies on the axis of this line segment.
- The isosceles triangle has one axis of symmetry which is the straight line perpendicular from its vertex to its base.
- The equilateral triangle has three axes of symmetry.
- The scalene triangle has no axes of symmetry.

Exams on Unit Four

Model 1

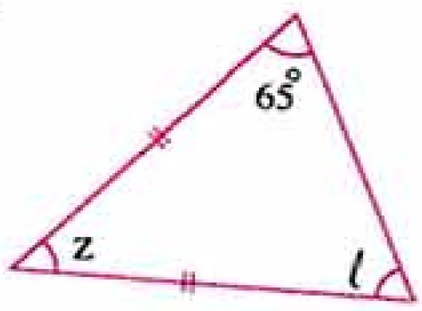
Answer the following questions :

1 Choose the correct answer from those given :

1. If M is the point of intersection of the medians in Δ ABC and $\overline{AD}$ is a median of length 6 cm. , then AM = ...
   (a) 1 cm. (b) 4 cm. (c) 3 cm. (d) 2 cm.
2. If the measure of a base angle of an isosceles triangle is 40° , then the measure of the vertex angle is ....
   (a) 40° (b) 50° (c) 80° (d) 100°
3. The measure of the exterior angle of the equilateral triangle equals ....
   (a) 30° (b) 60° (c) 90° (d) 120°
4. If the point A lies on the axis of symmetry of $\overline{XY}$, then $\overline{AX}$ .... $\overline{AY}$
   (a) // (b) $\perp$ (c) $\equiv$ (d) =
5. If ABC is a right-angled triangle at A and AB = AC , then m ($\angle$ B) = ...
   (a) 30° (b) 45° (c) 60° (d) 90°
6. The number of axes of symmetry of the isosceles triangle is ....
   (a) 0 (b) 1 (c) 2 (d) 3

2 Complete the following :

1. The point of intersection of the medians of the triangle divides each of them in the ratio .... : 2 from the vertex.
2. The length of the side opposite to the angle of measure 30° in the right-angled triangle equals ....
3. The median of the isosceles triangle drawn from the vertex .... , ...
4. If the length of the median of the triangle which is drawn from one of its vertices equals half the length of the opposite side to this vertex , then ....
5. **In the opposite figure :**
   ℓ = ...°
   z = ...°

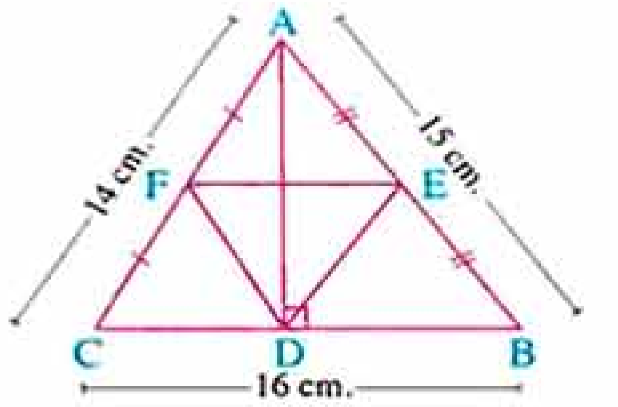


3 **[a] In the opposite figure :**

$\overline{AD} \perp \overline{BC}$, E is the midpoint of $\overline{AB}$

and F is the midpoint of $\overline{AC}$

Find : The perimeter of Δ DEF

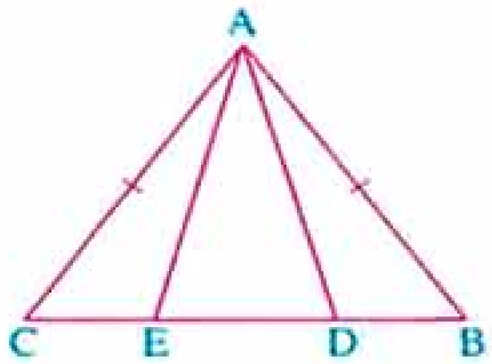


[b] In the opposite figure :

m ($\angle$ BAE) = m ($\angle$ CAD)

and AB = AC

Prove that : AE = AD



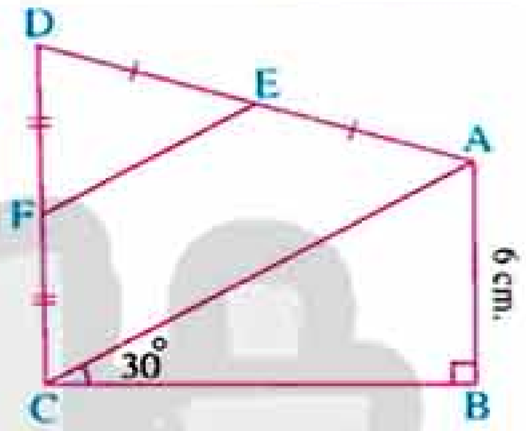
4 **[a] In the opposite figure :**

m ($\angle$ B) = 90° , m ($\angle$ ACB) = 30°

AB = 6 cm. , E is the midpoint of $\overline{AD}$

and F is the midpoint of $\overline{DC}$

Find : The length of $\overline{EF}$

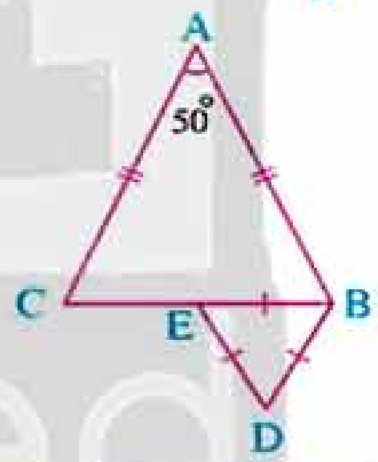


[b] In the opposite figure :

AB = AC , m ($\angle$ A) = 50°

and Δ BDE is an equilateral triangle.

Find : m ($\angle$ ABD)



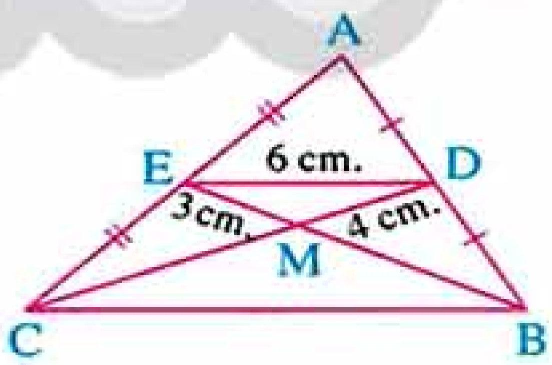
5 **[a] In the opposite figure :**

$\overline{BE}$ and $\overline{CD}$ are two medians of Δ ABC

intersecting at M , ME = 3 cm.

, MD = 4 cm. and DE = 6 cm.

Find : The perimeter of Δ MBC



[b] In the opposite figure :

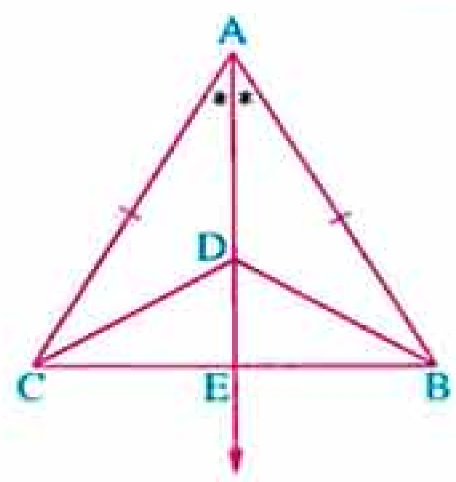
ABC is a triangle in which : AB = AC

, $\overrightarrow{AE}$ bisects $\angle$ BAC

, $\overrightarrow{AE} \cap \overline{BC} = \{E\}$ and D $\in \overrightarrow{AE}$

Prove that : **1** BE = $\frac{1}{2}$ BC

2 BD = CD



Unit 4

Model 2

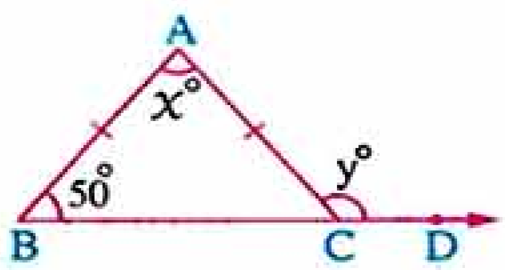
Answer the following questions :

1 **Choose the correct answer from those given :**

1. The base angles of the isosceles triangle are ....
   (a) complementary. (b) supplementary. (c) congruent. (d) straight.
2. If M is the point of intersection of the medians of Δ ABC , D is the midpoint of $\overline{BC}$, then AD = ...
   (a) 2 AM (b) $\frac{2}{3}$ MD (c) $\frac{3}{2}$ AM (d) 4 MD
3. If the measure of the vertex angle of an isosceles triangle is 50° , then the measure of each of the base angles is ....
   (a) 40° (b) 65° (c) 70° (d) 130°
4. ABC is a right-angled triangle at B , D is the midpoint of $\overline{AC}$, then BD = ...
   (a) $\frac{1}{2}$ AC (b) AC (c) $\frac{1}{2}$ BC (d) AB
5. The triangle which has three axes of symmetry is ....
   (a) isosceles. (b) equilateral. (c) right-angled. (d) obtuse-angled.
6. In Δ ABC , if AB = AC , m ($\angle$ A) = 2 m ($\angle$ B) , then m ($\angle$ C) = ...
   (a) 30° (b) 45° (c) 60° (d) 90°

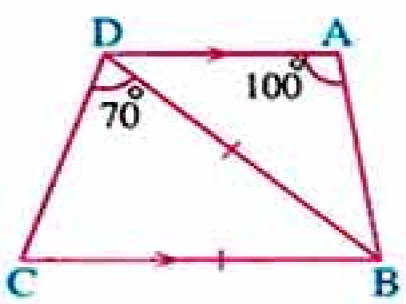
2 **Complete the following :**

1. The bisector of the vertex angle of an isosceles triangle is .... , ...
2. Any point on the axis of symmetry of a line segment is at .... distances from its two terminals.
3. ABC is a right-angled triangle at B , m ($\angle$ C) = 30° , AB = 4 cm. , then AC = ... cm.
4. **In the opposite figure :**
   AB = AC , D $\in \overrightarrow{BC}$
   , then x = ...
   , y = ...



3 **[a] In the opposite figure :**

$\overline{AD}$ // $\overline{BC}$, BD = BC
, m ($\angle$ A) = 100° and m ($\angle$ BDC) = 70°
Prove that : Δ ABD is isosceles.



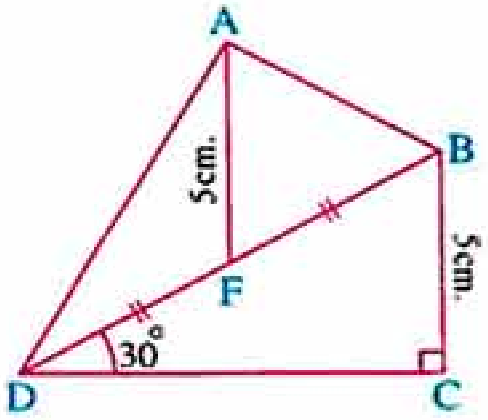
[b] In the opposite figure :

m (∠ C) = 90° , $\overline{AF}$ is a median in Δ ABD , m (∠ BDC) = 30°

and BC = AF = 5 cm.

1 **Find :** The length of $\overline{BD}$

2 **Prove that :** m (∠ BAD) = 90°



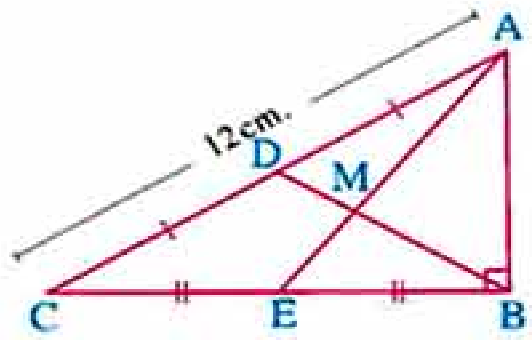
4 [a] In the opposite figure :

m (∠ ABC) = 90°

, AD = DC and BE = EC

If AC = 12 cm.

Find the length of each of : $\overline{BD}$ and $\overline{MD}$



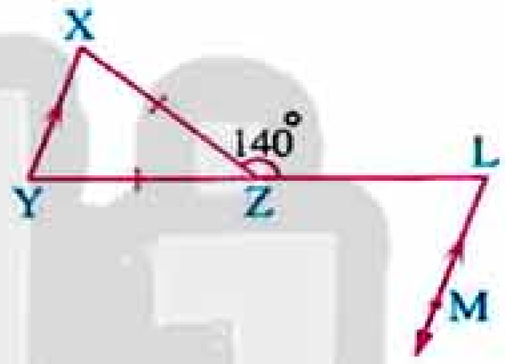
[b] In the opposite figure :

Z ∈ $\overline{LY}$, XZ = YZ

, m (∠ LZX) = 140°

and $\overrightarrow{LM}$ // $\overline{XY}$

Find : m (∠ MLY)

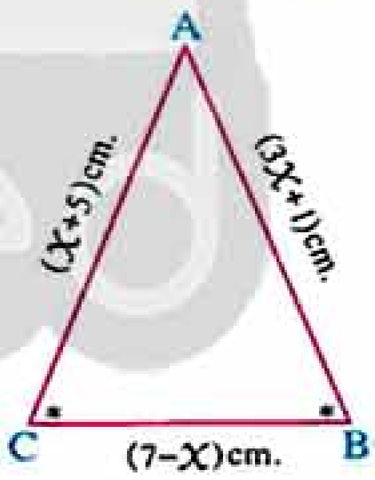


5 [a] In the opposite figure :

ABC is a triangle in which

m (∠ B) = m (∠ C)

Find : The perimeter of Δ ABC



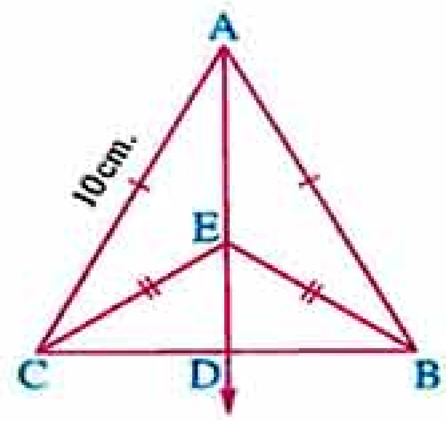
[b] In the opposite figure :

AB = AC = 10 cm.

, EB = EC and $\overrightarrow{AE}$ ∩ $\overline{BC}$ = {D}

1 **Prove that :** BD = DC

2 If BC = 6 cm. , **find the length of each of :** $\overline{CD}$ and $\overline{AD}$



UNIT 5

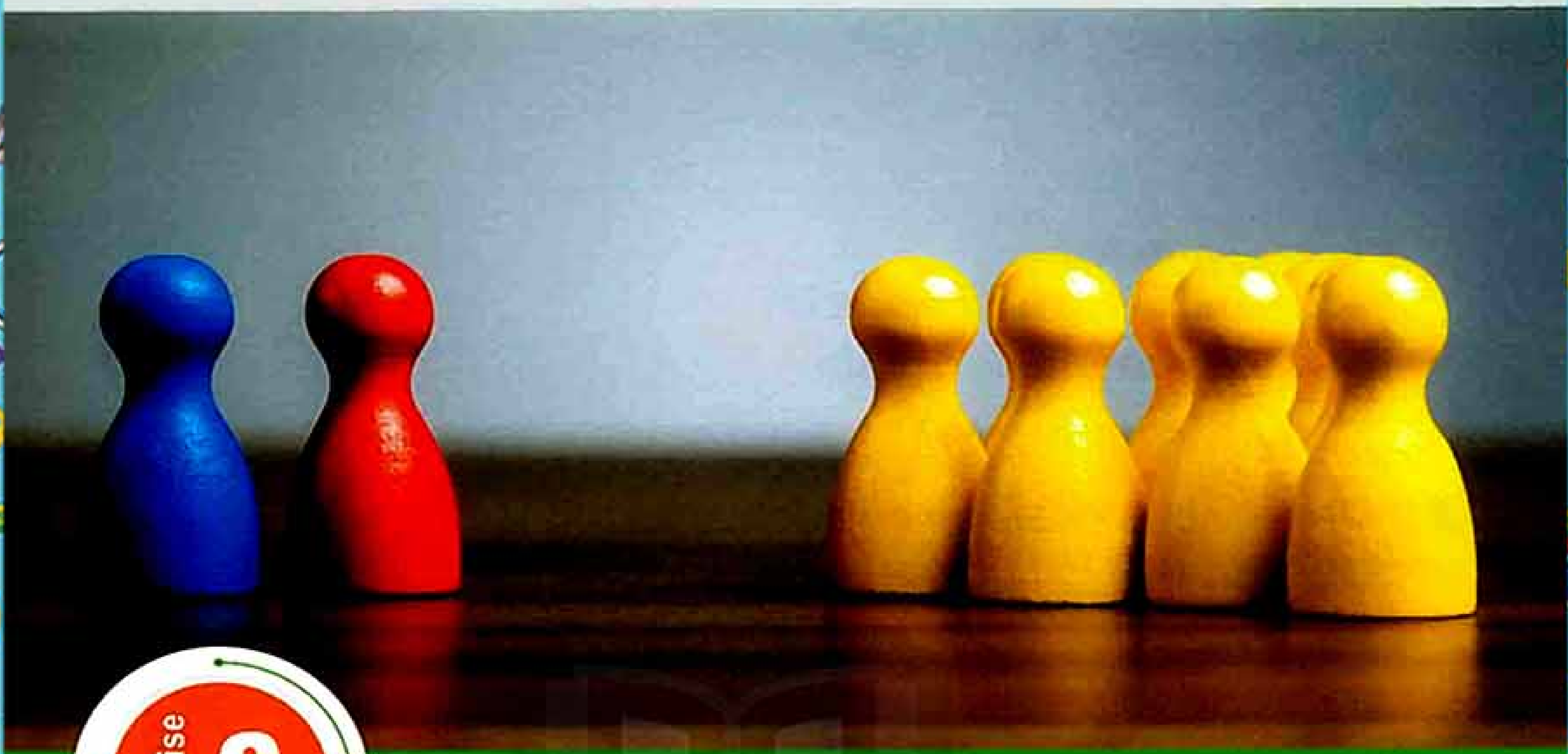
Inequality

Exercises of the unit :

6. Inequality.
7. Comparing the measures of angles in a triangle.
8. Comparing the lengths of sides in a triangle.
9. Triangle inequality.

✪ Summary of unit five.

✪ Unit exams.



Exercise 6

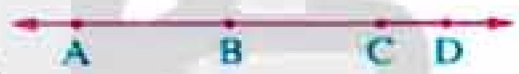
Inequality

From the school book

1 **Complete each of the following using > or < :**

1 **In the opposite figure :**

If C and B belong to $\overleftrightarrow{AD}$ such that DC < BA , then AC .... DB

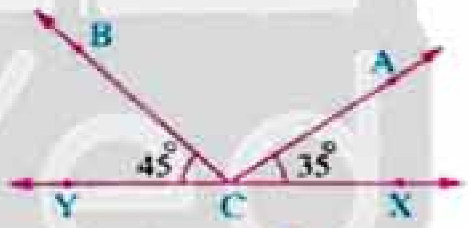


2 **In the opposite figure :**

If B and C belong to $\overleftrightarrow{AD}$ where AB > CD , then AC .... BD

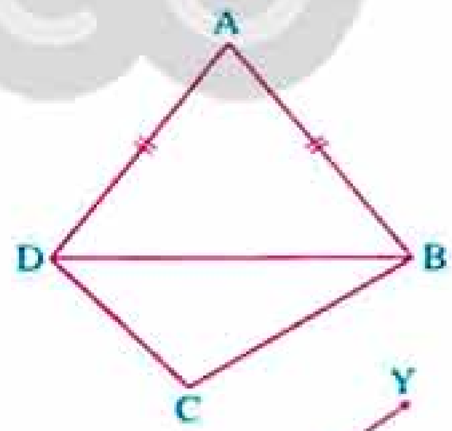
3 **In the opposite figure :**

If C $\in \overleftrightarrow{XY}$, m (∠ ACX) = 35°
and m (∠ BCY) = 45° , then m (∠ XCB) .... m (∠ ACY)



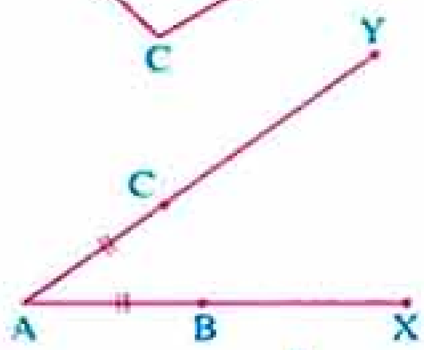
4 **In the opposite figure :**

AB = AD , m (∠ DBC) < m (∠ CDB)
, then m (∠ ABC) .... m (∠ ADC)



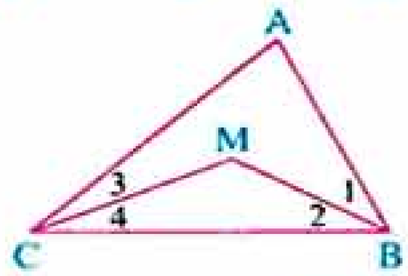
5 **In the opposite figure :**

If AB = AC and AY > AX , then BX .... CY



6 **In the opposite figure :**

m (∠ 1) > m (∠ 3) , m (∠ 2) > m (∠ 4)
, then m (∠ ABC) .... m (∠ ACB)



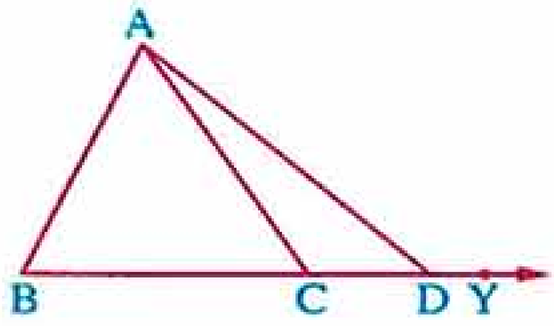
Unit 5

7 In the opposite figure :

ABC is a triangle , $C \in \overline{BD}$ and $Y \in \overrightarrow{CD}$

, then m (∠ ADY) ……… m (∠ DAC)

, m (∠ ABC) ……… m (∠ ADY)



2 **Use the opposite figure to arrange the given measures ascendingly , where B , C , D and E are collinear :**

1 m (∠ 1) , m (∠ 3)

2 m (∠ 2) , m (∠ 4)

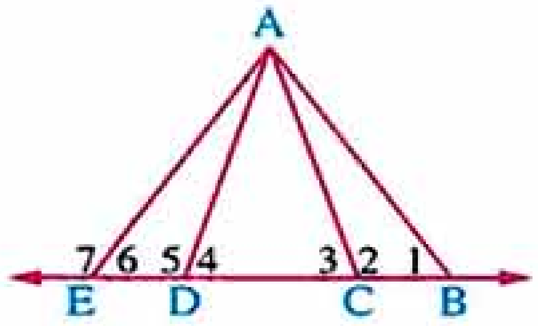
3 m (∠ 5) , m (∠ 3)

4 m (∠ 2) , m (∠ 6)

5 m (∠ 3) , m (∠ 1) , m (∠ 5)

6 m (∠ 3) , m (∠ 5) , m (∠ 7)

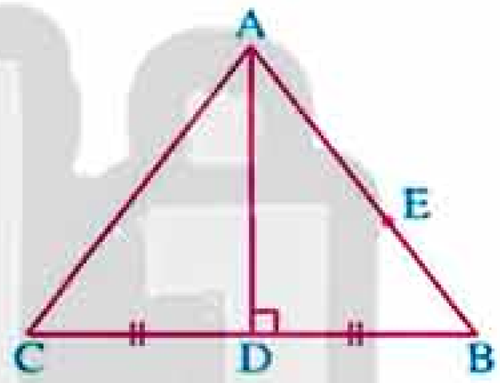
7 m (∠ 3) , m (∠ 1) , m (∠ 7) , m (∠ 5)



3 **In the opposite figure :**

$E \in \overline{AB}$, $\overline{AD} \perp \overline{BC}$ and D is the midpoint of $\overline{BC}$

Prove that : AC > AE



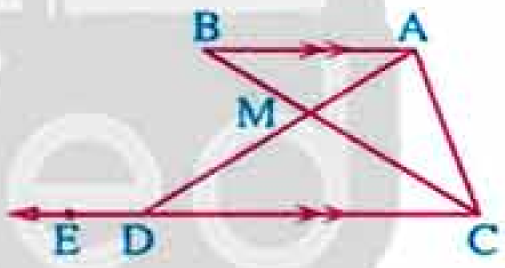
4 **In the opposite figure :**

$\overrightarrow{AB} \parallel \overrightarrow{CD}$, $\overline{AD} \cap \overline{BC} = \{M\}$, $E \in \overrightarrow{CD}$ and $E \notin \overline{CD}$

Prove that :

1 m (∠ ACD) > m (∠ ABC)

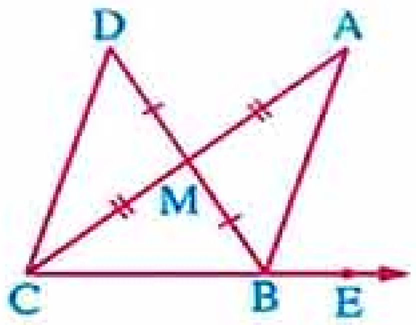
2 m (∠ ADE) > m (∠ ABC)



5 **In the opposite figure :**

$E \in \overrightarrow{CB}$ and M is the midpoint of each of $\overline{AC}$ and $\overline{BD}$

Prove that : m (∠ ABE) > m (∠ ACD)

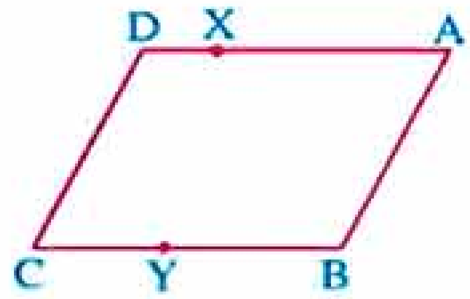


6 **In the opposite figure :**

ABCD is a parallelogram , $X \in \overline{AD}$ and $Y \in \overline{BC}$

such that DX < BY

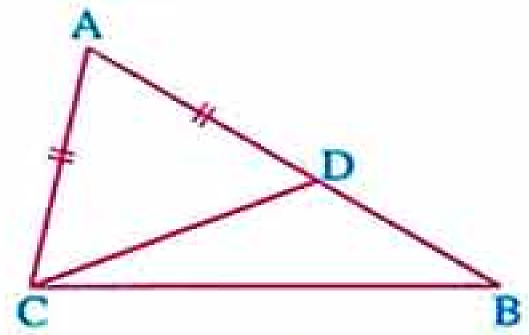
Prove that : AX + AB > CY + CD



7 **In the opposite figure :**

$D \in \overline{AB}$ where $AD = AC$

Prove that : $m(\angle ACB) > m(\angle B)$

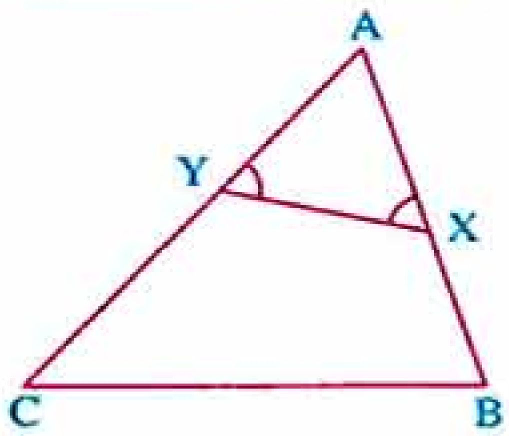


8 **In the opposite figure :**

ABC is a triangle in which : $AC > AB$, $X \in \overline{AB}$

and $Y \in \overline{AC}$ where $m(\angle AXY) = m(\angle AYX)$

Prove that : $YC > XB$

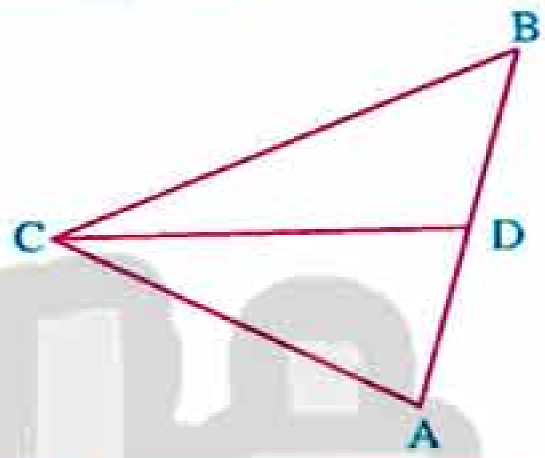


9 **In the opposite figure :**

ABC is a triangle in which :

$\overline{AB} \equiv \overline{AC}$ and $D \in \overline{AB}$

Prove that : $m(\angle ADC) > m(\angle ACB)$



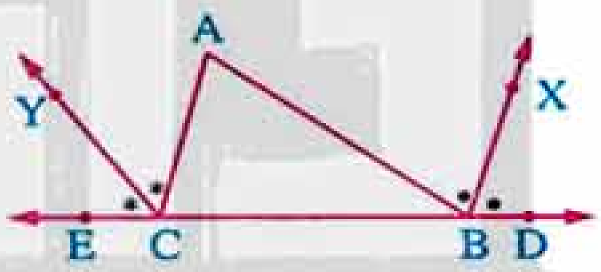
10 **In the opposite figure :**

$B \in \overleftrightarrow{DE}$, $C \in \overleftrightarrow{DE}$ such that

$m(\angle ACB) > m(\angle ABC)$

, $\overrightarrow{BX}$ bisects $\angle ABD$ and $\overrightarrow{CY}$ bisects $\angle ACE$

Prove that : $m(\angle ABX) > m(\angle ACY)$



11 M is a point inside the triangle ABC

Prove that : $m(\angle AMB) > m(\angle ACB)$

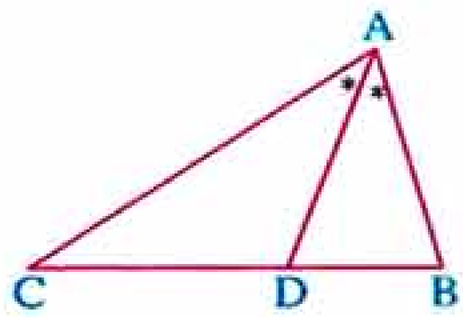
For excellent pupils

12 **In the opposite figure :**

ABC is a triangle in which : $m(\angle B) > m(\angle C)$, $D \in \overline{BC}$

such that $\overrightarrow{AD}$ bisects $\angle BAC$

Prove that : $\angle ADC$ is an obtuse angle.

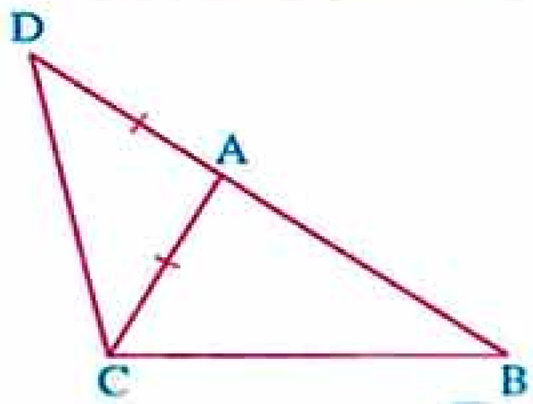


13 **In the opposite figure :**

ABC is a triangle in which : $m(\angle ACB) > m(\angle ABC)$

, $A \in \overline{BD}$ such that $AC = AD$

Prove that : $\angle BCD$ is an obtuse angle.



Exercise 7

Comparing the measures of angles in a triangle

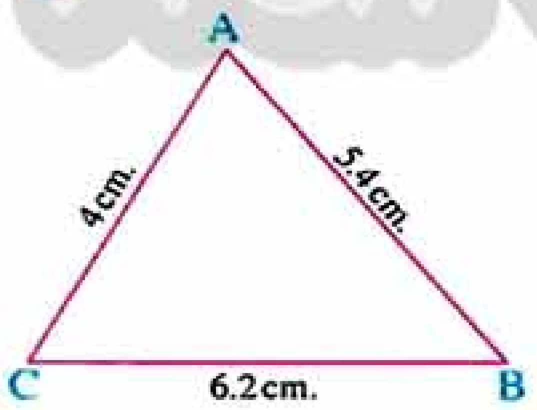
From the school book

1 Complete the following :

1. The lengths of two sides in a triangle are not equal , then the greater side in length is opposite to ....
2. In Δ ABC , AB = 7 cm. , BC = 5 cm. and AC = 6 cm. , then the smallest angle in measure is ....
3. In Δ DEF , if DE > EF , then m ($\angle$ F) > ....
4. In any triangle ABC , if AB > AC > BC , then m ($\angle$ ....) < m ($\angle$ ....) < m ($\angle$ ....)

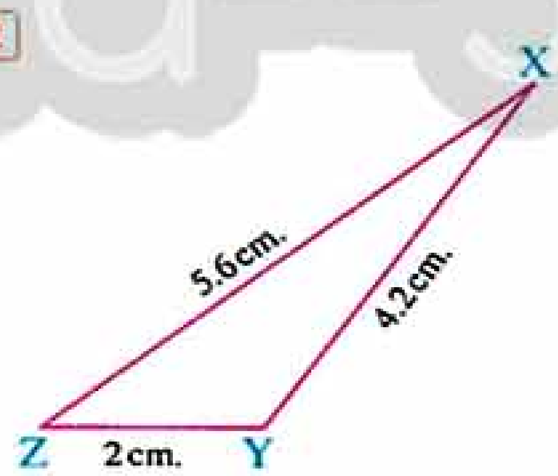
2 In each of the following figures , complete using (> or <) :

1.



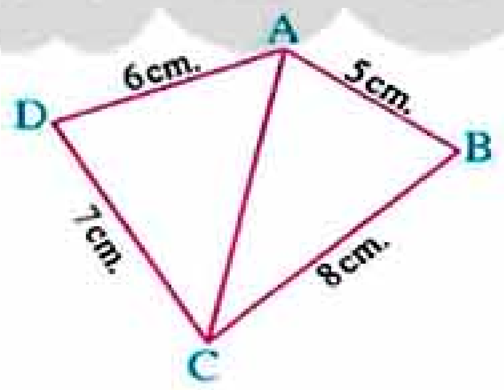
m ($\angle$ A) .... m ($\angle$ B)
m ($\angle$ A) .... m ($\angle$ C)
m ($\angle$ B) .... m ($\angle$ C)

2.



m ($\angle$ Z) .... m ($\angle$ Y)
m ($\angle$ X) .... m ($\angle$ Y)
m ($\angle$ Z) .... m ($\angle$ X)

3.



m ($\angle$ BAC) .... m ($\angle$ BCA)
m ($\angle$ DAC) .... m ($\angle$ DCA)
m ($\angle$ BAD) .... m ($\angle$ BCD)

3 Arrange the measures of the angles of Δ ABC in each of the following cases ascendingly :

1. If AB = 12 cm. , BC = 15 cm. and AC = 10 cm.
2. If AB = 5.7 cm. , BC = 8.5 cm. and AC = 6 cm.

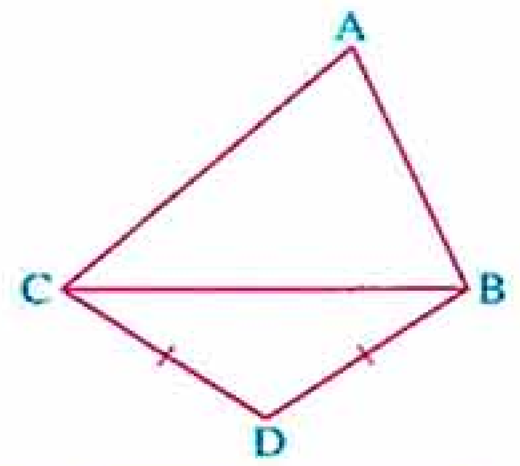
Exercise 7

4 **In the opposite figure :**

AC > AB and DB = DC

Prove that :

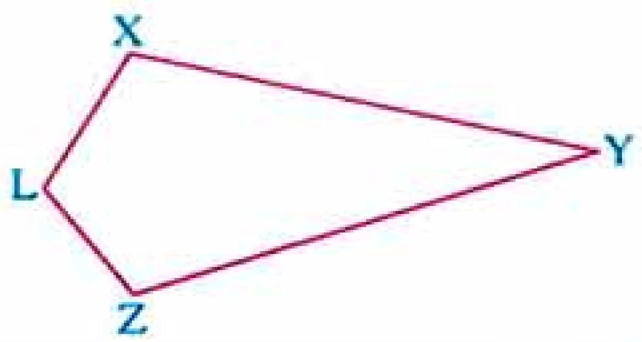
m (∠ ABD) > m (∠ ACD)



5 **In the opposite figure :**

XY > XL and YZ > ZL

Prove that : m (∠ XLZ) > m (∠ XYZ)

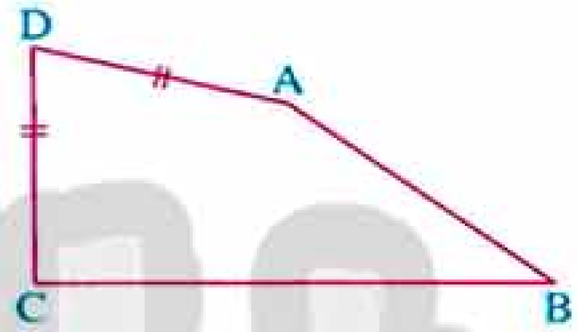


6 **In the opposite figure :**

ABCD is a quadrilateral in which :

AD = DC and BC > AB

Prove that : m (∠ A) > m (∠ C)



7 ABCD is a quadrilateral in which : $\overline{AB}$ is the longest side , $\overline{CD}$ is the shortest one

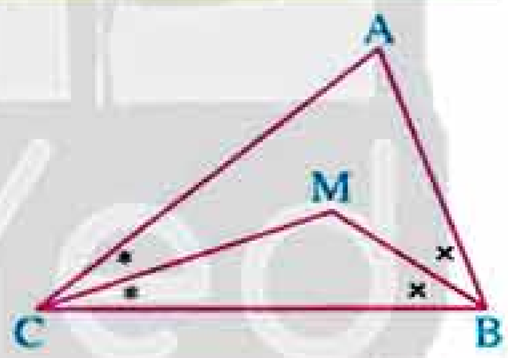
Prove that : m (∠ BCD) > m (∠ BAD)

8 **In the opposite figure :**

ABC is a triangle , $\overrightarrow{BM}$ bisects ∠ ABC and $\overrightarrow{CM}$ bisects ∠ ACB

If MC > MB

, **prove that :** m (∠ ABC) > m (∠ ACB)



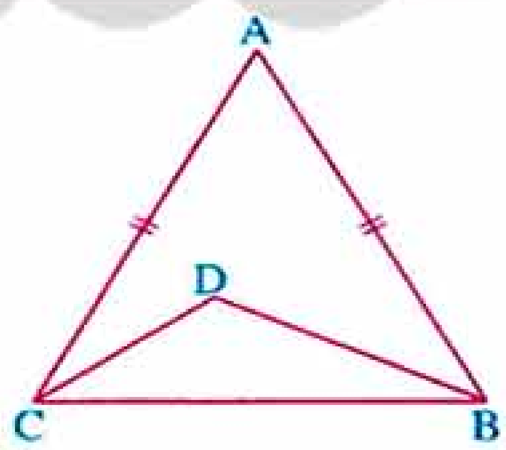
9 **In the opposite figure :**

ABC is a triangle in which :

AB = AC and DB > DC

Prove that :

m (∠ ABD) > m (∠ ACD)



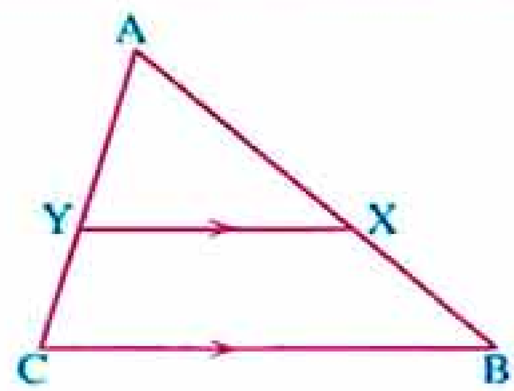
10 **In the opposite figure :**

ABC is a triangle ,

AB > AC and $\overline{XY}$ // $\overline{BC}$

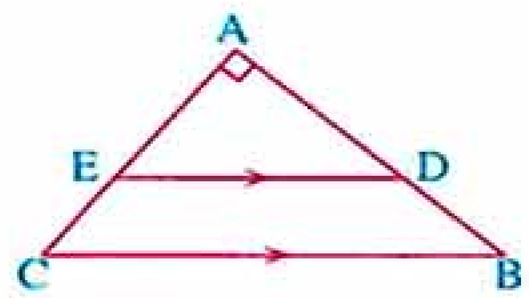
Prove that :

m (∠ AYX) > m (∠ AXY)



Unit 5

11 **In the opposite figure :**

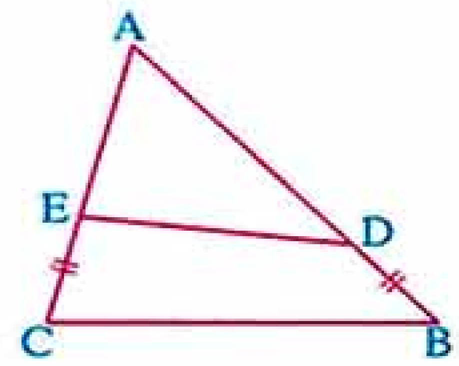


ABC is a triangle in which : m (∠ A) = 90° , AB > AC ,

D ∈ $\overline{AB}$, E ∈ $\overline{AC}$ and $\overline{DE}$ // $\overline{BC}$

Prove that : m (∠ AED) > 45°

12 **In the opposite figure :**



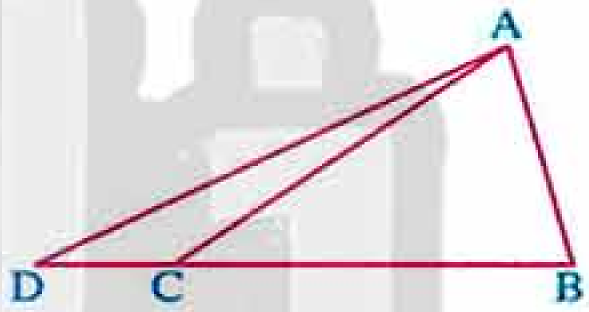
ABC is a triangle in which :

AB > AC , D ∈ $\overline{AB}$ and

E ∈ $\overline{AC}$ where BD = CE

Prove that : m (∠ AED) > m (∠ ADE)

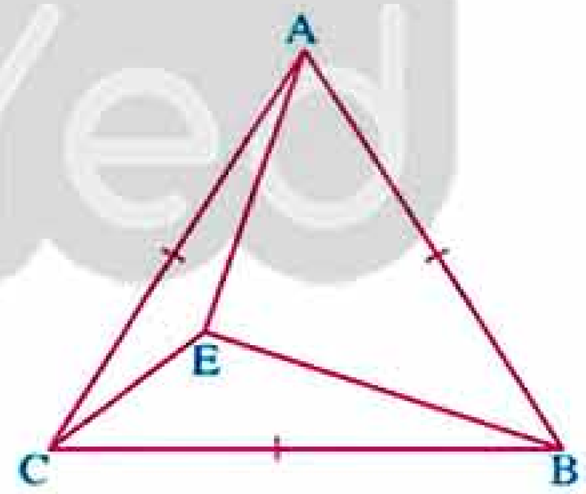
13 **In the opposite figure :**



C ∈ $\overline{BD}$ such that AC > AB

Prove that : m (∠ ABD) > m (∠ D)

14 **In the opposite figure :**



ABC is an equilateral triangle ,

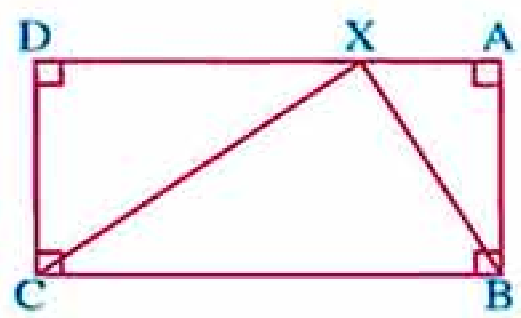
E is a point inside it ,

m (∠ ECB) > m (∠ EBC)

Prove that : 1 m (∠ ABE) > m (∠ ACE)

2 m (∠ A) > m (∠ ABE) > m (∠ ACE)

15 **In the opposite figure :**



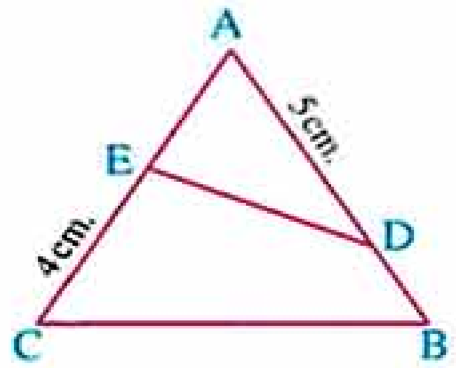
ABCD is a rectangle , X ∈ $\overline{AD}$

such that XC > XB

Prove that : m (∠ ABX) < m (∠ XCD)

Exercise 7

16 **In the opposite figure :**



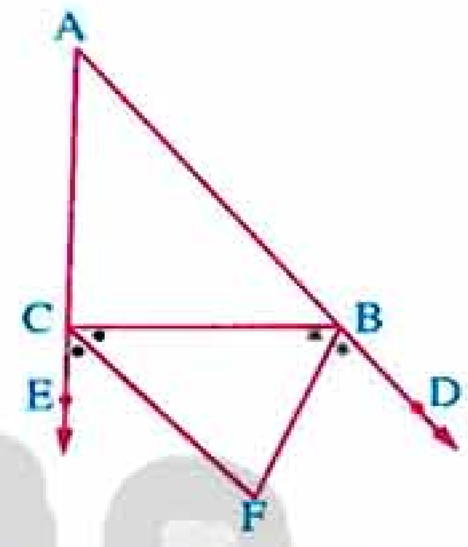
ABC is an equilateral triangle

whose side length = 7 cm. , D $\in \overline{AB}$ such that

AD = 5 cm. and E $\in \overline{AC}$ such that CE = 4 cm.

Prove that : m ($\angle$ AED) > 60°

17 **In the opposite figure :**



ABC is a triangle in which :

AB > AC , D $\in \overrightarrow{AB}$, E $\in \overrightarrow{AC}$,

$\overrightarrow{BF}$ bisects $\angle$ DBC and $\overrightarrow{CF}$ bisects $\angle$ BCE

Prove that : m ($\angle$ FBC) > m ($\angle$ BCF)

18 ABC is a triangle , D is a point inside it. If DA > DB > DC

, **prove that :** m ($\angle$ ACB) > m ($\angle$ DAC) + m ($\angle$ DBC)

19 ABC is a triangle , $\overline{AD}$, $\overline{CE}$ are two medians intersecting at M If MD > ME

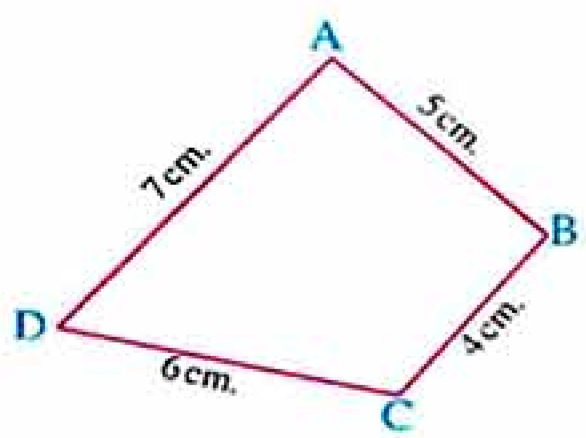
, **prove that :** m ($\angle$ CAM) < m ($\angle$ MCA)

20 ABC is a triangle in which : AB > AC , D is the midpoint of $\overline{AB}$

Draw $\overrightarrow{DE}$ // $\overline{AC}$ to meet $\overline{BC}$ at E

Prove that : m ($\angle$ CAE) > m ($\angle$ DAE)

21 **In the opposite figure :**



ABCD is a quadrilateral in which :

AB = 5 cm. , BC = 4 cm. , CD = 6 cm. and DA = 7 cm.

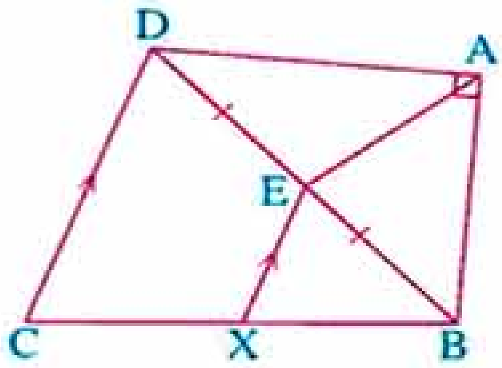
Prove that : 1 m ($\angle$ ABC) > m ($\angle$ ADC)

2 m ($\angle$ BCD) > m ($\angle$ BAD)

3 m ($\angle$ B) + m ($\angle$ C) > 180°

Unit 5

22 **In the opposite figure :**



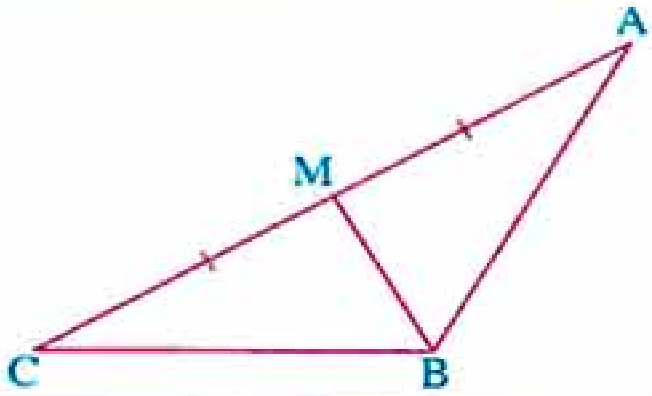
ABCD is a quadrilateral in which : m (∠ A) = 90° ,

$\overline{AE}$ is a median of Δ ABD , $\overrightarrow{EX}$ // $\overline{DC}$ and

$\overrightarrow{EX} \cap \overline{BC} = \{X\}$ If AE > EX

, prove that : m (∠ C) > m (∠ DBC)

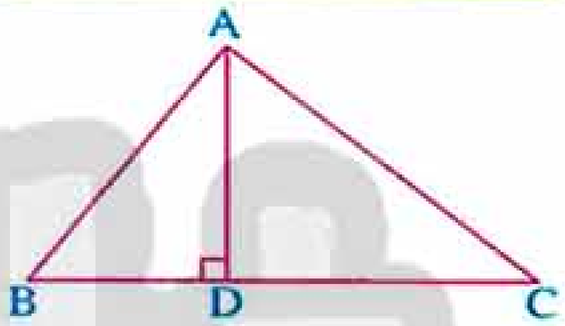
23 **In the opposite figure :**



$\overline{BM}$ is a median in the triangle ABC and BM < AM

Prove that : ∠ ABC is an obtuse angle.

24 **In the opposite figure :**



ABC is a triangle in which : AC > AB , $\overline{AD} \perp \overline{BC}$ and intersects it at D

Prove that : m (∠ BAD) < m (∠ CAD)

25 ABC is a triangle , $\overrightarrow{AD}$ bisects ∠ A and intersects $\overline{BC}$ at D , if AC > AB

, prove that : ∠ ADC is an obtuse angle.

26 ABCD is a parallelogram in which : AC > BD

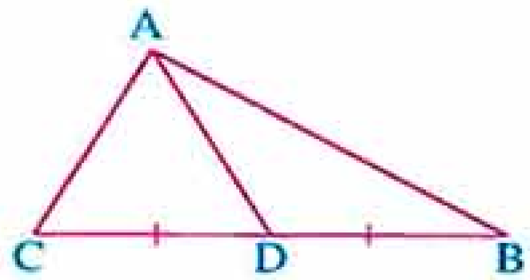
Prove that : ∠ D is an obtuse angle.

For excellent pupils

27 ABC is a triangle , D is the midpoint of $\overline{BC}$, if the perimeter of Δ ACD > the perimeter of Δ ABD

, prove that : m (∠ B) > m (∠ C)

28 **In the opposite figure :**



AB > AC and DB = DC

Prove that : m (∠ BAD) < m (∠ CAD)

Exercise 8

Comparing the lengths of sides in a triangle

From the school book

1 Complete the following :

1. If two angles in a triangle are unequal in measure , then the greater angle in measure is opposite to .... and if the two lengths of two sides in a triangle are unequal , then the greater side in length is opposite to the angle which is ....
2. The smallest angle of a triangle (in measure) is opposite to ....
3. The longest side in the right-angled triangle is ....
4. The shortest distance between a given point and a given straight line is ....
5. ABC is a triangle in which : $m(\angle C) = 110°$, then its longest side is ....
6. In ΔABC : If $m(\angle A) = 50°$, $m(\angle B) = 30°$, then the shortest side in the triangle is ....
7. In ΔABC : If $m(\angle A) = m(\angle B) + m(\angle C)$, then the longest side in the triangle is ....

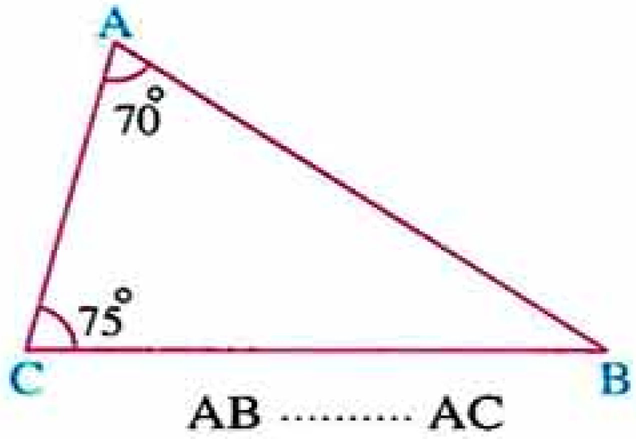
2 Choose the correct answer from those ones :

1. In ΔABC : If $m(\angle B) > m(\angle C)$, then ....
   (a) $AB > AC$ (b) $BC > AC$ (c) $AC > AB$ (d) $AB > BC$
2. In ΔABC : If $m(\angle B) = 90°$, then ....
   (a) $AC > CB$ (b) $AB > AC$ (c) $BC > AC$ (d) $AB = AC$
3. In ΔABC : If $m(\angle A) = 40°$ and $m(\angle B) = 70°$, then ....
   (a) $AB < AC$ (b) $AB > AC$ (c) $\overline{AB} \perp \overline{AC}$ (d) $AB = AC$

Unit 5

3 **In the following figures , complete using > , < or = :**

1

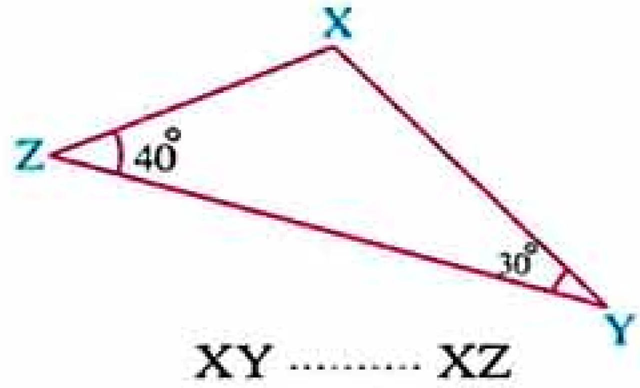


AB .... AC

AB .... BC

AC .... BC

2

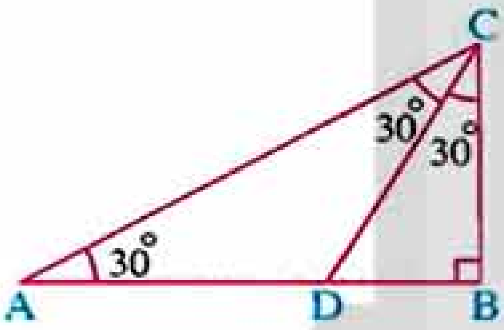


XY .... XZ

YZ .... XY

YZ .... XZ

3



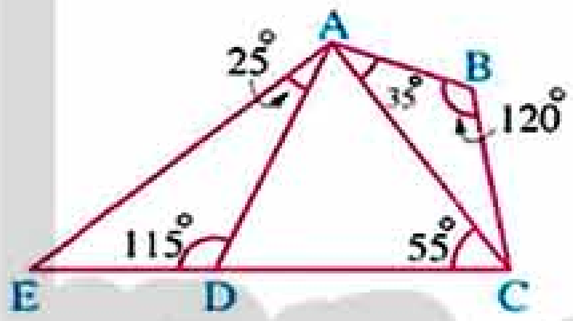
AC .... BC

BC .... DB

AC .... BD

CA .... DC

4



BC .... AB

CD .... CA

AD .... AE

CD .... AD

4 XYZ is a triangle in which : m ($\angle$ X) = 45° , m ($\angle$ Y) = 85° and m ($\angle$ Z) = 50°

Arrange the lengths of the sides of the triangle ascendingly.

5 ABC is a triangle in which : m ($\angle$ A) = 40° and m ($\angle$ B) = 75°

Order the lengths of the sides of the triangle descendingly.

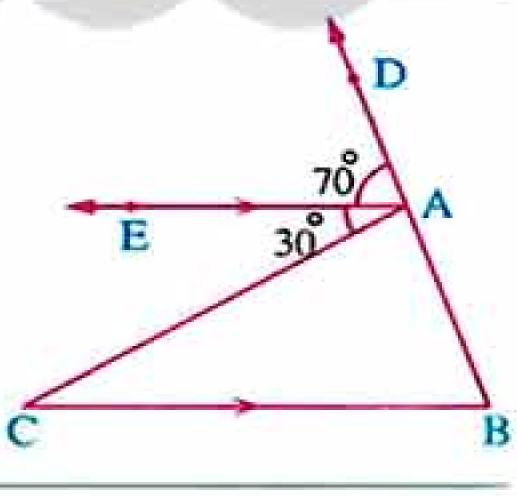
6 **In the opposite figure :**

$\overrightarrow{AE}$ // $\overline{BC}$,

m ($\angle$ DAE) = 70°

and m ($\angle$ EAC) = 30°

Prove that : AC > AB



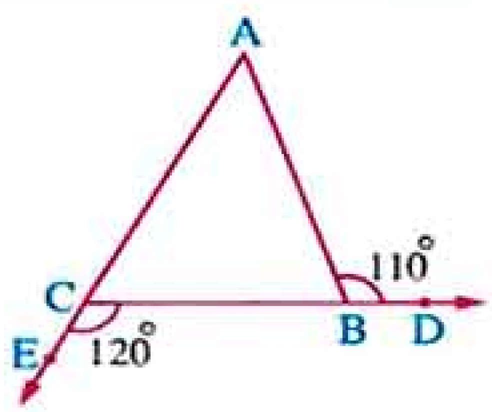
7 **In the opposite figure :**

ABC is a triangle , D $\in \overrightarrow{CB}$,

E $\in \overrightarrow{AC}$, m ($\angle$ ABD) = 110°

and m ($\angle$ BCE) = 120°

Prove that : AB > BC

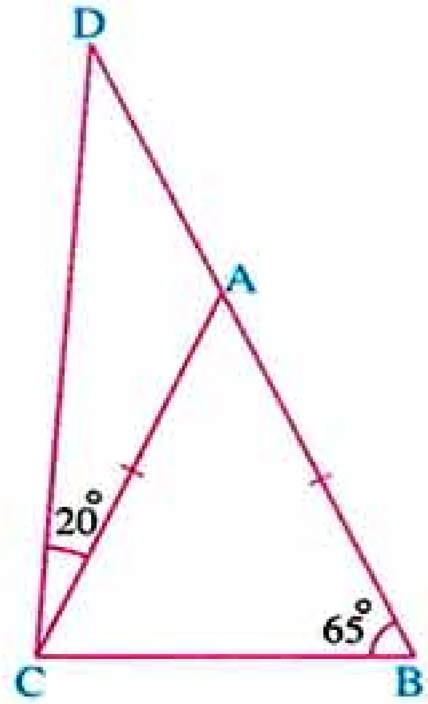


8 **In the opposite figure :**

$AB = AC$, $m(\angle ABC) = 65°$

, $m(\angle ACD) = 20°$, $A \in \overline{BD}$

Prove that : $AB > AD$

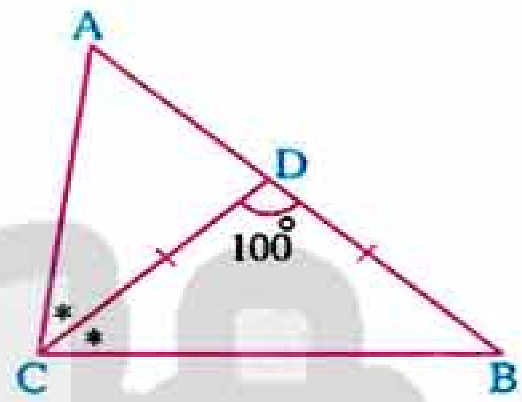


9 **In the opposite figure :**

ABC is a triangle , $\overrightarrow{CD}$ bisects $\angle C$ and intersects $\overline{AB}$ at point D

, $m(\angle BDC) = 100°$ and $DB = DC$

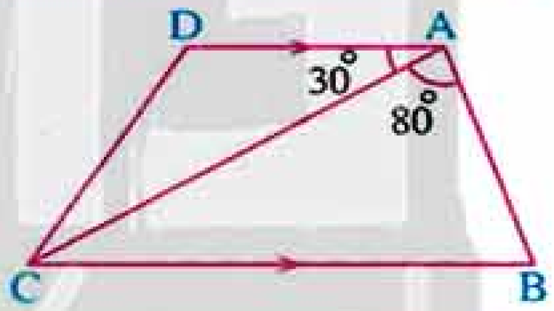
Prove that : $AC > DB$



10 **In the opposite figure :**

$\overline{AD} \mathbin{/\mkern-6mu/} \overline{BC}$, $m(\angle BAC) = 80°$ and $m(\angle DAC) = 30°$

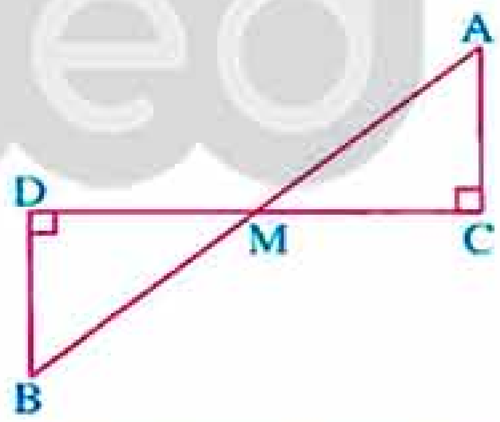
Prove that : $BC > AB$



11 **In the opposite figure :**

$\overline{AB} \cap \overline{CD} = \{M\}$, $\overline{AC} \perp \overline{CD}$ and $\overline{BD} \perp \overline{CD}$

Prove that : $AB > CD$

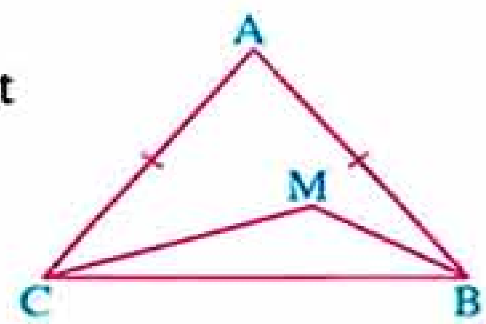


12 **In the opposite figure :**

ABC is a triangle in which : $AB = AC$, M is a point inside it such that

$m(\angle ABM) < m(\angle ACM)$

Prove that : $MC > MB$



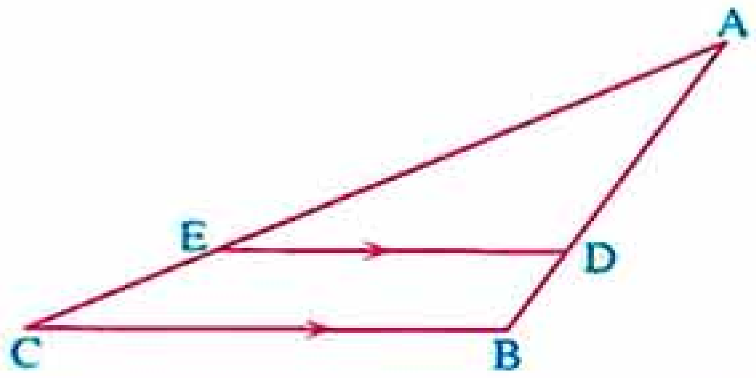
Unit 5

13 **In the opposite figure :**

ABC is an obtuse-angled triangle at B

, $\overline{DE}$ // $\overline{BC}$

Prove that : AE > AD

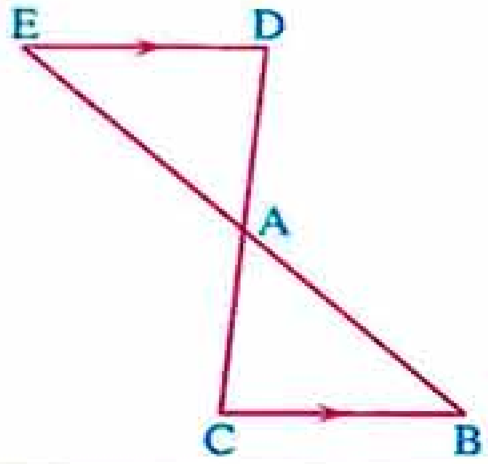


14 **In the opposite figure :**

AB > AC , $\overline{DE}$ // $\overline{BC}$ and

$\overline{DC} \cap \overline{BE} = \{A\}$

Prove that : AE > AD

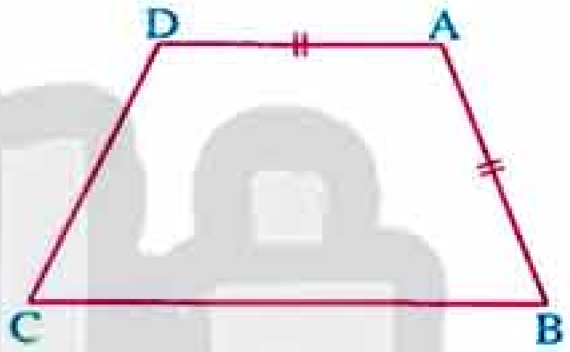


15 **In the opposite figure :**

ABCD is a quadrilateral , AB = AD

and m (∠ D) > m (∠ B)

Prove that : BC > CD



16 **In the opposite figure :**

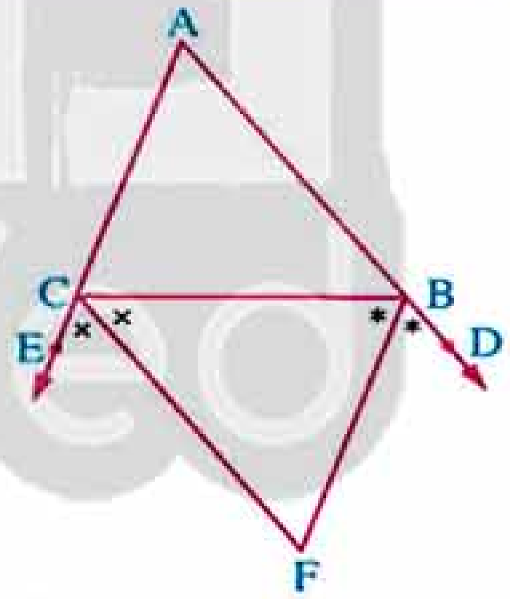
ABC is a triangle in which : AB > AC , D ∈ $\overrightarrow{AB}$, E ∈ $\overrightarrow{AC}$

, $\overrightarrow{BF}$ bisects ∠ DBC and $\overrightarrow{CF}$ bisects ∠ BCE

, $\overrightarrow{BF} \cap \overrightarrow{CF} = \{F\}$

Prove that :

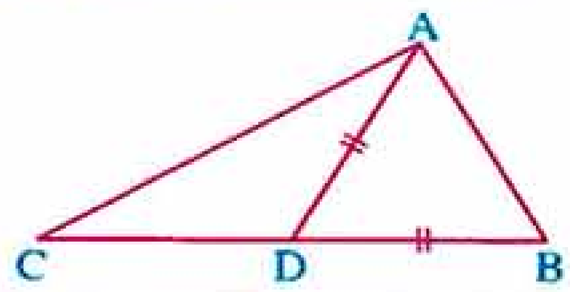
1 m (∠ FBC) > m (∠ BCF) 2 CF > BF



17 **In the opposite figure :**

ABC is a triangle and D ∈ $\overline{BC}$ where BD = AD

Prove that : BC > AC



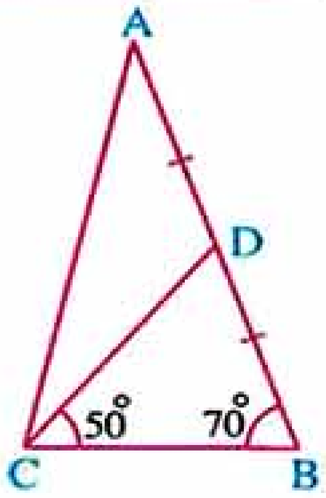
18 **In the opposite figure :**

D is the midpoint of $\overline{AB}$, m (∠ B) = 70° and m (∠ DCB) = 50°

Prove that :

1 m (∠ A) > m (∠ ACD)

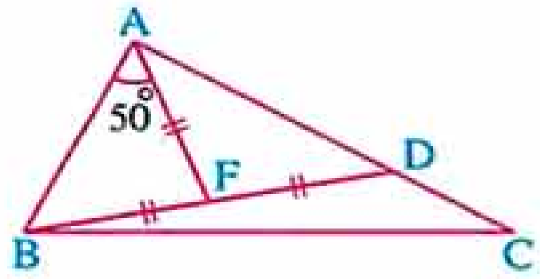
2 ∠ ACB is an acute angle.



19 **In the opposite figure :**

AF = BF = DF and m (∠ FAB) = 50°

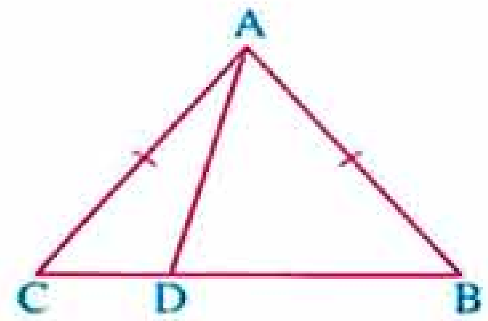
Prove that : 1 AD > AB 2 BC > AC



20 **In the opposite figure :**

ABC is a triangle in which : AB = AC and D ∈ $\overline{BC}$

Prove that : AB > AD

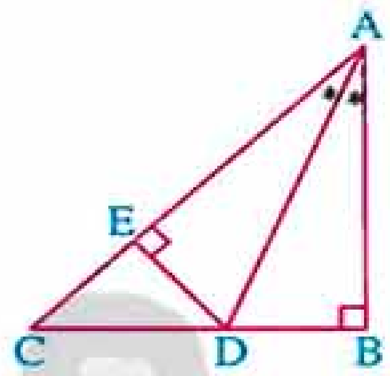


21 **In the opposite figure :**

m (∠ B) = 90° , $\overline{DE} \perp \overline{AC}$ and $\overrightarrow{AD}$ bisects ∠ BAE

Prove that : 1 BD = DE

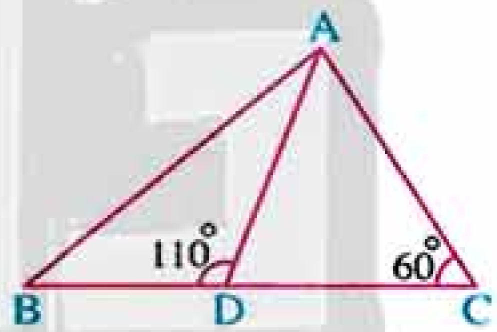
2 DC > DB



22 **In the opposite figure :**

m (∠ ADB) = 110° and m (∠ C) = 60°

Prove that : AB + AC > 2 AD



23 ABC is a right-angled triangle at B

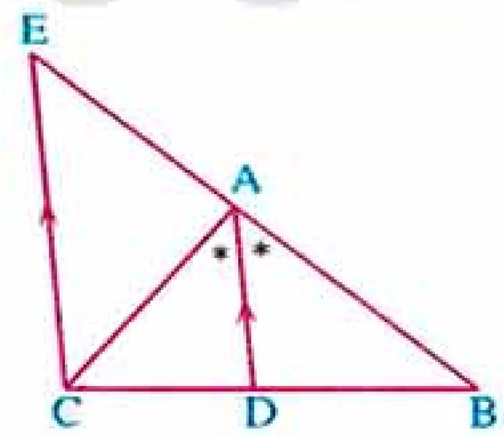
Prove that : AB + BC < 2 AC

24 **In the opposite figure :**

ABC is a triangle , $\overrightarrow{AD}$ bisects ∠ BAC

, $\overrightarrow{CE}$ // $\overrightarrow{DA}$ and cuts $\overrightarrow{BA}$ at E

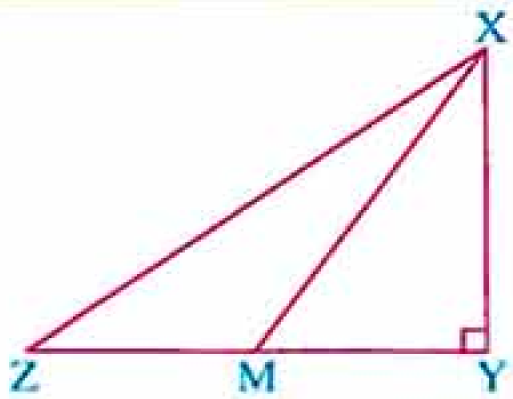
Prove that : BE > BC



25 **In the opposite figure :**

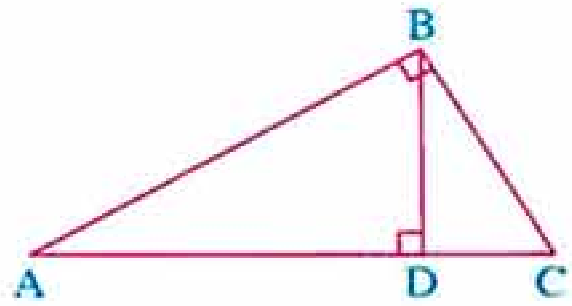
XYZ is a right-angled triangle at Y and M ∈ $\overline{YZ}$

Prove that : XZ > XM



Unit 5

26 **In the opposite figure :**



m (∠ ABC) = 90° , $\overline{BD} \perp \overline{AC}$

and AB > BC

Prove that : AD > BD

27 ABC is a triangle , $\overrightarrow{CD}$ bisects ∠ C , $\overrightarrow{CD} \cap \overline{AB} = \{D\}$

Prove that : BC > BD

28 ABC is a right-angled triangle at B , D ∈ $\overline{AC}$ and E ∈ $\overline{BC}$ where AD = BE

Prove that : m (∠ CED) > m (∠ CDE)

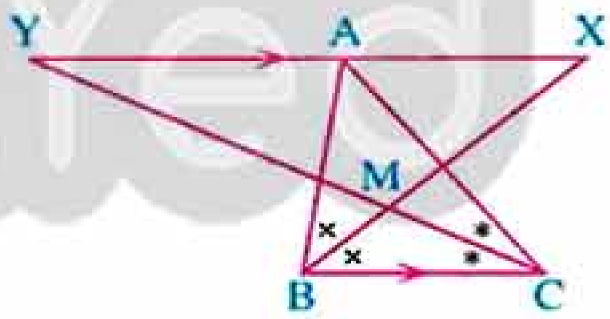
29 ABC is a triangle in which : AB = AC and X ∈ $\overline{AC}$, draw $\overleftrightarrow{XY}$ to cut $\overline{AB}$ at Y and cut $\overrightarrow{CB}$ at Z

Prove that : AY > AX

30 ABC is a triangle in which : m (∠ A) = $(5x + 2)°$,
m (∠ B) = $(6x - 10)°$ and m (∠ C) = $(x + 20)°$
Order the lengths of sides of the triangle ascendingly.

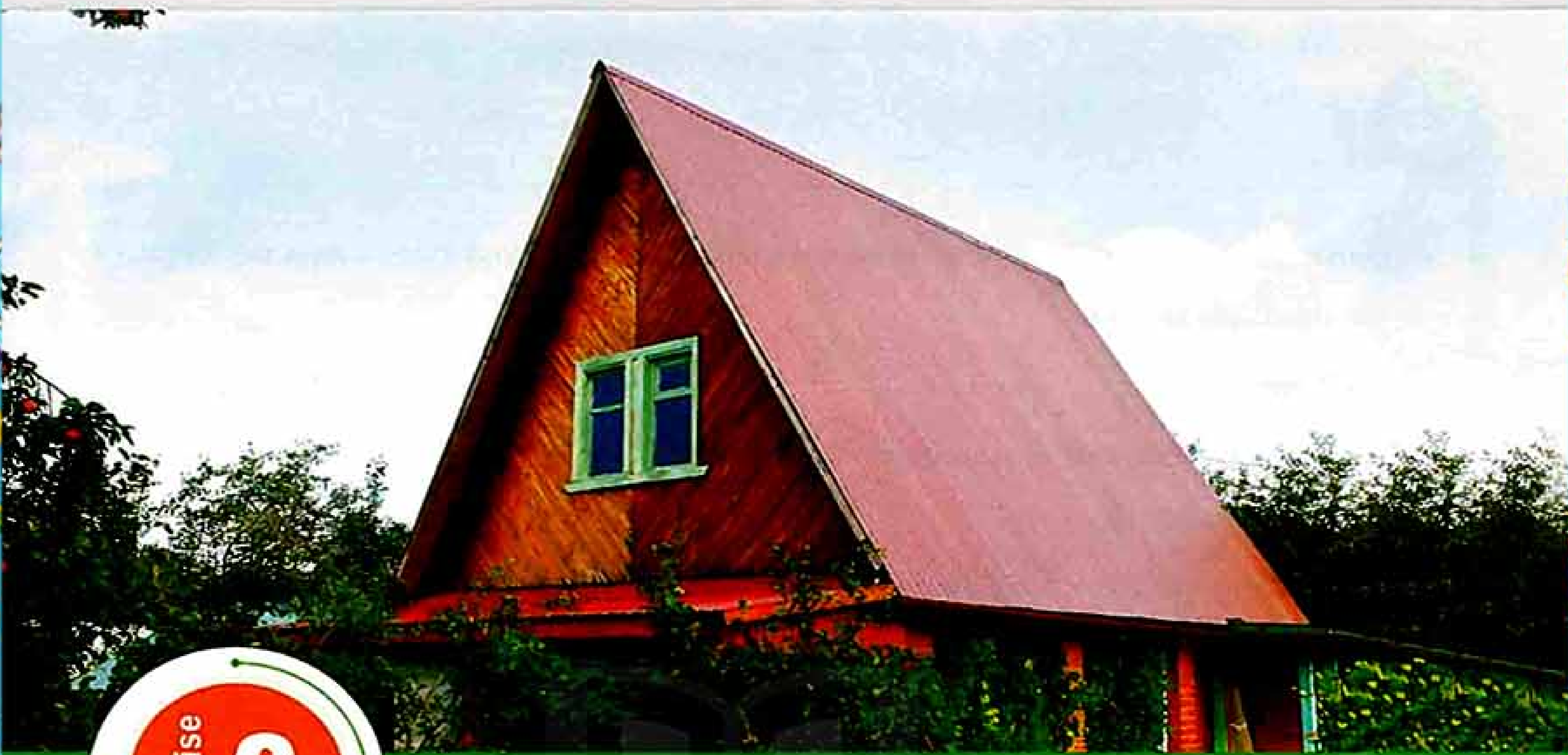
For excellent pupils

31 **In the opposite figure :**



AB < AC , ∠ B and ∠ C are bisected by two bisectors meeting at M , $\overrightarrow{BM}$ and $\overrightarrow{CM}$ intersect the straight line drawn from A parallel to $\overline{BC}$ at X and Y respectively.

Prove that : BX < CY



Exercise 9

Triangle inequality

From the school book

1 Is it possible to draw a triangle whose side lengths are as follows ? Give reasons :

1. 3 cm. , 4 cm. , 9 cm.
2. 📖 5 cm. , 7 cm. , 8 cm.
3. 📖 10 cm. , 6 cm. , 4 cm.
4. 13 cm. , 8 cm. , 6 cm.
5. 5 cm. , 3 cm. , 4 cm.
6. 9 cm. , 9 cm. , 19 cm.

2 Find the interval to which the length of the third side of the triangle belongs in each of the following triangles if the lengths of the two other sides are :

1. 📖 6 cm. , 9 cm.
2. 3 cm. , 3 cm.
3. 📖 2.9 cm. , 3.2 cm.
4. 5.7 cm. , 7.3 cm.

3 Choose the correct answer from those given :

1. The sum of lengths of any two sides in a triangle is .... the length of the third side.
   (a) less than (b) greater than (c) equal (d) half
2. The length of any side in a triangle .... the sum of lengths of the other two sides.
   (a) > (b) < (c) = (d) twice
3. Which of the following numbers cannot be the lengths of sides of a triangle ?
   (a) 7 , 7 , 5 (b) 9 , 9 , 9 (c) 3 , 6 , 12 (d) 3 , 4 , 5
4. If the lengths of two sides in a triangle are 7 cm. and 4 cm. , then the length of the third side can be ....
   (a) 1 cm. (b) 2 cm. (c) 3 cm. (d) 4 cm.

Unit 5

5 If the lengths of two sides of an isosceles triangle are 3 cm. and 7 cm. , then the length of the third side is ....

(a) 7 cm. (b) 3 cm. (c) 4 cm. (d) 10 cm.

6 A triangle has one axis of symmetry, the lengths of two sides in it are 4 cm. and 8 cm. , then its perimeter is ....

(a) 16 cm. (b) 20 cm. (c) 24 cm. (d) 30 cm.

7 In Δ ABC : If AB = 3 cm. , BC = 5 cm. and AC = x cm. , then $x \in$ ....

(a)] 3 , 5 [(b)] 2 , 5 [(c)] 5 , 8 [(d)] 2 , 8 [

8 If the lengths of two sides of a triangle are 5 cm. and 10 cm. , then the length of the third side belongs to ....

(a) [10 , 15 [(b)] 5 , 15 [(c)] 5 , 10] (d) [10 , 15]

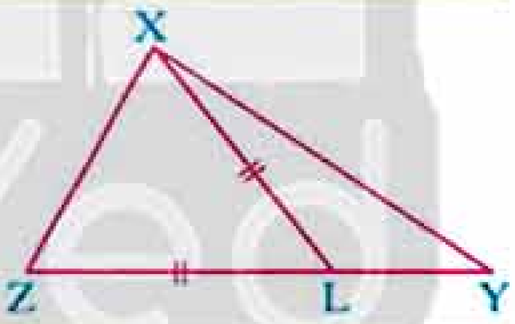
9 In Δ ABC : AB + BC – AC ....

(a) > zero (b) < zero
(c) = zero (d) = the perimeter of the triangle ABC

4 **In the opposite figure :**

XYZ is a triangle in which L $\in \overline{YZ}$ such that XL = LZ

Prove that : YZ > XY



5 ABC is a triangle in which $\overline{BC}$ is the longest side , D $\in \overline{BC}$ such that CD = CA

Prove that : AB > BD

6 ABC is a triangle , $\overrightarrow{AD}$ is drawn to cut $\overline{BC}$ at D

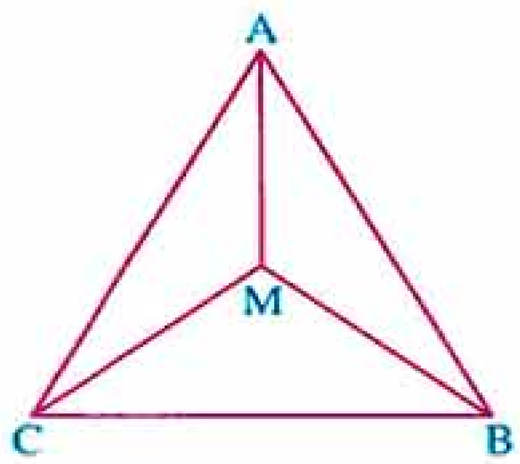
Prove that : BD + DC + 2 AD > AB + AC

7 **In the opposite figure :**

ABC is a triangle in which M is a point inside it.

Prove that :

MA + MB + MC > $\frac{1}{2}$ the perimeter of the triangle ABC



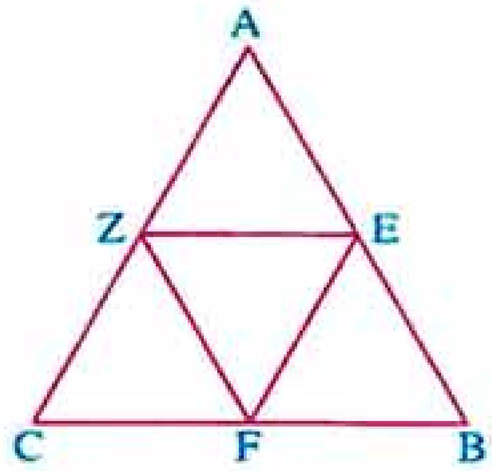
8 **In the opposite figure :**

ABC is a triangle in which $E \in \overline{AB}$

, $F \in \overline{BC}$ and $Z \in \overline{AC}$

Prove that :

The perimeter of Δ ABC > the perimeter of Δ EFZ

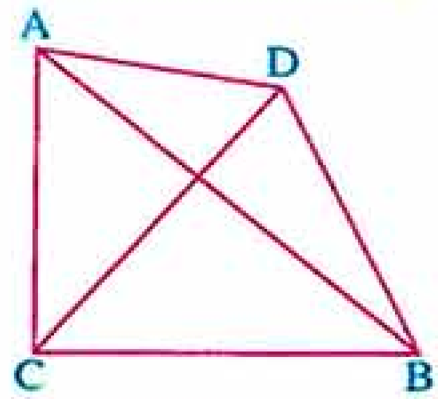


9 **In the opposite figure :**

ABC is a triangle and D is a point outside it.

Prove that :

The perimeter of Δ ABC < 2 (DA + DB + DC)



10 Prove that the length of any side in a triangle is less than half of the perimeter.

11 ABCD is a quadrilateral.

Prove that : AB + BC + CD > AD

12 Prove that the sum of the lengths of two diagonals in a convex quadrilateral is less than its perimeter.

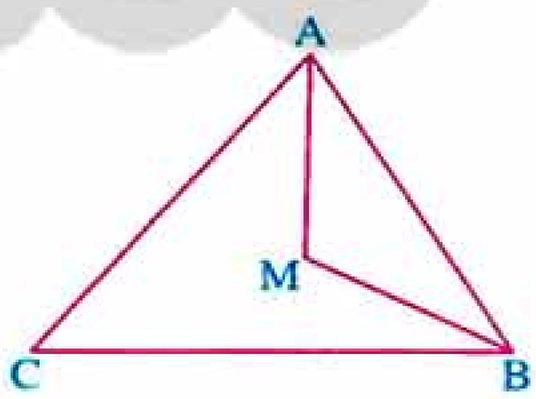
13 Prove that the perimeter of any quadrilateral is less than twice the sum of lengths of its diagonals.

For excellent pupils

14 **In the opposite figure :**

M is a point inside the triangle ABC

Prove that : AM + MB < AC + BC



15 ABC is a triangle and F is the midpoint of $\overline{BC}$

Prove that :

1. AB + AC > 2 AF
2. AB + AC > AF + BF

Summary of Unit 5

Summary
1.
2.
3.

- **Axioms of inequality relation :**

  For any four numbers a , b , c and d :

  1 If $a > b$, then $a + c > b + c$

  2 If $a > b$, then $a - c > b - c$

  3 If $a > b$, $c > 0$, then $a c > b c$

  4 If $a > b$, $b > c$, then $a > c$

  5 If $a > b$, $c > d$, then $a + c > b + d$

- In a triangle , if two sides have unequal lengths , then the longer is opposite to the angle of the greater measure.
- In a triangle , if two angles are unequal in measure , then the greater angle in measure is opposite to a side greater in length than that opposite to the other angle.
- In the right-angled triangle , the hypotenuse is the longest side.
- The length of the perpendicular line segment drawn from a point outside a straight line to this line is shorter than any line segment drawn from this point to the given straight line.
- The distance between any point and a given straight line is the length of the perpendicular line segment drawn from this point to the given line.
- **Triangle inequality :**

  In any triangle , the sum of the lengths of any two sides is greater than the length of the third side.
- The length of any side in a triangle is greater than the difference between the lengths of the other two sides and less than their sum.

Exams on Unit Five

Model 1

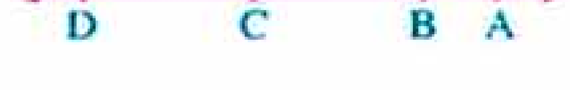
Answer the following questions :

1 **Choose the correct answer from those given :**

1. The sum of lengths of any two sides of a triangle .... the length of the third side.
   (a) is smaller than (b) is greater than (c) equals (d) equals twice
2. In Δ ABC , if m ($\angle$ B) > m ($\angle$ C) , then ....
   (a) AB < AC (b) AB = AC (c) AB > AC (d) $\overline{AB} \equiv \overline{BC}$
3. If the lengths of two sides in an isosceles triangle are 3 cm. and 7 cm. , then the length of the third side equals ....
   (a) 7 cm. (b) 3 cm. (c) 4 cm. (d) 10 cm.
4. Which of the following numbers can be lengths of sides of a triangle ?
   (a) 2 , 3 , 4 (b) 2 , 3 , 5 (c) 2 , 3 , 6 (d) 2 , 3 , 7
5. In Δ ABC , if m ($\angle$ C) = 65° and m ($\angle$ A) = 75° , then ....
   (a) AB > BC (b) AB < AC (c) BC > AB (d) AB = AC
6. In Δ ABC , if m ($\angle$ B) = 130° , then its longest side is ....
   (a) $\overline{BC}$ (b) $\overline{AC}$ (c) $\overline{AB}$ (d) its median.

2 **Complete the following :**

1. If two sides in a triangle are unequal in length , then the longer of them is opposite to an angle ....
2. The longest side of the right-angled triangle is ....
3. In Δ ABC , if AB < BC < AC , then the smallest angle in measure is ....
4. **In the opposite figure :**
   If B , C belong to $\overleftrightarrow{AD}$, such that DC > AB , then AC .... DB



5. ABC is a triangle in which : AB = 5 cm. and BC = 3 cm. , then AC ∈] .... , ... [

Unit 5

3 **[a]** In Δ ABC : m ($\angle$ A) = 30° and m ($\angle$ B) = 65°

Arrange the lengths of the sides of the triangle descendingly.

[b] ABCD is a quadrilateral in which : AB = 6 cm. , BC = 3 cm. , CD = 4 cm. and DA = 5 cm.

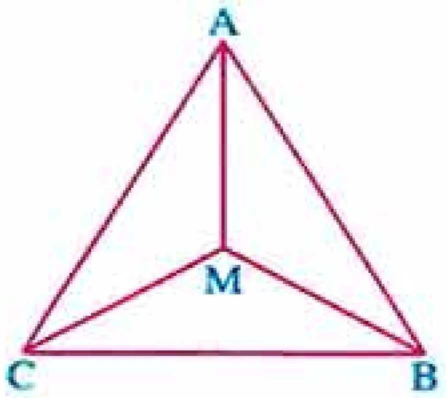
Prove that : m ($\angle$ DCB) > m ($\angle$ DAB)

4 **[a] In the opposite figure :**

ABC is a triangle

and M is a point inside it.

Prove that : MA + MB + MC > $\frac{1}{2}$ the perimeter of Δ ABC

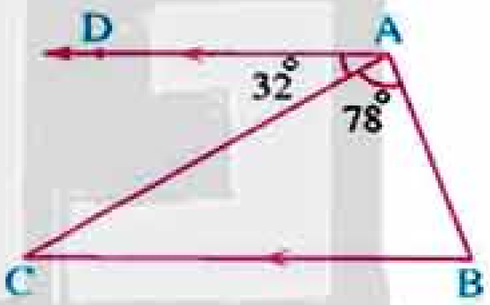


[b] In the opposite figure :

$\overrightarrow{AD}$ // $\overline{BC}$, m ($\angle$ BAC) = 78°

and m ($\angle$ CAD) = 32°

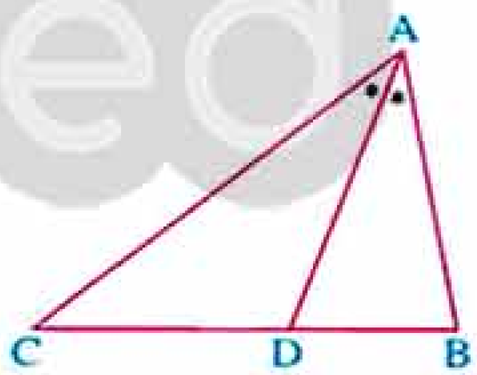
Prove that : AC > AB



5 **[a] In the opposite figure :**

$\overrightarrow{AD}$ bisects $\angle$ A

Prove that : AC > DC

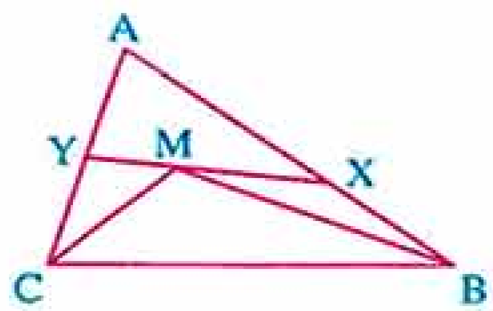


[b] In the opposite figure :

ABC is a triangle in which : X $\in \overline{AB}$

, Y $\in \overline{AC}$ and M $\in \overline{XY}$

Prove that : AB + AC > MB + MC



Model 2

Answer the following questions :

1 Choose the correct answer from those given :

1 If the triangle ABC is right-angled at B , then ....

(a) AC < AB (b) AC < BC (c) AB < AC (d) BC = AB

2 A triangle of two side lengths 4 cm. and 9 cm. , and it has one axis of symmetry , then the length of the third side equals ....

(a) 4 cm. (b) 5 cm. (c) 9 cm. (d) 13 cm.

3 The length of any side in a triangle .... the sum of lengths of the two other sides.

(a) is smaller than (b) is greater than (c) equals (d) is half

4 Δ ABD is an obtuse-angled triangle at B , C is the midpoint of $\overline{BD}$, then the greatest side in length is ....

(a) $\overline{AB}$ (b) $\overline{AC}$ (c) $\overline{BD}$ (d) $\overline{AD}$

5 Which of the following numbers can't be lengths of sides of a triangle ?

(a) 3 , 4 , 4 (b) 3 , 4 , 5 (c) 3 , 4 , 6 (d) 3 , 4 , 7

6 In Δ XYZ , XY + YZ – XZ ....

(a) > 0 (b) < 0

(c) = 0 (d) = the perimeter of Δ XYZ

2 Complete the following :

1 If two angles are unequal in measure in a triangle , then the greater angle in measure is opposite to ....

2 In the isosceles triangle ABC , if AB = AC , m ($\angle$ A) = 70° , then AB < ....

3 In the triangle ABC , if m ($\angle$ A) = 67° , m ($\angle$ B) = 33° , then AB > .... > ....

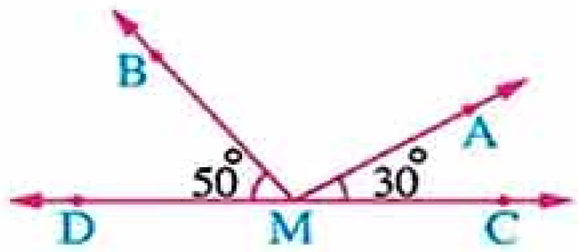
4 If ABC is a triangle in which m ($\angle$ A) = m ($\angle$ B) + m ($\angle$ C) , then the greatest side in length is ....

Unit 5

5 **In the opposite figure :**

M $\in \overleftrightarrow{CD}$

, then m ($\angle$ CMB) .... m ($\angle$ AMD)

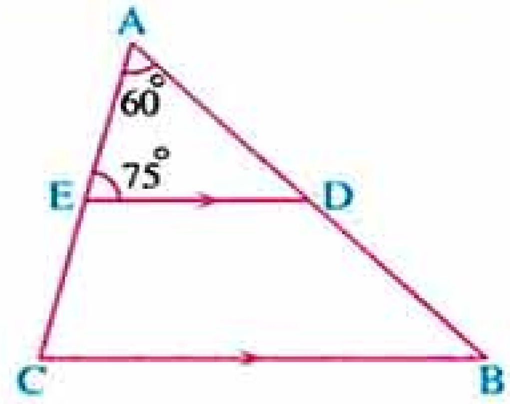


3 **[a] In the opposite figure :**

$\overline{ED}$ // $\overline{BC}$, m ($\angle$ A) = 60°

and m ($\angle$ AED) = 75°

Prove that : AB > AC

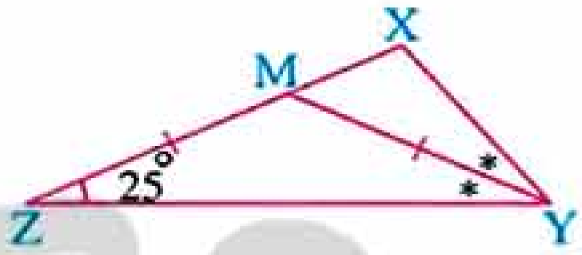


[b] In the opposite figure :

$\overrightarrow{YM}$ bisects $\angle$ XYZ

, MY = MZ and m ($\angle$ Z) = 25°

Prove that : YM > XY



4 [a] ABC is a triangle in which AB = 7 cm.

, BC = 4 cm. and CA = 5 cm.

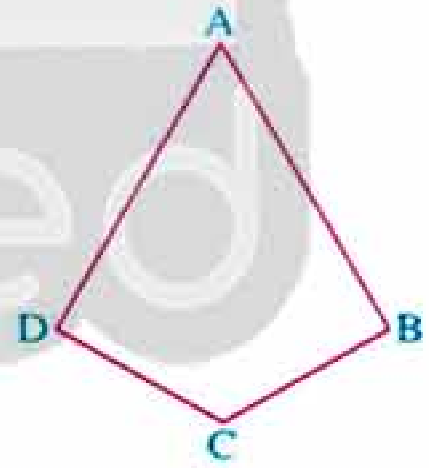
Arrange the angles of the triangle ascendingly due to their measures.

[b] In the opposite figure :

AB > BC

and AD > DC

Prove that : m ($\angle$ BCD) > m ($\angle$ BAD)



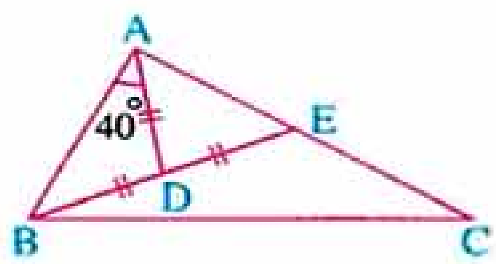
5 **[a] In the opposite figure :**

AD = BD = DE and m ($\angle$ DAB) = 40°

Prove that :

1 AD < AB

2 BC > AC

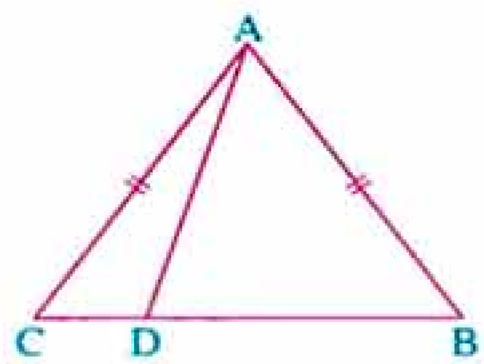


[b] In the opposite figure :

AB = AC

and D $\in \overline{BC}$

Prove that : AB > AD



TIMSS Problems

Accumulative basic skills

1 Complete the following :

1. A lamppost of height 4.5 metres is 2 metres far from a building of height 10.5 metres , then the distance between the top of the lamppost and the top of the building is .... metres.

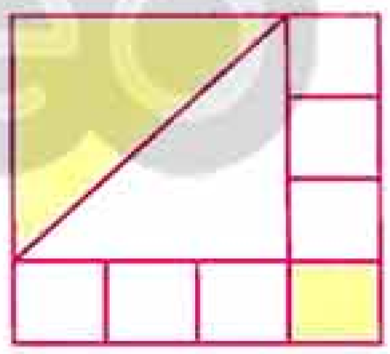
2. The ratio between the lateral and the total areas of a cube is ....

3. A cuboid is of lateral area 200 cm^2. and the dimensions of its base are 8 cm. and 12 cm. , then its height equals .... cm.

4. The measure of the angle between the two hands of the clock at 7 o'clock in degrees is ....°

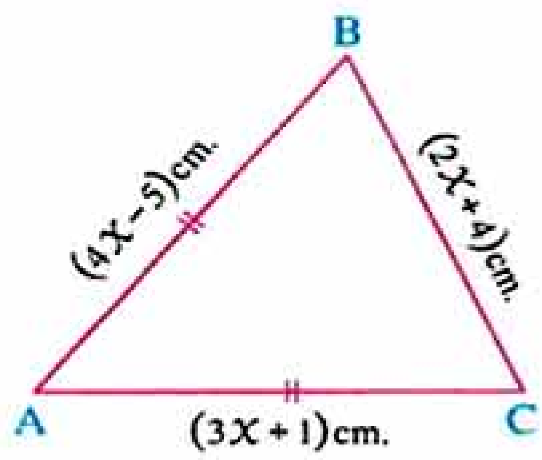
5. **In the opposite figure :**

   A square divided into 7 small congruent squares and two congruent triangles. If the area of the coloured square = 4 cm^2. , then the area of the coloured triangle is .... cm^2.



6. **In the opposite figure :**

   ABC is a triangle in which : AB = $(4x - 5)$ cm.
   , BC = $(2x + 4)$ cm. , AC = $(3x + 1)$ cm.
   , AB = AC
   , then the perimeter of Δ ABC = ... cm.



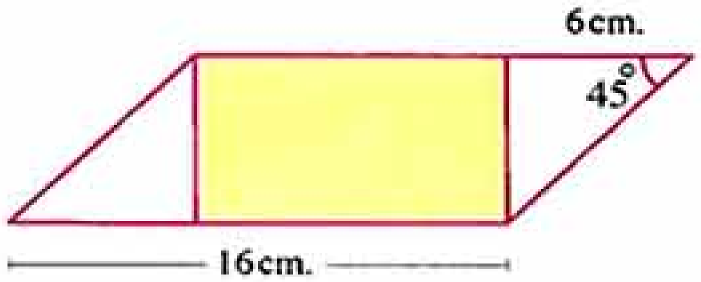
7. A rectangle its length is x cm. , its width is y cm. and its perimeter is P cm. , then the relation between x , y and P is x = ...

Basic skills

8 If the side length of an equilateral triangle is 10 cm. , then its height is .... cm.

9 The measure of the angle of the regular pentagon is .... °

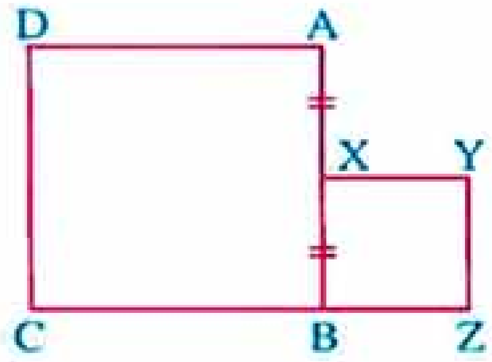
10 The opposite figure shows a coloured rectangle inside a parallelogram , then the area of the rectangle equals .... cm^2.



11 **In the opposite figure :**

If the perimeter of the square ABCD = 24 cm.

, then the area of the square XYZB is .... cm^2.



12 A cuboid is of total area 148 cm^2. and its lateral area is 110 cm^2. , then the area of its base is .... cm^2.

2 **Choose the correct answer from the given ones :**

1 The acute angle supplements .... angle.

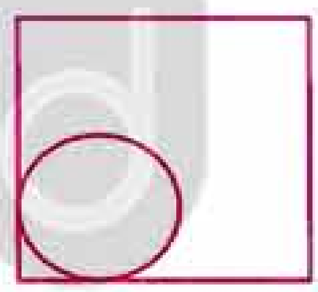
(a) an acute (b) an obtuse (c) a right (d) a reflex

2 The number of diagonals of the hexagon equals ....

(a) 3 (b) 6 (c) 9 (d) 12

3 **In the opposite figure :**

A circle of radius length 2 cm. touches two sides of a square , then the area of the coloured part is .... cm^2.



(a) $4 - \pi$ (b) $\pi - 2$ (c) $\frac{\pi}{2}$ (d) 2π

4 The ratio between the area of a square region of side length ℓ cm. and the area of a square region of side length 2ℓ cm. is ....

(a) 1 : 2 (b) ℓ : 4 (c) 1 : 4 (d) 4 : 1

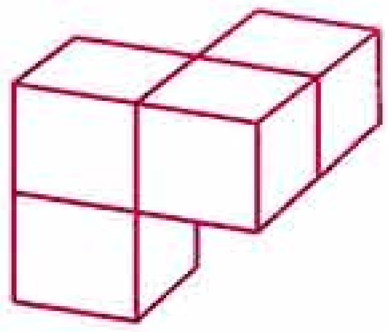
5 On a map , each 1 cm. represents 5 km. If the distance between two places is $\frac{1}{2}$ km. , then the distance between them on the map is ....

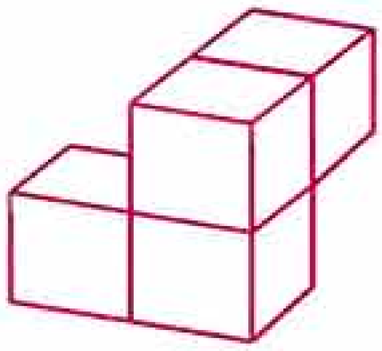
(a) 0.1 cm. (b) 10 cm. (c) 2.5 cm. (d) 0.4 cm.

6 If the area of the base of a cuboid is 12 cm^2. and the areas of two side faces are 6 cm^2. and 8 cm^2. , then the volume of the cuboid is .... cm^3.

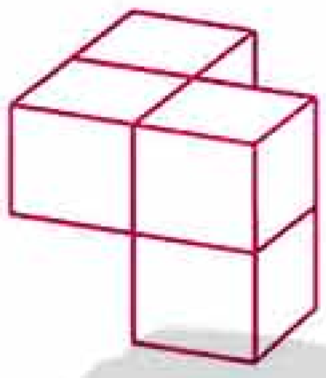
(a) 9 (b) 576 (c) 24 (d) 32

7 This solid will be rotated to another position. Which of the following may be a position of the solid after rotation ?

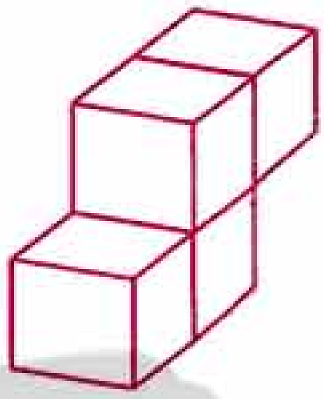




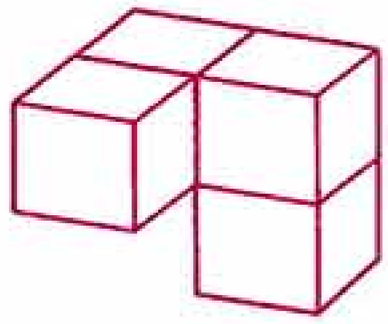
(a)



(b)



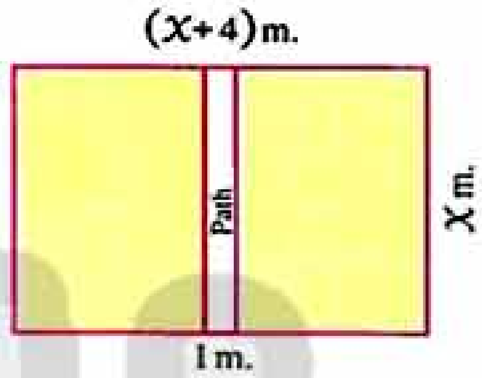
(c)



(d)

8 **In the opposite figure :**

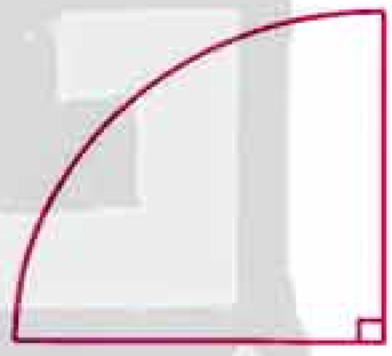
A rectangular garden with a rectangular path of width 1 metre. Which expression shows the area of the coloured part of the garden in square metres ?



(a) $x^2 + 3x$ (b) $x^2 + 4x$

(c) $x^2 + 4x - 1$ (d) $x^2 + 3x - 1$

9 The opposite figure represents a quarter of a circle of radius length 2 cm. , then the perimeter of the figure in centimetres is ....



(a) 2π (b) 5π

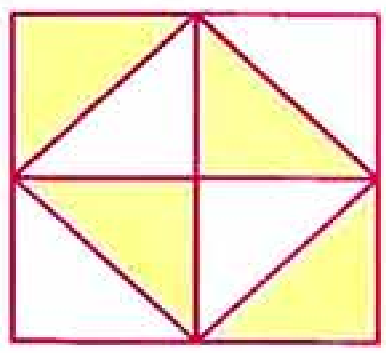
(c) $\pi + 4$ (d) $4\pi + 4$

10 The area of a square whose side length is an integer may be .... cm^2.

(a) 600 (b) 900 (c) 800 (d) 700

11 **In the opposite figure :**

A square of perimeter 32 cm. divided into 8 congruent triangles , then the area of the coloured region is .... cm^2.



(a) 4 (b) 8

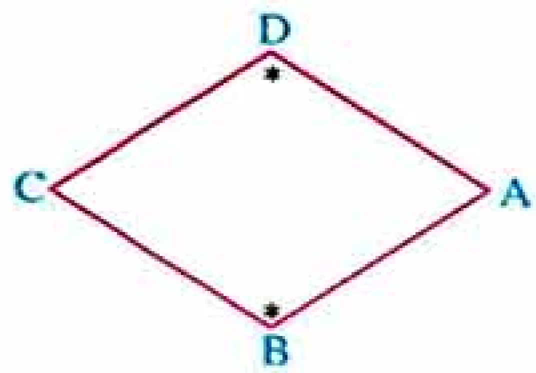
(c) 16 (d) 32

12 **In the opposite figure :**

If m (∠ A) + m (∠ C) = 140°

, m (∠ B) = m (∠ D)

, then m (∠ B) = ...



(a) 50° (b) 55°

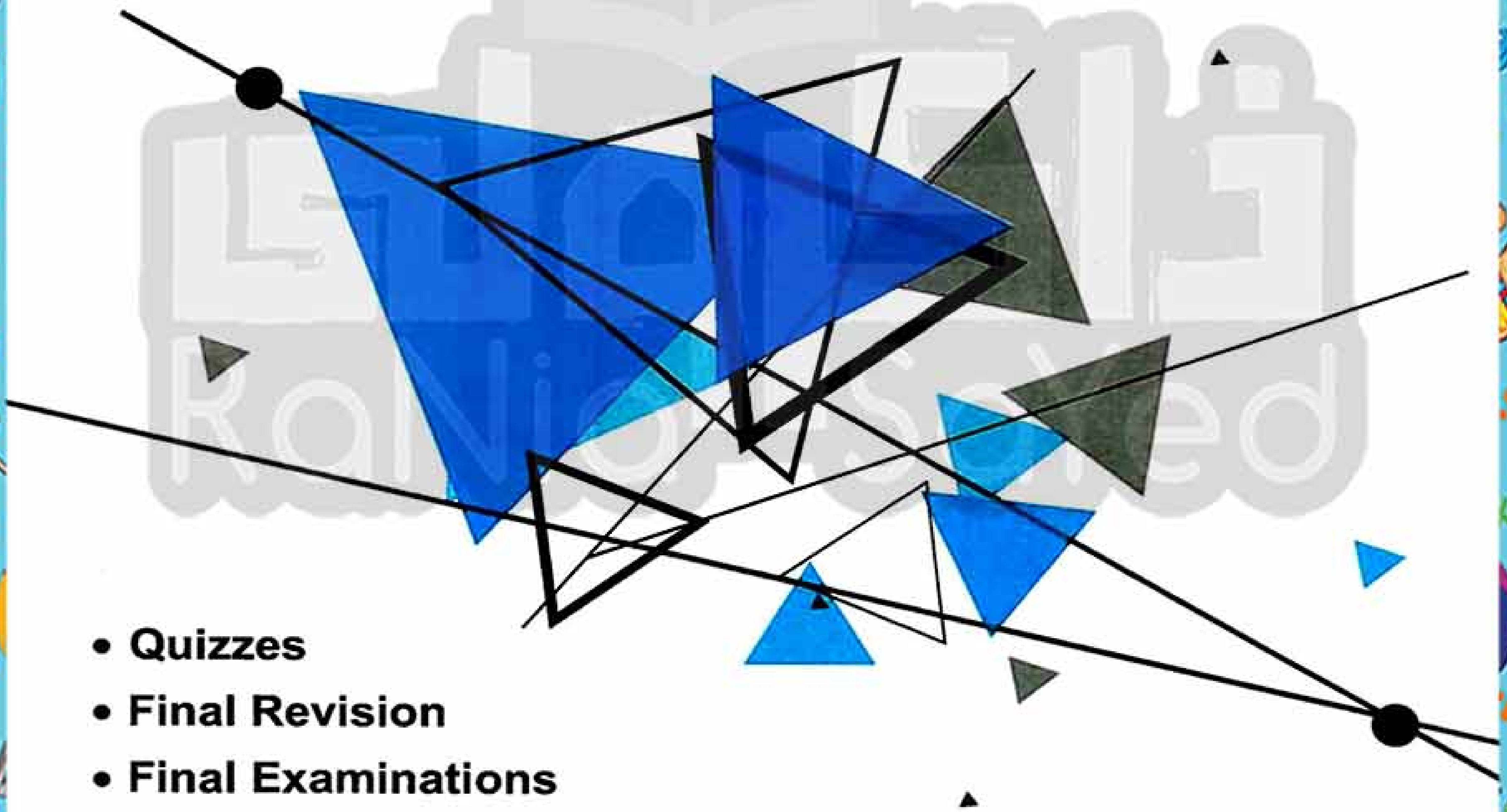
(c) 110° (d) 220°

EL-MOASSER

In Mathematics Notebook

For 2nd Prep.

First Term



- **Quizzes**
- **Final Revision**
- **Final Examinations**

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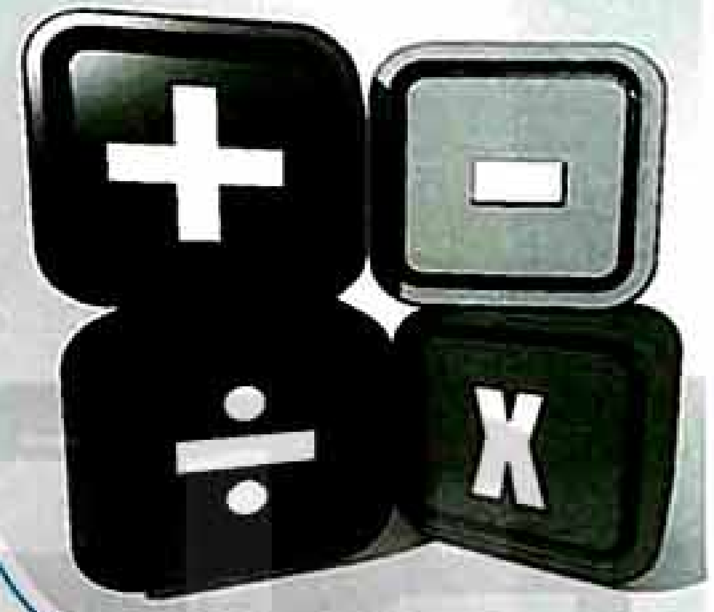
By
A group of supervisors

Contents

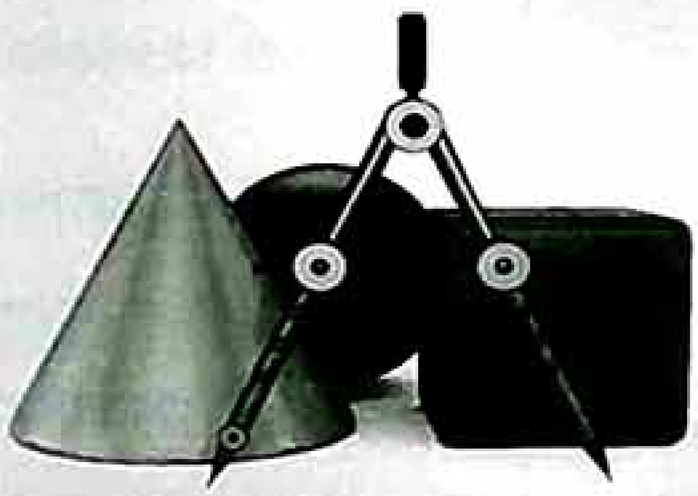
First Algebra and Statistics

- 18 Quizzes.
- Final revision.
- Final examinations :
  - School book examinations.
    (2 models examinations + model for the merge students)
  - 15 schools examinations.

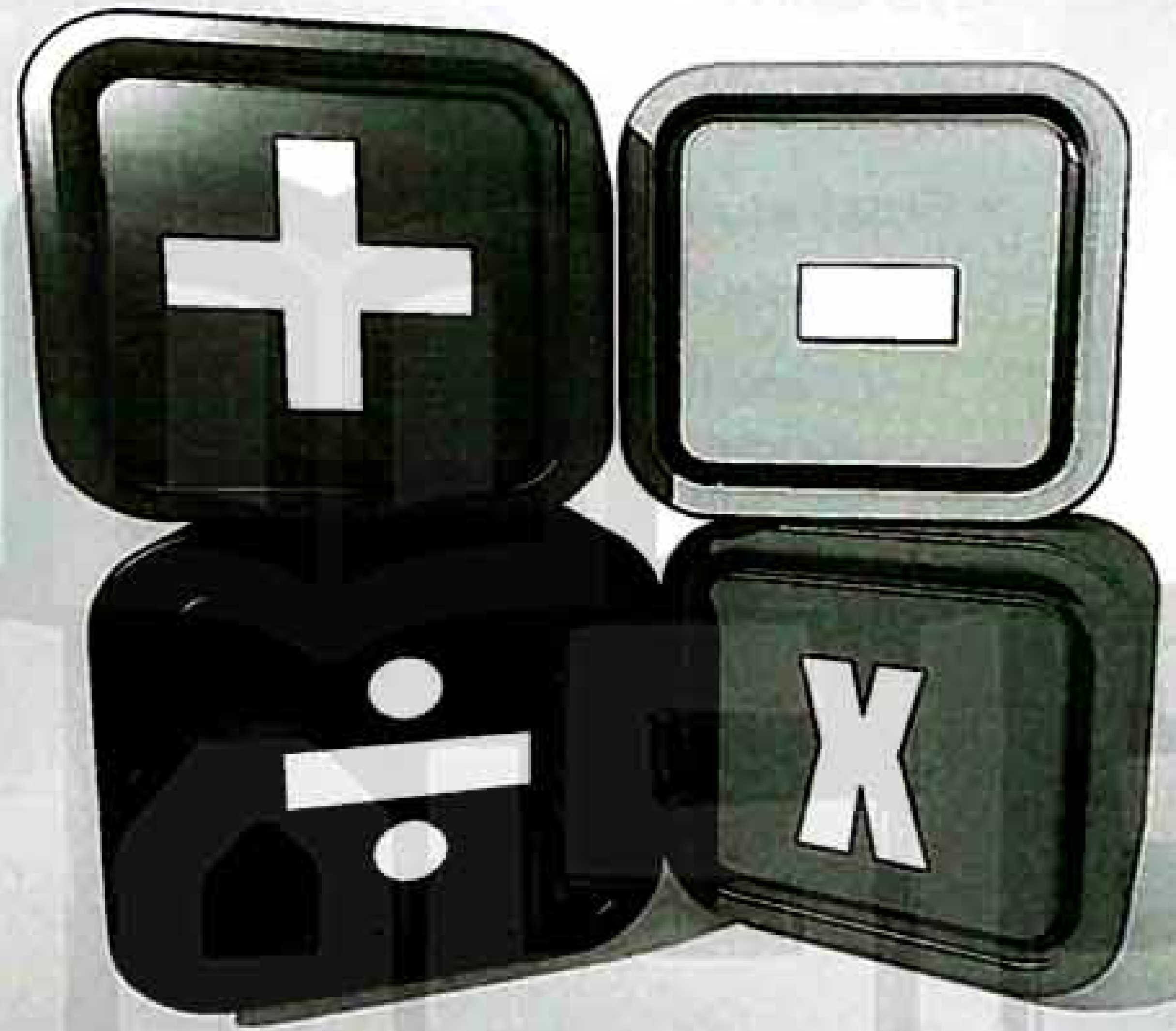


Second Geometry

- 9 Quizzes.
- Final revision.
- Final examinations :
  - School book examinations.
    (2 models examinations + model for the merge students)
  - 15 schools examinations.

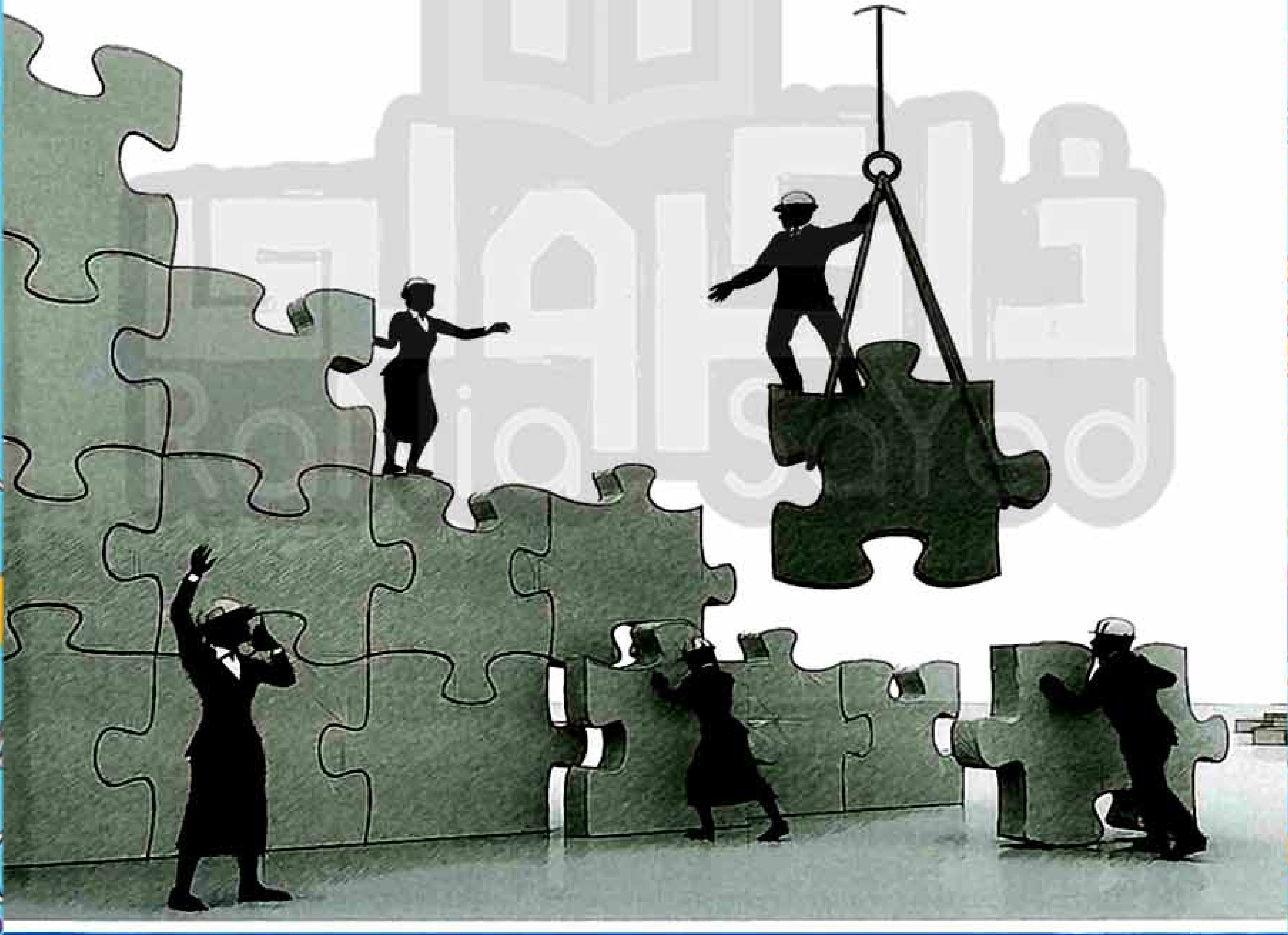


First Algebra and Statistics



Quizzes

on Algebra and Statistics



Quiz 1 on lesson 1 – unit 1

time 15 min.

1 Choose the correct answer from the given ones :

1 If $\sqrt[3]{x} = \frac{1}{4}$, then $x =$ ....

(a) $\frac{1}{2}$ (b) $\frac{1}{16}$ (c) $\frac{1}{12}$ (d) $\frac{1}{64}$

2 The diameter length of the sphere whose volume = 36π cm^3 is .... cm.

(The volume of the sphere = $\frac{4}{3}\pi r^3$)

(a) 3 (b) 6 (c) 9 (d) 27

3 $\sqrt[3]{64} = \sqrt{\ldots\ldots\ldots}$

(a) 4 (b) 8 (c) 16 (d) 36

2 [a] Find the solution set of each of the following equations in $\mathbb{Q}$:

1 $2x^3 - 1 = 53$ 2 $(5x - 3)^3 = 8$

[b] A cubic vessel has a capacity 8 litres. Calculate the length of its inner edge in cm.

Quiz 2 till lesson 2 – unit 1

time 15 min.

1 Choose the correct answer from the given ones :

1 The irrational number between 3 and 4 is ....

(a) 3.6 (b) $\sqrt{6}$ (c) $\sqrt{15}$ (d) $\sqrt{17}$

2 The square whose side length is $\sqrt{7}$ cm. , its area = ... cm^2.

(a) 28 (b) 49 (c) 7 (d) 14

3 If $x \in \mathbb{Z}$ and $x < \sqrt{11} < x + 1$, then $x =$ ....

(a) 3 (b) 2 (c) 4 (d) 10

2 [a] Prove that : $\sqrt{5}$ is included between 2.2 and 2.3

[b] Determine the point which represents the number $1 + \sqrt{3}$ on the number line.

Quiz 3 till lesson 3 – unit 1

time 15 min.

1 Complete the following :

1 The solution set in $\mathbb{R}$ of the equation : $x^2 + 9 = 0$ is ....

2 $\mathbb{R} - \mathbb{Q} =$ ....

3 If $\sqrt[3]{x} = -5$, then $x =$ ....

2 [a] Solve each of the following equations to the nearest one decimal given that $x \in \mathbb{R}$:

1 $\frac{1}{2} x^2 - 3 =$ zero

2 $(x - 3)^3 = 5$

[b] Write three irrational positive numbers less than 3

Quiz 4 till lesson 4 – unit 1

time 15 min.

1 Choose the correct answer from the given ones :

1 $[-3 , 2] - \{-3 , 6\} =$ ....

(a) $]-3 , 6[$ (b) $]-3 , 2[$ (c) $]-3 , 2]$ (d) $\varnothing$

2 If $\sqrt{7} \in]x , x + 1[$ where $x \in \mathbb{Z}$, then $x =$ ....

(a) 1 (b) 2 (c) 3 (d) 4

3 $\mathbb{R}_+ \cap [-1 , 3] =$ ....

(a) $]0 , 3[$ (b) $]0 , 3]$ (c) $[0 , 3]$ (d) $[0 , 3[$

2 [a] If $X =]-\infty , 1[$ and $Y = [-2 , 4[$, find each of the following as an interval using the number line :

1 $X \cup Y$

2 $X \cap Y$

3 $X - Y$

4 $\grave{X}$

[b] Find the solution set of the following equation in $\mathbb{R}$:

$3x^2 + 125 = 221$

Quiz 5 till lesson 5 – unit 1

time 15 min.

1 Choose the correct answer from the given ones :

1 If $x^3 + 9 = 1$ where $x \in \mathbb{R}$, then $x =$ ....

(a) -8 (b) -2 (c) 2 (d) 8

2 If $x = \sqrt{3} + 2$, then $x^2 =$ ....

(a) 5 (b) 7 (c) $7 + 2\sqrt{3}$ (d) $7 + 4\sqrt{3}$

3 If $x^2 - y^2 = 60$, $x + y = 5\sqrt{6}$, then $= x - y =$ ....

(a) $\sqrt{6}$ (b) $2\sqrt{6}$ (c) $3\sqrt{6}$ (d) $4\sqrt{6}$

2 [a] If $x = -\sqrt{3}$ and $y = 2\sqrt{3} - 3$, find the value of each of :

1 $x + y$ 2 $\frac{y}{x}$

[b] If $X = [-1 , 5[$, $Y =]1 , 7[$

, find each of the following as an interval using the number line :

1 $X \cap Y$ 2 $X \cup Y$ 3 $X - Y$

Quiz 6 till lesson 6 – unit 1

time 15 min.

1 Choose the correct answer from the given ones :

1 $(\sqrt{8} + \sqrt{2})^2 =$ ....

(a) $\sqrt{10}$ (b) 10 (c) 18 (d) $\sqrt{18}$

2 $\frac{1}{2}\sqrt{20} - \sqrt{5} =$ ....

(a) $\frac{1}{2}\sqrt{15}$ (b) $\sqrt{5}$ (c) zero (d) 1

3 If π is the ratio between the circumference of the circle and its diameter length , then $\pi \in$ ....

(a) $\mathbb{N}$ (b) $\mathbb{Z}$ (c) $\mathbb{Q}$ (d) $\grave{\mathbb{Q}}$

2 [a] Put in the simplest form : $\sqrt{75} - 2\sqrt{27} + 3\sqrt{\frac{1}{3}}$

[b] Find the solution set of the following equation in $\mathbb{R}$: $\frac{1}{6} x^2 + 6 = 6\frac{1}{2}$

Quiz 7 till lesson 7 – unit 1

time 15 min.

1 Complete the following :

1 The multiplicative inverse of the number $(\sqrt{3}+\sqrt{2})$ in the simplest form is ....

2 The rectangle whose dimensions are $(\sqrt{3}+1)$ cm. and $(\sqrt{3}-1)$ cm. has an area = ... cm^2.

3 $[1, 5] - \{1, 5\} =$ ....

2 [a] If $x=\sqrt{5}+\sqrt{3}$ and $y=\frac{2}{\sqrt{5}+\sqrt{3}}$,

find the value of the expression : $x^2-2xy+y^2$

[b] **Find in the shape of an interval using the number line :** $]-\infty, 1[\ \cup\ ]2, \infty[$

Quiz 8 till lesson 8 – unit 1

time 15 min.

1 Choose the correct answer from the given ones :

1 If $X =]-\infty, 0[$, then $\grave{X} =$ ....

(a) $\mathbb{R}_+$ (b) $[0, \infty[$ (c) $]-\infty, 0]$ (d) $\mathbb{R}_-$

2 $\frac{\sqrt[3]{24}}{\sqrt[3]{3}} =$ ....

(a) $2\sqrt[3]{2}$ (b) 8 (c) 3 (d) 2

3 If $x=\sqrt{3}+\sqrt{2}$ and $xy=1$, then $y =$ ....

(a) $\sqrt{2}-\sqrt{3}$ (b) $\sqrt{3}+\sqrt{2}$ (c) $\sqrt{3}-\sqrt{2}$ (d) 1

2 [a] If $x=2+\sqrt[3]{4}$ and $y=2-\sqrt[3]{4}$, **find the value of :** $\left(\frac{x-y}{x+y}\right)^3$

[b] **Put in the simplest form :** $\sqrt{18}-\sqrt[3]{72}-\sqrt{8}+\sqrt[3]{9}$

Quiz 9 till lesson 9 – unit 1

time 15 min.

1 Choose the correct answer from the given ones :

1. The circle whose radius length $= \sqrt{14}$ cm. has an area = ... cm^2.
   (a) 14π (b) $2\sqrt{14}\pi$ (c) 14 (d) $2\sqrt{14}$

2. The right circular cylinder whose base radius length = 3 cm. and its height = 5 cm. its volume = ... cm^3.
   (a) 15π (b) 75π (c) 45π (d) $\frac{3}{5}\pi$

3. The conjugate of the number $\frac{2}{\sqrt{5}+\sqrt{3}} =$ ....
   (a) $\frac{\sqrt{5}+\sqrt{3}}{2}$ (b) $\sqrt{5}-\sqrt{3}$ (c) $\sqrt{5}+\sqrt{3}$ (d) $\frac{\sqrt{5}-\sqrt{3}}{2}$

2 [a] The height of a right circular cylinder equals its radius length , and its volume $= 27\pi\ cm^3$. Calculate the lateral area of the cylinder.

[b] If $x = \sqrt{5}-\sqrt{3}$, $y = \frac{2}{\sqrt{5}-\sqrt{3}}$, **then find the value of :** $x^2 + 2xy + y^2$

Quiz 10 till lesson 10 – unit 1

time 15 min.

1 Choose the correct answer from the given ones :

1. The S.S. in $\mathbb{R}$ of the inequality $-1 < -x \le 1$ is ....
   (a) $]-1 , 1]$ (b) $[-1 , 1]$ (c) $[-1 , 1[$ (d) $]-1 , 1[$

2. If three quarters of the volume of a sphere is $8\pi\ cm^3$, then the length of its radius equals .... cm.
   (a) 64 (b) 8 (c) 4 (d) 2

3. $\sqrt[3]{2} + \sqrt[3]{2} =$ ....
   (a) $\sqrt[3]{4}$ (b) $\sqrt[3]{8}$ (c) $\sqrt[3]{16}$ (d) $\sqrt[3]{2}$

2 [a] **Find in $\mathbb{R}$ the S.S. of the following inequality and represent it on the number line :**

$-1 < 1 - 2x \le 5$

[b] **Simplify to the simplest form :** $2\sqrt{5} + 9\sqrt{\frac{1}{3}} - \sqrt{27} - 5\sqrt{\frac{1}{5}}$

Quiz 11 till lesson 1 – unit 2

time 15 min.

1 Choose the correct answer from the given ones :

1 The ordered pair that satisfies the relation : $3x - y = 1$ is ……………

(a) $(0, 5)$ (b) $(-1, 2)$ (c) $(1, 2)$ (d) $(2, 1)$

2 If $(2k, 3k)$ satisfies the relation : $x + y = 15$, then $k =$ ……………

(a) 5 (b) 3 (c) -5 (d) -3

3 If the ordered pair $(1, 3)$ satisfies the relation : $y = 3x + c$, then $c =$ ……………

(a) zero (b) 1 (c) 2 (d) 3

2 [a] Find three ordered pairs satisfying the relation :

$2x - 3y = 6$, then represent it graphically.

[b] If the straight line : $y - 3x = a$ intersects the x-axis at the point $(1, b)$, then find the value of each of : a and b

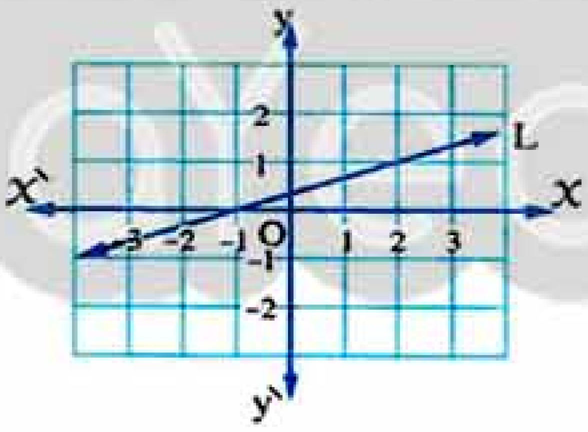
Quiz 12 till lesson 2 – unit 2

time 15 min.

1 Complete the following :

1 If $-\sqrt{25} = \sqrt[3]{x}$, then $x =$ ……………

2 The slope of the straight line L in the opposite graph is ……………



3 The slope of any straight line parallel to y-axis is ……………

2 [a] Represent the straight line that represents the relation : $2x + y = 4$, if this line intersects the x-axis at the point A and intersects the y-axis at the point B, then find the area of ΔAOB, where O is the origin point

[b] Prove that :

The points A, B and C are collinear where $A(1, 1)$, $B(-5, -11)$ and $C(4, 7)$

Quiz 13 till lesson 3 – unit 2

time 15 min.

1 Choose the correct answer from the given ones :

1 If the ordered pair (5 , 2) satisfies the relation $y = 2x - b$, then b = ...

(a) –1 (b) 3 (c) 8 (d) 4

2 The slope of the straight line passes through the two points (3 , 4) and (3 , – 5) is ....

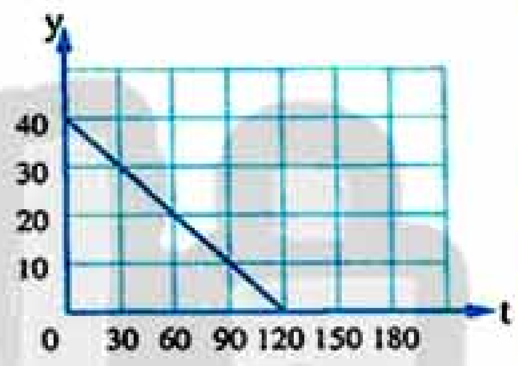
(a) zero (b) unknown (c) $\frac{9}{6}$ (d) $\frac{2}{3}$

3 The perimeter of a face of a cube is 12 cm. Its lateral area = ... cm^2.

(a) 144 (b) 64 (c) 36 (d) 54

2 [a] A right cylinder whose volume is 3080 cm^3 , and its height is 20 cm. , find the diameter length of its base. $\left(\pi = \frac{22}{7}\right)$

[b] Ahmed filled the tank of his car by fuel the opposite graph represent the relation between the time (t) in minutes and the amount of remained fuel in the tank (y) in litre , from the graph :



1 What is the greatest capacity of the tank ?

2 What is the average of the fuel consumption per minutes ?

3 When the tank get empty ?

Quiz 14 till lesson 1 – unit 3

time 15 min.

1 Choose the correct answer from the given ones :

1 The S.S. of the equation : $\sqrt{2}\,x - 1 = 3$ in $\mathbb{R}$ is ....

(a) $\{2\}$ (b) $\{\sqrt{2}\}$ (c) $\{2\sqrt{2}\}$ (d) $\{4\sqrt{2}\}$

2 If $x = \sqrt{3} + 2$, $y = \sqrt{3} - 2$, then $(xy , x + y)$ = ...

(a) $(1 , 2\sqrt{3})$ (b) $(-1 , 2\sqrt{3})$ (c) $(5 , 2\sqrt{3})$ (d) (5 , 9)

3 If (– 1 , 5) satisfies the relation : $3x + ky = 7$, then k = ...

(a) 2 (b) – 2 (c) 1 (d) 10

2 The following table shows the marks obtained by 30 students in an examination :

5	9	11	4	9	9	16	7	8	12	2	10	7	12	5
8	15	13	13	9	7	14	19	3	11	14	3	12	13	17

Form the frequency table to these data.

Quiz 15 till lesson 2 – unit 3

time 15 min.

1 Choose the correct answer from the given ones :

1 If $x^3 + 9 = 1$, where $x \in \mathbb{R}$, then $x =$ ....

(a) -8 (b) -2 (c) 2 (d) 8

2 The slope of the straight line passes through A (2 , 3) and B (0 , 1) is ....

(a) 2 (b) -2 (c) 1 (d) -1

3 $\sqrt[3]{54} + \sqrt[3]{-2} =$ ....

(a) $\sqrt[3]{52}$ (b) $\sqrt[3]{2}$ (c) $2\sqrt[3]{2}$ (d) $4\sqrt[3]{2}$

2 The following table shows the frequency distribution of wages of 100 workers weekly :

Sets	50 –	60 –	70 –	80 –	90 –	Total
Frequency	5	15	30	40	10	100

1 Find the number of workers whose wages are less than 70 pounds weekly.

2 Graph the ascending cummulative frequency curve.

Quiz 16 till lesson 3 – unit 3

time 15 min.

1 Complete the following :

1 If the mean of the values : 27 , 8 , 16 , 24 , 6 and k is 14 , then k = ...

2 The S.S. of the inequality : $-x > 3$ in $\mathbb{R}$ is ....

3 If the arithmetic mean of a frequency distribution is 12.3 and the sum of all frequences is 100 , then the sum of the products of each frequence and the centre of its set = ...

2 [a] The following table shows the frequency distribution of extra wages weekly for 100 workers in a factory :

Extra wages in pounds	20 –	30 –	40 –	50 –	60 –	70 –
Number of workers	10	14	k	k + 4	20	8

1 Calculate the value of k

2 Find the arithmetic mean of this distribution.

[b] If $x = \sqrt{7} + \sqrt{5}$ and $y = \frac{2}{x}$, find the value of the expression : $\frac{x + y}{x y}$ in its simplest form.

Revision for the important rules of algebra and statistics

First Real numbers

Remember that

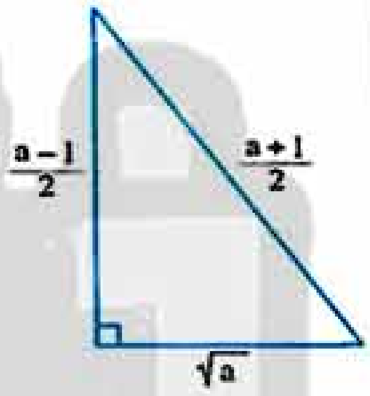
- $\mathbb{R} = \mathbb{Q} \cup \grave{\mathbb{Q}}$
- $\mathbb{Q} \cap \grave{\mathbb{Q}} = \varnothing$
- $\mathbb{R} - \mathbb{Q} = \grave{\mathbb{Q}}$
- $\mathbb{R} - \grave{\mathbb{Q}} = \mathbb{Q}$
- $\mathbb{R}_+ \cap \mathbb{R}_- = \varnothing$
- $\mathbb{R} = \mathbb{R}_+ \cup \{0\} \cup \mathbb{R}_-$
- $\pi \in \grave{\mathbb{Q}}$
- $\mathbb{R}^* = \mathbb{R} - \{0\}$

Remember The representing of the irrational number on the number line

Each irrational number can be represented by a point on the number line.

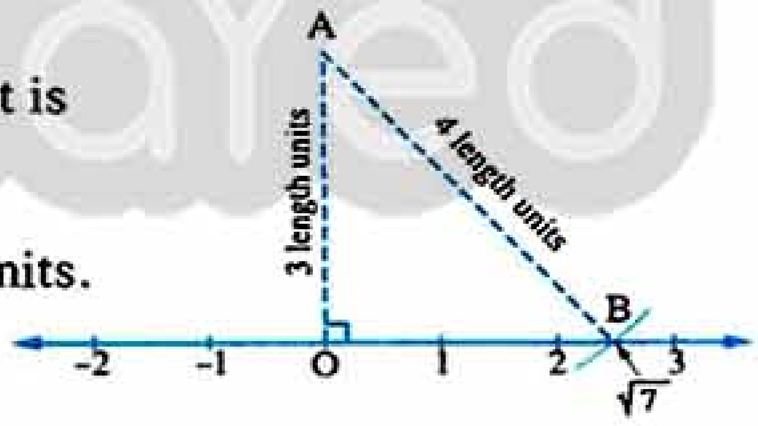
and to draw a line segment with length $= \sqrt{a}$ length unit where $a > 1$

Draw a right-angled triangle in which :



- The length of one side of the right-angle $= \frac{a-1}{2}$ length unit.
- The length of the hypotenuse $= \frac{a+1}{2}$ length unit.

and we can apply this to represent the irrational number $\sqrt{7}$ on the number line as the following :



- From the point which represents the number zero on the number line , we draw a perpendicular line segment and it is $\overline{OA}$ where $OA = \frac{7-1}{2} = 3$ length units.
- Using the compasses with a distance $= \frac{7+1}{2} = 4$ length units. and centre at A then draw an arc to cut the number line on the right side of the point O at the point B , then B is the point which represents $\sqrt{7}$ as in the figure.
- Notice that : To represent the number $(-\sqrt{7})$, we draw the arc which cuts the number line on its left side , not on its right side.
- Notice that : To represent the number $(1 + \sqrt{7})$, we follow the same previous steps but we draw the perpendicular line segment $\overline{OA}$ from the point which represents the number 1 , not the number 0

Remember — The operations on intervals

Intervals	Intersection	Union	Difference	Complement
$X = [-1, 5[$, $Y =]-3, 2[$	$X \cap Y = [-1, 2[$	$X \cup Y =]-3, 5[$	$X - Y = [2, 5[$, $Y - X =]-3, -1[$	$\grave{X} = \mathbb{R} - [-1, 5[$ $=]-\infty, -1[\cup [5, \infty[$
$X =]-\infty, 1]$, $Y = [-2, 1[$	$X \cap Y = [-2, 1[$	$X \cup Y =]-\infty, 1]$	$X - Y =]-\infty, -2[\cup \{1\}$, $Y - X = \varnothing$	$\grave{X} =]1, \infty[$
$X = [-1, 5]$, $Y =]-1, 5[$	$X \cap Y =]-1, 5[$	$X \cup Y = [-1, 5]$	$X - Y = \{-1, 5\}$, $Y - X = \varnothing$	$\grave{Y} = \mathbb{R} -]-1, 5[$ $=]-\infty, -1] \cup [5, \infty[$
$X =]-3, 4]$, $Y = \{-3, 4\}$	$X \cap Y = \{4\}$	$X \cup Y = [-3, 4]$	$X - Y =]-3, 4[$, $Y - X = \{-3\}$	$\grave{Y} = \mathbb{R} - \{-3, 4\}$

Remember — The operations on the square roots and the cube roots

The square roots

1. $\sqrt{a}\times\sqrt{b}=\sqrt{ab}$ — ***For Example :*** $\sqrt{3}\times\sqrt{12}=\sqrt{3\times12}=\sqrt{36}=6$
2. $\frac{\sqrt{a}}{\sqrt{b}}=\sqrt{\frac{a}{b}}$ (where $b\neq0$) — ***For Example :*** $\frac{\sqrt{8}}{\sqrt{2}}=\sqrt{\frac{8}{2}}=\sqrt{4}=2$
3. $\frac{\sqrt{a}}{\sqrt{b}}=\frac{\sqrt{a}}{\sqrt{b}}\times\frac{\sqrt{b}}{\sqrt{b}}=\frac{\sqrt{ab}}{b}$ (where $b\neq0$) — ***For Example :*** $\frac{\sqrt{2}}{\sqrt{5}}=\frac{\sqrt{2}}{\sqrt{5}}\times\frac{\sqrt{5}}{\sqrt{5}}=\frac{\sqrt{10}}{5}$

The cube roots

1. $\sqrt[3]{a}\times\sqrt[3]{b}=\sqrt[3]{ab}$ — ***For Example :*** $\sqrt[3]{3}\times\sqrt[3]{9}=\sqrt[3]{3\times9}=\sqrt[3]{27}=3$
2. $\frac{\sqrt[3]{a}}{\sqrt[3]{b}}=\sqrt[3]{\frac{a}{b}}$ (where $b\neq0$) — ***For Example :*** $\frac{\sqrt[3]{32}}{\sqrt[3]{4}}=\sqrt[3]{\frac{32}{4}}=\sqrt[3]{8}=2$

Example Simplify to the simplest form :

1. $\sqrt{32}-\sqrt{72}+6\sqrt{\frac{1}{2}}$
2. $\sqrt{18}-\frac{\sqrt{12}}{\sqrt{6}}$
3. $5\sqrt{2}\left(2\sqrt{2}+\sqrt{12}\right)$
4. $\sqrt[3]{54}+6\sqrt[3]{16}-6\sqrt[3]{\frac{1}{4}}$
5. $\sqrt[3]{72}+\sqrt[3]{\frac{1}{3}}+\sqrt[3]{-9}$

Solution

1. $\sqrt{32}-\sqrt{72}+6\sqrt{\frac{1}{2}}=\sqrt{2\times16}-\sqrt{2\times36}+3\times2\sqrt{\frac{1}{2}}$
$=4\sqrt{2}-6\sqrt{2}+3\sqrt{\frac{1}{2}\times4}=4\sqrt{2}-6\sqrt{2}+3\sqrt{2}=\sqrt{2}$

2. $\sqrt{18}-\frac{\sqrt{12}}{\sqrt{6}}=\sqrt{2\times9}-\sqrt{2}=3\sqrt{2}-\sqrt{2}=2\sqrt{2}$

3. $5\sqrt{2}\left(2\sqrt{2}+\sqrt{12}\right)=5\sqrt{2}\times2\sqrt{2}+5\sqrt{2}\times\sqrt{12}=10\sqrt{4}+5\sqrt{24}=10\times2+5\sqrt{4\times6}$
$=20+5\times2\sqrt{6}=20+10\sqrt{6}$

4. $\sqrt[3]{54}+6\sqrt[3]{16}-6\sqrt[3]{\frac{1}{4}}=\sqrt[3]{2\times27}+6\sqrt[3]{8\times2}-3\times2\sqrt[3]{\frac{1}{4}}$
$=3\sqrt[3]{2}+6\times2\times\sqrt[3]{2}-3\times\sqrt[3]{8\times\frac{1}{4}}=3\sqrt[3]{2}+12\sqrt[3]{2}-3\sqrt[3]{2}=12\sqrt[3]{2}$

5. $\sqrt[3]{72}+\sqrt[3]{\frac{1}{3}}+\sqrt[3]{-9}=\sqrt[3]{8\times9}+\sqrt[3]{\frac{1}{3}\times\frac{9}{9}}-\sqrt[3]{9}$
$=2\sqrt[3]{9}+\sqrt[3]{\frac{9}{27}}-\sqrt[3]{9}=2\sqrt[3]{9}+\frac{1}{3}\sqrt[3]{9}-\sqrt[3]{9}=\frac{4}{3}\sqrt[3]{9}$

Remember — The two conjugate numbers

If a and b are two positive rational numbers :
then each of the two numbers $\left(\sqrt{a}+\sqrt{b}\right)$ and $\left(\sqrt{a}-\sqrt{b}\right)$
is conjugate to the other one and we find that :

- Their sum $=\left(\sqrt{a}+\sqrt{b}\right)+\left(\sqrt{a}-\sqrt{b}\right)=2\sqrt{a}$ = twice the first term
- Their product $=\left(\sqrt{a}+\sqrt{b}\right)\left(\sqrt{a}-\sqrt{b}\right)=\left(\sqrt{a}\right)^2-\left(\sqrt{b}\right)^2=a-b$
  = The square of the first term – the square of the second term

For example : The number $\left(\sqrt{3}-\sqrt{2}\right)$ its conjugate is $\left(\sqrt{3}+\sqrt{2}\right)$, then we find that :

- Their sum $=2\sqrt{3}$
- Their product $=3-2=1$

Remark

If we have a real number whose denominator is written in the form $\left(\sqrt{a}+\sqrt{b}\right)$ or $\left(\sqrt{a}-\sqrt{b}\right)$, we should put it in the simplest form by multiplying both the numerator and denominator by the conjugate of the denominator.

For example :

For writting the number $\frac{12}{\sqrt{6}-\sqrt{2}}$ in the simplest form , we multiply the two terms of the number by the conjugate of the denominator which is $\left(\sqrt{6}+\sqrt{2}\right)$

$$\therefore \frac{12}{\sqrt{6}-\sqrt{2}}\times\frac{\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}}=\frac{12\left(\sqrt{6}+\sqrt{2}\right)}{6-2}=3\left(\sqrt{6}+\sqrt{2}\right)=3\sqrt{6}+3\sqrt{2}$$

Important remarks from multiplying by inspection

- **We know that :** $(x-y)(x+y)=x^2-y^2$
- **And we know also :**

$(x+y)^2=x^2+2xy+y^2$

Then

- $x^2+xy+y^2=(x+y)^2-xy$
- $x^2+y^2=(x+y)^2-2xy$

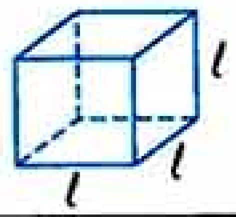
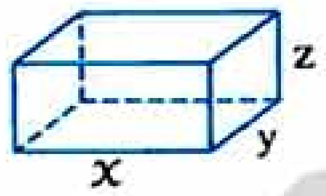
$(x-y)^2=x^2-2xy+y^2$

Then

- $x^2-xy+y^2=(x-y)^2+xy$

or $x^2+y^2=(x-y)^2+2xy$

Summary of rules of areas and volumes of some solids

The solid		The lateral area	Total area	The volume
The cube		$4\ell^2$	$6\ell^2$	ℓ^3
The cuboid		$2(x+y)\times z$	$2(xy+yz+zx)$	xyz
The cylinder		$2\pi rh$	$2\pi rh+2\pi r^2$ $=2\pi r(h+r)$	$\pi r^2 h$
The sphere			$4\pi r^2$	$\frac{4}{3}\pi r^3$

Remember that : The circumference of the circle $= 2\pi r$, the area of the circle $= \pi r^2$

Remember — Solving equation of the first degree in one unknown in $\mathbb{R}$

- Solving the equation of the first degree in one unknown in $\mathbb{R}$ means finding the real number which satisfies this equation.

And the following example shows how to solve an equation of the first degree in one unknown.

Example

Find in $\mathbb{R}$ the solution set of each of the following equations , then represent the solution on the number line :

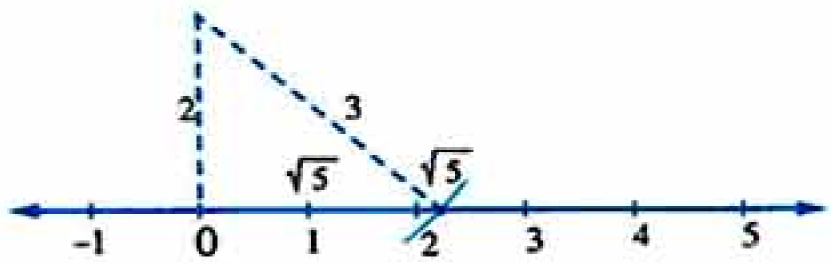
1. $\sqrt{5}\,x - 1 = 4$
2. $x - \sqrt{3} = 2$

Solution

1. $\because \sqrt{5}\,x - 1 = 4 \qquad \therefore \sqrt{5}\,x = 4 + 1 = 5$

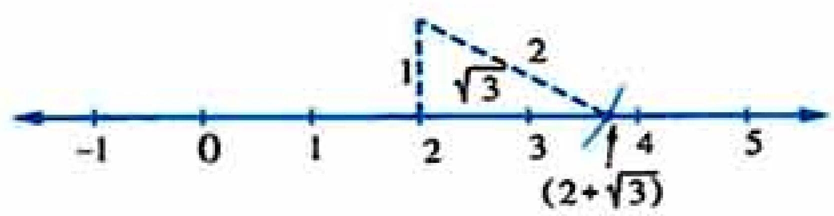
$\therefore x = \frac{5}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{5\sqrt{5}}{5} = \sqrt{5}$

$\therefore$ The S.S. $= \{\sqrt{5}\}$



2. $\because x - \sqrt{3} = 2 \qquad \therefore x = 2 + \sqrt{3}$

$\therefore$ The S.S. $= \{2 + \sqrt{3}\}$



Remember — Solving inequality of the first degree in one unknown in $\mathbb{R}$

- Solving the inequality means finding all values of the unknown which satisfy this inequality.
- The solution set of the inequality in $\mathbb{R}$ will be written as an interval

And the following example shows how to solve an inequality of the first degree in one unknown in $\mathbb{R}$

Example

Find in $\mathbb{R}$ the solution set of each of the following inequalities , then represent the solution on the number line :

1. $2x+6<2$
2. $5-4x\leq -3$
3. $3<3-5x<13$
4. $x-2\geq 3x-5$

Solution

1 $\because 2x+6<2$ $\quad\therefore 2x<2-6$

$\therefore 2x<-4$ $\quad\therefore x<\frac{-4}{2}$

$\therefore x<-2$ $\quad\therefore$ The S.S. $=]-\infty , -2[$

2 $\because 5-4x\leq -3$ $\quad\therefore -4x\leq -8$

$\therefore x\geq \frac{-8}{-4}$

(Notice the change in the direction of the symbol of the inequality because we divided by a negative number)

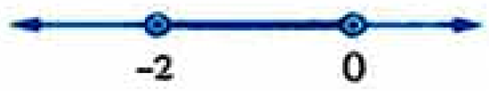
$\therefore x\geq 2$ $\quad\therefore$ The S.S. $= [2 , \infty[$

3 $\because 3<3-5x<13$ (adding -3 to all sides)

$\therefore 0<-5x<10$ (dividing all sides by -5) $\quad\therefore 0>x>-2$

(Notice the change in the direction of the symbol of the inequality because we divided by a negative number)

$\therefore$ The S.S. $=]-2 , 0[$



4 $\because x-2\geq 3x-5$ $\quad\therefore x-3x\geq -5+2$

$\therefore -2x\geq -3$ $\quad\therefore x\leq \frac{3}{2}$

$\therefore$ The S.S. $=]-\infty , \frac{3}{2}]$

Second Relation between two variables

Remember The linear relation

It is a relation of the first degree between two variables x and y , it is in the form :
$a x + b y = c$, where a , b and c are real numbers , a and $b \neq 0$ together.
And there is an infinite number of ordered pairs which satisfy this relation , and it is enough to get three ordered pairs satisfying the relation at the graphical representation.

Example 1

Find three ordered pairs satisfying the relation : $3x - 2y = 6$

Solution

$\because 3x - 2y = 6 \qquad \therefore -2y = 6 - 3x \qquad \therefore y = \frac{3x - 6}{2}$

- Putting $x = 0 \qquad \therefore y = -3 \qquad \therefore (0, -3)$ satisfies the relation.
- Putting $x = 1 \qquad \therefore y = -\frac{3}{2} \qquad \therefore \left(1, -\frac{3}{2}\right)$ satisfies the relation.
- Putting $x = 2 \qquad \therefore y = 0 \qquad \therefore (2, 0)$ satisfies the relation.

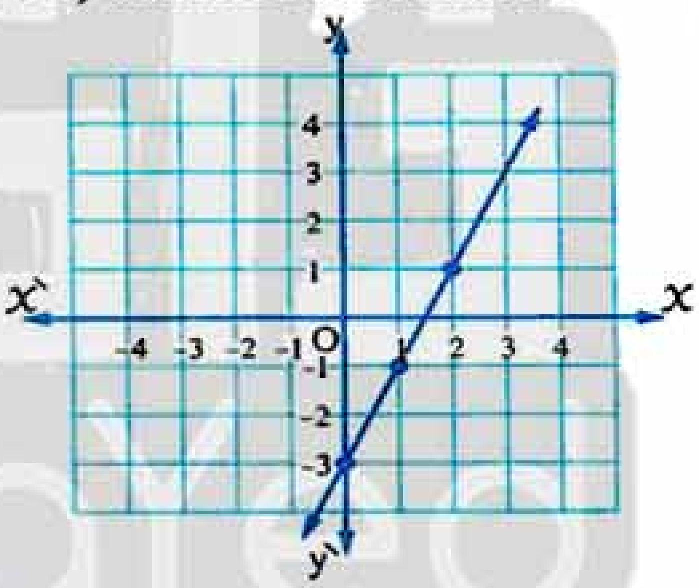
Example 2

Represent graphically the relation : $2x - y = 3$

Solution

$\because 2x - y = 3 \qquad \therefore y = 2x - 3$

x	0	1	2
y	−3	−1	1



Remember The slope of the straight line

The slope of the straight line = $\frac{\text{the change in y-coordinates}}{\text{the change in } x\text{-coordinates}} = \frac{\text{the vertical change}}{\text{the horizontal change}}$

i.e. $S = \frac{y_2 - y_1}{x_2 - x_1}$, where $x_1 \neq x_2$

For example : The slope of the straight line passing through the two points : (2 , 3) , (− 5 , 2) is :

$S = \frac{2 - 3}{-5 - 2} = \frac{-1}{-7} = \frac{1}{7}$

Notice that

- The slope of the straight line parallel to x-axis = 0
- The slope of the straight line parallel to y-axis is undefined.

Third Statistics

Remember The tables and cumulative frequency curves

The following frequency table shows the weekly wages in pounds of 50 workers in a factory :

Sets of wages	54 –	58 –	62 –	66 –	70 –	Total
No. of workers (frequency)	5	12	22	7	4	50

1 Forming the ascending cumulative frequency table and graphing the curve

Sets of wages	54 –	58 –	62 –	66 –	70 –
Number of workers (frequency)	5	12	22	7	4

Less than 54 = 0
Less than 58 = 5 + 0 = 5
Less than 62 = 5 + 12 = 17
Less than 66 = 5 + 12 + 22 = 39
Less than 70 = 5 + 12 + 22 + 7 = 46
Less than 74 = 5 + 12 + 22 + 7 + 4 = 50

The upper boundaries of sets	Frequency
Less than 54	zero
Less than 58	5
Less than 62	17
Less than 66	39
Less than 70	46
Less than 74	50

"The ascending cumulative frequency table"

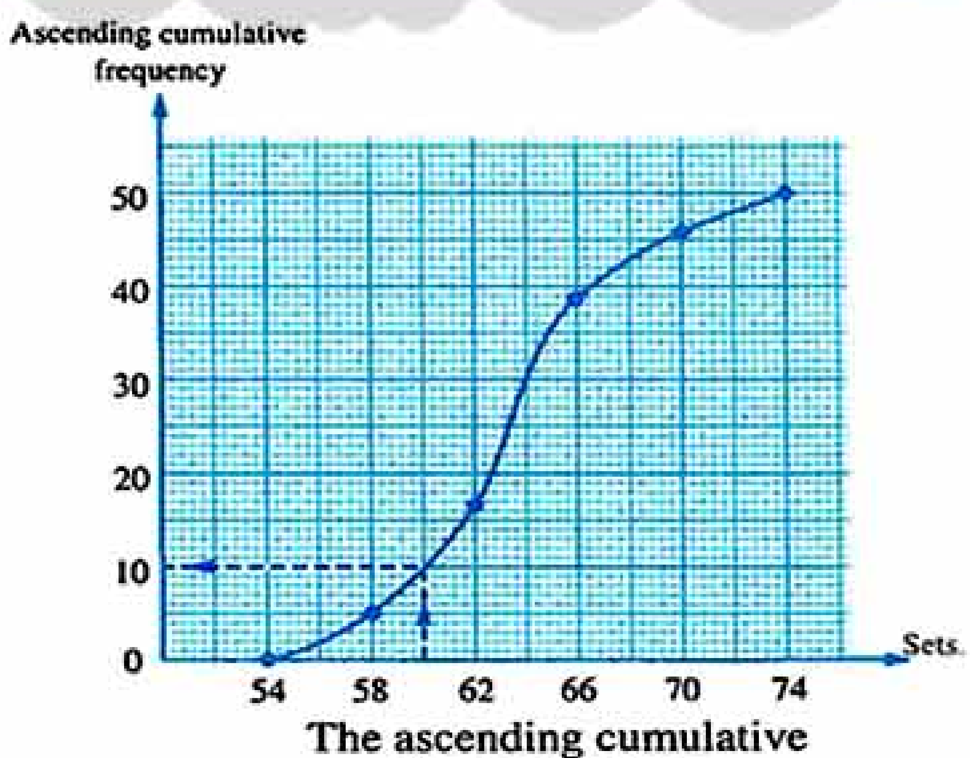
Notice that

The ascending cumulative frequency begins with zero and ends at the total frequency.

- From the opposite graph , we can find the number of individuals which less than a certain value.

For Example :

The number of workers whose wages are less than 60 pounds is 10 workers



The ascending cumulative frequency curve

2 Forming the descending cumulative frequency table and graphing the curve

Sets of wages	54 –	58 –	62 –	66 –	70 –
Number of workers (frequency)	5	12	22	7	4

54 and more = $5 + 12 + 22 + 7 + 4 = 50$

58 and more = $12 + 22 + 7 + 4 = 45$

62 and more = $22 + 7 + 4 = 33$

66 and more = $7 + 4 = 11$

70 and more = 4

74 and more = 0

The lower boundaries of sets	Frequency
54 and more	50
58 and more	45
62 and more	33
66 and more	11
70 and more	4
74 and more	zero

"The descending cumulative frequency table"

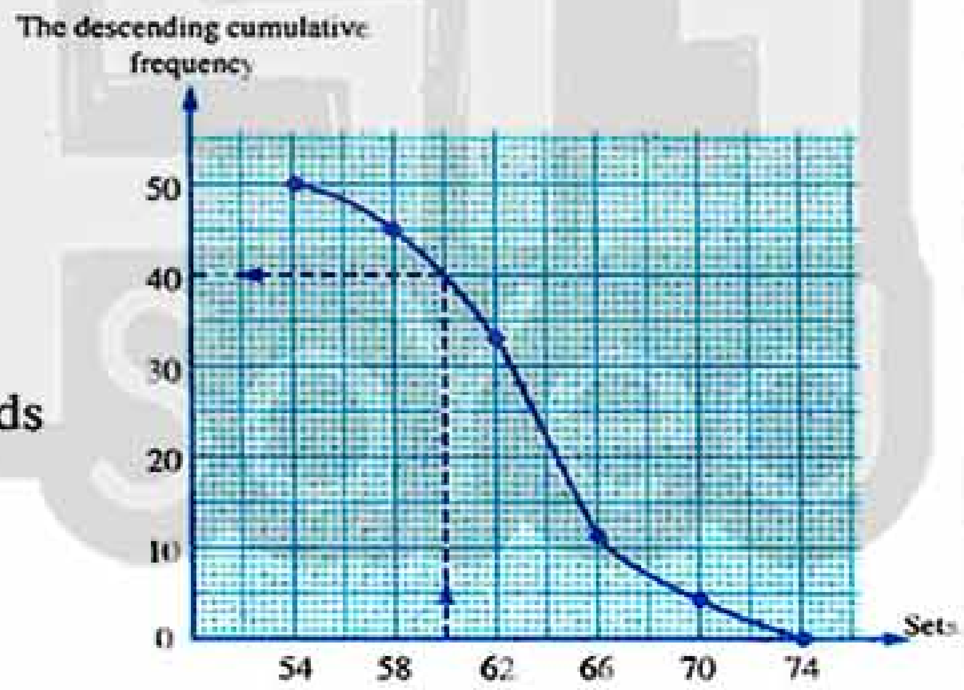
Notice that

The descending cumulative frequency begins with the total frequency and ends with zero.

- From the opposite graph , we can find the number of individuals which more than or equal to a certain value.

For example :

The number of workers whose wages are 60 pounds or more = 40 workers.



The descending cumulative frequency curve

Remember The measures of the central tendency

(1) The mean. (2) The median. (3) The mode.

1 The mean

[a] The mean of a set of values (simple frequency distribution)

The mean of a set of values = $\frac{\text{The total of values}}{\text{Number of values}}$

For example : The mean of the numbers : 5 , 3 , 7 , 9 = $\frac{5+3+7+9}{4} = 6$

[b] The mean of a frequency distribution with sets

Example

The following table shows the distribution of the marks of 50 pupils in mathematics :

Sets	10 –	20 –	30 –	40 –	50 –	Total
Frequency	8	12	14	9	7	50

and the required is finding the mean of these marks.

Solution

1. Determine the centres of sets according to the rule :

$$\text{The centre of a set} = \frac{\text{the lower limit + the upper limit}}{2}$$

$\therefore$ The centre of the first set = $\frac{10+20}{2} = 15$ ... and so on.

Since the lengths of the subsets are equal and each of them = 10 therefore we consider the upper limit of the last set = 60

, then its centre = $\frac{50+60}{2} = 55$

2. Form the following table :

Set	Centre of the set « X »	Frequency « f »	$X \times f$
10 –	15	8	120
20 –	25	12	300
30 –	35	14	490
40 –	45	9	405
50 –	55	7	385
Total		50	1700

The mean = $\frac{\text{The sum of } (X \times f)}{\text{The sum of } f} = \frac{1700}{50} = 34$ marks.

2 The median

[a] The median of a set of values

The median is the middle value in a set of values after arranging it ascendingly or descendingly. such that the number of values which are less than it is equal to the number of values which are greater than it.

We arrange the values ascendingly or descendingly

If the values number is odd , then

The median is the value lying in the middle exactly.

If the values number is even , then

The median $= \frac{\text{The sum of the two values lying in the middle}}{2}$

For example :

If the values are

42 , 23 , 17 , 30 and 20

We arrange them ascendingly as follows

17 , 20 , 23 , 30 , 42

The median = 23

For example :

If the values are

27 , 13 , 23 , 24 , 13 , 21

We arrange them ascendingly as follows

13 , 13 , $\boxed{21 , 23}$, 24 , 27

The median $= \frac{21 + 23}{2} = 22$

[b] Finding the median of a frequency distribution with sets graphically

For finding the median of a frequency distribution with sets graphically , do the following steps :

1. Form the ascending or the descending cumulative frequency table , then draw the cumulative frequency curve of it.
2. We find the order of the median $= \frac{\text{The total of frequency}}{2}$
3. Determine the point which represents the order of the median on the vertical axis, from this point , draw a horizontal straight line to cut the curve at a point, then from this point , draw a perpendicular to the horizontal axis to intersect it at a point which represents the median.
   The following example shows how to find the median using the two curves (the ascending or the descending cumulative frequency curve).

Example

The following table shows the frequency distribution of marks of 50 students in math exam :

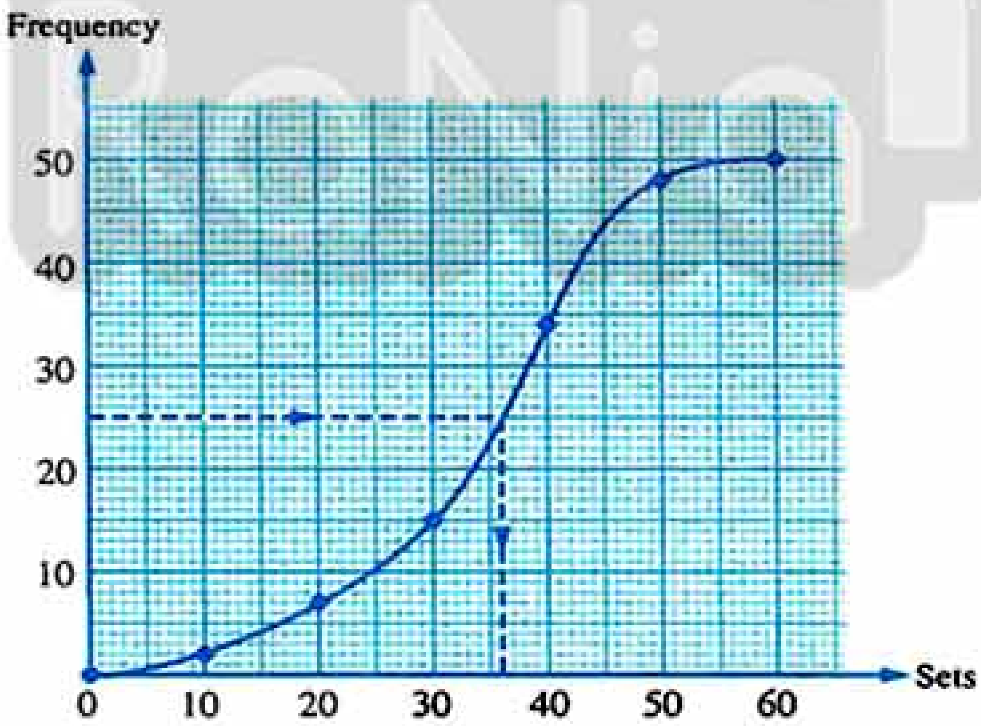
Sets of marks	0 –	10 –	20 –	30 –	40 –	50 –	Total
Number of students	2	5	8	19	14	2	50

Find the median mark of the student.

Solution

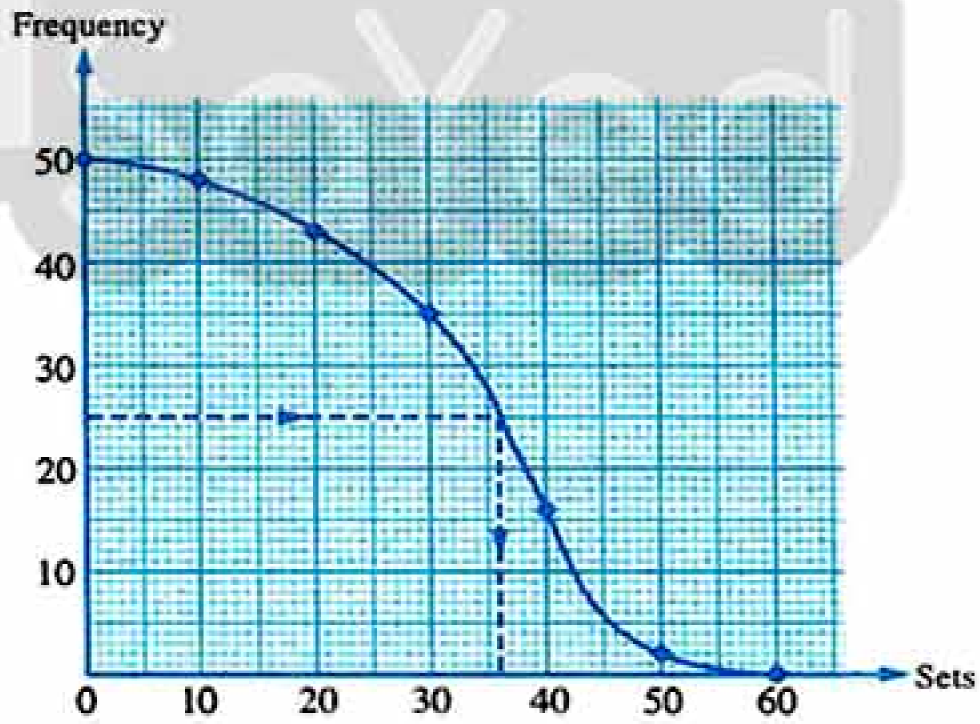
Using the ascending cumulative frequency curve :

The upper boundaries of sets	Frequency
Less than 0	0
Less than 10	2
Less than 20	7
Less than 30	15
Less than 40	34
Less than 50	48
Less than 60	50



Using the descending cumulative frequency curve :

The lower boundaries of sets	Frequency
0 and more	50
10 and more	48
20 and more	43
30 and more	35
40 and more	16
50 and more	2
60 and more	0



$\because$ The order of the median $= \frac{50}{2} = 25$

$\therefore$ From the two previous graphs , the median = 36 approximately

3 The mode

[a] The mode of a set of values

The mode of a set of values is the most common value in the set, or in other words, it is the value which is repeated more than any other values.

For example : The mode of the set of the values : 7 , 3 , 4 , 1 , 7 , 9 , 7 , 4 is 7

[b] The mode of a frequency distribution with sets

Example

The following is the frequency distribution of marks of 100 pupils in one of the exams :

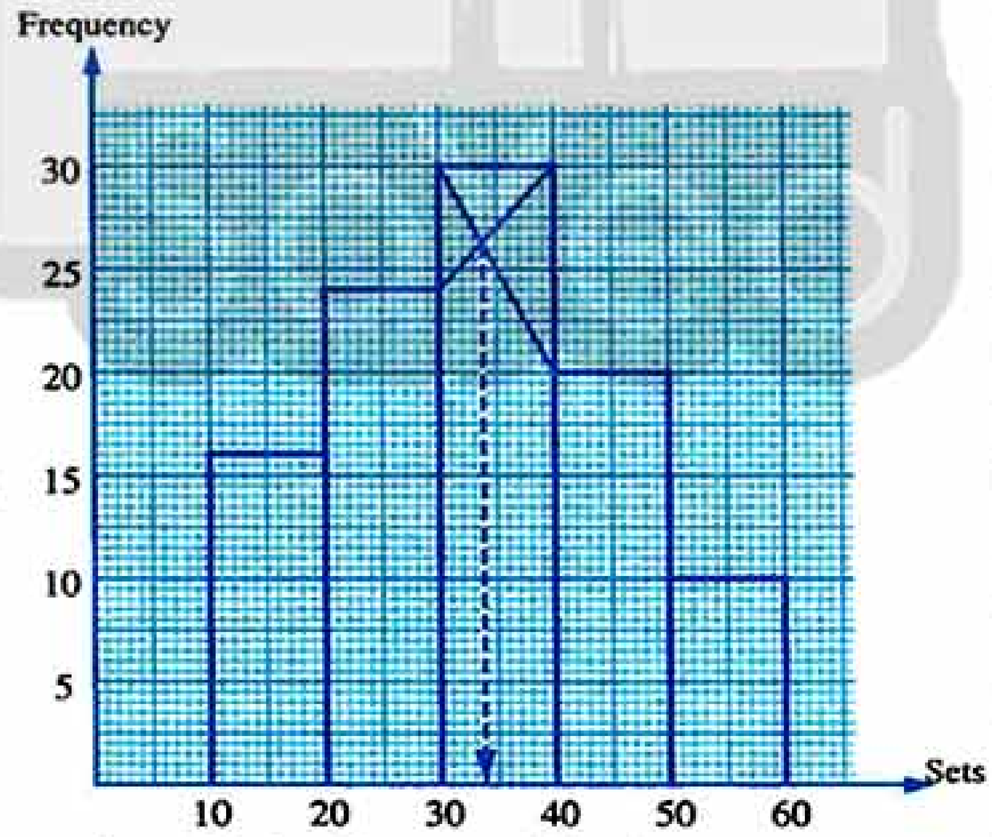
Set of marks	10 –	20 –	30 –	40 –	50 –	Total
Number of pupils	16	24	30	20	10	100

Find the mode mark for these pupils.

Solution

We can find the mode of that distribution graphically using the histogram as follows :

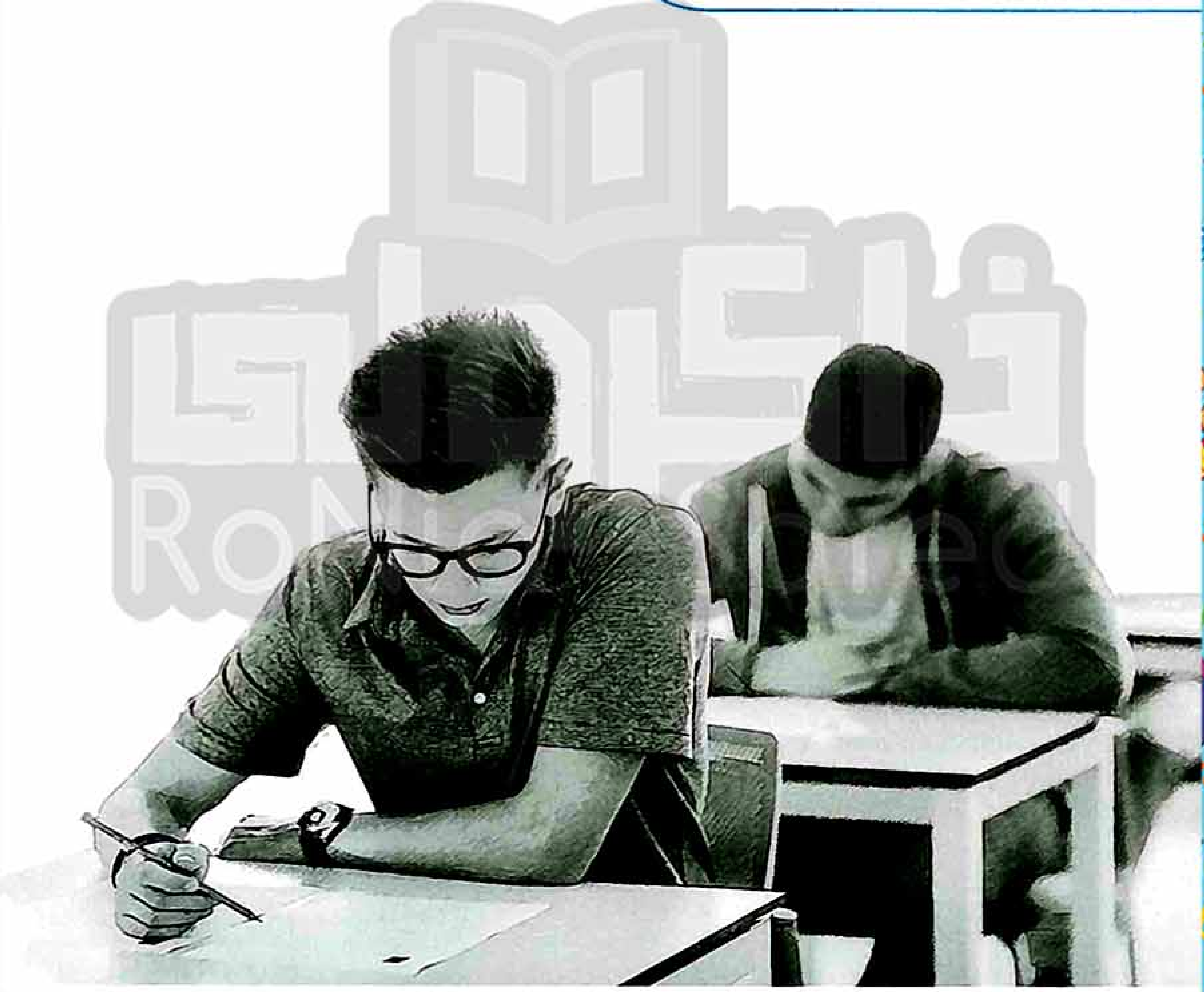
1. Draw two orthogonal axes : one of them is horizontal and the other is vertical to represent the frequency of each set.
2. Divide the horizontal axis into a number of equal parts with a suitable drawing scale to represent the sets.
3. Divide the vertical axis into a number of equal parts with a suitable drawing scale to represent the greatest frequency in the sets.
4. Draw a rectangle whose base is set (10 –) and its height equals the frequency (16)
5. Draw a second rectangle adjacent to the first one whose base is set (20 –) and its height equals the frequency (24)



6. Repeat drawing the remained adjacent rectangles till the last set (50 –)
7. Determine the set which has the greatest frequency then draw two lines as shown in the histogram to intersect at a point.
   From this point , draw a vertical line to intersect the horizontal axis at a point which represents the value of the mode.
   i.e. The mode mark is 34 approximately.

Final Examinations

on Algebra and Statistics



Model Examinations of the School Book on Algebra and Statistics

Model 1

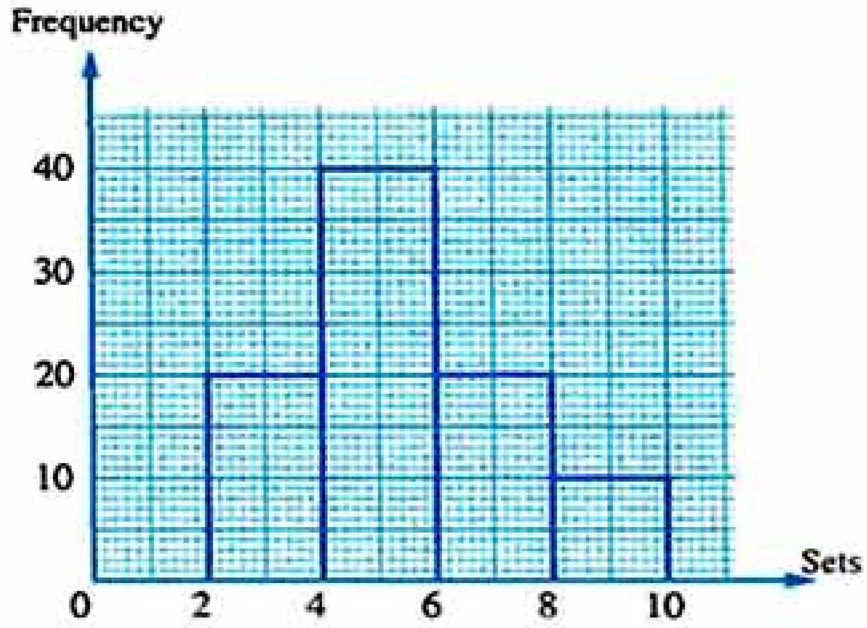
Answer the following questions :

1 Complete the following :

1. The S.S. of the equation : $(x^2+3)(x^3+1)=0$ is .... , $x \in \mathbb{R}$
2. If the lower boundary of a set is 10 and the upper boundary is x and its centre is 15 , then $x =$ ....
3. $]-2 , 2] \cup \{-2 , 0\} =$ ....
4. The cube whose volume is 8 cm^3. , then the sum of all its edge lengths = ... cm.
5. The multiplicative inverse of the number $(\sqrt{3}+\sqrt{2})$ is .... in the simplest form.

2 Choose the correct answer from the given ones :

1. If the radius length of a sphere is 6 cm. , then its volume is ....
   (a) 6π cm^3.   (b) 36π cm^3.   (c) 72π cm^3.   (d) 288π cm^3.
2. If the point (a , 1) satisfies the relation $x + y = 5$, then a = ...
   (a) 1   (b) -4 (c) 4   (d) 5
3. $(2\sqrt[3]{2})^3 =$ ....
   (a) 4   (b) 8   (c) 16   (d) 40
4. The median of the values : 34 , 23 , 25 , 40 , 22 , 4 is ....
   (a) 22   (b) 23   (c) 24   (d) 25
5. If the arithmetic mean of the values : 27 , 8 , 16 , 24 , 6 , k is 14 , then k = ...
   (a) 3   (b) 6   (c) 27   (d) 84
6. **In the opposite figure :**
   The value of the mode = ...
   (a) 4   (b) 5
   (c) 6   (d) 40



3 **[a] Find the value of :** $\sqrt{18}+\sqrt[3]{54}-3\sqrt{2}-\frac{1}{2}\sqrt[3]{16}$

[b] If $x=\frac{3}{\sqrt{5}-\sqrt{2}}$ and $y=\sqrt{5}-\sqrt{2}$

, prove that : x and y are two conjugate numbers.

4 **[a]** The area of a square is 1089 cm^2. Find the length of its diagonal.

[b] Find the S.S. of the inequality : $\frac{3x+1}{6}<x+1<\frac{x+4}{2}$ in $\mathbb{R}$

, then represent it on the number line.

5 **[a]** The radius length of the base of a right circular cylinder is $4\sqrt{2}$ cm. and its height is 9 cm. Find its volume in terms of π and if its volume equals the volume of a sphere , find the radius length of the sphere.

[b] Find the arithmetic mean of the following frequency distribution :

The sets	5 –	15 –	25 –	35 –	45 –	Total
Frequency	7	10	12	13	8	50

Model 2

Answer the following questions :

1 **Complete the following :**

1. The additive inverse of the number : $-\sqrt{3}-\sqrt{5}$ is ....
2. $(\sqrt{8}+\sqrt{2})(\sqrt{8}-\sqrt{2})=$ ....
3. The conjugate of the number $\frac{2\sqrt{5}-3\sqrt{2}}{\sqrt{2}}$ is ....
4. If the volume of a sphere is $\frac{9}{2}\pi$ cm^3 , then its diameter length is .... cm.
5. $[3 , 4]-\{3 , 5\}=$ ....

2 **Choose the correct answer from the given ones :**

1. If the volume of a cube is 27 cm^3 , then the area of one of its faces is ....
   (a) 3 cm^2 (b) 9 cm^2 (c) 36 cm^2 (d) 54 cm^2
2. If the mode of the values 4 , 11 , 8 , $2x$ is 4 , then $x=$ ....
   (a) 2 (b) 4 (c) 6 (d) 8

3 If the arithmetic mean of the values 18 , 23 , 29 , 2 k – 1 , k is 18 , then k = ...

(a) 1 (b) 7 (c) 29 (d) 90

4 If the lower limit of a set is 4 and the upper limit is 8 , then its centre is ....

(a) 2 (b) 4 (c) 6 (d) 8

5 A right circular cylinder the radius length of its base is r cm. and its height equals its diameter length , then its volume = ... cm^3.

(a) πr^3 (b) πr^2 (c) $2 \pi r^3$ (d) $2 r^3$

6 The solution set of the equation : $x(x^2 - 1) = 0$, $x \in \mathbb{R}$ is ....

(a) {0} (b) {1} (c) {– 1} (d) {0 , – 1 , 1}

3 [a] **Reduce to the simplest form :** $\frac{\sqrt{3}}{\sqrt{5}-\sqrt{3}} + \frac{\sqrt{5}}{\sqrt{5}+\sqrt{3}}$

[b] **Prove that :** $\sqrt[3]{128} + \sqrt[3]{16} - 2\sqrt[3]{54} = 0$

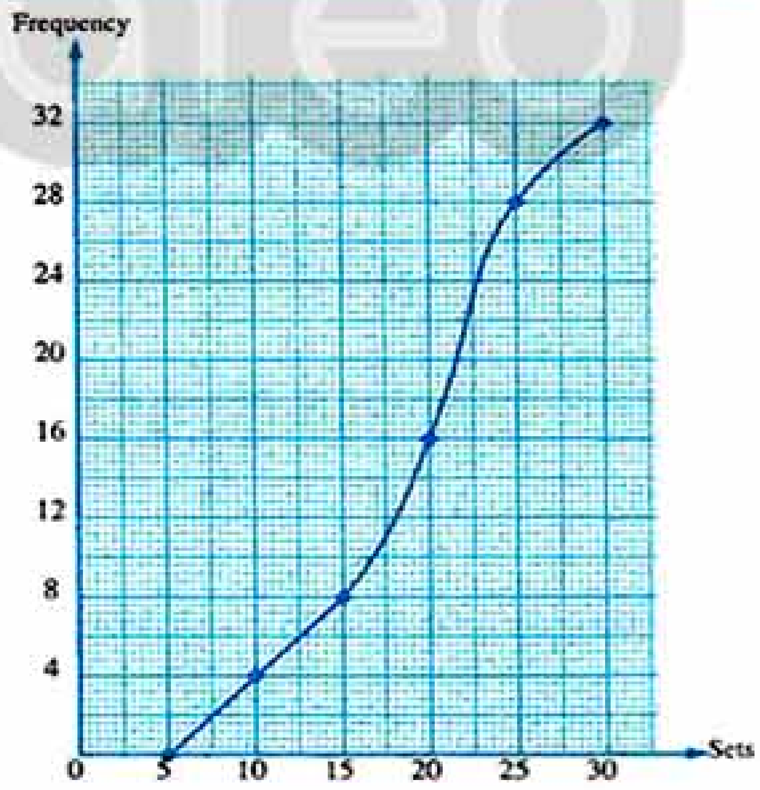
4 [a] **Find the S.S. of the inequality :** $-2 < 3x + 7 \le 10$ in $\mathbb{R}$, then represent the interval of solution on the number line.

[b] If $x = \sqrt{2 + \sqrt{3}}$, **find the value of :** $x^4 - 2x^2 + 1$

5 [a] The opposite graph represents the marks of 32 pupils in an exam.

Complete :

The median mark = ...



[b] **Find the arithmetic mean of the following frequency distribution :**

The sets	5 –	15 –	25 –	35 –	45 –	Total
Frequency	4	5	6	3	2	20

Model for the merge students

Answer the following questions :

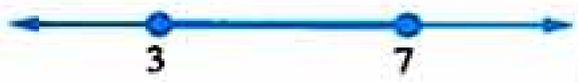
1 Complete each of the following :

1. The conjugate of the number $\sqrt{3}+\sqrt{2}$ is ....
2. $\sqrt{18}+\sqrt{54}-3\sqrt{2}=$ ....
3. The mode for the numbers : 3 , 5 , 3 , 4 , 3 is ....
4. The median of the values : 2 , 3 , 5 , 7 , 9 is ....
5. The solution set of the equation : $x^2+9=0$ in $\mathbb{R}$ is ....

2 Choose the correct answer from those given :

1. The arithmetic mean for the values : 9 , 6 , 5 , 14 , 1 is ....
   (a) 7 (b) 3 (c) 5 (d) 9
2. The simplest form of the expression : $(\sqrt{3}-\sqrt{2})(\sqrt{3}+\sqrt{2})$ is ....
   (a) $\sqrt{3}$ (b) 1 (c) $\sqrt{2}$ (d) $2\sqrt{3}$
3. The additive inverse of the number $-\sqrt{5}$ is ....
   (a) $\sqrt{5}$ (b) 5 (c) $\sqrt{2}$ (d) -5
4. $[3 , 5]-\{3 , 5\}=$ ....
   (a) $]3 , 5[$ (b) $[3 , 5[$ (c) $\varnothing$ (d) $]3 , 5]$
5. A cube is of volume 64 cm^3 , then its edge length is .... cm.
   (a) 4 (b) 8 (c) 16 (d) 64

3 Match from the column (A) to the suitable one from the column (B) :

(A)	(B)
1. The S.S. of the equation : $x^2-25=0$ in $\mathbb{R}$ is	$[0 , 2]$
2. $[-3 , 2]\cap[0 , 2]=$	7
3. If the order of the median is fourth , then the number of values is	$\{5 , -5\}$
4. $\sqrt{3}$ is a .... number.	
5. The S.S. of the inequality : $3\le x\le 7$ on the number line is	irrational

4 **Put (✔) for the correct statements and (✘) for the incorrect ones :**

1. The arithmetic mean of a set of values = sum of values ÷ its number. ( )
2. If $x=\sqrt{13}-\sqrt{7}$, $y=\sqrt{13}+\sqrt{7}$, then x , y are two conjugate numbers. ( )
3. The irrational number $\sqrt{7}$ lies between 2 and 3 ( )
4. $\sqrt{75}-2\sqrt{27}=7\sqrt{3}$ ( )
5. The simplest form of the number $\frac{1}{\sqrt{5}}$ is $\frac{\sqrt{5}}{5}$ ( )

5 **[a] Complete :** If the lower limit of a set is 4 and the upper limit is 8

, then its centre $= \frac{\text{..........} + \text{..........}}{2} =$ ....

[b] Complete the following table to obtain the arithmetic mean of the following frequency distribution :

Sets	5 –	15 –	25 –	35 –	45 –	Total
Frequency	7	10	12	13	8	50

Sets	The centre of the set « x »	Frequency « f »	$x \times f$
5 –	10	7	10 × 7 = 70
15 –	20	10	20 × 10 = ...
25 –	....	....	.... × 12 = ...
35 –	....	....	.... × 13 = ...
45 –	....	....	.... × 8 = ...
Total		50	....

The arithmetic mean $= \frac{\Sigma(x \times f)}{\Sigma(f)} = \frac{\text{..........}}{\text{..........}} =$ ....

Some Schools Examinations on Algebra and Statistics

1 Cairo Governorate

Nasr City Educ. Administration
St. Fatima Language School

Answer the following questions :

1 Choose the correct answer :

1. $[0, 5] \cup [3, 8[=$ ....
   (a) $]3, 5]$ (b) $[3, 5]$ (c) $[0, 8]$ (d) $[0, 8[$
2. $\sqrt{12} - \sqrt{3} =$ ....
   (a) 3 (b) $\sqrt{3}$ (c) $2\sqrt{3}$ (d) $3\sqrt{3}$
3. The S.S. in $\mathbb{R}$ of the equation $x(x^2 - 1) = 0$ is ....
   (a) $\{0\}$ (b) $\{1\}$ (c) $\{-1\}$ (d) $\{0, -1, 1\}$
4. The arithmetic mean of the values 27 , 8 , 16 , 24 , 6 , k is 14 , then k = ...
   (a) 3 (b) 6 (c) 27 (d) 84
5. The additive inverse of the number $-\sqrt{5}$ is ....
   (a) $\sqrt{5}$ (b) 5 (c) $\sqrt{2}$ (d) -5
6. The radius length of a sphere is 6 cm. , then its volume is ....
   (a) 6π cm^3. (b) 36π cm^3. (c) 72π cm^3. (d) 288π cm^3.

2 Complete :

1. $[1, 5] \cap]-2, 3] =$ ....
2. The mode of the set of the values 3 , 4 , 7 , 4 , 2 is ....
3. The volume of the cuboid whose dimensions are $\sqrt{2}$, $\sqrt{3}$, $\sqrt{6}$ cm. is .... cm^3.
4. The S.S. in $\mathbb{R}$ of $3 < 2x - 1 < 5$ as an interval is ....
5. The slope of any line parallel to x-axis is ....

3 **[a]** If $a = \sqrt{3} + \sqrt{2}$, $b = \sqrt{3} - \sqrt{2}$, **find the value of :** $a^2 - ab + b^2$

[b] Find the S.S. for each of the following inequalities in $\mathbb{R}$, in the form of an interval , then represent the S.S. on the number line :

1. $5x - 3 < 2x + 9$
2. $1 \le 3 - 2x < 5$

4 **[a]** If $M = [2, \infty[$, $J =]-2, 3[$, **find each of the following using the number line :**

1. $M \cap J$
2. $M - J$

[b] Simplify : $\dfrac{\sqrt{3}}{\sqrt{5} - \sqrt{3}} + \dfrac{\sqrt{5}}{\sqrt{5} + \sqrt{3}}$

5 **[a] Reduce to the simplest form :** $2\sqrt{18}+\sqrt{50}+\frac{1}{3}\sqrt{162}$

[b] Find the arithmetic mean of the following frequency distribution :

The Set	5 –	15 –	25 –	35 –	45 –	Total
Frequency	4	5	6	3	2	20

2 Cairo Governorate

El-Maadi Zone
Directing Mathematics

Answer the following questions :

1 **Choose the correct answer :**

1. The multiplicative inverse of $\frac{\sqrt{3}}{12}$ is ……………
   (a) $4\sqrt{3}$ (b) 2 (c) $2\sqrt{3}$ (d) $6\sqrt{3}$
2. The conjugate of the number $2-\sqrt{3}$ is ……………
   (a) $\sqrt{3}-2$ (b) $2-\sqrt{3}$ (c) $\sqrt{2}-3$ (d) $2+\sqrt{3}$
3. The volume of the cuboid whose dimensions are $\sqrt{8}$, $\sqrt{3}$, $\sqrt{6}$ is ……………
   (a) 144 (b) 12 (c) $\sqrt{120}$ (d) 20
4. The median for the values 7 , 8 , 9 , 6 and 5 is ……………
   (a) 7 (b) 8 (c) 9 (d) 10
5. $4^3+4^3+4^3+4^3=$ ……………
   (a) 4^{20} (b) 4^4 (c) 4^{12} (d) 16^3
6. If (2 k , k) satisfies the relation $2x+y=15$, then k = ……………
   (a) 1 (b) 2 (c) 3 (d) 4

2 **Complete :**

1. $[2 , 7]-]2 , 7[=$ ……………
2. If the mode of the values 8 , 11 , 4 , $2x$ is 4 , then $x=$ ……………
3. $\mathbb{R}\cap\mathbb{R}_-=$ ……………
4. The slope of the straight line passing through the two points A (5 , 3) , B (2 , 1) is ……………
5. The solution set in $\mathbb{R}$ for $x^2+4=16$ is ……………

3 **[a] Put in the simplest form :** $2\sqrt{8}+\sqrt{50}-\sqrt{32}$

[b] Find the solution set in $\mathbb{R}$ for : $3x-4\le 5$ and represent it on the number line.

4 [a] If $x = \frac{2}{\sqrt{7} - \sqrt{5}}$, $y = \sqrt{7} - \sqrt{5}$, **find :** $(x + y)^2$

[b] **Represent graphically the relation :** $y = 3x - 2$

5 [a] If the volume of a sphere equals $\frac{500}{3} \pi$ cm³, find the length of its radius.

[b] **The following table shows the frequency of marks of 50 students :**

Sets	5 –	15 –	25 –	35 –	45 –	Total
Frequency	7	10	12	13	8	50

Find the mean of the marks of the students.

3 Cairo Governorate

El-Khalifa and El-Mokatam Zone
El-Helmia Exper. Lang. School

Answer the following questions :

1 **Choose the correct answer :**

1. The S.S. in $\mathbb{R}$ for the equation : $x^3 + 8 = 0$ is ....
   (a) $\{4\}$ (b) $\{2\}$ (c) $\varnothing$ (d) $\{-2\}$
2. If the mode of the values 3 , 5 , $x + 1$, 5 , 3 , 1 is 5 , then $x =$ ....
   (a) 5 (b) 4 (c) 3 (d) 6
3. The cube whose volume is 8 cm³, the area of one of its faces is .... cm².
   (a) 4 (b) 8 (c) 16 (d) 64
4. If $x < \sqrt{15} < x + 1$, $x \in \mathbb{Z}$, then $x =$ ....
   (a) 3 (b) 4 (c) 5 (d) $\varnothing$
5. $\sqrt{3} + \sqrt{3} =$ ....
   (a) -3 (b) $\sqrt{12}$ (c) 12 (d) 3
6. Which of the following ordered pairs satisfies the relation $2x + y = 5$?
   (a) $(-1 , 3)$ (b) $(1 , 3)$ (c) $(3 , 1)$ (d) $(2 , 2)$

2 **Complete :**

1. $\sqrt[3]{\text{..............}} = -\sqrt{9}$
2. If $(-1 , 5)$ satisfies the relation $3x + ky = 7$, then k = ...
3. If the order of the median of some values is fifth , then the number of these values is ....
4. $[-2 , 5] \cap [3 , 7] =$ ....
5. If the lower limit of a set is 4 and the upper limit of the same set is 10 , then the centre of this set is ....

3 **[a]** The volume of a sphere is $562.5\,\pi\ cm^3$, find its surface area.

[b] If $x = \frac{4}{\sqrt{7}+\sqrt{3}}$, $y = \sqrt{7}+\sqrt{3}$, **then find the numerical value of :** $x^2 - 2xy + y^2$

4 **[a]** **Find in $\mathbb{R}$ the S.S. of :** $-1 < 3x + 5 \leq 14$ and represent it on the number line.

[b] **Graph the relation :** $2x + y = 1$

[c] If $A =]-\infty , 3[$, $B = [-1 , 5]$

, find the following using the number line : **1** $A \cap B$ **2** $A - B$

5 **[a]** Find the slope of $\overleftrightarrow{AB}$ where $A(-1 , 3)$, $B(2 , 5)$

Is the point $C(8 , 1) \in \overleftrightarrow{AB}$?

[b] **The following table shows the marks of 50 students in an examination :**

Sets	5 –	15 –	25 –	35 –	45 –	Total
Frequency	7	10	12	13	8	50

Find the arithmetic mean of this frequency distribution.

4 Giza Governorate

El-Haram Directorate
Al Maarefa Exp. Language School

Answer the following questions :

1 **Complete the following :**

1 $\sqrt{4} = \sqrt[3]{\cdots\cdots}$

2 $]-3 , 4[\cup \{-3\} = \cdots\cdots$

3 The mode of the values 7 , 3 , 8 , 2 , 3 , 4 , 3 , 7 is ……………

4 If $(3k , 2k)$ satisfies the relation $2x - y + 2 = 12$, then $k = \cdots\cdots$

5 The slope of the straight line which passes through $A(2 , -5)$, $B(3 , -2)$ is ……………

2 **Choose the correct answer :**

1 The multiplicative inverse of $\frac{\sqrt{2}}{4}$ is ……………

(a) $\sqrt{2}$ (b) $2\sqrt{2}$ (c) $4\sqrt{2}$ (d) 2

2 $[2 , 5] -]2 , 5[= \cdots\cdots$

(a) $\{2 , 5\}$ (b) $[2 , 5[$ (c) $]2 , 5]$ (d) $\varnothing$

3 The mean of the values 4 , 7 , 3 , 9 , 2 is ……………

(a) 2 (b) 3 (c) 5 (d) 7

4 The S.S. of the equation $x^2 + 36 = 0$ in $\mathbb{R}$ is ……………

(a) $\{6\}$ (b) $\{-6\}$ (c) $\{6 , -6\}$ (d) $\varnothing$

5 If $5x = 35$, then $2x + 1 =$ ....

(a) 9 (b) 15 (c) 8 (d) 7

6 The order of the median of 5 , 2 , 3 , 9 , 7 , 1 , 6 is ....

(a) 9 (b) 5 (c) 4 (d) 2

3 **[a]** If $X = [-2, 4]$, $Y =]1, 6]$

, find by using the number line : 1 $\acute{X}$ 2 $X \cap Y$ 3 $X - Y$

[b] Find in $\mathbb{R}$ the S.S. of the inequality : $2x + 1 < 7$

4 **[a] Find in the simplest form :** $2\sqrt{18} + \sqrt{50} - \sqrt{162}$

[b] If $x = 3 + \sqrt{5}$, $y = \frac{4}{3 + \sqrt{5}}$

, prove that : x , y are conjugate numbers **and find the value of :** $x^2 - 2xy + y^2$

5 **[a]** A lead cuboid in which its dimensions are 77 cm. , 24 cm. and 21 cm. It was melted to form a sphere. Find the radius length of that sphere $\left(\pi = \frac{22}{7}\right)$

[b] Find the median by using the ascending cumulative frequency curve :

Sets	5 –	15 –	25 –	35 –	45 –	Total
Frequency	4	5	6	3	2	20

5 Giza Governorate

Abo El-Nomros Educational Zone
Royal House Language Schools

Answer the following questions :

1 **Choose the correct answer :**

1 $\left(\sqrt{8} + \sqrt{2}\right)^2 =$ ....

(a) $\sqrt{10}$ (b) 10 (c) 18 (d) $\sqrt{18}$

2 The slope of any line // x-axis is ....

(a) 1 (b) undefined (c) – 1 (d) zero

3 The multiplicative inverse of $\left(-2\frac{1}{3}\right)$ is ....

(a) $\frac{1}{3}$ (b) $-\frac{7}{3}$ (c) $\frac{3}{7}$ (d) $-\frac{3}{7}$

4 The median of the values 34 , 23 , 25 , 40 , 22 is ....

(a) 22 (b) 23 (c) 24 (d) 25

5 $2a^2b \times$ .... $= 12a^3b$

(a) 6 a b (b) 6 a (c) 6 b (d) $6ab^2$

6 The mode of the values 8 , 5 , $x+3$, 5 , 8 is 8 , then $x =$ ....

(a) 5 (b) 8 (c) 3 (d) – 5

2 Complete :

1 The point (3 , ...) satisfies $2x + y = 10$

2 The mean of x , $2x$, $3x$ is ....

3 If $2x = y$, then $x : y =$ .... : ....

4 If the centre of a set is 4 and the upper limit of this set is 8 , then the lower limit of this set is ....

5 $[2 , 3] - \{2 , 3\} =$ ....

3

[a] If $x = \sqrt{7} - \sqrt{6}$, $y = \frac{1}{x}$, **find the value of :** $(x + y)^2$ (Show the steps).

[b] **Find in $\mathbb{R}$ the S.S. of :** $-15 \le 2x - 3 \le 5$

[c] **Simplify :** $\sqrt[3]{54} + 8\sqrt[3]{\frac{1}{4}} + 5\sqrt[3]{16}$

4

[a] If $X =]-\infty , 5]$ and $Y =]1 , 9[$, **find by using the number line :**

1 $X \cap Y$ 2 $X \cup Y$ 3 $X - Y$

[b] Find the slope of the straight line passing through the two points (2 , 4) , (4 , 5)

5

[a] **Find the S.S. in $\mathbb{R}$:** $125x^3 - 7 = 20$

[b] **Find the mode of the following distribution :**

The Set	2 –	6 –	10 –	14 –	18 –	22 –	26 –	Total
Frequency	3	5	8	10	7	5	2	40

6 Alexandria Governorate

East Educational Zone
Maths Supervision

Answer the following questions :

1 Choose the correct answer from the given ones :

1 The arithmetic mean for the values : 9 , 6 , 5 , 14 , 1 is ....

(a) 7 (b) 3 (c) 5 (d) 9

2 The additive inverse of the number $-\sqrt{5}$ is ....

(a) $\sqrt{5}$ (b) 5 (c) $\sqrt{2}$ (d) – 5

3 If the lower limit of a set is 4 and the upper limit is 8 , then its centre is ....

(a) 2 (b) 4 (c) 6 (d) 8

4 The simplest form of the expression : $(\sqrt{3}-\sqrt{2})(\sqrt{3}+\sqrt{2})$ is ....

(a) $\sqrt{3}$ (b) 1 (c) $\sqrt{2}$ (d) $2\sqrt{3}$

5 If the radius length of a sphere is 6 cm. , then its volume is .... π cm^3.

(a) 6 (b) 36 (c) 72 (d) 288

6 $(2\sqrt[3]{2})^3$ = ...

(a) 4 (b) 8 (c) 16 (d) 40

2 Complete the following :

1 If $3^x = 1$, then x = ...

2 The median of the values 2 , 9 , 3 , 7 , 5 is ....

3 $]-2 , 2] \cup \{-2 , 0\}$ = ...

4 The mode for the numbers : 3 , 5 , 3 , 4 , 3 is ....

5 A cube whose volume is 8 cm^3, then the sum of lengths of all its edges is ....

3 [a] Find the value of : $\sqrt{18}+\sqrt[3]{54}-3\sqrt{2}-\frac{1}{2}\sqrt[3]{16}$ (with steps).

[b] Represent graphically the relation : $y = 2 - x$

4 [a] Find the S.S. of the inequality : $-2 < 3x + 7 \le 10$ in $\mathbb{R}$, then represent the interval of solution on the number line.

[b] Reduce to the simplest form : $\frac{\sqrt{3}}{\sqrt{5}-\sqrt{3}}+\frac{\sqrt{5}}{\sqrt{5}+\sqrt{3}}$ (with steps).

5 [a] If $(\sqrt{3})^x = (2\sqrt{2}-\sqrt{5})(2\sqrt{2}+\sqrt{5})$, then what is the value of x ?

[b] Find the arithmetic mean of the following frequency distribution :

The Sets	5 –	15 –	25 –	35 –	45 –	Total
Frequency	7	10	12	13	8	50

7 Alexandria Governorate

El-Montazah Educational Zone
Math's Supervision

Answer the following questions :

1 Choose the correct answer :

1 $\frac{3}{4}$ = ... %

(a) 70 (b) 50 (c) 75 (d) 25

2 $[2, 7] -]2, 7[=$ ……………

(a) $]2, 7]$ (b) $[2, 7[$ (c) $\{2, 7\}$ (d) $[2, \infty[$

3 The median of the values 3 , 7 , 2 , 9 , 5 , 11 is ……………

(a) 9 (b) 6 (c) 8 (d) 11

4 The remainder of subtracting $-5x$ from $3x$ equals ……………

(a) $-2x$ (b) $8x$ (c) $2x$ (d) $8x^2$

5 If (a , 4) satisfies the relation $x - y = -1$, then the value of a is ……………

(a) $\sqrt{3}$ (b) 5 (c) 27 (d) 3

6 If the lower limit of a set is 4 and its centre is 9 , then its upper limit is ……………

(a) 36 (b) 5 (c) 13 (d) 14

2 Complete :

1 $\sqrt[3]{5} +$ …………… = zero

2 $\mathbb{R}^+ \cup \mathbb{R}^- =$ ……………

3 $\sqrt{a} + \sqrt{b}$ its conjugate is …………… and their sum is ……………

4 The mode of the set of values 4 , 5 , k + 1 , 3 is 3 , then k = ……………

5 The slope of the straight line parallel to x-axis equals ……………

3 [a] Simplify :

1 $\sqrt{32} - \sqrt{50} + 4\sqrt{\frac{1}{2}}$ 2 $\sqrt[3]{16} - \frac{1}{3}\sqrt[3]{54}$

[b] If $x = \sqrt{7} + \sqrt{5}$, $y = \frac{2}{x}$, find the value of $\frac{x + y}{x y}$ in the simplest form.

4 [a] Find in $\mathbb{R}$ the S.S. of the following inequality : $-1 \le 3 - 2x < 5$,

then represent the interval of solution on the number line.

[b] Find the height of a right circular cylinder whose height is equal to its base radius length and its volume is 72π cm^3.

[c] Graph the relation : $x + 2y = 3$

5 [a] Find the slope of $\overleftrightarrow{AB}$, where A (– 1 , 3) and B (2 , 5). Is the point C (8 , 1) $\in \overleftrightarrow{AB}$?

[b] Find the mean of the following frequency data :

Sets	8 –	12 –	16 –	20 –	24 –	Total
Frequency	4	10	16	12	8	50

8 El-Kalyoubia Governorate

Directorate of Education
Inspection of Mathematics

Answer the following questions :

1 Choose the correct answer :

1 Let A (3 , 5) and B (5 , – 1) , then the slope of $\overleftrightarrow{AB}$ = ...

(a) $-\frac{1}{3}$ (b) – 3 (c) 3 (d) $\frac{1}{3}$

2 If the point (a , 1) satisfies the relation $x + y = 5$, then a = ...

(a) 1 (b) – 4 (c) 4 (d) 5

3 The median of the values 34 , 23 , 25 , 40 , 22 , 4 is ....

(a) 22 (b) 23 (c) 24 (d) 25

4 If the mode of the set of values 4 , 11 , 8 , 2 x is 4 , then x = ...

(a) 2 (b) 4 (c) 6 (d) 8

5 The arithmetic mean for the values 9 , 6 , 5 , 14 , 1 is ....

(a) 7 (b) 3 (c) 5 (d) 9

6 The mode for the values 3 , 5 , 3 , 4 , 3 is ....

(a) 3 (b) 4 (c) 5 (d) 12

2 Complete :

1 25% = ... $\left(\text{in the form of } \frac{a}{b} \text{ in the simplest form}\right)$

2 The sum of the two square roots of the number $2\frac{1}{4}$ is ....

3 $|-0.75|$ = ...

4 $\sqrt[3]{-125}$ = ...

5 The multiplicative inverse for $\left(\sqrt{3}+\sqrt{2}\right)$ in its simplest form is ....

3 [a] Find the value of x if : $x^3 - 1000 = 0$

[b] Find the circumference of the circle whose area is 3π cm^2.

4 [a] Find : [2 , ∞[∩]– 2 , 3[(by using the number line)

[b] Simplify the following to the simplest form : $\left(\sqrt{2}+5\right)\left(3+\sqrt{2}\right)$

5 [a] Graph the straight line that represents the relation : $x + 2y = 3$

[b] Find the arithmetic mean of the following frequency distribution :

The Set	5 –	15 –	25 –	35 –	45 –	Total
Frequency	4	5	6	3	2	20

9 El-Gharbia Governorate

Central Mathematics Supervision
Official Languages Schools

Answer the following questions :

1 Choose the correct answer :

1. If the radius length of a sphere is 6 cm. , then its volume is ....
   (a) 6π cm³. (b) 36π cm³. (c) 72π cm³. (d) 288π cm³.
2. If the point (a , 1) satisfies the relation $x + y = 5$, then a = ...
   (a) 1 (b) -4 (c) 4 (d) 5
3. The median of the values 34 , 23 , 25 , 40 , 22 , 4 is ....
   (a) 22 (b) 23 (c) 24 (d) 25
4. The solution set of the equation $x(x^2 - 1) = 0$, $x \in \mathbb{R}$ is ....
   (a) $\{1\}$ (b) $\{0\}$ (c) $\{-1\}$ (d) $\{0 , 1 , -1\}$
5. If the arithmetic mean of the values 18 , 21 , 29 , 2 k + 1 , k is 18 , then k = ...
   (a) 1 (b) 7 (c) 29 (d) 90
6. $\sqrt{3\frac{3}{8}} = \frac{3}{2}\sqrt{\frac{\cdots\cdots}{\cdots\cdots}}$
   (a) $\frac{3}{8}$ (b) $\frac{3}{2}$ (c) $\frac{27}{8}$ (d) $\frac{729}{64}$

2 Complete the following :

1. If the lower boundary of a set is 10 and the upper boundary is x and its centre is 15 , then x = ...
2. The multiplicative inverse of the number $\left(\sqrt{3}+\sqrt{2}\right)$ is .... (in the simplest form).
3. $[3 , 4] - \{3 , 5\}$ = ...
4. $\sqrt{64} - \sqrt[3]{64}$ = ...
5. The slope of the straight line passing through (2 , 3) and (5 , – 1) is ....

3 **[a]** If $x = \sqrt{7} + \sqrt{5}$, $y = \frac{2}{\sqrt{7}+\sqrt{5}}$

1. **Prove that :** x and y are two conjugate numbers.
2. **Find :** xy , $(x + y)^2$

[b] Find in the simplest form : $\sqrt{12} + \sqrt[3]{54} - \sqrt{3} - \sqrt[3]{16}$

4 **[a]** Graph the relation : $2x + 3y = 6$, if the straight line representing this relation intersects the x-axis at A and the y-axis at B , find the area of the triangle OAB where O is the origin point.

[b] Find the solution set in $\mathbb{R}$: $8x^3 + 7 = 8$

5 **[a] Find the solution set for the inequality :** $2x - 1 \geq 5$ in $\mathbb{R}$

[b] Find the arithmetic mean of the following frequency distribution :

The Set	5 –	15 –	25 –	35 –	45 –	Total
Frequency	4	5	6	3	2	20

10 El-Dakahlia Governorate

Talkha Educational Directorate
A.M.D.L School

Answer the following questions :

1 **Choose the correct answer from the given ones :**

1. If $x = 3 + \sqrt{3}$ and $y = 3 - \sqrt{3}$, then $x - y = \cdots\cdots$
   (a) $6\sqrt{3}$ (b) -6 (c) $\sqrt{6}$ (d) $2\sqrt{3}$
2. If the order of the median of a set of values is the fifth , then the number of these values is $\cdots\cdots$
   (a) 6 (b) 10 (c) 11 (d) 9
3. The result of $\left(1 + \sqrt{5}\right)\left(1 - \sqrt{5}\right) = \cdots\cdots$
   (a) 2 (b) -4 (c) $-2\sqrt{5}$ (d) $2\sqrt{5}$
4. If A $(3 , -2)$, B $(0 , 4)$, then the slope of $\overleftrightarrow{AB} = \cdots\cdots$
   (a) -2 (b) 2 (c) $\frac{1}{2}$ (d) $-\frac{1}{2}$
5. The mean of the values 2 , 8 , 6 , 4 is $\cdots\cdots$
   (a) 3 (b) 4 (c) 5 (d) 6
6. The multiplicative inverse of $\frac{\sqrt{3}}{6}$ is $\cdots\cdots$
   (a) $-\frac{\sqrt{3}}{6}$ (b) $6\sqrt{3}$ (c) $2\sqrt{3}$ (d) $-2\sqrt{3}$

2 **Complete the following :**

1. $[-3 , 7] - \{-3 , 7\} = \cdots\cdots$
2. The S.S. of the equation $x^2 + 9 = 0$ in $\mathbb{R}$ is $\cdots\cdots$
3. If the mode of 14 , 8 , $x + 5$, 8 and 14 is 8 , then $x = \cdots\cdots$
4. The slope of the straight line perpendicular to y-axis is $\cdots\cdots$
5. If the volume of a sphere is $\frac{9}{2}\pi$ cm^3 , then its radius length is $\cdots\cdots$

3 **[a] Find in the simplest form :** $\sqrt{18} + \sqrt[3]{54} - 3\sqrt{2} - \frac{1}{2}\sqrt[3]{16}$

[b] If $X = [-3 , 4]$, $Y =]1 , \infty[$ **, find each of the following using the number line :**

1. $X \cap Y$
2. $X - Y$

4 **[a] Find in $\mathbb{R}$ the S.S. of the inequality :** $-7 \le -3x + 1 < 13$ and represent it on the number line.

[b] If $x = \sqrt{6} + \sqrt{5}$, $y = \frac{1}{\sqrt{6} + \sqrt{5}}$:

1 **Prove that :** x , y are two conjugate numbers.

2 **Find :** the numerical value of $(x - y)^2$

5 **[a]** Graph the relation $y + 3x = 6$ and find the slope of the straight line.

[b] Find the arithmetic mean of the following frequency distribution :

Sets	10 –	20 –	30 –	40 –	50 –	Total
Frequency	5	15	20	25	10	75

11 Ismailia Governorate

Directorate of Education
Math's Supervision

Answer the following questions :

1 **Choose the correct answer :**

1 A (2 , 5) , B (3 , 7) , then the slope of $\overleftrightarrow{AB}$ = ...

(a) $\frac{1}{2}$ (b) 2 (c) -2 (d) 5

2 $]3 , 5[\cup \{3 , 5\}$ = ...

(a) $]3 , 5[$ (b) $\{3 , 5\}$ (c) $[3 , 5]$ (d) $[3 , 5[$

3 The median of 4 , 11 , 8 , 16 , 9 , 14 is ....

(a) 10 (b) 8 (c) 16 (d) 9

4 $\mathbb{Q} \cup \mathbb{Q}'$ = ...

(a) $\varnothing$ (b) $\mathbb{R}$ (c) $\mathbb{Z}$ (d) $\mathbb{N}$

5 The slope of x-axis is ....

(a) negative. (b) positive. (c) undefined. (d) zero.

6 $\mathbb{Z}^+ \cap \mathbb{Z}^-$ = ...

(a) zero (b) $\varnothing$ (c) $\mathbb{Z}$ (d) $\mathbb{N}$

2 **Complete :**

1 The mean of 12 , 13 , 10 , 11 , 14 is ....

2 The multiplicative inverse of $\sqrt{3} - \sqrt{2}$ is ....

3 The mode of 5 , 11 , 6 , 2 , 11 , 7 is ....

4 If $\frac{x}{y} = 1$, then $x - y$ = ...

5 $\sqrt{5^2 - 4^2}$ = ...

3 **[a] Find the S.S. in $\mathbb{R}$ of :** $8 \leq 3x + 2 \leq 17$ and represent it on the number line.

[b] Simplify : $\sqrt{72} + 3\sqrt{18} - 2\sqrt{\frac{1}{2}}$

4 **[a]** The volume of a cylinder is 1540 cm^3. , if its height is 10 cm. , find its diameter length. $\left(\pi = \frac{22}{7}\right)$

[b] Graph the relation : $y = -3$

5 **[a]** If $X = [-1, \infty[$, $Y =]-4, 3]$ **, using the number line find :**

1. $X \cap Y$
2. $X \cup Y$
3. $\grave{X}$

[b] Find the mean of the following frequency distribution :

Sets	10 –	20 –	30 –	40 –	50 –	Total
Frequency	8	12	14	9	7	50

12 Damietta Governorate

Damietta Inspection of mathematics
Official Language Schools

Answer the following questions :

1 **Choose the correct answer from those given :**

1. $\sqrt{25} - \sqrt[3]{-125} = \cdots\cdots$
   (a) zero (b) 10 (c) 5 (d) ± 5
2. The multiplicative inverse of $\frac{\sqrt{2}}{6}$ is $\cdots\cdots$
   (a) $\sqrt{2}$ (b) $2\sqrt{2}$ (c) $3\sqrt{6}$ (d) $3\sqrt{2}$
3. If the lower limit of a set is 4 and the upper limit is 8 , then its centre is $\cdots\cdots$
   (a) 8 (b) 6 (c) 4 (d) 2
4. The solution set of the equation $x^2 + 9 = 0$ in $\mathbb{R}$ is $\cdots\cdots$
   (a) $\{3\}$ (b) $\{-3\}$ (c) $\varnothing$ (d) $\{-3, 3\}$
5. The arithmetic mean of the values $6 - k$, 12 , 18 and $k + 4$ is $\cdots\cdots$
   (a) 9 (b) 10 (c) 15 (d) 40
6. If the volume of a cube is 27 cm^3 , then the perimeter of one of its faces is $\cdots\cdots$ cm.
   (a) 12 (b) 9 (c) 36 (d) 3

2 **Complete each of the following :**

1. The slope of the straight line passing through the points $(1, -1)$ and $(-3, 7)$ is $\cdots\cdots$
2. If the ordered pair $(k, 2k)$ satisfies the relation $x + y = 15$, then $k = \cdots\cdots$
3. The point of intersection of the ascending and descending cumulative frequency curves determines $\cdots\cdots$ on the set-axis.

4 If three times of a number is 60 , then $\frac{1}{5}$ of this number equals ....

5 If the mode of the values 5 , 9 , 5 , $x+3$, 9 is 9 , then $x=$ ....

3 [a] If $x=\sqrt{5}+\sqrt{2}$, $y=\frac{3}{x}$, **then find the value of :** $\frac{x+y}{xy}$ in its simplest form.

[b] **Find in $\mathbb{R}$ the solution set of the inequality :** $-3 \leq 4x-7 \leq 5$

[c] A right circular cylinder whose height is 8 cm. and its volume is 72π cm^3.
Find the length of the radius of its base.

4 [a] **Find in its simplest form :** $\sqrt{50}+\sqrt[3]{54}-10\sqrt{\frac{1}{2}}-\sqrt[3]{16}$

[b] If X = [– 1 , 5[and Y = [2 , ∞[, **find using the number line :**

1 $X \cup Y$ 2 $X \cap Y$ 3 $X - Y$

5 [a] Find three ordered pairs satisfying the relation $2x+y=7$, then represent it graphically.

[b] **Find the arithmetic mean of the following frequency distribution :**

Sets	5 –	15 –	25 –	35 –	45 –	Total
Frequency	4	5	6	3	2	20

13 Kafr El-Sheikh Governorate

Directorate of Education
Math's Supervision

Answer the following questions :

1 **Choose the correct answer :**

1 $\left(\sqrt{5}+\sqrt{3}\right)^2\left(\sqrt{5}-\sqrt{3}\right)^2=$ ....

(a) 2 (b) 3 (c) 4 (d) 8

2 If the lower limit of a set is 4 and the upper limit is 8 , then its centre is ....

(a) 8 (b) 6 (c) 4 (d) 2

3 $2 \in$ ....

(a)]– 1 , ∞[(b)]2 , 5[(c)]– ∞ , 1[(d) {22}

4 If (– 1 , 5) satisfies the relation $3x+ky=7$, then k = ...

(a) 7 (b) 4 (c) 3 (d) 2

5 If the slope of the straight line $ax+by+1=$ zero is undefined , then = ...

(a) a = b (b) a = zero (c) b = zero (d) a = – b

6 The intersection point of the ascending and descending cumulative frequency curves determines the .... on the sets axis.

(a) mode (b) median (c) mean (d) centre

2 Complete :

1 The slope of the straight line passing through the two points (2 , 6) and (– 1 , 3) equals ....

2 If the mode of the values 4 , 11 , 8 , 2 x is 4 , then x = ...

3 If the mean of the values 9 , 6 , 5 , 14 is k , then k = ...

4 If the volume of a sphere = 36 π cm^3 , then its diameter length = ... cm.

5 The degree of the algebraic term 3 $x^2 y^2$ is ....

3 [a] Find the volume of the right circular cylinder whose diameter length of its base is 10 cm. and its height is 7 cm. $\left(\pi = \frac{22}{7}\right)$

[b] If X =]– ∞ , 5] , Y =]1 , 7]

, find by using the number line : 1 X ∩ Y 2 X ∪ Y 3 Y – X

[c] **Find the S.S. of the equation :** $8 x^3 + 7 = 8$ in $\mathbb{R}$

4 [a] Represent graphically the relation y = x + 2 and if (– 4 , a) satisfies the relation , find the value of a

[b] **Simplify :** $\sqrt{18} + \sqrt{50} - 2\sqrt{8}$

[c] **Find in $\mathbb{R}$ the S.S. of the inequality :** $-8 < 3x + 1 \le 4$

5 [a] If $x = \sqrt{3} + \sqrt{2}$, $y = \frac{1}{\sqrt{3} + \sqrt{2}}$ **, then find the value of :** $\frac{x + y}{x y}$

[b] **From the following frequency table with equal sets :**

The Set	10 –	20 –	30 –	40 –	50 –	60 – 70	Total
Frequency	12	15	25	27	k + 4	4	100

1 Find the value of k

2 Calculate the median.

14 Souhag Governorate

Maths Supervision

Answer the following questions :

1 Choose the correct answer from those given :

1 If the mode of the values 5 , 8 , 6 + x , 9 is 9 , then x = ...

(a) 5 (b) 6 (c) 3 (d) 8

2 The volume of a cube is 27 cm^3 , then the area of one of its faces is ....

(a) 3 cm^2 (b) 9 cm^2 (c) 36 cm^2 (d) 54 cm^2

3 The slope of any line parallel to x-axis equals ....

(a) 1 (b) undefined (c) -1 (d) zero

4 The multiplicative inverse of $\frac{2\sqrt{3}}{6}$ is ....

(a) $\sqrt{2}$ (b) 6 (c) $\sqrt{3}$ (d) zero

5 $\mathbb{Q} \cup \mathbb{Q}' =$ ....

(a) $\varnothing$ (b) 0 (c) $\mathbb{R}$ (d) $\mathbb{Z}$

6 If $(-1, 5)$ satisfies the relation $3x + ky = 7$, then $k =$ ....

(a) 5 (b) 6 (c) 2 (d) 7

2 Complete the following :

1 $[1, 5] - \{1, 5\} =$ ....

2 The S.S. of the equation : $x(x^2 - 1) = 0$ in $\mathbb{R}$ is ....

3 $(2x^2 y) \times (\text{...............}) = 12x^3 y$

4 The arithmetic mean of the values 8, 6, 3, 7, 1 is ....

5 $\sqrt[3]{64} + \sqrt{16} =$ ....

3

[a] Use the following table to find the relation between x, y :

x	-1	0	1	2
y	-1	1	3	5

[b] Find the S.S. of the inequality : $-2 < 3x + 7 \le 10$ in $\mathbb{R}$, then represent the interval of the S.S. on the number line.

4

[a] If $x = \sqrt{3} + \sqrt{2}$, $y = \frac{1}{\sqrt{3} + \sqrt{2}}$, then find the value of : $\frac{x + y}{xy}$

[b] If $X =]-2, 1]$, $Y = [0, 3[$, use the number line to find :

1 $X \cap Y$ 2 $X \cup Y$ 3 $X - Y$

5

[a] Simplify : 1 $\sqrt{50} + \sqrt{18} - \sqrt{32}$ 2 $\sqrt[3]{54} + 8\sqrt[3]{\frac{1}{4}} + 5\sqrt[3]{16}$

[b] Find the arithmetic mean of the following frequency distribution :

Sets	5 –	15 –	25 –	35 –	45 –	Total
Frequency	4	5	6	3	2	20

15 Luxor Governorate

Luxor Directorate
El-Salam Private Language School

Answer the following questions :

1 Choose the correct answer :

1. The smallest prime number is ……………
   (a) 0 (b) 1 (c) 2 (d) 3
2. If the mode of the set of values 4 , 11 , 8 , 2 x is 4 , then x = ……………
   (a) 2 (b) 4 (c) 6 (d) 8
3. If (2 , 5) satisfies the relation $3x + y = c$, then c = ……………
   (a) 1 (b) − 1 (c) 11 (d) − 11
4. The solution set of the equation $x^2 + 9 = 0$ in $\mathbb{R}$ is ……………
   (a) $\varnothing$ (b) $\{-3\}$ (c) $\{3\}$ (d) $\{3, -3\}$
5. The lower limit of a set is 4 and the upper limit is 8 , then its centre is ……………
   (a) 2 (b) 4 (c) 6 (d) 8
6. $4.274 \simeq$ …………… $\left(\text{to the nearest } \frac{1}{10}\right)$
   (a) 4 (b) 4.2 (c) 4.3 (d) 4.27

2 Complete :

1. $[2, 7] - \{2, 7\} =$ ……………
2. The coefficient of the algebraic term $5\,a^3\,b^2$ is ……………
3. The mean of 3 , 5 , 7 , 4 , 1 is ……………
4. The slope of any line parallel to y-axis is ……………
5. The median of the values 3 , 7 , 6 , 9 , 2 is ……………

3 [a] **Simplify to the simplest form :** $\sqrt{27} - \sqrt{12} + \sqrt{300}$

[b] If $a = \sqrt{5} + \sqrt{3}$, $b = \sqrt{5} - \sqrt{3}$, **find :** $a^2 + 2\,ab + b^2$

4 [a] **Find the S.S. in $\mathbb{R}$ of the inequality :** $2x + 1 \le 7$, then represent it on the number line.

[b] Find the volume of the sphere whose diameter length is 4.2 cm. $\left(\pi = \frac{22}{7}\right)$

5 [a] Let A (2 , − 1) , B (10 , 3) and C (2 , 3). Find the slope of each of $\overleftrightarrow{AB}$ and $\overleftrightarrow{BC}$

[b] **Find the arithmetic mean of the following distribution :**

Sets	5 –	15 –	25 –	35 –	45 –	Total
Frequency	4	5	6	3	2	20

Final Examinations of Algebra and Statistics

2019

Some Schools Examinations on Algebra and Statistics

1 Cairo Governorate

Al-Nozha Administration
Al Farouk Islamic Language School

Answer the following questions :

1 Choose the correct answer from the given ones :

(1) The irrational number lies between 3 and 4 is ……………

(a) 3.5 (b) $3\frac{1}{8}$ (c) $\sqrt{13}$ (d) $\sqrt{20}$

(2) $]-2, 1] \cap \{-2, 0, 1\} =$ ……………

(a) $\{-2, 0, 1\}$ (b) $\{1\}$ (c) $\{0, 1\}$ (d) $[-2, 1]$

(3) If $x = \sqrt{3} + 2$ and $y = \sqrt{3} - 2$, then $(x y, x + y) =$ ……………

(a) $(5, 2\sqrt{3})$ (b) $(5, 9)$ (c) $(1, 2\sqrt{3})$ (d) $(-1, 2\sqrt{3})$

(4) The line represented the relation : $3x + 8y = 24$ intersects the y-axis at the point ……………

(a) $(0, 8)$ (b) $(8, 0)$ (c) $(0, 3)$ (d) $(3, 0)$

(5) If the arithmetic mean of the set of the values $m, m+5, m+4, m+3$ is 9, then $m =$ ……………

(a) 2 (b) 6 (c) 9 (d) 10

2 Complete each of the following :

(1) The slope of a straight line which passes through $(-3, 1)$ and $(-2, 5)$ is ……………

(2) If the mode of the set of the values 17, 8, k + 5, 8, 17 is 8, then k = ……………

(3) The multiplicative inverse of $\frac{\sqrt{13}-\sqrt{10}}{3}$ is …………… (In the simplest form)

(4) The radius length of a sphere whose volume is $\frac{9}{2}\pi$ cm^3 is …………… cm.

(5) If the order of the median of the set of values is fifth, then the number of these values equals ……………

3 [a] If $A =]-1, 3]$ and $B = [0, 5[$, then find :

(1) $A \cap B$ (2) $B - A$ (3) $\mathbb{R}_+ \cap B$

[b] Simplify : $2\sqrt{27} + \frac{1}{3}\sqrt[3]{54} - \sqrt{75} + \sqrt[3]{16}$

4 [a] Find in $\mathbb{R}$ the S.S. of each of the following :

(1) $\frac{(2x-1)^3}{3} = 9$ (2) $-1 < 3 - 2x \le 5$

[b] If $x = 2\sqrt{3} - \sqrt{2}$ and $y = \sqrt{12} + \sqrt{2}$ **Find the value of :** $\frac{x+y}{xy+2}$

5 [a] If (a , 3) and (3 , b) satisfies the relation $2x - y = 1$

(1) Find the value of a and b

(2) Find the slope of the straight line which represented the relation : $2x - y = 1$

[b] From the following frequency table :

Sets	10 –	20 –	30 –	40 –	50 –	60 –	Total
Frequency	10	17	20	32	k + 2	4	**100**

(1) Find the value of k

(2) Graph the frequency histogram , then find the mode.

2 Cairo Governorate

Western Cairo Educational Zone
Mathematics Inspection

Answer the following questions :

1 Choose the correct answer :

(1) If the volume of a cube is 64 cm^3 , then its edge length is ....

(a) 32 cm. (b) 16 cm. (c) 8 cm. (d) 4 cm.

(2) The figure  represents the solution of the inequality .... in $\mathbb{R}$

(a) $x > -3$ (b) $x \geq -3$ (c) $x < -3$ (d) $x \leq -3$

(3) $\sqrt{3}\,(\sqrt{11} + \sqrt{3}) =$ ....

(a) $3\sqrt{11} + 2$ (b) $\sqrt{33} + 3$ (c) $11\sqrt{3} + 2$ (d) $2\sqrt{11} + 3$

(4) (3 , 2) does not satisfy the relation ....

(a) $y + x = 5$ (b) $3y - x = 3$ (c) $y + x = 7$ (d) $x - y = 1$

(5) The arithmetic mean of the values : 5 , 12 , 17 , 6 is ....

(a) 10 (b) 12 (c) 4 (d) 17

2 Complete each of the following :

(1) $\sqrt[3]{-64} + \sqrt{16} =$ ....

(2) If the mode of the set of the values : 15 , 9 , $x + 1$, 9 and 15 is 9 , then $x =$ ....

(3) The multiplicative inverse of the number $\frac{3}{\sqrt{3}}$ is $\frac{\cdots\cdots}{\sqrt{3}}$

(4) If the volume of a sphere = $\frac{9}{16}\pi$ cm^3 , then its radius length = ... cm.

(5) If the order of the median of the set of values is fourth , then the number of these values is ....

Algebra and Statistics

3 [a] If $x=\sqrt{3}-2$ and $y=\sqrt{3}+2$, **find the value of :** $\left(\frac{x-y}{x+y}\right)^2$

[b] Simplify the following to the simplest form : $\sqrt{98}-\sqrt{128}-\sqrt{18}+4\sqrt{2}$

4 [a] If $X =]-\infty , 2[$ and $Y = [-1 , 5]$, **find using the number line :**

(1) $X \cap Y$ (2) $X - Y$

[b] Find the slope of the straight line passing through the two points : A (1 , 3) and B (2 , 3)

5 **[a] Find the solution set for the following equation in $\mathbb{R}$, then represent the solution on the number line :** $-8 \le 3x + 1 \le 4$

[b] Find the mean of the following frequency distribution :

Sets	5 –	15 –	25 –	35 –	45 –	Total
Frequency	3	10	12	10	5	40

3 Cairo Governorate

New Cairo Educational Zone
Akhnaton Egyptian College

Answer the following questions :

1 **Complete the following :**

(1) The S.S. of the equation : $x^3 - 27 = 0$ in $\mathbb{R}$ is ....

(2) $[1 , 5] - \{1 , 5\} =$ ....

(3) The slope of the straight line which passes through the two points (2 , – 2) and (4 , 2) is ....

(4) A cube whose volume is 8 cm^3 , the length of its edge = ... cm.

(5) The arithmetic mean of 10 , 6 , 5 , 14 , 15 is ....

2 **Choose the correct answer :**

(1) If $x=\sqrt{3}+2$ and $y=\sqrt{3}-2$, then xy = ...

(a) 1 (b) –1 (c) – 4 (d) 3

(2) $]-1 , 3[\cap [-3 , -1]$ = ...

(a) $\varnothing$ (b) $\{-3\}$ (c) $\{-1\}$ (d) $\{3\}$

(3) If the lower limit of a set is 6 and the upper limit is 10 , then its centre is ....

(a) 4 (b) 6 (c) 10 (d) 8

(4) The multiplicative inverse of $\frac{\sqrt{5}}{10}$ is ....

(a) $\sqrt{10}$ (b) $\sqrt{5}$ (c) $2\sqrt{5}$ (d) $-2\sqrt{5}$

(5) The S.S. of $x + 2 \geq 1$ in $\mathbb{R}$ is ....

(a) $[-1, \infty[$ (b) $]-1, \infty[$ (c) $[1, 2]$ (d) $[1, 2[$

---

3 **[a] Simplify :** $\sqrt[3]{16} - \frac{1}{3}\sqrt[3]{54} + \sqrt[3]{-2}$

[b] Find the S.S. of : $-2 < 3x + 7 \leq 10$ in $\mathbb{R}$ **, then represent the interval of the solution set on the number line.**

---

4 **[a]** If $x = \sqrt{5} + \sqrt{2}$ and $y = \sqrt{5} - \sqrt{2}$ **, then find the value of :** $\frac{x + y}{x y - 1}$

[b] If $X = [-2, 1]$ and $Y = [0, \infty[$ **Find :**

(1) $X \cap Y$ (2) $X \cup Y$ (3) $Y - X$

---

5 **[a] Find the arithmetic mean of the following frequency distribution :**

Sets	5 –	15 –	25 –	35 –	45 –	Total
Frequency	4	5	6	3	2	20

[b] Represent graphically the relation : $2y - x = 2$

4 Giza Governorate

Al-Agoza Directorate
Supervision of math.

Answer the following questions :

1 **Complete :**

(1) The S.S. of the equation $x^2 + 9 = 0$ in $\mathbb{R}$ is ....

(2) $\sqrt{16} = \sqrt[3]{\ldots\ldots\ldots\ldots}$

(3) The multiplicative inverse of the number $2\sqrt{3}$ is ....

(4) $\{8, 9, 10\} \cap]8, 10[=$ ....

(5) The length of the edge of a cube of volume $15\frac{5}{8}$ cm^3 is ....

---

2 **Choose the correct answer :**

(1) The mean of the set of numbers : 5 , 12 , 17 , 6 is ....

(a) 40 (b) 20 (c) 5 (d) 10

(2) The S.S. of the equation : $x^2 - 1 = 8$ in $\mathbb{R}$ is ....

(a) $\varnothing$ (b) $\{3\}$ (c) $\{-3\}$ (d) $\{-3, 3\}$

(3) The conjugate of $\frac{1}{\sqrt{3}-\sqrt{2}}$ is ....

(a) $\sqrt{3}-\sqrt{2}$ (b) $3-\sqrt{2}$ (c) $3+\sqrt{2}$ (d) $\sqrt{3}+\sqrt{2}$

(4) The value of b that makes $(-2, 3)$ satisfies the relation : $3x + by = 3$ is ....

(a) 3 (b) 2 (c) 1 (d) -3

(5) If the mode of the values : 5 , $x + 3$, 9 , 4 is 9 , then $x =$ ....

(a) 5 (b) 4 (c) 6 (d) 3

---

3 **[a] Represent graphically the relation :** $y = 2x - 3$

[b] If $X =]-\infty, 2]$ and $Y = [-1, 8]$ **, using the number line , find :**

(1) $X \cup Y$ (2) $X - Y$ (3) $X \cap Y$

---

4 **[a] Simplify :**

(1) $\sqrt{50} + \sqrt{18} - \sqrt{32}$ (2) $\sqrt[3]{54} + 8\sqrt[3]{\frac{1}{4}} + 5\sqrt[3]{16}$

[b] Find the slope of the straight line passing through the two points : A $(5, -3)$ and B $(6, 2)$

---

5 **[a] Write two ordered pairs satisfying the relation :** $y = x + 1$

[b] Find the arithmetic mean of the following frequency distributive :

Sets	10 –	20 –	30 –	40 –	50 –	Total
Frequency	10	20	25	30	15	100

5 Giza Governorate

El-Haram Educational Zone
Pyramids Language School

Answer the following questions :

1 **Complete the following :**

(1) $\sqrt[3]{64} = \sqrt{\ldots\ldots\ldots\ldots}$

(2) If $a = \sqrt{5} - 2$, $b = \sqrt{5} + 2$, then $a^2 b^2 =$ ....

(3) The S.S. of the equation $x^2 + 5 = 0$ in $\mathbb{R}$ is ....

(4) $[-1, 5] \cap [3, 7] =$ ....

(5) If $a^2 + b^2 = 25$ and $ab = 5$, then $\frac{a}{b} + \frac{b}{a} =$ ....

2 Choose the correct answer :

(1) $\left(\sqrt{2}+\sqrt{8}\right)^2 =$ ....

(a) 18 (b) $\sqrt{10}$ (c) 4 (d) 10

(2) The sum of the real numbers of the interval $[-150, 150]$ is ....

(a) 300 (b) -300 (c) zero (d) 150

(3) The volume of a cuboid whose dimensions $\sqrt{2}$ cm. , $\sqrt{3}$ cm. , $\sqrt{6}$ cm. is ....

(a) 6 cm^3 (b) 36 cm^3 (c) $6\sqrt{6}$ cm^3 (d) $18\sqrt{2}$ cm^3

(4) $\sqrt{(10)^2-(6)^2} =$ ....

(a) 4 (b) 8 (c) ± 4 (d) ± 8

(5) $\sqrt[3]{3\sqrt{3}} =$ ....

(a) 3 (b) $\frac{1}{2}$ (c) $\sqrt[3]{3}$ (d) $\sqrt{3}$

3 [a] Simplify the following :

(1) $6\sqrt{\frac{5}{2}} + 20\sqrt{\frac{2}{5}}$ (2) $4\sqrt[3]{\frac{1}{2}} + 3\sqrt[3]{32} - \sqrt[3]{4}$

[b] Find the S.S. in $\mathbb{R}$: $(x-1)^2 = 4$

4 [a] If (3 , 2) satisfies the relation $x + 2y = m$, then find the value of m

[b] Find the slope of the straight line passes through the two points (3 , 5) and (4 , 7)

[c] Represent graphically : $y = x + 2$

5 [a] Find the median of : 28 , 25 , 24 , 26 , 27

[b] Find the arithmetic mean of the following frequency distribution :

Sets	10 –	20 –	30 –	40 –	50 –	Sum
Frequency	4	6	8	7	5	30

6 Alexandria Governorate

Middle Educational Zone
Math's Supervision

Answer the following questions :

1 Complete each of the following :

(1) If $3^x = 1$, then $x =$ ....

(2) The S.S. of the equation : $x(x^3 - 1) = 0$ in $\mathbb{R}$ is ....

(3) $]5, 7[\cup \{5, 7\} =$ ....

(4) If the arithmetic mean of the values : 9 , 6 , 5 , 14 , k is 7 , then k = ...

(5) If the slope of the straight line : $k x + 2 y = 5$ is zero , then k = ...

2 Choose the correct answer from the given ones :

(1) $\left(2\sqrt[3]{2}\right)^3 =$ ....

(a) 4 (b) 8 (c) 16 (d) 40

(2) If the volume of a cube is 27 cm^3 , then the area of its face is .... cm^2

(a) 3 (b) 9 (c) 36 (d) 54

(3) If the order of the median of a set of values is the fourth , then the number of values is ....

(a) 3 (b) 5 (c) 7 (d) 9

(4) If the mode of the set of values : 5 , 9 , 5 , $x - 2$, 9 is 9 , then $x =$ ....

(a) 5 (b) 57 (c) 9 (d) 11

(5) If (– 1 , 5) satisfies the relation : $3x + k y = 7$, then k = ...

(a) 2 (b) – 2 (c) 1 (d) 10

3 [a] Find the value of : $\sqrt{18} + \sqrt[3]{54} - 3\sqrt{2} - \frac{1}{2}\sqrt[3]{16}$

[b] If $x = \sqrt{5} + \sqrt{2}$ and $y = \sqrt{5} - \sqrt{2}$ **, find the value of :** $\frac{x + y}{x y - 1}$

4 [a] Write in the form of an interval the S.S. of the inequality : $x + 4 \geq 2x - 3 > x + 1$

[b] Represent graphically the relation : $y = 2 - x$

5 [a] The volume of a sphere is $\frac{99000}{7}$ cm^3 Calculate its radius length. $\left(\pi = \frac{22}{7}\right)$

[b] Find the arithmetic mean of the following frequency distribution :

Sets	5 –	15 –	25 –	35 –	45 –	Total
Frequency	7	10	12	13	8	50

7 Alexandria Governorate

El-Montazah Educational Zone
Math's Supervision

Answer the following questions :

1 Complete each of the following :

(1) The multiplicative inverse for $\frac{-\sqrt{2}}{6}$ is ....

(2) If $5x - 3y = 0$, then $x : y =$ .... : ....

(3) The slope of any line parallel to x-axis = ...

(4) $\sqrt{5}+\sqrt{2}$ its conjugate is .... and their product is ....

(5) If $(-1\ ,5)$ satisfies the relation $3x+ky=7$, then k = ...

2 Choose the correct answer :

(1) If $|a|=5$, then a = ...

(a) 5 (b) -5 (c) ± 5 (d) $\sqrt{5}$

(2) The order of the median of the set of values : 4 , 5 , 6 , 7 , 8 is ....

(a) third. (b) fourth. (c) fifth. (d) sixth.

(3) The S.S. of the inequality $-2x \geq 6$ in $\mathbb{R}$ is ....

(a) $]-\infty\ ,-3[$ (b) $]-\infty\ ,-3]$ (c) $[-3\ ,\infty[$ (d) $]-3\ ,\infty[$

(4) $\{8\ ,9\ ,10\}-]8\ ,10[$ = ...

(a) $\varnothing$ (b) $\{9\}$ (c) $\mathbb{N}$ (d) $\{8\ ,10\}$

(5) The mode of the set of values : 5 , 9 , 5 , $x-2$, 9 is 9 , then x = ...

(a) 5 (b) 57 (c) 9 (d) 11

3 [a] Find in the simplest form : $2\sqrt{18}+\sqrt{50}+\frac{1}{3}\sqrt{162}$

[b] If $a-b=2\sqrt{7}$ **, then find the value of :** $a(a-b)^2-b(a-b)^2$

[c] Find the slope of line $\overleftrightarrow{AB}$, where A $(-1\ ,3)$ and B $(2\ ,5)$ Is the point C $(8\ ,1)\in\overleftrightarrow{AB}$?

4 [a] Find the S.S. of the inequality : $-1<2x-3\leq 5$ in $\mathbb{R}$ and represent the interval of solution on the number line.

[b] Find the lateral area for right circular cylinder of volume 924 cm^3 , and its height 6 cm. $\left(\pi=\frac{22}{7}\right)$

5 [a] If $\left(\sqrt{3}\right)^{x}=\left(2\sqrt{2}-\sqrt{5}\right)\left(2\sqrt{2}+\sqrt{5}\right)$, then what is the value of x ?

[b] By using the following distribution :

Sets	5 –	15 –	25 –	35 –	45 –	Total
Frequency	3	10	k – 2	10	5	40

(1) Find the value of k

(2) Find the arithmetic mean.

8 El-Kalyoubia Governorate

Mathematics Inspection

Answer the following questions :

1 Choose the correct answer :

(1) $\mathbb{Q} \cap \acute{\mathbb{Q}} =$ ……………

(a) $\mathbb{R}$ (b) $\mathbb{R}_+$ (c) $\mathbb{R}_-$ (d) $\varnothing$

(2) The S.S. of the equation : $x^3 + 27 = 0$ in $\mathbb{R}$ is ……………

(a) $\{3\}$ (b) $\{-3\}$ (c) $\varnothing$ (d) $\{3\sqrt{3} , -3\sqrt{3}\}$

(3) $\{x : x \in \mathbb{R} , x < 1\} =$ ……………

(a) $\{0 , -1 , -2\}$ (b) $]-\infty , 1]$ (c) $]-\infty , 1[$ (d) $]1 , \infty[$

(4) The mode of values : 3 , 5 , 3 , 6 , 5 , 3 , 7 is ……………

(a) 3 (b) 5 (c) 7 (d) 6

(5) The arithmetic mean of the values : 6 , 19 , 32 , 25 , 8 is ……………

(a) 90 (b) 32 (c) 18 (d) 6

2 Complete the following :

(1) If $3^x = 1$, then $x =$ ……………

(2) The conjugate of the number $\frac{4}{\sqrt{7}-\sqrt{3}}$ is ……………

(3) The total area of a cube of edge length 4 cm. is …………… cm^2.

(4) If the point (6 , a) lies on the straight line whose equation is $x + y = 3$, then a = ……………

(5) The median of the set of the values : 2 , 9 , 3 , 7 , 5 is ……………

3 **[a]** If $x = \sqrt{5} + \sqrt{2}$ and $y = \sqrt{5} - \sqrt{2}$ **Find the value of :** $\frac{x + y}{x y + 1}$

[b] If X = [− 1 , 2] and Y = [1 , ∞[**Find :**

(1) $X \cap Y$ (2) $X \cup Y$

4 **[a] Find the S.S. of the inequality :** $7 \geq 2x + 1 > 3$

[b] The radius length of the base of a right cylinder is $4\sqrt{2}$ cm. and its height is 9 cm. Find its volume in terms of π .

5 [a] Find the slope of $\overleftrightarrow{AB}$ where A $(2 , -1)$ and B $(-1 , 3)$, then draw $\overleftrightarrow{AB}$ on 2-dimensions coordinate.

[b] Find the arithmetic mean of the following frequency distribution :

The sets	5 –	15 –	25 –	35 –	45 –	Total
Frequency	3	4	7	4	2	20

9 El-Sharkia Governorate

Directorate of Education
Dept. of Governmental L. Schools

Answer the following questions :

1 Complete each of the following :

(1) $[2 , 7[\cup \{2 , 7\} =$ ....

(2) If the volume of a cube is 64 cm^3 , then its lateral area = ... cm^2

(3) If (k , 4) satisfies the relation $x + 2y = 15$, then k = ...

(4) If $a = \sqrt{5} + 1$ and $b = \sqrt{5} - 1$, then $a - b =$ ....

(5) The mean of the numbers 3 , 4 , 6 , 7 is ....

2 Choose the correct answer :

(1) The additive inverse of $\sqrt{5} - \sqrt{3}$ is ....

(a) $\sqrt{5} - \sqrt{3}$ (b) $\sqrt{3} + \sqrt{5}$ (c) $-\sqrt{5} - \sqrt{3}$ (d) $\sqrt{3} - \sqrt{5}$

(2) The S.S. of the equation $x^2 + 16 = 0$ in $\mathbb{R}$ is ....

(a) $\{4\}$ (b) $\varnothing$ (c) $\{4 , -4\}$ (d) $\{-4\}$

(3) $\left(\sqrt{5} + \sqrt{3}\right)^2 \left(\sqrt{5} - \sqrt{3}\right)^2 =$ ....

(a) 4 (b) 2 (c) 8 (d) 3

(4) The slope of any line parallel to x-axis equals ....

(a) 1 (b) undefined (c) – 1 (d) zero

(5) If $5x = 35$, then $2x + 1 =$ ....

(a) 7 (b) 15 (c) 8 (d) 71

3 **[a] Find the value of :** $\sqrt{50} - \sqrt{8} + 2\sqrt{\frac{1}{2}} - \sqrt{18}$

[b] If $x = \dfrac{4}{3 + \sqrt{5}}$ and $y = 3 + \sqrt{5}$ **Prove that :** x and y are conjugate numbers

, then find the value of : $(x + y)^2$

4 [a] If $A =]-2, 6]$ and $B = [4, \infty[$ **, use the number line to find :**

(1) $A \cup B$ (2) $A \cap B$

[b] If the volume of a sphere is 36π cm³. Find the length of its radius , then calculate its total area ($\pi = 3.14$)

---

5 **[a] Graph the linear relation :** $y = 2x - 1$

[b] Solve in $\mathbb{R}$ the inequality : $x + 2 \leq 3x + 2 < x + 16$

[c] Find the mean of the following data :

Sets	20 –	30 –	40 –	50 –	60 –	70 –	Total
Frequency	10	15	22	25	20	8	100

10 El-Dakahlia Governorate

Math's Supervision (E.L.S)

Answer the following questions :

1 **Complete the following :**

(1) $[-5, 9] - \{-5, 9\} =$ ....

(2) The S.S. of the equation : $x^3 + 8 = 0$ in $\mathbb{R}$ is ....

(3) If the mode of 14 , 9 , $x + 5$, 9 and 14 is 9 , then $x =$ ....

(4) The slope of the straight line parallel to x-axis is ....

(5) If the volume of the sphere is $\frac{1}{6}\pi$ cm³ , then its radius length = ...

---

2 **Choose the correct answer :**

(1) If $x = 5 + \sqrt{3}$ and $y = 5 - \sqrt{3}$, then $x - y =$ ....

(a) 10 (b) – 10 (c) $\sqrt{6}$ (d) $2\sqrt{3}$

(2) If the order of the median of the set of values is the fourth , then the number of values is ....

(a) 8 (b) 10 (c) 7 (d) 9

(3) $(1 + \sqrt{7})(1 - \sqrt{7}) =$ ....

(a) 2 (b) – 4 (c) $-2\sqrt{7}$ (d) – 6

(4) If A (2 , – 2) and B (1 , 4) , then the slope of $\overleftrightarrow{AB} =$ ....

(a) – 2 (b) 2 (c) – 6 (d) $-\frac{1}{2}$

(5) The mean of the values 3 , 7 , 8 , 2 is ....

(a) 2 (b) 4 (c) 5 (d) 6

3 **[a] Simplify to the simplest form :** $2\sqrt{18}+\sqrt[3]{54}-12\sqrt{\frac{1}{2}}-5\sqrt[3]{16}$

[b] If $X=[-2\ ,5]$ and $Y=]2\ ,\infty[$

Find : (1) $X\cap Y$ (2) $Y-X$

---

4 **[a] Find in $\mathbb{R}$ the S.S. of the inequality :** $-9\le -3x+2<17$

[b] If $x=\sqrt{7}+\sqrt{6}$ and $y=\dfrac{1}{\sqrt{7}+\sqrt{6}}$

(1) **Prove that :** x and y are conjugate. (2) **Find :** the numerical value of x^2-y^2

---

5 **[a] Graph :** $y+2x=4$ Does the point $(-1\ ,6)$ belong to the straight line ?

[b] Using the following distribution , find the arithmetic mean :

Sets	10 –	20 –	30 –	40 –	50 –
Frequency	6	14	21	24	10

11 Ismailia Governorate

Directorate of Education
El-Manar Language School

Answer the following questions :

1 **Complete the following :**

(1) $[-1\ ,5]-]-1\ ,5[=$ ....

(2) If (k , 5) satisfies the relation : $2y+2x=8$, then k = ...

(3) The S.S. of the equation $x^3+125=0$ in $\mathbb{R}$ is ....

(4) The additive inverse of $\sqrt{7}+\sqrt{3}$ is ....

(5) If the dimensions of a rectangle is $(\sqrt{11}+2)$ cm. and $(\sqrt{11}-2)$ cm.
, then its area = ... cm^2.

---

2 **Choose the correct answer :**

(1) If the mode of the values 8 , 7 , 8 , 5 , $x-5$, 5 is 8 , then $x=$ ....

(a) 8 (b) 10 (c) 5 (d) 13

(2) The slope of the straight line passing through the two points $(-2\ ,2)$ and $(-8\ ,5)$ is ....

(a) $\frac{-7}{10}$ (b) $\frac{10}{7}$ (c) $\frac{-6}{12}$ (d) -2

(3) If the volume of a cube is 27 cm^3, then the sum of edges of this cube is .... cm.

(a) 36 (b) 3 (c) 12 (d) 27

(4) The median of the values 31 , 13 , 9 , 60 , 1 , 45 , 4 is ....

(a) 60 (b) 13 (c) 31 (d) 163

(5) $]-\infty , 0]$ = ...

(a) $\mathbb{R}_+$ (b) $\mathbb{R}_-$

(c) set of non positive real numbers. (d) set of non negative real numbers.

---

3 **[a] Find the simplest form of :** $\sqrt[3]{54} - \frac{1}{2}\sqrt[3]{16} + \sqrt[3]{-2}$

[b] If $x = \sqrt{5} + \sqrt{3}$ and $y = \frac{2}{\sqrt{5}+\sqrt{3}}$ **, find the value of :** $\frac{x+y}{xy}$

---

4 **[a] Find the S.S. in $\mathbb{R}$ of the inequality :**

$-2 < 3x + 7 \leq 10$ and represent it on the number line.

[b] If $X =]-\infty , 5]$ and $Y =]1 , 9[$ **Find using the number line :**

(1) $X \cap Y$ (2) $X \cup Y$ (3) $X - Y$ (4) $\grave{X}$

---

5 **[a]** If the volume of a sphere is 288π cm^3 find its area.

[b] The following table shows the frequency distribution of marks of 40 students in an algebra exam :

Sets	5 –	15 –	25 –	35 –	45 –	Total
Frequency	7	9	12	x	4	40

(1) Find the value of x (2) Find the arithmetic mean.

12 Port Said Governorate

Educational Directorate
Math inspection

Answer the following questions :

1 **Choose the correct answer :**

(1) The multiplicative inverse to the number $\frac{3}{\sqrt{2}}$ is ....

(a) $\frac{\sqrt{2}}{3}$ (b) $\frac{\sqrt{3}}{2}$ (c) $\frac{\sqrt{2}}{2}$ (d) $2\sqrt{3}$

(2) The solution set of the equation : $x^3 = 8$ in $\mathbb{R}$ is ....

(a) $\varnothing$ (b) $\{2\}$ (c) $\{-2\}$ (d) $\{0\}$

(3) $\mathbb{Q} \cup \acute{\mathbb{Q}} =$ ....

(a) $\varnothing$ (b) 0 (c) $\mathbb{R}$ (d) $\mathbb{Z}$

(4) The conjugate of the number $\sqrt{2}-\sqrt{3}$ is ....

(a) $\sqrt{2}+\sqrt{3}$ (b) $\sqrt{3}-2$ (c) $2-\sqrt{3}$ (d) $-\sqrt{2}+\sqrt{3}$

(5) The arithmetic mean of the values 2 , 5 , 8 is ....

(a) 5 (b) 4 (c) 3 (d) 2

2 Complete each of the following :

(1) The mode of the values 5 , 5 , 6 , 4 , 5 is ....

(2) The slope of the straight line which parallel to the x-axis = ...

(3) $[2 , 8[\cup \{8\} =$ ....

(4) $\sqrt[3]{\text{...............}} = \sqrt{4}$

(5) A cube of side length 3 cm. , then its volume = ... cm^3.

3 [a] Find the solution set in $\mathbb{R}$ to the following inequality in the form of an interval :

$x - 2 > 3$

[b] If $x = \sqrt{3}+\sqrt{2}$ and $y = \sqrt{3}-\sqrt{2}$ **Find the value of :** $x \times y$

4 [a] Without using calculator , simplify : $\sqrt{2}+\sqrt{8}-\sqrt{18}$

[b] Find the slope of the straight line which passes through the two points (2 , 3) and (1 , 2)

5 [a] Write three ordered pairs satisfy the relation : $x + y = 5$

[b] Find the arithmetic mean for the following frequency distribution :

Sets	2 –	4 –	6 –	Total
Frequency	2	4	2	8

13 Kafr El-Sheikh Governorate

General Maths Supervision

Answer the following questions :

1 Choose the correct answer :

(1) The mean of the values : 21 , 19 , 27 , 3 , 5 is ....

(a) 90 (b) 32 (c) 18 (d) 15

(2) If $x = \sqrt{7} - \sqrt{5}$ and $y = \sqrt{7} + \sqrt{5}$, then $(x y)^3 =$ ....

(a) 4 (b) 6 (c) 8 (d) 9

(3) $[1, 3] - \{1, 3\} =$ ....

(a) $]1, 3[$ (b) $]-1, -3[$ (c) $[1, 3[$ (d) $]-1, 3[$

(4) $\mathbb{R} =$ ....

(a) $[0, \infty]$ (b) $]-\infty, \infty[$ (c) $[0, \infty[$ (d) $]-\infty, 0]$

(5) If A $(2, 7)$ and B $(5, -2)$, then the slope of $\overleftrightarrow{AB} =$ ....

(a) -2 (b) 2 (c) -3 (d) 3

2 Complete :

(1) The volume of a sphere whose diameter length is 6 cm. = ... π cm^3.

(2) The S.S. for the equation $x^3 + 8 = 0$ in $\mathbb{R}$ is ....

(3) If $(k, 2k)$ satisfies $x + y = 15$, then $k =$ ....

(4) The slope of any line parallel to the x-axis = ...

(5) If the area of one face of a cube = 9 cm^2, then its volume = ... cm^3.

3 [a] Simplify : $\sqrt{18} + \sqrt[3]{54} - 3\sqrt{2} - \sqrt[3]{16}$

[b] Find in $\mathbb{R}$ the S.S. of the following inequality : $-1 \le 5x + 4 \le 14$

, then represent the S.S. on the number line.

4 [a] If $x = \sqrt{6} + \sqrt{5}$ and $y = \sqrt{6} - \sqrt{5}$ **Find :** $(x + y)^2$

[b] If $X =]-3, 2]$ and $Y =]-1, 5]$, **then find :**

(1) $X \cap Y$ (2) $X \cup Y$

5 [a] Represent the relation $x + y = 3$ on the coordinate plane.

[b] Find the mean for the following frequency distribution :

Sets	5 –	15 –	25 –	35 –	45 –	Total
Frequency	4	5	6	3	2	20

14 Beni Suef Governorate

Directorate Of Official Language School
Education administration

Answer the following questions :

1 Choose the correct answer :

① The irrational number lies between – 2 and – 1 is ....

(a) – 3 (b) $-1\frac{1}{2}$ (c) $-\sqrt{3}$ (d) $\sqrt{2}$

② $\sqrt[3]{x^6} = \sqrt{\dots\dots}$

(a) x^3 (b) x^2 (c) x (d) x^4

③ $|-5|-|5| =$ ....

(a) – 10 (b) – 5 (c) 0 (d) 10

④ (3 , 2) does not satisfy the relation ....

(a) $y + x = 5$ (b) $3y - x = 3$ (c) $y + x = 7$ (d) $x - y = 1$

⑤ If the volume of a right circular cylinder is 90π cm^3 and its height is 10 cm. , then the radius length of its base equals .... cm.

(a) 3 (b) 4.5 (c) 5 (d) 9

2 Complete :

① If (a , 3) satisfies the relation $2x - y = 7$, then a = ...

② $\left(\frac{-5}{7}\right) \times \left(\frac{-7}{5}\right) =$ ....

③ If the arithmetic mean of the values 9 , 6 , 5 , 14 , x is 7 , then $x =$ ....

④ The point of intersection of the ascending and descending cumulative frequency curves determines .... on the set-axis.

⑤ If the sum of five numbers equals 30 , then the arithmetic mean of these numbers is ....

3 [a] Simplify to the simplest form : $\sqrt[3]{-16} + \frac{14}{\sqrt{7}} - \sqrt{28} + \sqrt[3]{54}$

[b] If $x = \frac{4}{3+\sqrt{5}}$ and $y = 3 + \sqrt{5}$, **Find the value of :** $x^2 + y^2$

4 [a] If X = [– 1 , 4] , Y = [3 , ∞[and Z = {3 , 4}

, find each of the following using the number line :

① X – Y ② Y ∩ Z

[b] Find the solution set ot the inequality $3 - 2x \leq 7$ in $\mathbb{R}$ in the form of an interval , then represent the solution on the number line.

5 **[a]** Let A (2 , – 1) , B (10 , 3) and C (2 , 3) , **find the slope of each of :** $\overleftrightarrow{AB}$ and $\overleftrightarrow{AC}$

[b] The following table shows the frequency distribution of the weekly bonus of 100 workers in a factory :

Bonus in L.E.	20 –	30 –	40 –	50 –	m –	70 –
Number of workers	10	k	22	26	20	8

① Find the value of each of k and m

② Graph the frequency histogram , then find the mode value of the weekly bonus.

15 Assiut Governorate

Badr Language School

Answer the following questions :

1 **Choose the correct answer from those given :**

① If the volume of a cube is 27 cm^3 , then the area of one of its faces is ....

(a) 3 cm^2 (b) 9 cm^2 (c) 36 cm^2 (d) 54 cm^2

② The S.S. of the equation : $x^2 + 3 = 0$ in $\mathbb{R}$ is = ...

(a) $\varnothing$ (b) $\{-\sqrt{3}\}$ (c) $\{\sqrt{3}\}$ (d) $\{-\sqrt{3} , \sqrt{3}\}$

③ If $x = \sqrt{3} + 2$ and $y = \sqrt{3} - 2$, then $(x y , x + y)$ = ...

(a) $(1 , 2\sqrt{3})$ (b) $(-1 , 2\sqrt{3})$ (c) $(5 , 2\sqrt{3})$ (d) (5 , 9)

④ If the median of the set of the values : k + 1 , k + 2 , k + 5 , k + 4 , k + 3 where is k is a positive number is 13 , then k = ...

(a) 2 (b) 5 (c) 10 (d) 13

⑤ If the mode of the set of values : 4 , 11 , 8 , $2x$ is 4 , then x = ...

(a) 2 (b) 4 (c) 6 (d) 8

2 **Complete :**

① If (– 1 , 5) satisfies the relation $3x + k y = 7$, then k = ...

② $[2 , 6] - \{2 , 6\}$ = ...

③ If the arithmetic mean of the values 9 , 6 , 5 , 14 , k is 7 , then k = ...

④ The slope of the straight line passing through the two points (2 , 6) and (– 1 , 3) is ....

⑤ The multiplicative inverse of the number $\sqrt{3} - \sqrt{2}$ is .... (in the simplest form)

3 **[a]** If $x=\sqrt{5}+\sqrt{2}$ and $y=\sqrt{5}-\sqrt{2}$ **, find the value of :** $\frac{x+y}{xy-1}$

[b] Find the S.S. of the inequality : $-5 \le 2x-3<5$ in $\mathbb{R}$ **,** then represent it on the number line.

4 **[a] Prove that :** $\sqrt[3]{128}+\sqrt[3]{16}-2\sqrt[3]{54}=0$

[b] Represent graphically the relation : $y=2-x$

5 **[a]** If $X =]-\infty , 2[$ and $Y = [-1 , 5]$ **find as an intervals using the number line :**

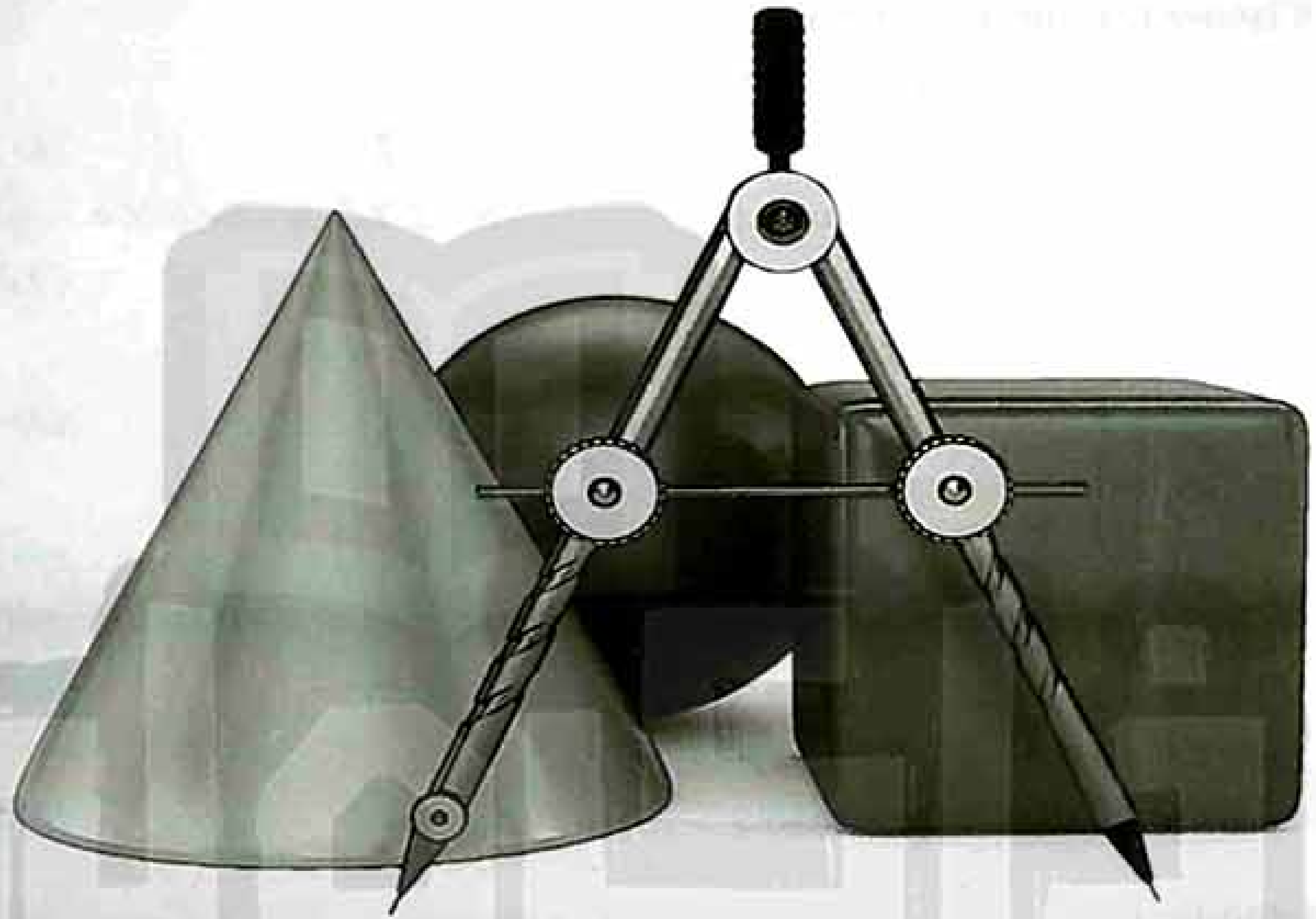
(1) $X \cup Y$ (2) $X \cap Y$ (3) $X - Y$

[b] Find the arithmetic mean of the following frequency distribution :

Sets	5 –	15 –	25 –	35 –	45 –	Total
Frequency	7	10	12	13	8	**50**

Second

Geometry



Quizzes

on Geometry

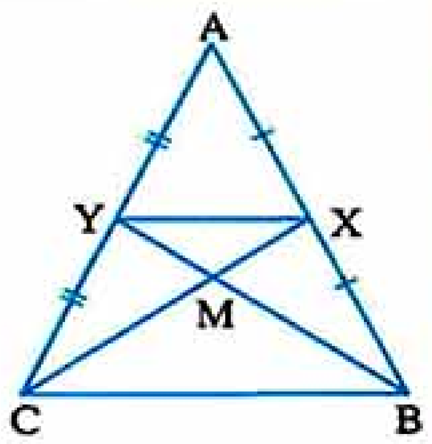
Quiz 1 on lesson 1 – unit 4

time 20 min.

1 Complete the following :

1. The medians of the triangle intersect at ....
2. The point of intersection of the medians of the triangle divides each of them by the ratio .... : .... from the vertex.
3. If $\overline{AD}$ is a median in Δ ABC and M is the point of intersection of its medians , AM = 6 cm. , then AD = ... cm.

2 [a] In the opposite figure :



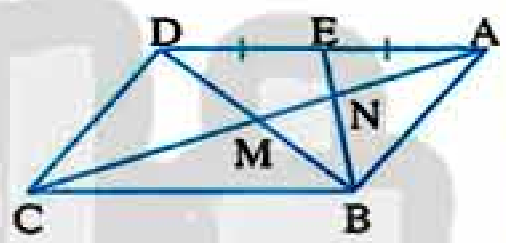
ABC is a triangle , X is the midpoint of $\overline{AB}$

, Y is the midpoint of $\overline{AC}$

, XM = 4 cm. , XY = 5 cm. , BY = 12 cm.

Find : The perimeter of Δ MBC

[b] In the opposite figure :



ABCD is a parallelogram whose diagonals intersect at M

, E is the midpoint of $\overline{AD}$

, $\overline{BE} \cap \overline{AC} = \{N\}$

Prove that : AN = $\frac{1}{3}$ AC

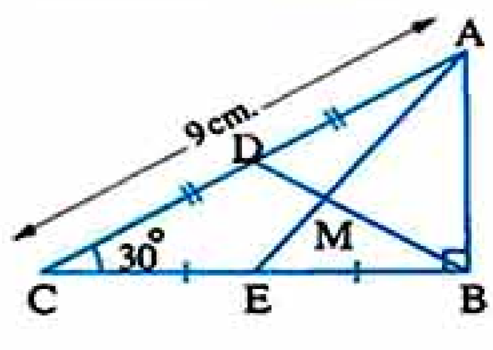
Quiz 2 till lesson 2 – unit 4

time 20 min.

1 Complete the following :

1. The length of the median drawn from the vertex of the right angle of the right-angled triangle = ...
2. In Δ ABC if $\overline{AD}$ is a median of length 12 cm. , M is the point of intersection of medians , then AM = ... cm.
3. The length of the side opposite to the angle whose measure = 30° in the right-angled triangle = ...

2 [a] In the opposite figure :



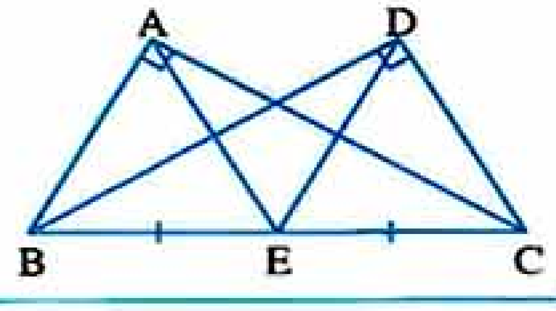
ABC is a triangle in which :

m (∠ B) = 90° , m (∠ C) = 30° , AC = 9 cm.

, $\overline{AE}$ and $\overline{BD}$ are two medians intersecting at M

Find : The length of each of $\overline{BD}$, $\overline{BM}$ and $\overline{AB}$

[b] In the opposite figure :



m (∠ BAC) = m (∠ BDC) = 90° , E is the midpoint of $\overline{BC}$

Prove that : AE = DE

Quiz 3 till lesson 3 – unit 4

time 20 min.

1 Complete the following :

1. The measure of any exterior angle of the equilateral triangle = ……………°
2. ABC is an isosceles triangle in which AB = AC , m (∠ A) = 110° , then m (∠ B) = ……………°
3. If the length of the median which is drawn from a vertex of a triangle equals half the length of the opposite side to this vertex , then the angle at this vertex is ……………

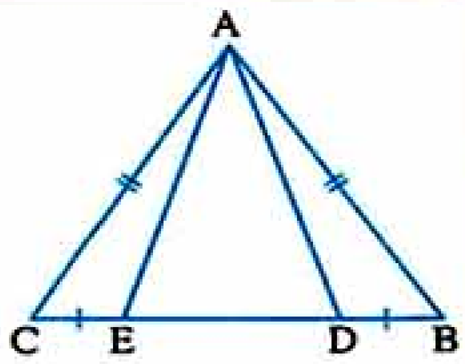
2 [a] In the opposite figure :

ABC is a triangle in which : AB = AC

, D ∈ $\overline{BC}$ and E ∈ $\overline{BC}$

such that : BD = EC

Prove that : AD = AE

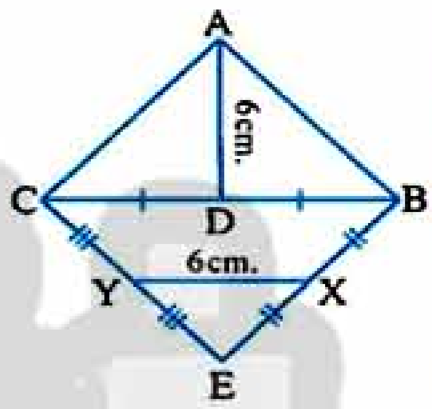


[b] In the opposite figure :

AD = XY = 6 cm. , D is the midpoint of $\overline{BC}$

, X is the midpoint of $\overline{BE}$, Y is the midpoint of $\overline{CE}$

Prove that : m (∠ BAC) = 90°



Quiz 4 till lesson 4 – unit 4

time 20 min.

1 Complete the following :

1. The isosceles triangle in which the measure of one of its angles = 60° is ……………
2. If ABC is a triangle in which : m (∠ B) = 50° and m (∠ C) = 80° , then BC = ……………
3. In Δ ABC , if m (∠ A) = 30° , m (∠ B) = 90° , then : BC = …………… AC

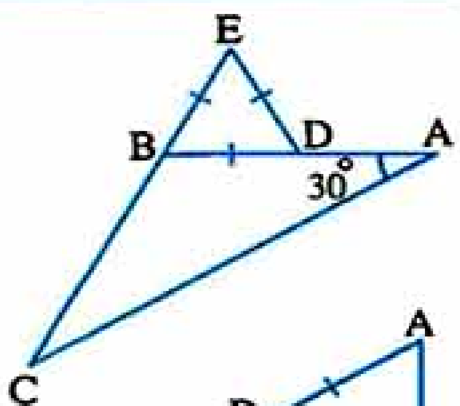
2 [a] In the opposite figure :

E ∈ $\overrightarrow{CB}$, D ∈ $\overline{AB}$,

ED = DB = EB and m (∠ A) = 30°

Prove that :

ABC is an isosceles triangle.



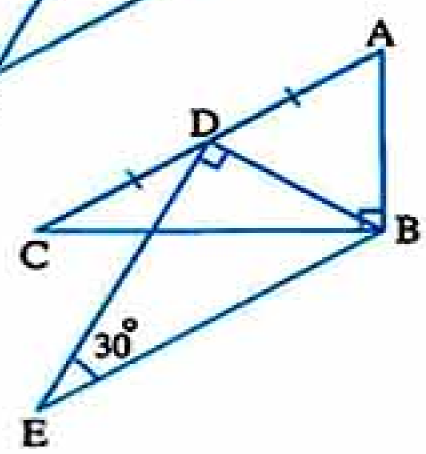
[b] In the opposite figure :

m (∠ ABC) = m (∠ BDE) = 90°

, m (∠ E) = 30°

, D is the midpoint of $\overline{AC}$

Prove that : AC = BE



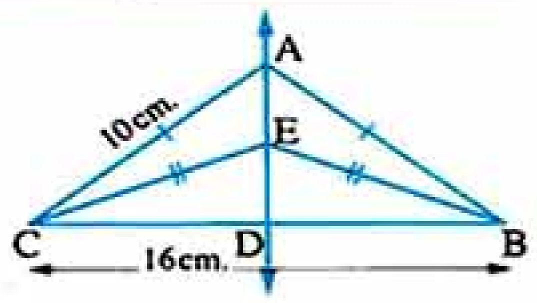
Quiz 5 till lesson 5 – unit 4

time 20 min.

1 Complete the following :

1. The bisector of the vertex angle of the isosceles triangle ....
2. If $\overline{AD}$ is a median in Δ ABC , M is the point of intersection of its medians , then DM = ... AD
3. Any point on the axis of symmetry of a line segment is .... from its terminals.

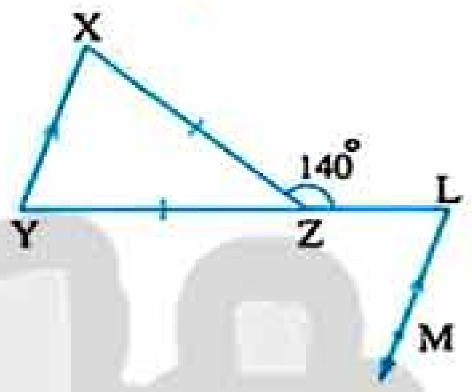
2 [a] In the opposite figure :



ABC is a triangle in which : AB = AC = 10 cm. , BE = EC , BC = 16 cm. and $\overleftrightarrow{AE} \cap \overline{BC} = \{D\}$

Find : The length of $\overline{AD}$ ABC is an isosceles triangle.

[b] In the opposite figure :



$Z \in \overline{LY}$, XZ = ZY

, m ($\angle$ LZX) = 140°

, $\overrightarrow{LM} \parallel \overline{XY}$

Find : m ($\angle$ MLY)

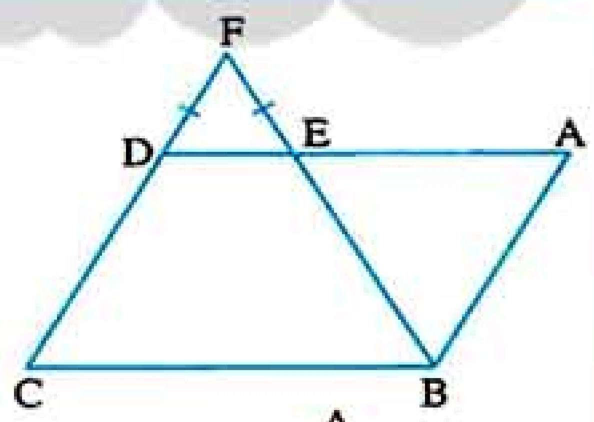
Quiz 6 till lesson 1 – unit 5

time 20 min.

1 Complete the following :

1. The measure of any exterior angle of a triangle is greater than ....
2. In Δ ABC if $\overline{AD}$ is a median , M is the point of intersection of medians , then AM = ... AD
3. If $x > y$, $z < y$, then x .... z

2 [a] In the opposite figure :



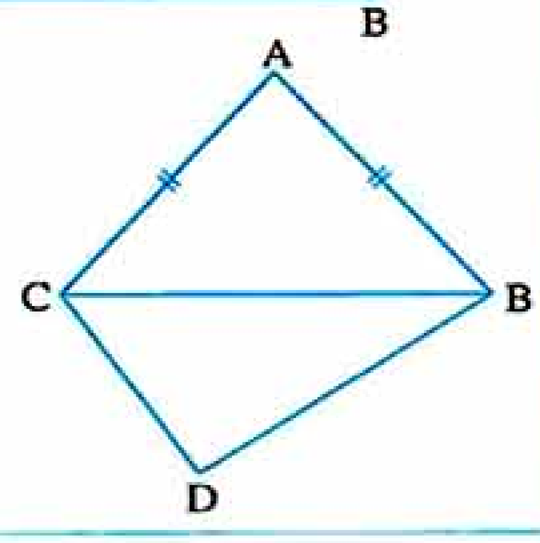
ABCD is a parallelogram ,

$E \in \overline{AD}$, $\overrightarrow{BE} \cap \overrightarrow{CD} = \{F\}$

in which EF = DF

Prove that : Δ BAE is an isosceles triangle.

[b] In the opposite figure :



AB = AC and m ($\angle$ BCD) > m ($\angle$ CBD)

Prove that :

m ($\angle$ ACD) > m ($\angle$ ABD)

Quiz 7 till lesson 2 – unit 5

time 20 min.

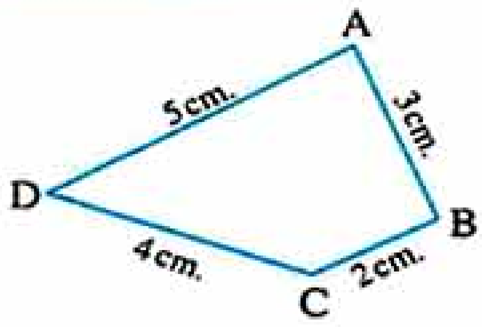
1 Complete the following :

1. In a triangle , if two sides have unequal lengths , the longer is opposite ....
2. The perpendicular to a line segment from its midpoint is .... to it.
3. If ABC is a triangle in which : AB = 4 cm. , BC = 5 cm. and AC = 6 cm. , then : m (∠ ....) > m (∠ ....) > m (∠ ....)

2 [a] In the opposite figure :

ABCD is a quadrilateral

Prove that : m (∠ ABC) > m (∠ ADC)



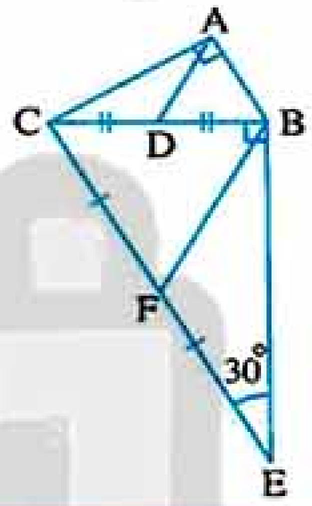
[b] In the opposite figure :

m (∠ BAC) = m (∠ CBE) = 90°

, m (∠ BEC) = 30°

, D and F are the midpoints of $\overline{BC}$ and $\overline{CE}$ respectively.

Prove that : AD = $\frac{1}{2}$ BF



Quiz 8 till lesson 3 – unit 5

time 20 min.

1 Complete the following :

1. The longest side in the right-angled triangle is ....
2. In Δ ABC : If m (∠ A) = 60° and m (∠ B) = 70° , then the shortest side is ....
3. In Δ ABC , if AB = AC , m (∠ A) = 2 m (∠ B) , then m (∠ C) = ...°

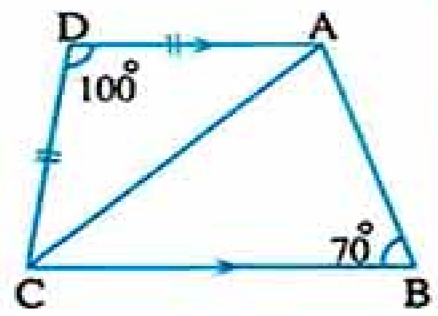
2 [a] In the opposite figure :

$\overline{AD}$ // $\overline{BC}$, AD = DC ,

m (∠ B) = 70° and m (∠ D) = 100°

Prove that :

1. AC > AB
2. Δ ABC is an isosceles triangle.



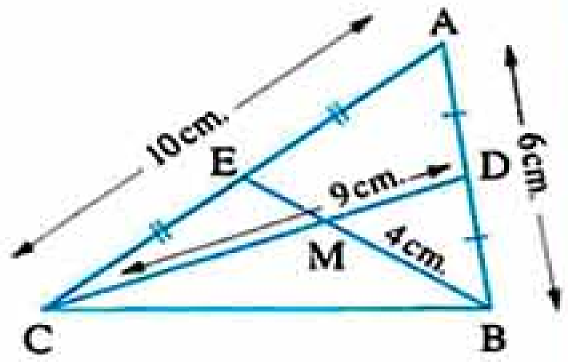
[b] In the opposite figure :

AB = 6 cm. , AC = 10 cm.

, BM = 4 cm. , CD = 9 cm.

, D and E are the midpoints of $\overline{AB}$ and $\overline{AC}$ respectively

Find : The perimeter of the figure ADME



Quiz 9 till lesson 4 – unit 5

time 20 min.

1 Choose the correct answer from the given ones :

1 In Δ ABC : If AB = 6 cm. and AC = 7 cm. then BC $\in$ ……………

(a) $]6 , 13]$ (b) $[6 , 7]$ (c) $]1 , 13[$ (d) $[1 , 7[$

2 An isosceles triangle in which the measure of the vertex angle is 100° , then the measure of one of the two base angles = ……………

(a) 80° (b) 40° (c) 50° (d) 100°

3 The numbers that can be lengths of sides of a triangle are ……………

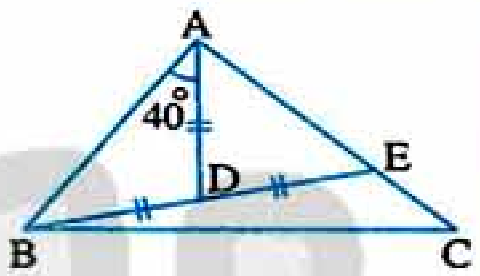
(a) 7 , 7 , 14 (b) 3 , 4 , 9 (c) 4 , 5 , 12 (d) 5 , 5 , 5

2 [a] In the opposite figure :

AD = BD = ED , m ($\angle$ DAB) = 40°

Prove that :

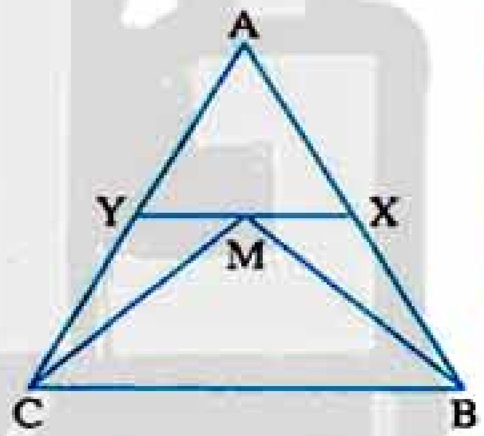
1 AD < AB 2 BC > AC



[b] In the opposite figure :

ABC is a triangle in which X $\in \overline{AB}$, Y $\in \overline{AC}$, M $\in \overline{XY}$

Prove that : AB + AC > MB + MC



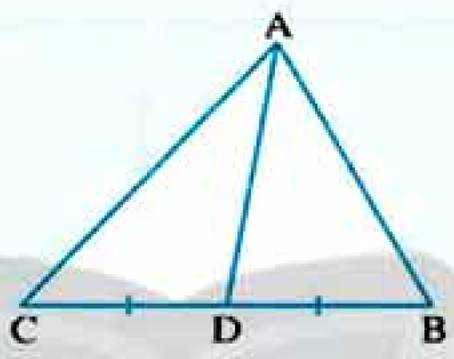
Final Revision

of Geometry

Revision for the important theorems , corollaries and rules of geometry

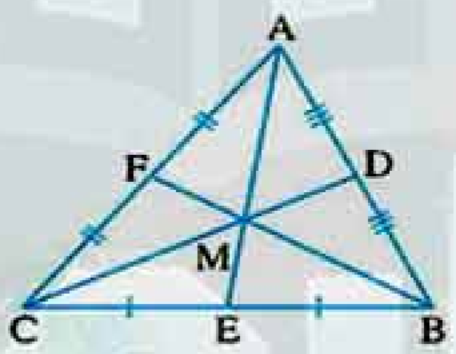
Medians of triangle

The median of the triangle is the line segment drawn from any vertex of the triangle vertices to the midpoint of the opposite side of this vertex.



If D is the midpoint of $\overline{BC}$, then $\overline{AD}$ is a median in Δ ABC

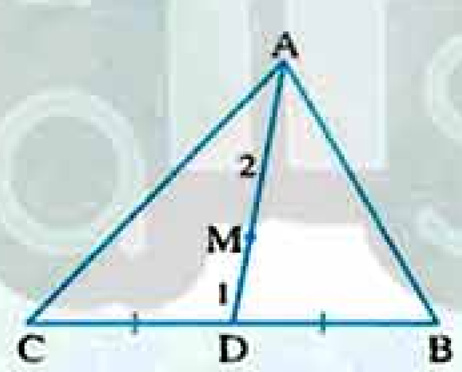
The medians of a triangle are concurrent.



If $\overline{CD}$, $\overline{BF}$ and $\overline{AE}$ are the medians of Δ ABC where $\overline{CD} \cap \overline{BF} \cap \overline{AE} = \{M\}$, then M is the intersection point of medians of Δ ABC

The point of concurrence of the medians of the triangle divides each median in the ratio of :

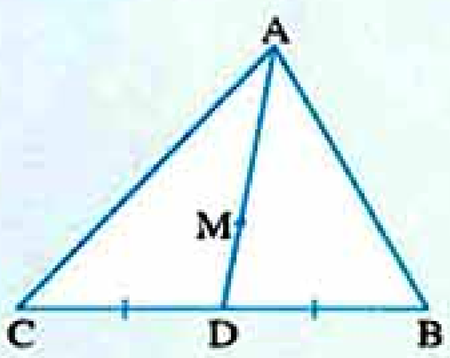
- 1 : 2 from the base.
- 2 : 1 from the vertex.



If M is the intersection point of medians of Δ ABC , then :

- $DM = \frac{1}{2}$ AM
- AM = 2 DM
- $DM = \frac{1}{3}$ AD
- $AM = \frac{2}{3}$ AD

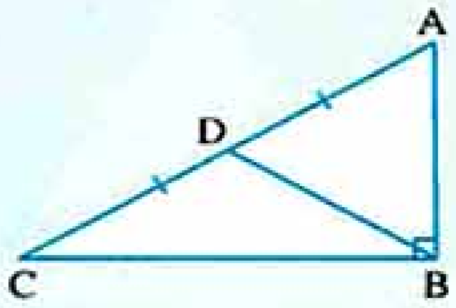
The point which divides the median in a triangle by the ratio 1 : 2 from the base is the point of the intersection of the medians of the triangle.



If DM : MA = 1 : 2 , then M is the intersection point of medians of Δ ABC

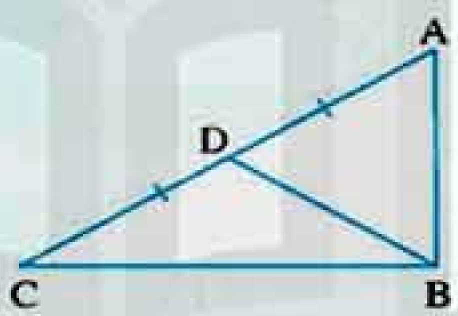
Right-angled triangle

The length of the median from the vertex of the right angle equals half the length of the hypotenuse.



If Δ ABC is right-angled at B , $\overline{BD}$ is a median in it , then $BD = \frac{1}{2} AC$

If the length of the median drawn from a vertex of a triangle equals half the length of the opposite side to this vertex , then the angle at this vertex is right.

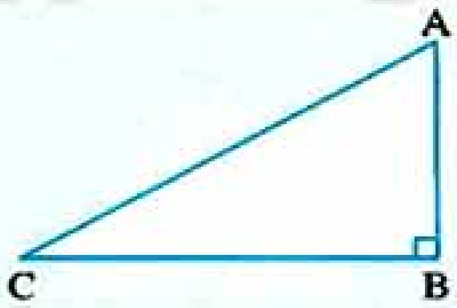


If $\overline{BD}$ is a median in Δ ABC , $BD = \frac{1}{2} AC$
$\therefore m(\angle ABC) = 90°$

The length of the side opposite to the angle of measure 30° in the right-angled triangle equals half the length of the hypotenuse.

If Δ ABC is a right-angled at B in which : $m(\angle C) = 30°$, then $AB = \frac{1}{2} AC$

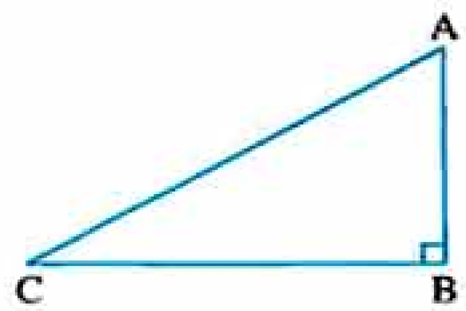
In the right-angled triangle, the hypotenuse is the longest side of the triangle.



If Δ ABC is a right-angled at B , then $AC > AB$, $AC > BC$

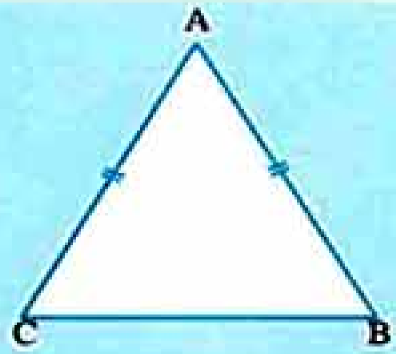
If Δ ABC is a right-angled at B , then :

- $(AC)^2 = (AB)^2 + (BC)^2$
- $(AB)^2 = (AC)^2 - (BC)^2$
- $(BC)^2 = (AC)^2 - (AB)^2$



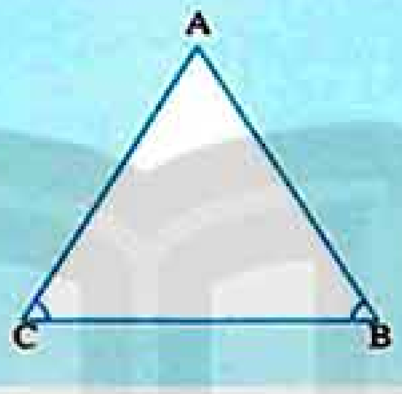
The isosceles triangle

The base angles of the isosceles triangle are congruent.



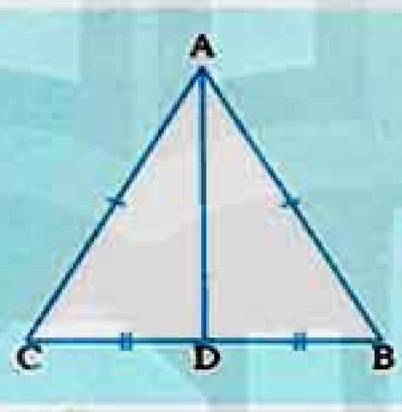
If Δ ABC in which :
AB = AC , then
m ($\angle$ B) = m ($\angle$ C)

If two angles of a triangle are congruent , then the two sides opposite to these two angles are congruent and the triangle is isosceles.



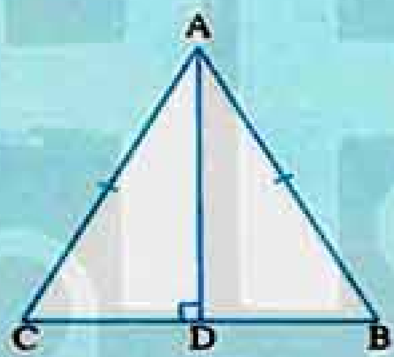
If Δ ABC in which :
m ($\angle$ B) = m ($\angle$ C)
, then AB = AC

The median of an isosceles triangle from the vertex angle bisects it and is perpendicular to the base.



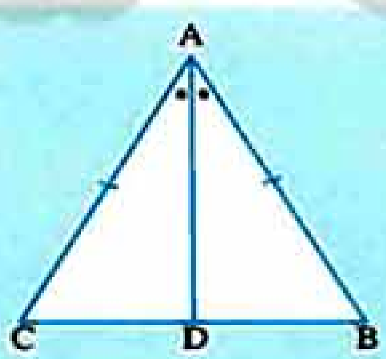
If Δ ABC in which :
AB = AC , $\overline{AD}$ is a median
, then $\overrightarrow{AD}$ bisects $\angle$ BAC
, $\overline{AD} \perp \overline{BC}$

The straight line drawn passing through the vertex angle of an isosceles triangle perpendicular to the base bisects each of the base and the vertex angle.



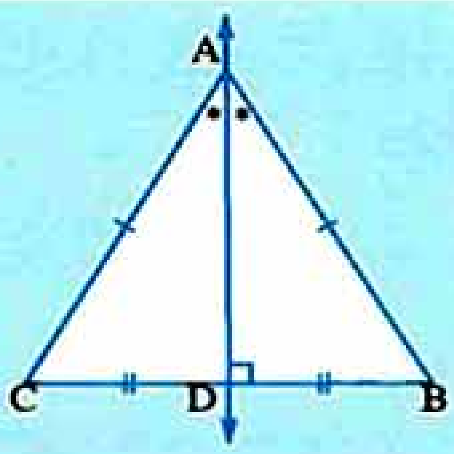
If Δ ABC in which :
AB = AC , $\overline{AD} \perp \overline{BC}$
, then D is the midpoint of $\overline{BC}$,
$\overrightarrow{AD}$ bisects $\angle$ BAC

The bisector of the vertex angle of an isosceles triangle bisects the base and is perpendicular to it.



If Δ ABC in which :
AB = AC , $\overrightarrow{AD}$ bisects $\angle$ BAC , then D is the midpoint of $\overline{BC}$, $\overline{AD} \perp \overline{BC}$

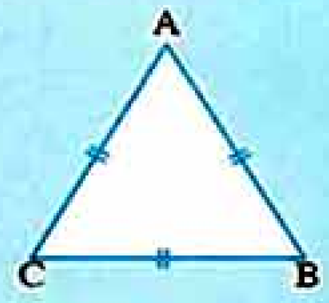
The number of axes of symmetry of the isosceles triangle = 1



If Δ ABC in which :
AB = AC , $\overleftrightarrow{AD} \perp \overline{BC}$ and intersect it at D
, then $\overleftrightarrow{AD}$ is the axis of symmetry of the triangle ABC

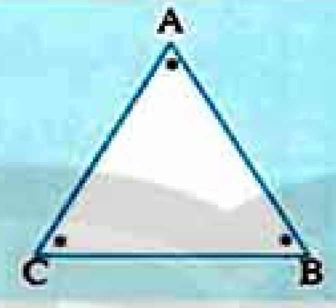
The equilateral triangle

If the triangle is an equilateral , then it is equiangular where each angle measure is 60°



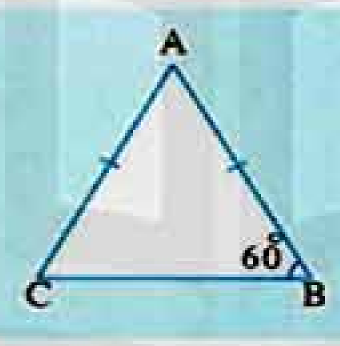
If Δ ABC in which :
AB = BC = CA , then
m ($\angle$ A) = m ($\angle$ B) = m ($\angle$ C) = 60°

If the angles of a triangle are congruent , then the triangle is equilateral.



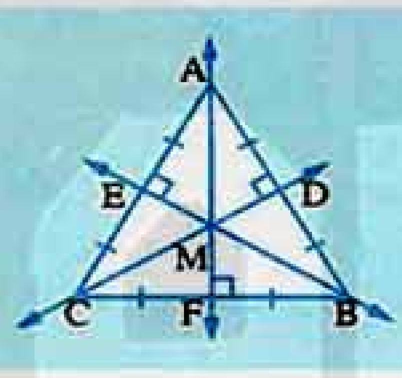
If Δ ABC in which :
m ($\angle$ A) = m ($\angle$ B) = m ($\angle$ C)
, then AB = BC = CA

The isosceles triangle in which the measure of one of its angles = 60° is an equilateral triangle.



If Δ ABC in which :
AB = AC , m ($\angle$ B) = 60°
, then Δ ABC is an equilateral triangle.

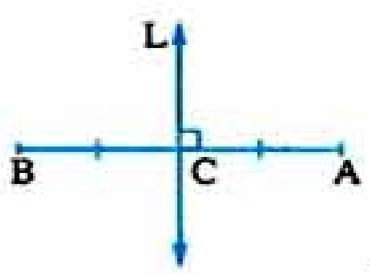
The equilateral triangle has three axes of symmetry.



If Δ ABC is an equilateral triangle
, $\overleftrightarrow{AF} \perp \overline{BC}$, $\overleftrightarrow{CD} \perp \overline{AB}$, $\overleftrightarrow{BE} \perp \overline{AC}$
, then $\overleftrightarrow{AF}$, $\overleftrightarrow{CD}$ and $\overleftrightarrow{BE}$ are the axes of symmetry of the triangle ABC

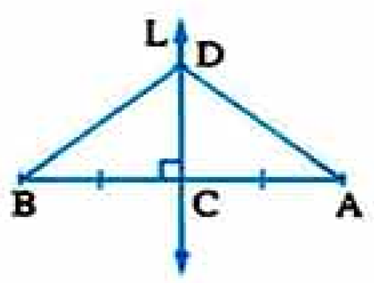
The axis of symmetry

The axis of symmetry of a line segment is the straight line perpendicular to it from its middle.



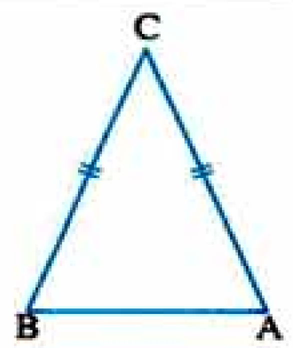
If the straight line L $\perp \overline{AB}$, C $\in \overline{AB}$ where CA = CB , C $\in$ the straight line L , then L is the axis of $\overline{AB}$

Any point on the axis of symmetry of a line segment is at equal distances from its terminals (end points).



If the straight line L is the axis of $\overline{AB}$, D $\in$ the straight line L , then DA = DB

If a point is at equal distances from the two terminals of a line segment , then this point lies on the axis of this line segment.

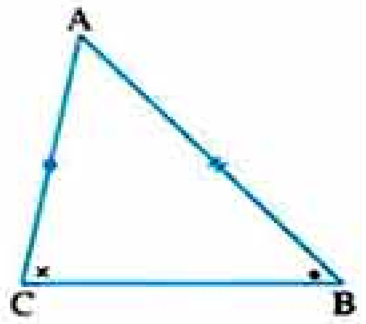


If CA = CB , then C lies on the axis of $\overline{AB}$

Inequality relations in the triangle

Comparing the measures of angles in a triangle

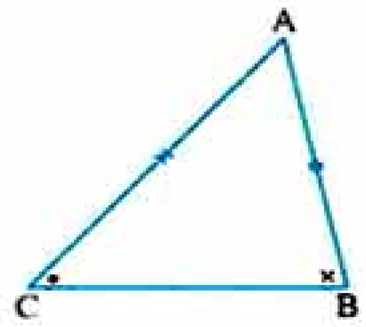
If two sides have unequal lengths , the longer is opposite to the angle of the greater measure



If AB > AC , then m (∠ C) > m (∠ B)

Comparing the lengths of sides in a triangle

If two angles are unequal in measure , then the greater angle in measure is opposite to a side greater in length than that opposite to the other angle.



If m (∠ B) > m (∠ C) , then AC > AB

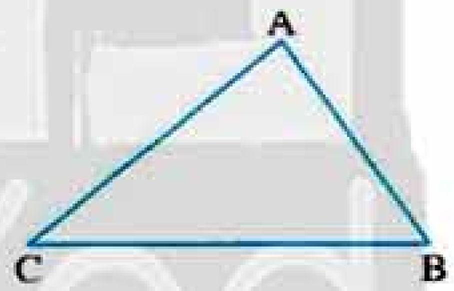
Triangle inequality

In any triangle , the sum of the lengths of any two sides is greater than the length of the third side.

AB + BC > AC

, BC + CA > AB

, CA + AB > BC

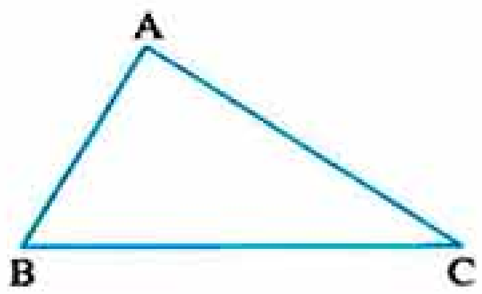


Notice that

- The length of any side in a triangle is greater than the difference between the lengths of the two other sides and less than their sum.

  In Δ ABC :

  AC – AB < BC < AC + AB

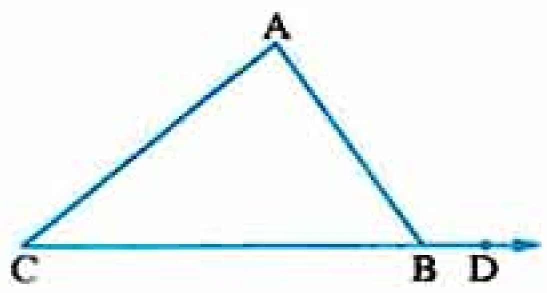


- The measure of any exterior angle of a triangle is greater than the measure of any interior angle of the triangle except its adjacent angle.

  In Δ ABC :

  m (∠ ABD) > m (∠ A)

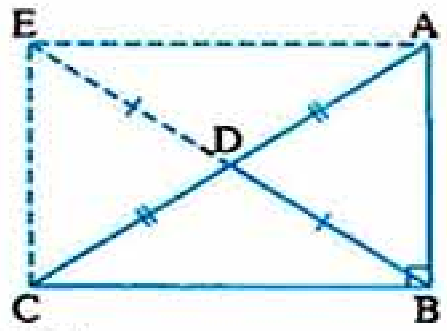
  , m (∠ ABD) > m (∠ C)



Proofs of the important theorems

Theorem

In the right-angled triangle , the length of the median from the vertex of the right angle equals half the length of the hypotenuse.



Given ABC is a triangle in which m (∠ ABC) = 90° ,
$\overline{BD}$ is a median in the triangle ABC

R.T.P. $BD = \frac{1}{2} AC$

Construction Draw $\overrightarrow{BD}$ and take the point $E \in \overrightarrow{BD}$ such that BD = DE

Proof In the figure ABCE : ∵ $\overline{AC}$ and $\overline{BE}$ bisect each other

∴ The figure ABCE is a parallelogram.

∵ m (∠ ABC) = 90°

∴ The figure ABCE is a rectangle.

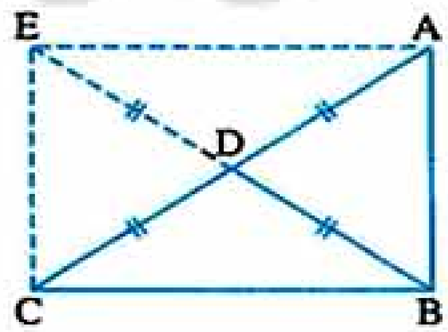
∴ BE = AC

, ∵ $BD = \frac{1}{2} BE$

∴ $BD = \frac{1}{2} AC$ (Q.E.D.)

Theorem

If the length of the median drawn from a vertex of a triangle equals half the length of the opposite side to this vertex , then the angle at this vertex is right.



Given In Δ ABC , $\overline{BD}$ is a median and DA = DB = DC

R.T.P. m (∠ ABC) = 90°

Construction Draw $\overrightarrow{BD}$, then take the point $E \in \overrightarrow{BD}$ such that BD = DE

Proof ∵ $BD = \frac{1}{2} BE = \frac{1}{2} AC$ ∴ BE = AC

∴ In the figure ABCE :

$\overline{AC}$ and $\overline{BE}$ are equal in length and bisect each other.

∴ The figure ABCE is a rectangle.

∴ m (∠ ABC) = 90° (Q.E.D.)

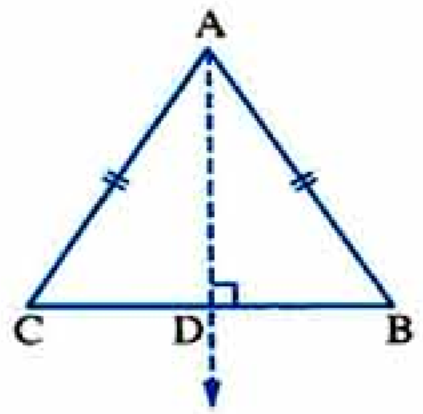
Theorem

The base angles of the isosceles triangle are congruent.

Given ABC is a triangle in which $\overline{AB} \equiv \overline{AC}$

R.T.P. $\angle B \equiv \angle C$

Construction Draw $\overrightarrow{AD} \perp \overline{BC}$ where $\overrightarrow{AD} \cap \overline{BC} = \{D\}$



Proof $\because \Delta\Delta$ ADB , ADC in which :

$$\begin{cases} m(\angle ADB) = m(\angle ADC) = 90^\circ & \text{(const.)} \\ \overline{AB} \equiv \overline{AC} & \text{(given)} \\ \overline{AD} \text{ is a common side} & \end{cases}$$

$\therefore \Delta ADB \equiv \Delta ADC$, then we deduce that $\angle B \equiv \angle C$ **(Q.E.D.)**

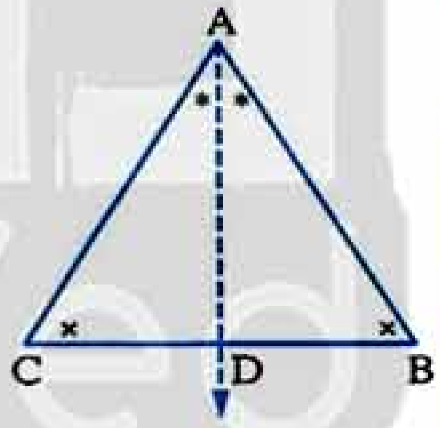
Theorem

If two angles of a triangle are congruent , then the two sides opposite to these two angles are congruent and the triangle is isosceles.

Given ΔABC in which $\angle B \equiv \angle C$

R.T.P. $\overline{AB} \equiv \overline{AC}$

Construction bisect $\angle BAC$ by $\overrightarrow{AD}$ to intersect $\overline{BC}$ at D



Proof $\because \angle B \equiv \angle C$

$\therefore m(\angle B) = m(\angle C)$

$\because \overrightarrow{AD}$ bisects $\angle BAC$ $\therefore m(\angle BAD) = m(\angle CAD)$

$\because$ The sum of measures of the interior angles of the triangle = 180°

$\therefore m(\angle ADB) = m(\angle ADC)$

$\therefore$ In $\Delta\Delta$ ABD and ACD :

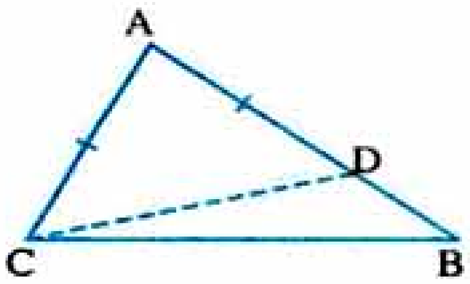
$$\begin{cases} \overline{AD} \text{ is a common side} \\ m(\angle BAD) = m(\angle CAD) \text{ (const.)} \\ m(\angle ADB) = m(\angle ADC) \text{ (by proof)} \end{cases}$$

$\therefore \Delta ABD \equiv \Delta ACD$, then we deduce that

$\overline{AB} \equiv \overline{AC}$, then ΔABC is an isosceles triangle. **(Q.E.D.)**

Theorem

In a triangle , if two sides have unequal lengths , the longer is opposite to the angle of the greater measure.



Given ABC is a triangle in which AB > AC

R.T.P. m (∠ ACB) > m (∠ ABC)

Construction Take D ∈ $\overline{AB}$ such that AD = AC

Proof In Δ ACD : ∵ AD = AC ∴ m (∠ ADC) = m (∠ ACD) (1)

∵ ∠ ADC is an exterior angle of Δ DBC

∴ m (∠ ADC) > m (∠ B) (2)

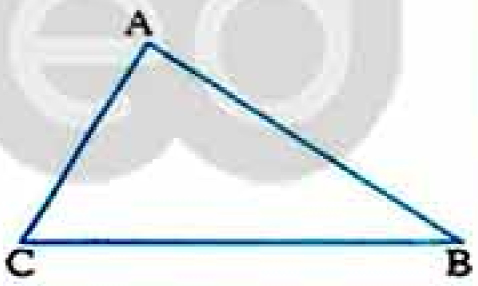
From (1) and (2) : ∴ m (∠ ACD) > m (∠ B)

, ∵ m (∠ ACB) > m (∠ ACD)

∴ m (∠ ACB) > m (∠ ABC) (Q.E.D.)

Theorem

In a triangle , if two angles are unequal in measure , then the greater angle in measure is opposite to a side greater in length than that opposite to the other angle.



Given ABC is a triangle in which m (∠ C) > m (∠ B)

R.T.P. AB > AC

Proof ∵ $\overline{AB}$ and $\overline{AC}$ are two line segments.

∴ One of the following cases should be verified.

1. AB > AC 2. AB = AC 3. AB < AC

Unless AB > AC , then either AB = AC **or** AB < AC

- If : AB = AC , then m (∠ C) = m (∠ B) and this contradicts the given where m (∠ C) > m (∠ B)
- If : AB < AC , then m (∠ C) < m (∠ B) according to the preceding theorem.

Again this contradicts the given , where m (∠ C) > m (∠ B)

∴ It should be that AB > AC (Q.E.D.)

Final Examinations

on Geometry

Model Examinations of the School Book on Geometry

Model 1

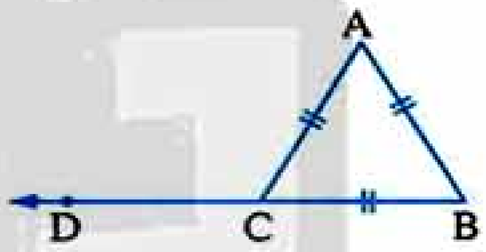
Answer the following questions :

1 Complete the following :

1. The longest side in the right-angled triangle is ....
2. If the lengths of two sides in a triangle are 2 cm. and 7 cm. , then : .... < the length of the third side < ....
3. If the measures of two angles in a triangle are different , then the greater in measure of them is opposite to ....
4. If the length of the median drawn from a vertex of a triangle equals half the opposite side to this vertex in length , then ....
5. If the measure of an angle in the isosceles triangle equals 60° , then the triangle is ....

2 Choose the correct answer from those given :

1. **In the opposite figure :**



Δ ABC is equilateral , then m ($\angle$ ACD) = ...

(a) 45° (b) 60°
(c) 120° (d) 135°

2. In Δ ABC which is right-angled at B , if AC = 20 cm. , then the length of the median of the triangle drawn from B equals ....

(a) 10 cm. (b) 8 cm. (c) 6 cm. (d) 5 cm.

3. XYZ is a triangle in which : m ($\angle$ Z) = 70° and m ($\angle$ Y) = 60° , then YZ .... XY

(a) > (b) < (c) = (d) twice

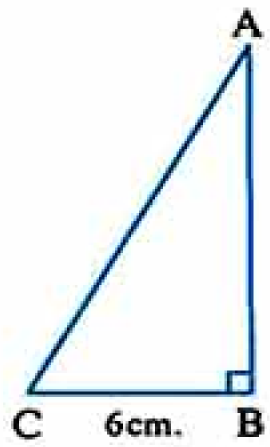
4. The lengths which can be lengths of sides of a triangle are ....

(a) 0 , 3 , 5 (b) 3 , 3 , 5 (c) 3 , 3 , 6 (d) 3 , 3 , 7

5. The triangle in which the measures of two angles of it are 42° and 69° is ....

(a) an isosceles triangle. (b) an equilateral triangle.
(c) a scalene triangle. (d) a right-angled triangle.

6. **In the opposite figure :**

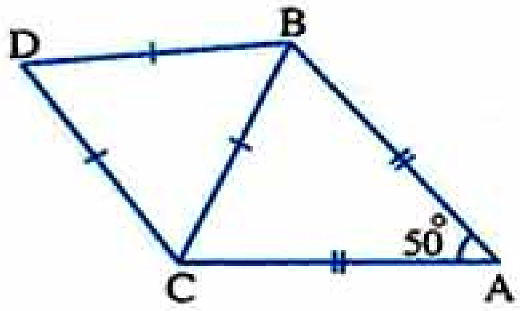


m ($\angle$ C) = 2 m ($\angle$ A)
, BC = 6 cm.
, then AC = ... cm.

(a) 3 (b) 6
(c) 9 (d) 12

3 [a] **Complete :** ABC is a triangle in which AB > AC , then m (∠ C) .... m (∠ B)

[b] **In the opposite figure :**

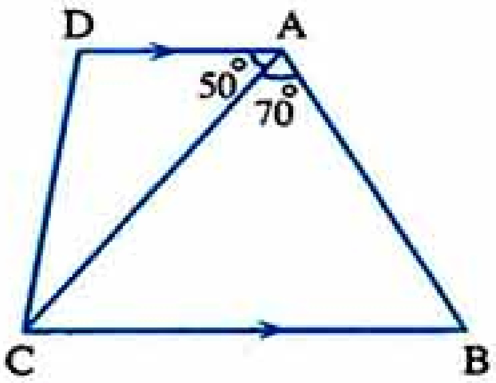


m (∠ A) = 50° , AB = AC

and Δ DBC is equilateral

Find : m (∠ ABD)

[c] **In the opposite figure :**



$\overline{AD}$ // $\overline{BC}$

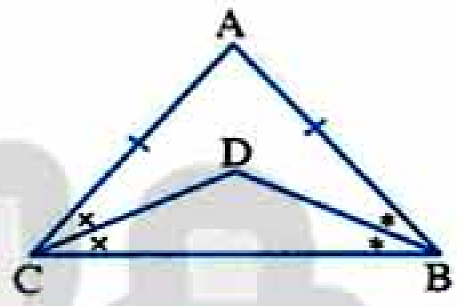
, m (∠ BAC) = 70°

and m (∠ DAC) = 50°

Prove that : BC > AC

4 [a] **Prove that :** The two base angles of the isosceles triangle are congruent.

[b] **In the opposite figure :**

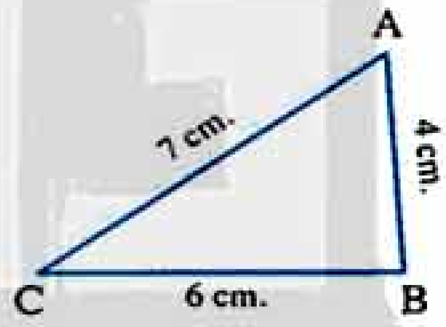


AB = AC , $\overrightarrow{BD}$ bisects ∠ B

and $\overrightarrow{CD}$ bisects ∠ C

Prove that : Δ DBC is isosceles.

5 [a] **In the opposite figure :**

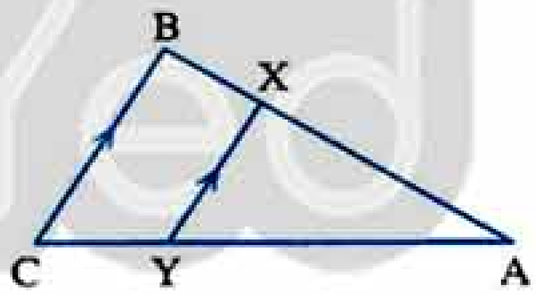


Arrange the angles

of Δ ABC descendingly

due to their measures

[b] **In the opposite figure :**



AB > BC , $\overline{XY}$ // $\overline{BC}$

Prove that : AX > XY

Model 2

Answer the following questions :

1 **Choose the correct answer from those given :**

1 The triangle which has three axes of symmetry is .... triangle.

(a) scalene (b) isosceles (c) right-angled (d) equilateral

2 The sum of lengths of two sides in a triangle is .... the length of the third side.

(a) greater than (b) smaller than (c) equals to (d) twice

3 If the lengths of two sides in an isosceles triangle are 8 cm. and 4 cm. , then the length of the third side is .... cm.

(a) 4 (b) 8 (c) 3 (d) 12

4 In Δ ABC if m ($\angle$ B) = 130° , then the longest side of it is ....
(a) $\overline{BC}$ (b) $\overline{AC}$ (c) $\overline{AB}$ (d) its median.

5 Δ XYZ is an isosceles triangle in which : m ($\angle$ X) = 100° , then m ($\angle$ Y) = ...
(a) 100° (b) 80° (c) 60° (d) 40°

6 **In the opposite figure :**

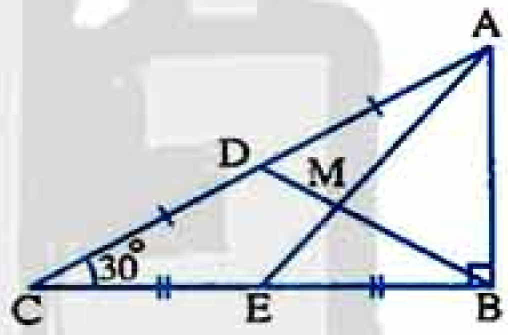
$x + y =$ ....
(a) 100° (b) 140°
(c) 180° (d) 280°

2 Complete the following :

1 If the measure of an angle in a right-angled triangle is 45° , then the triangle is ....
2 The length of any side in a triangle .... the sum of lengths of the two other sides.
3 If $\overline{AB} \equiv \overline{XY}$, then AB = ...
4 In Δ ABC , if m ($\angle$ A) = 30° and m ($\angle$ B) = 90° , then BC = ... AC
5 The axis of symmetry of a line segment is the straight line which .... at its midpoint.

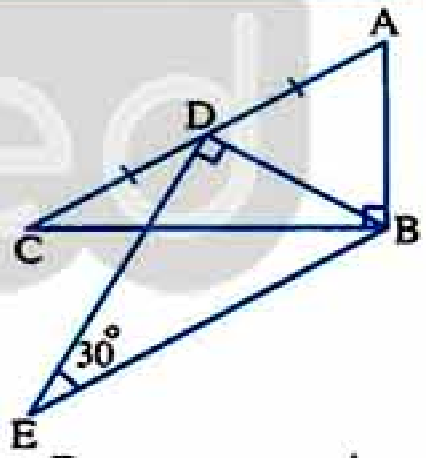
3 [a] In Δ ABC : AB = 7 cm. , BC = 5 cm. and AC = 6 cm.
Arrange its angles ascendingly due to their measures.

[b] In the opposite figure :



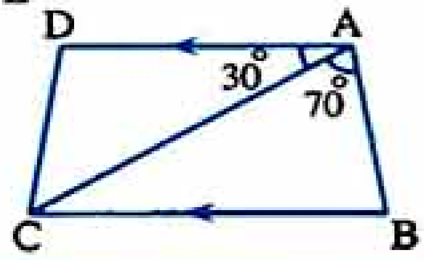
Δ ABC is right-angled at B
, m ($\angle$ C) = 30° , D is the midpoint of $\overline{AC}$
, E is the midpoint of $\overline{BC}$, AC = 9 cm.
Find the length of each of : $\overline{BD}$, $\overline{BM}$ and $\overline{AB}$

4 **[a] In the opposite figure :**



m ($\angle$ ABC) = m ($\angle$ BDE) = 90°
, m ($\angle$ E) = 30°
, D is the midpoint of $\overline{AC}$
Prove that : AC = BE

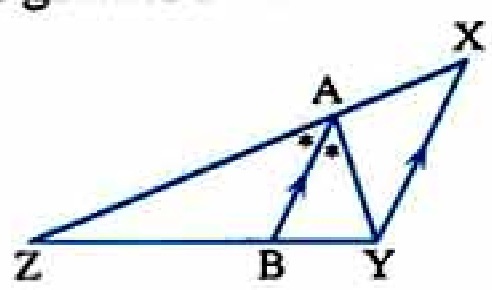
[b] In the opposite figure :



$\overleftrightarrow{AD}$ // $\overline{BC}$, m ($\angle$ BAC) = 70°
, m ($\angle$ DAC) = 30°
Prove that : AC > BC

5 **[a] Complete :**
If the measures of two angles of a triangle are different , then their greater in measure is opposite to ....

[b] In the opposite figure :



$\overleftrightarrow{AB}$ // $\overline{XY}$ and $\overrightarrow{AB}$ bisects $\angle$ YAZ
Prove that : XZ > YZ

Model for the merge students

Answer the following questions :

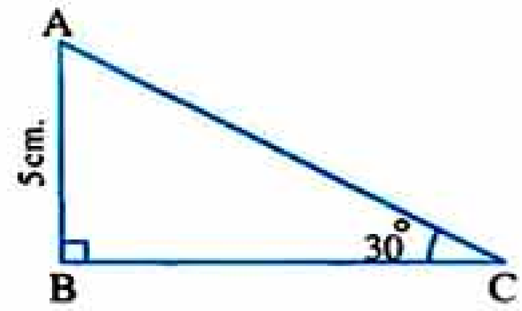
1 Complete each of the following :

1. The point of concurrence of the medians of the triangle divides each median in the ratio .... : .... from the base.
2. In the right-angled triangle , the length of the median drawn from the vertex of the right angle equals ....
3. The base angles of the isosceles triangle are ....
4. In Δ ABC : m ($\angle$ B) = 70° , m ($\angle$ C) = 50° , then AC .... AB
5. The median of the isosceles triangle from the vertex angle .... , ...

2 Choose the correct answer from those given :

1. If ABC is an equilateral triangle , then m ($\angle$ B) = ...
   (a) 30° (b) 60° (c) 70° (d) 90°
2. The length of the side opposite to the angle of measure 30° in the right-angled triangle equals .... the length of the hypotenuse.
   (a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{1}{4}$ (d) 2
3. If the measure of the vertex angle of an isosceles triangle is 80° , then the measure of one of the base angles equals ....
   (a) 60° (b) 40° (c) 30° (d) 50°
4. The number of axes of symmetry of the isosceles triangle is ....
   (a) 1 (b) 2 (c) 3 (d) zero
5. In Δ ABC : m ($\angle$ A) = 50° , m ($\angle$ B) = 60° , then the longest side is ....
   (a) $\overline{AB}$ (b) $\overline{BC}$ (c) $\overline{AC}$

3 In the opposite figure , complete :



Δ ABC is a right-angled triangle at B , m ($\angle$ C) = 30° , AB = 5 cm.

Find : The length of $\overline{AC}$

$\because$ m ($\angle$ B) = ... , m ($\angle$ C) = ...

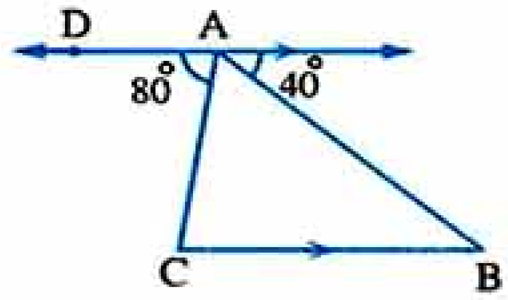
$\therefore$ AB = $\frac{1}{2}$ × .... $\therefore$ AC = ... cm.

4 **[a]** In ΔABC : $m(\angle A) = 40°$, $m(\angle B) = 75°$, $m(\angle C) = 65°$

Arrange the lengths of the sides of the triangle descendingly.

The order is : .... , ... , ...

[b] In the opposite figure :



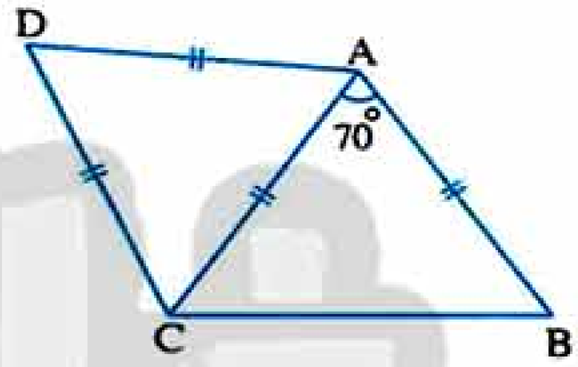
$\overleftrightarrow{AD} \parallel \overline{BC}$

Complete :

1 $m(\angle B) =$ ....°

2 The side .... is the longest side of ΔABC

5 **In the opposite figure :**



$AB = AC = CD = AD = 10$ cm.

, $m(\angle BAC) = 70°$

Put (✓) or (×) :

1 $m(\angle B) = 55°$ (   )

2 $m(\angle D) = 70°$ (   )

3 $m(\angle DCB) = 120°$ (   )

4 $AB + AD = 20$ cm. (   )

5 $AB + BC = BC + CD$ (   )

Some Schools Examinations on Geometry

1 Cairo Governorate

Centre Cairo Educative Zone
Saint Joseph College Khoronfish

Answer the following questions :

1 Choose the correct answer from the given ones :

1 In ΔABC , if $AB = 6$ cm. and $AC = 7$ cm. , then $BC \in$ ....

(a) $]6 , 13]$ (b) $[6 , 7]$ (c) $]1 , 13[$ (d) $[1 , 7[$

2 The point of intersection of the medians of the triangle divides each of them in the ratio of .... from the vertex.

(a) 1 : 2 (b) 1 : 3 (c) 2 : 1 (d) 2 : 3

3 The measure of any exterior angle of the equilateral triangle equals ....°

(a) 60 (b) 100 (c) 120 (d) 150

4 In ΔABC , if $\overline{AD}$ is a median , M is the point of intersection of its medians , then AM = ... AD

(a) $\frac{1}{2}$ (b) 2 (c) $\frac{2}{3}$ (d) $\frac{3}{2}$

5 ΔXYZ is an isosceles triangle in which $m(\angle X) = 110°$, then $m(\angle Y) =$ ....°

(a) 110 (b) 35 (c) 60 (d) 45

6 In ΔABC , if $\overline{AB} \perp \overline{BC}$ and $AB = BC$, then $m(\angle A) =$ ....°

(a) 30 (b) 45 (c) 60 (d) 90

2 Complete the following :

1 The number of axes of symmetry of the equilateral triangle equals ....

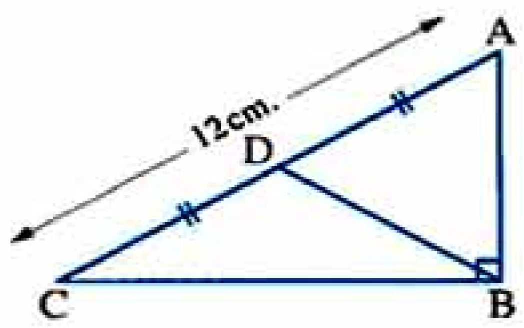
2 The base angles in an isosceles triangle are ....

3 The longest side in the right-angled triangle is ....

4 The bisector of the vertex angle of the isosceles triangle ....

5 **In the opposite figure :**

$AC = 12$ cm. , then $BD =$ .... cm.

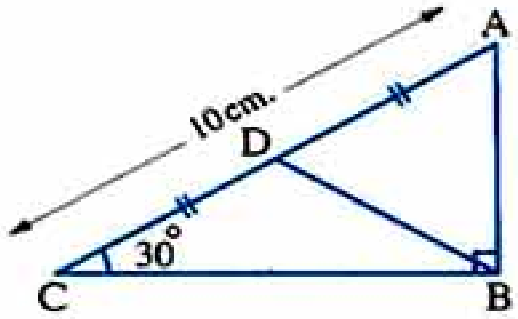


3 **[a]** In Δ ABC , if m ($\angle$ A) = $(6\,X)^\circ$, m ($\angle$ B) = $(4\,X - 9)^\circ$

and m ($\angle$ C) = $3\,(X - 2)^\circ$

Arrange the side lengths of Δ ABC ascendingly.

[b] In the opposite figure :



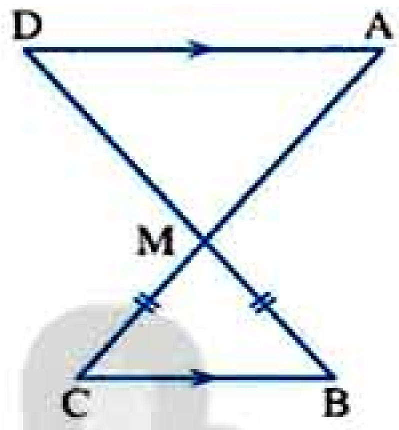
m ($\angle$ ABC) = 90° , m ($\angle$ C) = 30°

, AD = DC and AC = 10 cm.

Find : The perimeter of Δ ABD

---

4 **[a] In the opposite figure :**



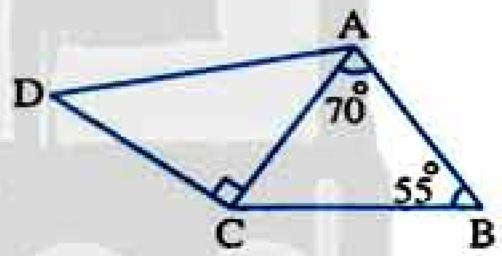
If $\overline{AC} \cap \overline{BD} = \{M\}$

, $\overline{AD} \parallel \overline{BC}$ and MB = MC

, prove that :

Δ MAD is isosceles.

[b] In the opposite figure :



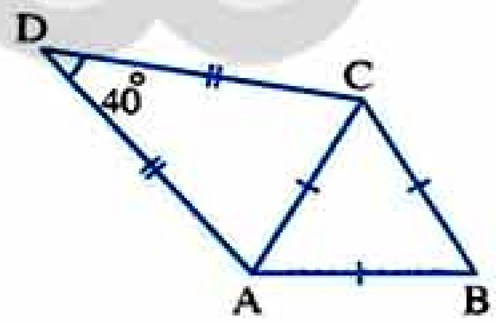
m ($\angle$ BAC) = 70° , m ($\angle$ B) = 55°

and m ($\angle$ ACD) = 90°

Prove that : AD > AB

---

5 **[a] In the opposite figure :**



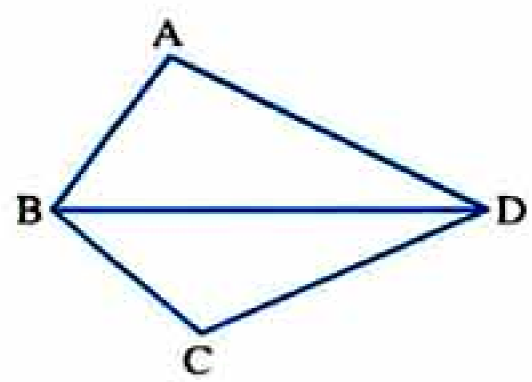
m ($\angle$ D) = 40°

, DA = DC

and ΔABC is an equilateral triangle.

Find : m ($\angle$ DCB)

[b] In the opposite figure :



AB < AD and BC < CD

Prove that :

m ($\angle$ ABC) > m ($\angle$ ADC)

2 Cairo Governorate

Hadaik El-Kobba Educational Zone

Answer the following questions :

1 Complete :

1 The median of an isosceles triangle from the vertex angle bisects .... and is perpendicular to ....

2 The measure of the exterior angle at any vertex of the equilateral triangle is ....°

3 The base angles of the isosceles triangle are ....

4 ABC is a triangle in which AB = 4 cm. , BC = 6 cm. , then AC ∈]............... , ...[

5 The longest side in the right-angled triangle is ....

2 Choose the correct answer :

1 In Δ ABC , if AC = 4 cm. , BC = 3 cm. , then m ($\angle$ B) .... m ($\angle$ A)

(a) > (b) < (c) = (d) ≤

2 The length of the side opposite to the angle of measure 30° in the right-angled triangle equals .... the length of the hypotenuse.

(a) half (b) twice (c) third (d) quarter

3 In Δ ABC , if m ($\angle$ A) = 100° and AB = AC , then m ($\angle$ ABC) = ...

(a) 80° (b) 60° (c) 40° (d) 30°

4 The point of intersection of the medians of the triangle divides each of them in the ratio .... from the base.

(a) 1 : 3 (b) 3 : 1 (c) 1 : 2 (d) 2 : 1

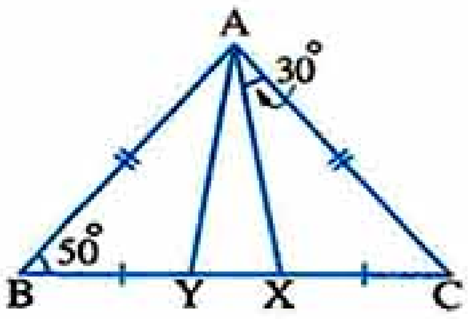
5 If Δ ABD is obtuse-angled at B and C is the midpoint of $\overline{BD}$, then the longest side is ....

(a) $\overline{AB}$ (b) $\overline{AC}$ (c) $\overline{AD}$ (d) $\overline{BD}$

6 The triangle whose side lengths are 2 cm. , $(x + 3)$ cm. and 5 cm. becomes an isosceles triangle when x = ... cm.

(a) 1 (b) 2 (c) 3 (d) 4

3 **[a] In the opposite figure :**



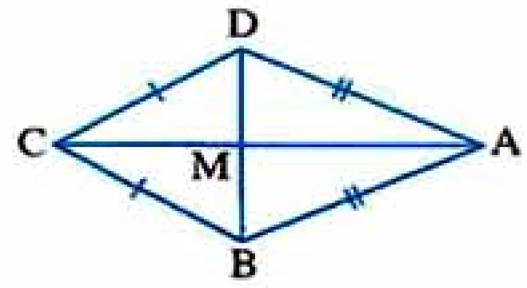
ABC is a triangle , AB = AC , XC = YB

, m (∠ B) = 50° , m (∠ CAX) = 30°

1 **Prove that :** Δ AXY is an isosceles triangle.

2 **Find :** m (∠ AYB)

[b] In the opposite figure :

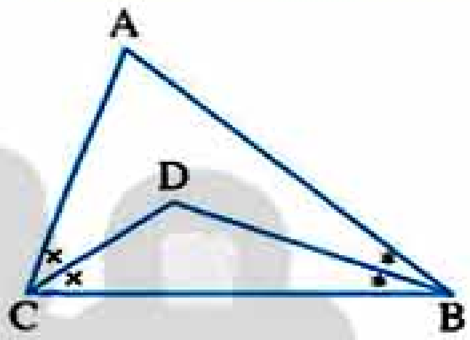


$\overline{BD} \cap \overline{AC} = \{M\}$

, AB = AD and BC = DC

Prove that : M is the midpoint of $\overline{BD}$

4 **[a] In the opposite figure :**

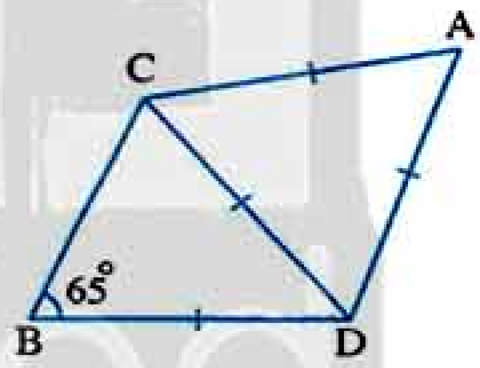


ABC is a triangle in which AB > AC , $\overrightarrow{BD}$ bisects ∠ ABC

, $\overrightarrow{CD}$ bisects ∠ ACB

Prove that : BD > CD

[b] In the opposite figure :

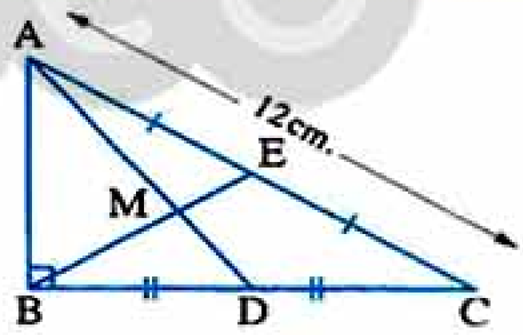


AD = DC = AC = BD

, m (∠ B) = 65°

Find with proof : m (∠ BDA)

5 **[a] In the opposite figure :**



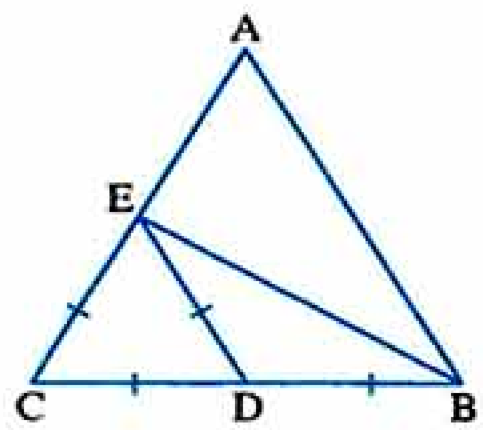
Δ ABC is right-angled at B

, E and D are the midpoints of $\overline{AC}$ and $\overline{BC}$ respectively

, AC = 12 cm.

Find the length of each of : $\overline{BE}$ and $\overline{ME}$

[b] In the opposite figure :



ABC is a triangle , D ∈ $\overline{BC}$ and E ∈ $\overline{AC}$

such that BD = CD = CE = DE

Prove that : **1** BC > BE

2 AB + BD > AE

3 Cairo Governorate

Rod El-Farag Educational Zone
S.T. Mary's School

Answer the following questions :

1 Choose the correct answer from the given ones :

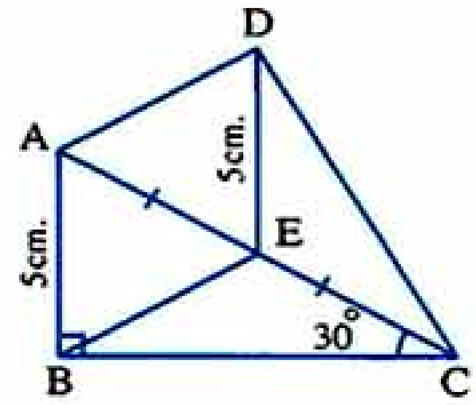
1. In the triangle XYZ , if m (∠ Z) = 70° and m (∠ Y) = 60° , then YZ .... XY
   (a) > (b) = (c) < (d) twice
2. The measure of the exterior angle of the equilateral triangle equals ....
   (a) 45° (b) 60° (c) 90° (d) 120°
3. The intersection point of the medians of a triangle divides each of them from the direction of the base in the ratio ....
   (a) 1 : 2 (b) 2 : 1 (c) 1 : 3 (d) 2 : 3
4. ABCD is a rectangle , M is the point of intersection of its diagonals , if the length of the diagonal is 6 cm. , then the length of the median $\overline{AM}$ equals .... cm.
   (a) 3 (b) 6 (c) 9 (d) 12
5. ABC is an isosceles triangle where AB = AC and m (∠ A) = 100° , then m (∠ B) = ...
   (a) 60° (b) 50° (c) 40° (d) 30°
6. The number of axes of symmetry of the isosceles triangle equals ....
   (a) 0 (b) 1 (c) 2 (d) 3

2 Complete :

1. If the measures of two angles of a triangle are different , then the greater in measure is opposite to ....
2. The bisector of the vertex angle of the isosceles triangle .... , ...
3. The base angles of the isosceles triangle are ....
4. In any triangle , the sum of the lengths of any two sides .... the length of the third side.
5. Δ ABC is right-angled at B , m (∠ A) = 30° , AC = 10 cm. , then CB = ... cm.

3 **[a]** ABC is a triangle in which AB = AC , $\overrightarrow{BD}$ bisects ∠ ABC , $\overrightarrow{CD}$ bisects ∠ ACB , $\overrightarrow{BD} \cap \overrightarrow{CD} = \{D\}$ **Prove that :** Δ DBC is an isosceles triangle.

[b] In the opposite figure :



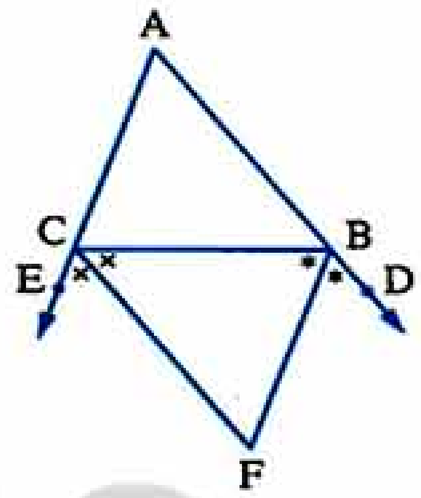
ABC is a right-angled triangle at B

, m (∠ ACB) = 30° , AB = 5 cm.

, E is the midpoint of $\overline{AC}$, if DE = 5 cm.

, prove that : m (∠ ADC) = 90°

4 **[a] In the opposite figure :**



ABC is a triangle in which AB > AC , D ∈ $\overrightarrow{AB}$, E ∈ $\overrightarrow{AC}$

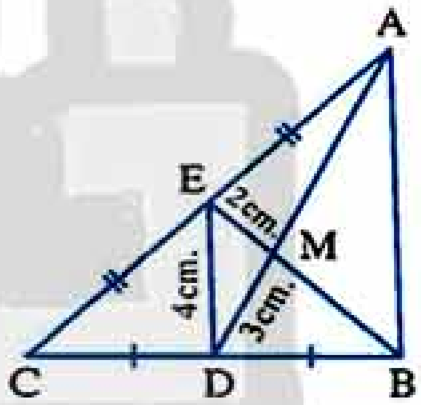
, $\overrightarrow{BF}$ bisects ∠ DBC , $\overrightarrow{CF}$ bisects ∠ BCE

, $\overrightarrow{BF} \cap \overrightarrow{CF} = \{F\}$

Prove that : 1 m (∠ FBC) > m (∠ BCF)

2 CF > BF

[b] In the opposite figure :

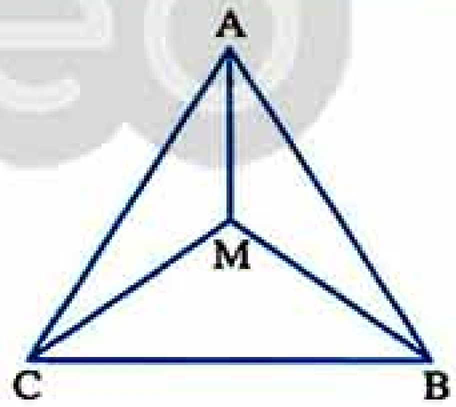


ABC is a triangle in which ME = 2 cm. , MD = 3 cm.

, DE = 4 cm. , D and E are the midpoints of $\overline{BC}$, $\overline{AC}$ respectively

Find : The perimeter of Δ MAB

5 **[a] In the opposite figure :**

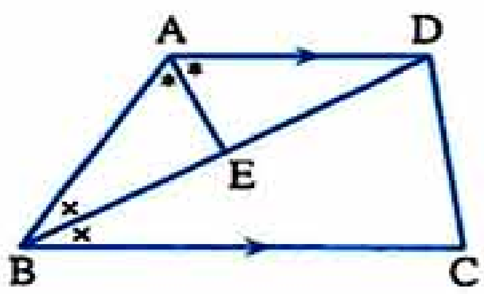


ABC is a triangle in which

M is a point inside it.

Prove that : MA + MB + MC > $\frac{1}{2}$ the perimeter of Δ ABC

[b] In the opposite figure :



ABCD is a quadrilateral in which $\overline{AD}$ // $\overline{BC}$

, $\overrightarrow{BD}$ bisects ∠ ABC , $\overrightarrow{AE}$ bisects ∠ BAD

Prove that : 1 AB = AD 2 $\overline{AE} \perp \overline{BD}$

3 BE = ED

4 Giza Governorate

Boulaq El Dakrour Directorate of Education
Dar El-Hanan Lang. Sch. for Girls

Answer the following questions :

1 Choose the correct answer :

1. The number of axes of symmetry of the isosceles triangle equals ....
   (a) 3 (b) 2 (c) 1 (d) 0
2. The point of intersection of the medians of the triangle divides each of them in the ratio of .... from the base.
   (a) 2 : 1 (b) 3 : 1 (c) 3 : 2 (d) 1 : 2
3. Δ XYZ is right-angled at Y , then XZ .... YZ
   (a) > (b) < (c) = (d) ≤
4. If 10 cm. , 5 cm. and x cm. are side lengths of an isosceles triangle , then x = ...
   (a) 10 (b) 5 (c) 15 (d) 4
5. The measure of the exterior angle of an equilateral triangle equals ....°
   (a) 30 (b) 60 (c) 90 (d) 120
6. **In the opposite figure :**
   $x + y$ = ...
   (a) 100° (b) 140°
   (c) 180° (d) 280°

2 Complete the following :

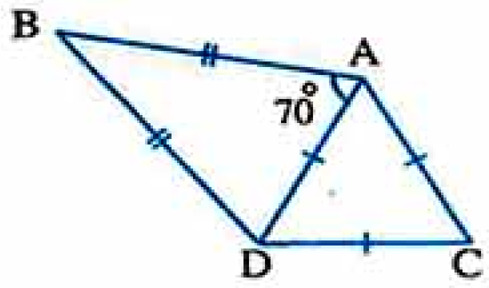
1. In Δ ABC , if m ($\angle$ B) = 70° , m ($\angle$ C) = 50° , then AC .... AB
2. In Δ ABC , if m ($\angle$ A) = m ($\angle$ B) + m ($\angle$ C) , then the longest side is ....
3. The axis of symmetry of a line segment is the straight line which .... from its midpoint.
4. ABC is a triangle in which AB = 4 cm. , CB = 7 cm.
   , then AC ∈] .... , ... [
5. If $\overline{AD}$ is a median in Δ ABC , and M is the point of intersection of its medians and AM = 12 cm. , then AD = ...

3 **[a] In the opposite figure :**

$AB = BD$, $m(\angle BAD) = 70°$

, ΔADC is an equilateral triangle.

Find : $m(\angle BDC)$

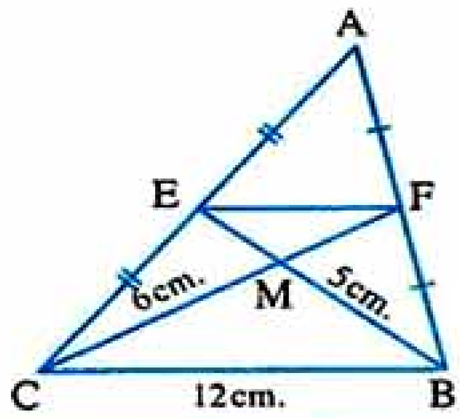


[b] In the opposite figure :

ABC is a triangle , F and E are the midpoints of $\overline{AB}$ and $\overline{AC}$ respectively.

If $BM = 5$ cm. , $CM = 6$ cm. , $BC = 12$ cm.

, **then find :** The perimeter of ΔMEF



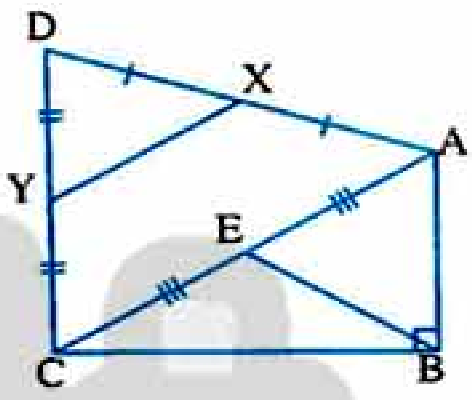
4 **[a] In the opposite figure :**

$m(\angle ABC) = 90°$

, E is the midpoint of $\overline{AC}$

and X , Y are the midpoints of $\overline{DA}$ and $\overline{DC}$

Prove that : $XY = BE$



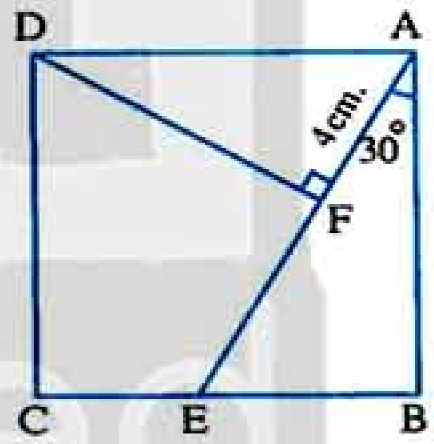
[b] In the opposite figure :

ABCD is a square , $E \in \overline{BC}$

where $m(\angle BAE) = 30°$ and $\overline{DF} \perp \overline{AE}$

, if $AF = 4$ cm.

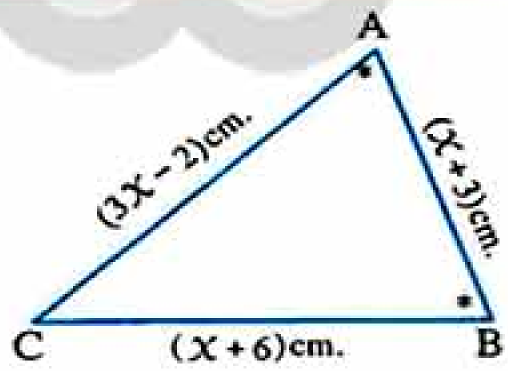
, **calculate :** The area of the square ABCD



5 **[a] In the opposite figure :**

$m(\angle A) = m(\angle B)$

Find : The perimeter of ΔABC



[b] In the opposite figure :

ABC is a triangle in which :

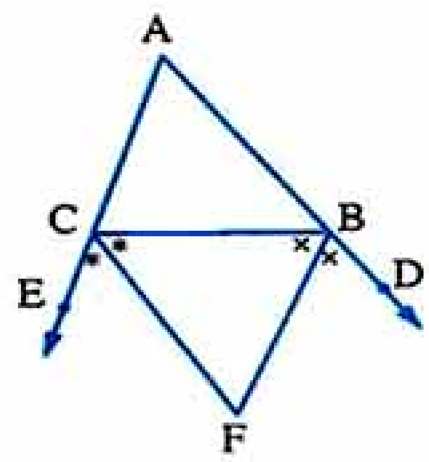
$AB > AC$, $D \in \overrightarrow{AB}$, $E \in \overrightarrow{AC}$

, $\overrightarrow{BF}$ bisects $\angle DBC$, $\overrightarrow{CF}$ bisects $\angle BCE$

, $\overrightarrow{BF} \cap \overrightarrow{CF} = \{F\}$

Prove that : $\boxed{1}$ $m(\angle FBC) > m(\angle BCF)$

$\boxed{2}$ $CF > BF$



5 Giza Governorate

6th October Directorate
Om El-Moamneen Lang. School

Answer the following questions :

1 Choose the correct answer :

1. If ABC is an isosceles triangle , m (∠ A) = 60° , AB = 4 cm.
, then its perimeter = ... cm.
(a) 4 (b) 12 (c) 6 (d) 9

2. XYZ is a triangle in which m (∠ Z) = 70° , m (∠ Y) = 60° , then YZ .... XY
(a) > (b) < (c) = (d) ≥

3. In Δ ABC , if m (∠ B) = 90° , then the longest side is ....
(a) $\overline{BC}$ (b) $\overline{AB}$ (c) $\overline{AC}$ (d) its median.

4. A triangle has one axis of symmetry , the lengths of two sides are 4 cm. and 8 cm.
, then the length of the third side is .... cm.
(a) 3 (b) 6 (c) 4 (d) 8

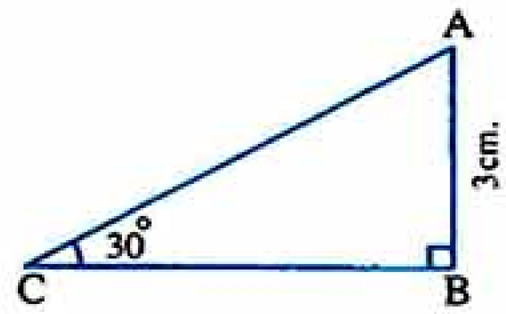
5. The point of intersection of the medians of the triangle divides each of the medians in the ratio .... from the base.
(a) 2 : 1 (b) 3 : 2 (c) 2 : 4 (d) 3 : 4

6. If the length of any side of a triangle = $\frac{1}{3}$ the perimeter of the triangle , then the number of axes of symmetry of the triangle equals ....
(a) 3 (b) 1 (c) 2 (d) zero

2 Complete :

1. The bisector of the vertex angle of the isosceles triangle .... and ....

2. **In the opposite figure :**
The length of $\overline{AC}$ = ...

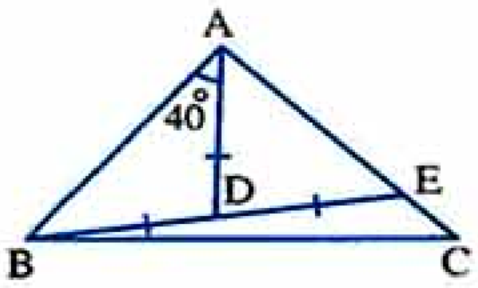


3. In Δ ABC , m (∠ A) = m (∠ B) = m (∠ C) , then the measure of the exterior angle equals ....

4. If the lengths of two sides of a triangle are 4 cm. , 7 cm. , then the length of the third side belongs to] .... , ... [

5. If ∠ X and ∠ Y are two supplementary angles , ∠ X ≡ ∠ Y , then m (∠ X) = ...°

3 **[a] In the opposite figure :**

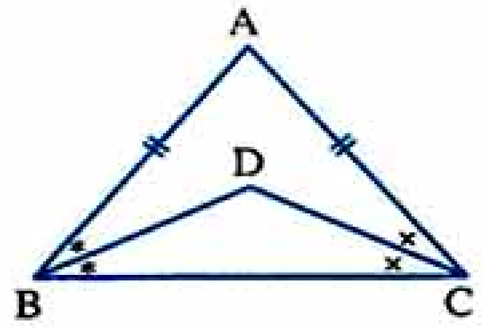


AD = BD = ED , m (∠ DAB) = 40°

Prove that :

1 AD < AB **2** BC > AC

[b] In the opposite figure :



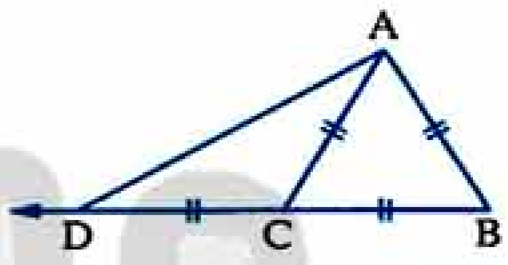
AB = AC , $\overrightarrow{BD}$ bisects ∠ ABC

and $\overrightarrow{CD}$ bisects ∠ ACB

Prove that : Δ DBC is an isosceles triangle.

4 **[a]** ABC is a triangle in which m (∠ A) = (6 x)° , m (∠ B) = (4 x – 9) °
, m (∠ C) = 3 (x – 2)° Arrange the lengths of the sides of the triangle ascendingly.

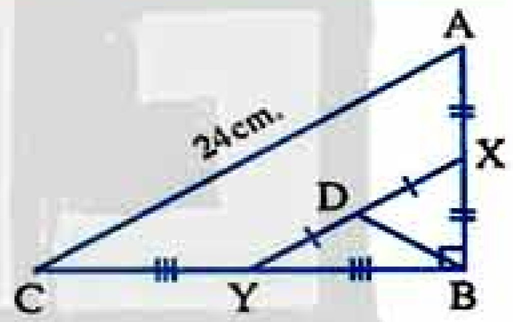
[b] In the opposite figure :



AB = AC = CB = CD

Prove that : $\overline{AB} \perp \overline{AD}$

5 **[a] In the opposite figure :**



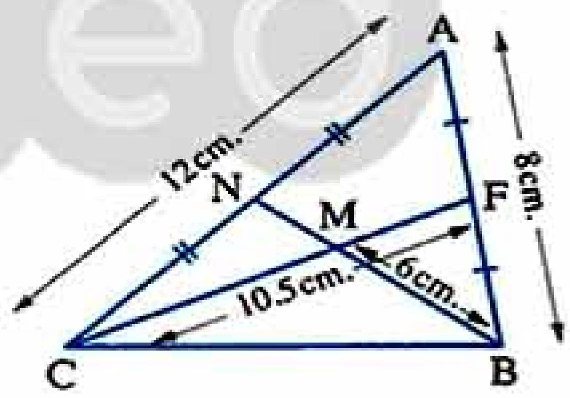
m (∠ ABC) = 90° , X is the midpoint of $\overline{AB}$

, Y is the midpoint of $\overline{BC}$

, D is the midpoint of $\overline{XY}$, AC = 24 cm.

Find : The length of $\overline{BD}$

[b] In the opposite figure :



F and N are the midpoints of $\overline{AB}$ and $\overline{AC}$ respectively

, AB = 8 cm. , AC = 12 cm. , BM = 6 cm.

, CF = 10.5 cm.

Find : The perimeter of the figure AFMN

6 Alexandria Governorate

Middle Educational Zone
Math Supervision

Answer the following questions :

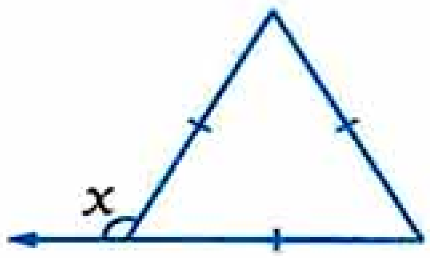
1 **Complete each of the following :**

1 If m (∠ A) = 65° , then m (complementary ∠ A) = ...°

2 In Δ ABC , m (∠ A) = 50° , m (∠ C) = 80° , then CB = ...

3 **In the opposite figure :**

$x =$ .... °



4 The number of axes of symmetry for the rectangle equals ....

5 In Δ ABC , m ($\angle$ B) = 70° , m ($\angle$ C) = 45° , then BC .... AC

6 The medians of the triangle are ....

2 **Choose the correct answer :**

1 The sum of lengths of two sides in a triangle is .... the length of the third side.

(a) > (b) < (c) = (d) twice

2 The triangle which has no axis of symmetry is ....

(a) scalene. (b) isosceles. (c) equilateral. (d) right-angled.

3 The numbers which can not be side lengths of a triangle are ....

(a) 3 , 3 , 3 (b) 3 , 3 , 4 (c) 3 , 3 , 5 (d) 3 , 3 , 6

4 $\overline{BE}$ is a median in Δ ABC , M is the point of concurrence of the medians

If BM = 6 cm. , then ME = ... cm.

(a) 2 (b) 3 (c) 4 (d) 9

5 The angle whose measure is 180° is called .... angle.

(a) an acute (b) an obtuse (c) a straight (d) a reflex

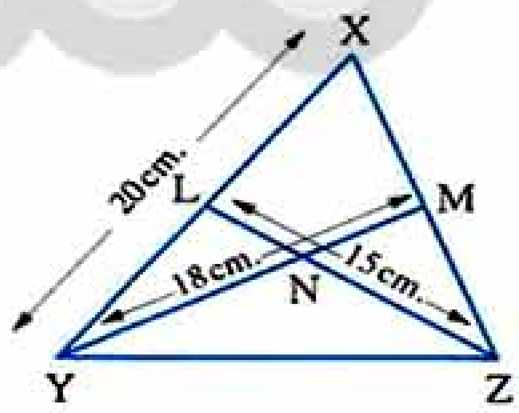
3 **[a]** Δ ABC is right-angled at B , if m ($\angle$ A) = 75° , arrange the lengths of its sides descendingly.

[b] **In the opposite figure :**

N is the point of concurrence of the medians of Δ XYZ

, LZ = 15 cm. , YM = 18 cm. , XY = 20 cm.

Find : The perimeter of Δ NLY



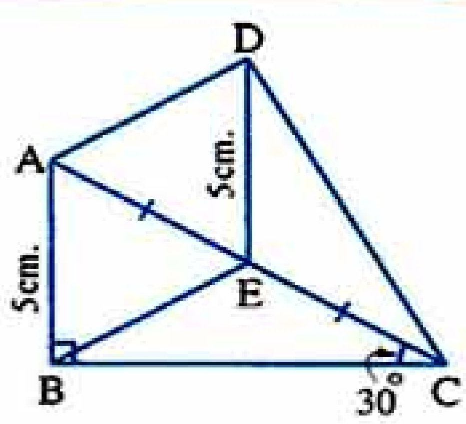
4 **[a]** **In the opposite figure :**

m ($\angle$ ABC) = 90° , E is the midpoint of $\overline{AC}$

, m ($\angle$ ACB) = 30°

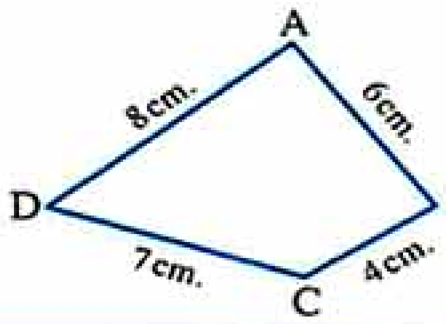
, AB = DE = 5 cm.

Prove that : m ($\angle$ ADC) = 90°



[b] In the opposite figure :

Prove that : m (∠ BCD) > m (∠ BAD)



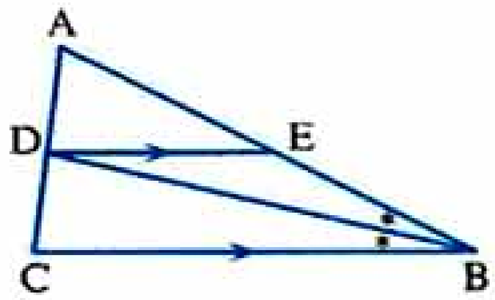
5 **[a] In the opposite figure :**

$\overrightarrow{BD}$ bisects ∠ ABC

, $\overline{DE}$ // $\overline{BC}$

Prove that :

Δ EBD is an isosceles triangle.

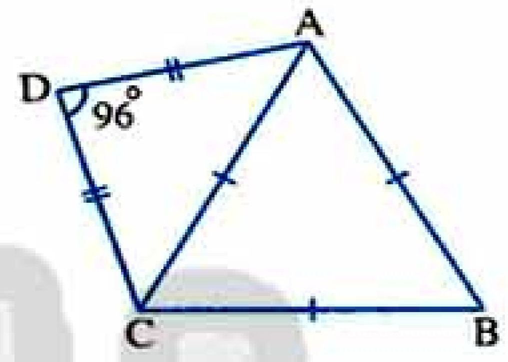


[b] In the opposite figure :

Δ ABC is equilateral , DA = DC

, m (∠ ADC) = 96°

Find : m (∠ DAB)



7 Alexandria Governorate

Agemy Educational Zone
Inspector of Maths

Answer the following questions :

1 **Choose the correct answer :**

1. XYZ is a triangle in which m (∠ Z) = 70° , m (∠ Y) = 60° , then YZ .... XY
   (a) > (b) < (c) = (d) twice
2. The two diagonals are perpendicular in the ....
   (a) rectangle. (b) rhombus. (c) trapezium. (d) triangle.
3. The measure of the exterior angle of the equilateral triangle equals ....°
   (a) 360 (b) 120 (c) 60 (d) 180
4. If the lengths of two sides in an isosceles triangle are 3 cm. , 7 cm. , then the length of the third side is .... cm.
   (a) 3 (b) 7 (c) 10 (d) 4
5. The point of concurrence of the medians of the triangle divides each median in the ratio .... from its base.
   (a) 2 : 1 (b) 1 : 3 (c) 1 : 4 (d) 1 : 2
6. If the side length of an equilateral triangle is 10 cm. , then its height equals .... cm.
   (a) 5 (b) 10 (c) $5\sqrt{3}$ (d) 30

2 **Complete :**

1 If the isosceles triangle has an angle of measure 45° , then the triangle is …………… - angled triangle.

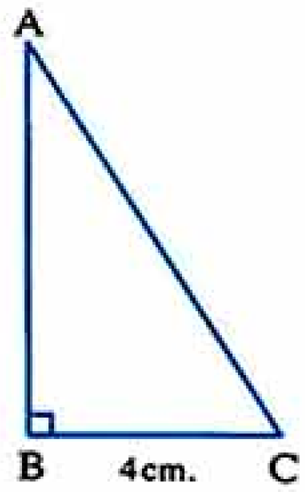
2 The sum of lengths of any two sides of a triangle is …………… the length of the third side.

3 **In the opposite figure :**

If m (∠ C) = 2 m (∠ A)

, CB = 4 cm.

, then AC = …………… cm.



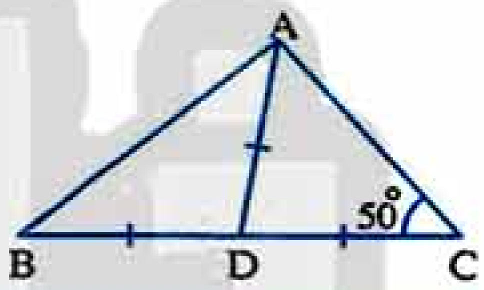
4 If the two side lengths in a triangle are 4 cm. , 7 cm. , then the length of the third side ∈]…………… , ……………[

5 **In the opposite figure :**

AD = DC = BD

, m (∠ C) = 50°

, then m (∠ B) = ……………°

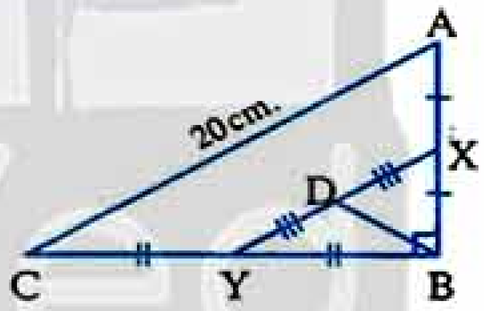


3 **[a] In the opposite figure :**

m (∠ ABC) = 90° , D is the midpoint of $\overline{XY}$

, X , Y are the midpoints of $\overline{AB}$, $\overline{BC}$ respectively , AC = 20 cm.

Find : The length of $\overline{BD}$



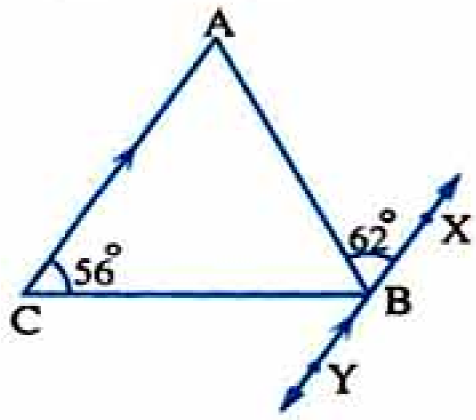
[b] In the opposite figure :

B ∈ $\overleftrightarrow{XY}$, $\overleftrightarrow{XY}$ // $\overline{AC}$

, m (∠ ABX) = 62°

and m (∠ C) = 56°

Prove that : AC = BC

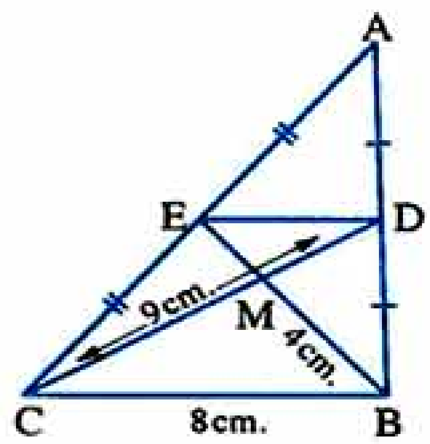


4 **[a] In the opposite figure :**

D , E are the midpoints of $\overline{AB}$ and $\overline{AC}$ respectively

, DC = 9 cm. , MB = 4 cm. and BC = 8 cm.

Find : The perimeter of Δ DME



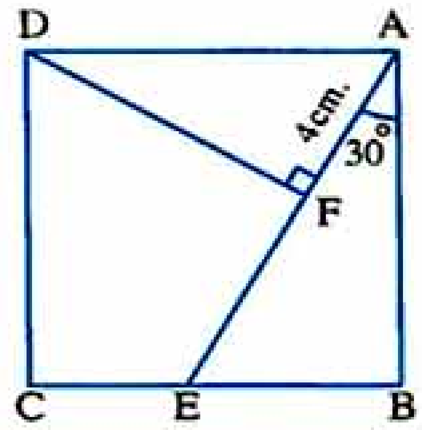
[b] In the opposite figure :

ABCD is a square , E $\in \overline{BC}$

, where m ($\angle$ BAE) = 30° and $\overline{DF} \perp \overline{AE}$

, if AF = 4 cm.

, **calculate :** The area of the square ABCD

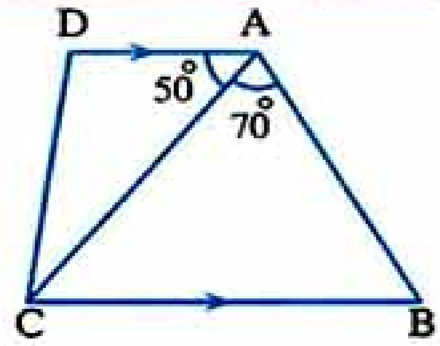


5 **[a] In the opposite figure :**

$\overline{AD}$ // $\overline{BC}$, m ($\angle$ CAB) = 70°

, m ($\angle$ DAC) = 50°

Prove that : BC > AC

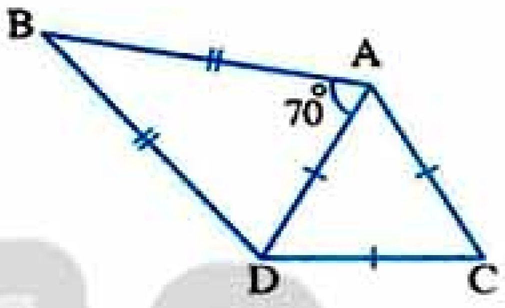


[b] In the opposite figure :

AB = BD , m ($\angle$ BAD) = 70°

, Δ ADC is equilateral

Find : m ($\angle$ BDC)



8 El-Kalyoubia Governorate

Directorate of Education
Inspection of Mathematics

Answer the following questions :

1 **Choose the correct answer :**

1. ABC is an equilateral triangle , then m ($\angle$ A) = ...°

   (a) 45 (b) 60 (c) 120 (d) 35

2. Δ XYZ is an isosceles triangle , m ($\angle$ X) = 100° , then m ($\angle$ Y) = ...°

   (a) 100 (b) 80 (c) 60 (d) 40

3. The length of the side opposite to the angle of measure 30° in the right-angled triangle equals .... the length of the hypotenuse.

   (a) $\frac{1}{2}$ (b) $\frac{2}{3}$ (c) $\frac{1}{4}$ (d) 2

4. The number of axes of symmetry of the isosceles triangle equals ....

   (a) 0 (b) 1 (c) 2 (d) 3

5. If the lengths of two sides of an isosceles triangle are 2 cm. , 5 cm. , then the length of the third side equals .... cm.

   (a) 2 (b) 3 (c) 4 (d) 5

6. In the triangle ABC , if m ($\angle$ A) = 50° , m ($\angle$ B) = 60° , then the longest side is ....

   (a) $\overline{AB}$ (b) $\overline{BC}$ (c) $\overline{AC}$ (d) 110 cm.

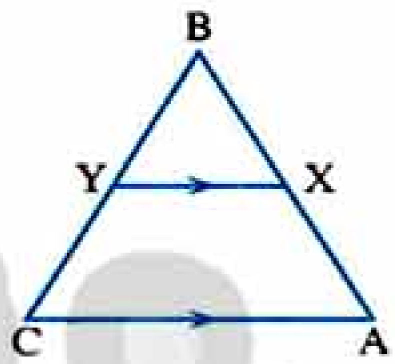
2 Complete :

1. The medians of a triangle are ……………
2. The longest side of the right-angled triangle is the ……………
3. If AB = AC in the triangle ABC , then ABC is …………… triangle.
4. XYZ is a triangle , m (∠ Z) = 40° , m (∠ Y) = 30° , then XY …………… XZ
5. If the lengths of two sides of a triangle are 6 cm. and 9 cm. , then the length of the third side ∈]…………… , ……………[

3 **[a]** In Δ ABC , m (∠ A) = 40° , m (∠ B) = 75° , m (∠ C) = 65°

Arrange the lengths of the sides of this triangle descendingly.

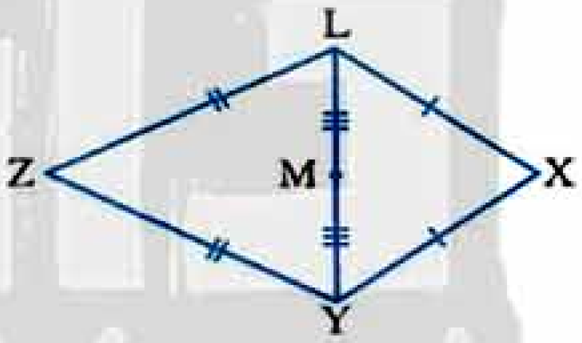
[b] In the opposite figure :



AB = BC , $\overline{XY}$ // $\overline{AC}$

Prove that : BX = BY

4 **[a] In the opposite figure :**

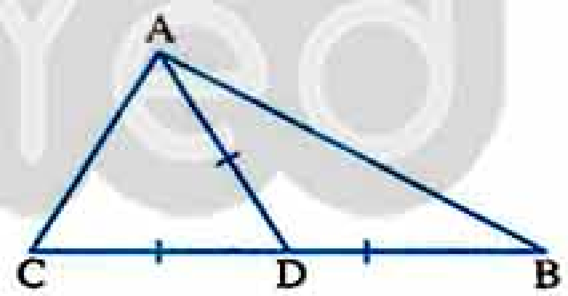


XY = XL , ZY = ZL

, LM = MY

Prove that : X , M , Z are on the same straight line.

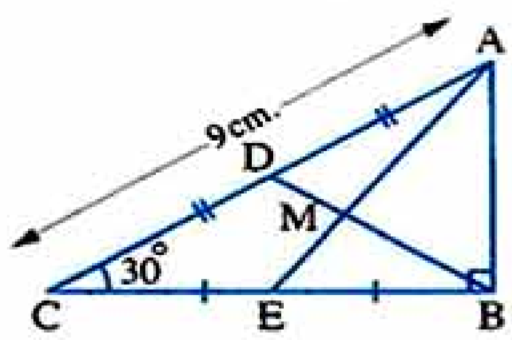
[b] In the opposite figure :



AB > AC , DB = DC = AD

Prove that : m (∠ BAD) < m (∠ CAD)

5 **[a] In the opposite figure :**



Δ ABC is a right-angled triangle at B

, m (∠ C) = 30° , D is the midpoint of $\overline{AC}$

, E is the midpoint of $\overline{BC}$, AC = 9 cm.

Find the length of each of : $\overline{BD}$, $\overline{BM}$, $\overline{AB}$, $\overline{MD}$

[b] ABC is a triangle such that

m (∠ A) = $(2x)°$, m (∠ C) = $(x + 40)°$, m (∠ B) = $(3x - 10)°$

Prove that : AB = AC

9 El-Sharkia Governorate

Zagazig English Language School for Girls

Answer the following questions :

1 Choose the correct answer :

1. In Δ ABC , m ($\angle$ A) = 60° , m ($\angle$ C) = 45° , then ....
 (a) AB < AC (b) AB = AC (c) AB > AC (d) AB = BC

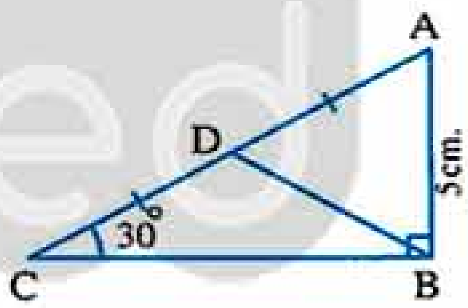
2. If M is the point of concurrence of the medians of Δ ABC , $\overline{AD}$ is a median , then MA = ...
 (a) 2 AD (b) $\frac{2}{3}$ AD (c) $\frac{3}{2}$ AD (d) $\frac{1}{2}$ MD

3. In Δ ABC , AB = 4 cm. , BC = 6 cm. , then AC $\in$ ....
 (a)]2 , 4[(b) [2 , 10] (c)]2 , 10[(d) [0 , 10]

4. The number of axes of symmetry of the equilateral triangle equals ....
 (a) zero (b) 1 (c) 2 (d) 3

5. In Δ ABC , AB = AC , m ($\angle$ B) = x + 30° , m ($\angle$ C) = 2 x + 5° , then x = ...
 (a) 25° (b) 20° (c) 35° (d) 3°

6. **In the opposite figure :**
 AD = DC , m ($\angle$ C) = 30° , m ($\angle$ ABC) = 90° , AB = 5 cm. , then the perimeter of Δ ABD = ... cm.



 (a) 5 (b) 15
 (c) 20 (d) 25

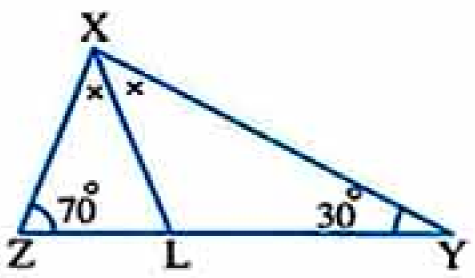
2 Complete :

1. ABCD is a rectangle , AB = 3 cm. , BC = 4 cm. , then BD = ... cm.
2. In Δ ABC , if D is the midpoint of $\overline{BC}$ and AD = $\frac{1}{2}$ BC , then m ($\angle$ CAB) = ...°
3. The longest side in the right-angled triangle is ....
4. If Δ ABC $\equiv$ Δ XYZ , then AC – XZ = ...
5. The median that is drawn from the vertex angle of an isosceles triangle .... and ....

3 [a] **In the opposite figure :**

$\overrightarrow{XL}$ bisects $\angle$ YXZ , m ($\angle$ Y) = 30°

, m ($\angle$ Z) = 70°



1 **Find :** m ($\angle$ LXZ) and m ($\angle$ XLZ)

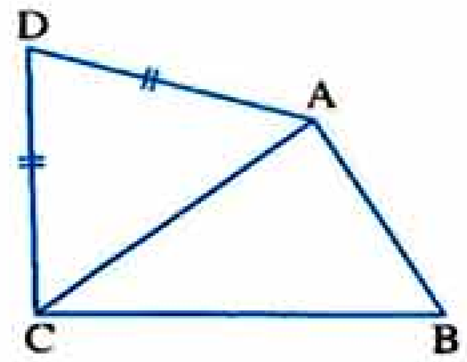
2 **Prove that :** Δ XLZ is an isosceles triangle.

[b] **In the opposite figure :**

ABCD is a quadrilateral

, AD = DC , BC > AB

Prove that : m ($\angle$ BAD) > m ($\angle$ BCD)

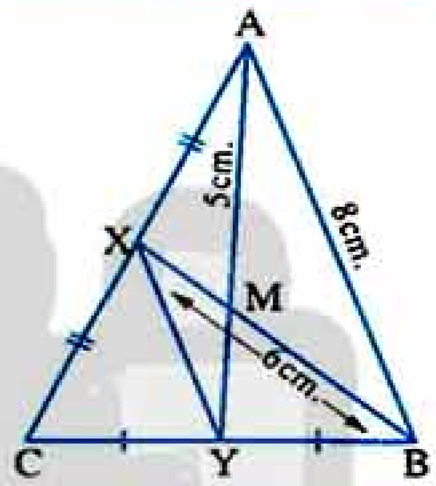


4 [a] **In the opposite figure :**

X is the midpoint of $\overline{AC}$, AB = 8 cm.

, Y is the midpoint of $\overline{BC}$, AM = 5 cm. , BX = 6 cm.

Find : The perimeter of Δ XMY



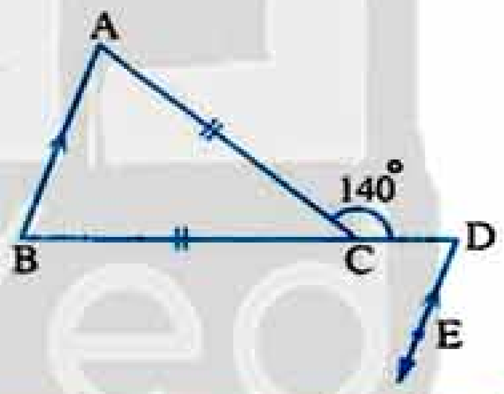
[b] **In the opposite figure :**

C $\in \overrightarrow{BD}$, CA = CB

, $\overrightarrow{AB}$ // $\overrightarrow{DE}$

, m ($\angle$ ACD) = 140°

Find : m ($\angle$ A) and m ($\angle$ BDE)



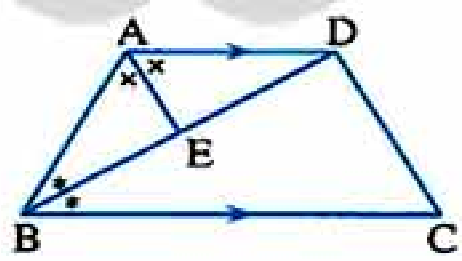
5 [a] **In the opposite figure :**

ABCD is a quadrilateral , $\overline{AD}$ // $\overline{BC}$

, $\overrightarrow{BD}$ bisects $\angle$ ABC

, $\overrightarrow{AE}$ bisects $\angle$ BAD

Prove that : **1** AD = AB **2** $\overline{AE} \perp \overline{BD}$



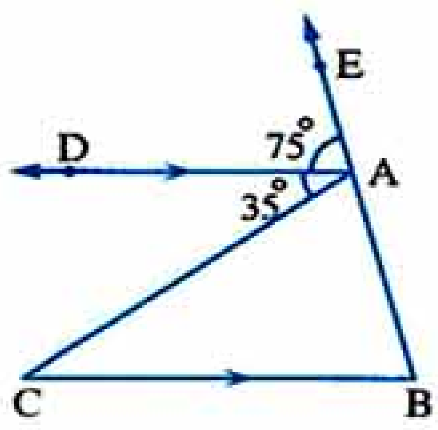
[b] **In the opposite figure :**

E $\in \overrightarrow{BA}$, $\overrightarrow{AD}$ // $\overrightarrow{BC}$

, m ($\angle$ DAE) = 75°

, m ($\angle$ DAC) = 35°

Prove that : BC > AB



10 El-Monofia Governorate

El-Shohadaa Directorate
Maths Supervision

Answer the following questions :

1 Choose the correct answer :

1. The intersecting point of the medians of the triangle divides each median in the ratio of .... from its base.
   (a) 1 : 2 (b) 2 : 1 (c) 3 : 1 (d) 1 : 3
2. The number of symmetry axes of the isosceles triangle is ....
   (a) 1 (b) 2 (c) 3 (d) 4
3. The sum of lengths of any two sides of a triangle .... the length of the third side.
   (a) < (b) > (c) = (d) ≡
4. The diagonals are perpendicular in the ....
   (a) trapezium. (b) parallelogram. (c) square. (d) rectangle.
5. If Δ ABC is right-angled at B , AB = 6 cm. , BC = 8 cm. , then the length of the median drawn from B equals .... cm.
   (a) 3 (b) 4 (c) 5 (d) 6
6. If 4 cm. , $(X+3)$ cm. and 8 cm. are side lengths of an isosceles triangle , then X = ...
   (a) 3 (b) 4 (c) 5 (d) 6

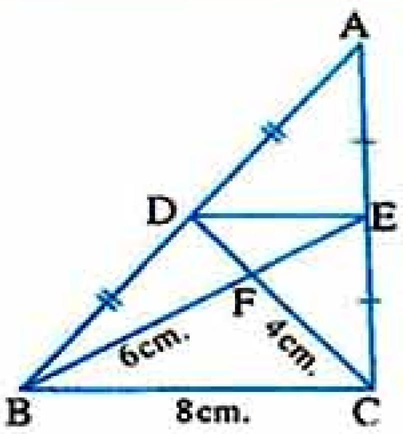
2 Complete each of the following :

1. The base angles in an isosceles triangle are ....
2. If m ($\angle$ A) = 100° , then m (reflex $\angle$ A) = ...°
3. The number of medians of the isosceles triangle is ....
4. In Δ ABC , if AB > BC , then m ($\angle$ A) .... m ($\angle$ C)
5. The bisector of the vertex angle of an isosceles triangle bisects the base and ....

3 [a] In the opposite figure :

ABC is a triangle in which D , E are the midpoints of $\overline{AB}$, $\overline{AC}$, FC = 4 cm. , FB = 6 cm. and BC = 8 cm.

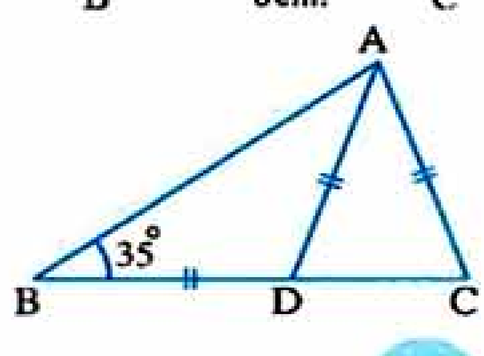
Find : The perimeter of Δ DFE



[b] In the opposite figure :

AC = AD = BD
, m ($\angle$ B) = 35°

Find : m ($\angle$ BAC)

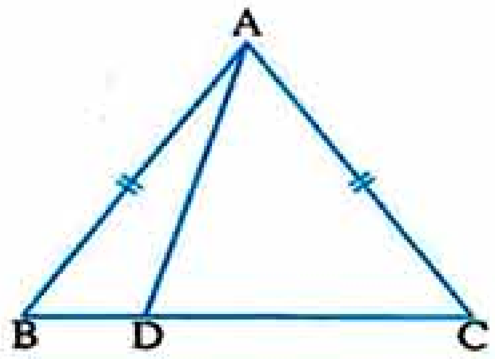


4 **[a] In the opposite figure :**

AC = AB

Prove that :

AB > AD



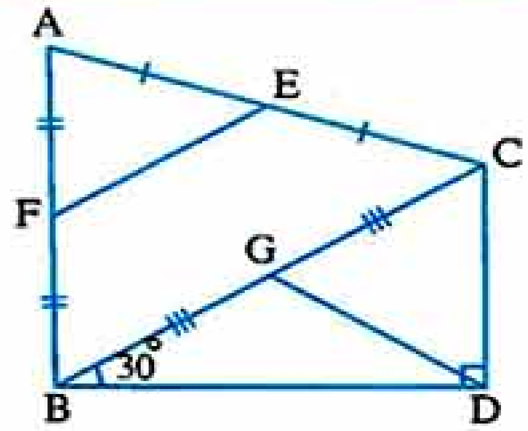
[b] ABC is a triangle in which m (∠ A) = 40° , m (∠ B) = 80° Arrange the lengths of the sides of the triangle descendingly.

5 **In the opposite figure :**

F , E , G are the midpoints of $\overline{AB}$, $\overline{AC}$, $\overline{BC}$

, m (∠ BDC) = 90° , m (∠ CBD) = 30°

, BC = 10 cm.



1 **Prove that :** FE = DC = GD

2 **Find :** The perimeter of Δ GCD

11 El-Dakahlia Governorate

Talkha Educational Directorate
A.M.D.L School

Answer the following questions :

1 **Choose the correct answer from the given ones :**

1 The numbers 4 , x + 4 , 8 can be lengths of sides of an isosceles triangle if x = ...

(a) 4 (b) 0 (c) 3 (d) 8

2 In Δ LMN , if m (∠ M) = 55° , m (∠ N) = 80° , then LM .... MN

(a) < (b) > (c) = (d) twice

3 The measure of the exterior angle of the equilateral triangle equals ....

(a) 30° (b) 60° (c) 90° (d) 120°

4 If $\overline{AD}$ is a median of Δ ABC , and M is the point of concurrence of the medians , then AD = ... AM

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) $\frac{1}{2}$ (d) $\frac{3}{2}$

5 The base angles of the isosceles triangle are ....

(a) alternate (b) corresponding (c) congruent (d) supplementary

6 If XA = XB , YA = YB , then $\overleftrightarrow{XY}$ .... $\overline{AB}$

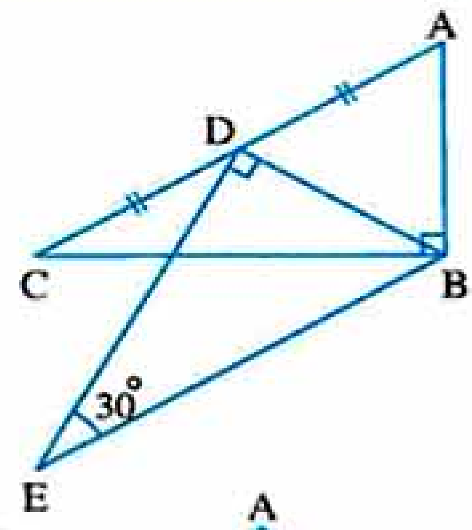
(a) ⊥ (b) ≡ (c) // (d) =

2 **Complete the following :**

1. The number of axes of symmetry of the isosceles triangle is ....
2. The bisector of the vertex angle of the isosceles triangle ....
3. The medians of the triangle intersect at ....
4. The longest side in the right-angled triangle is the ....
5. In Δ ABC , if AB = AC , m ($\angle$ C) = 40° , then m ($\angle$ A) = ...°

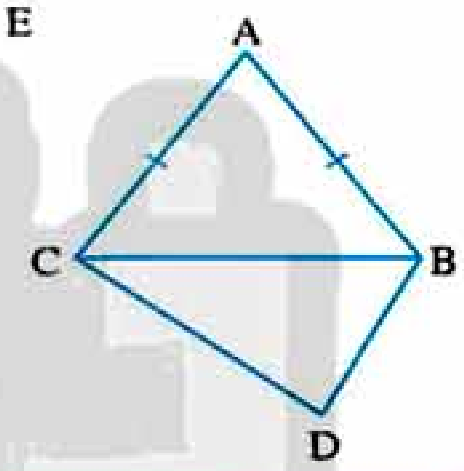
3 **[a] In the opposite figure :**

m ($\angle$ ABC) = m ($\angle$ BDE) = 90°
, m ($\angle$ E) = 30°
, D is the midpoint of $\overline{AC}$
Prove that : AC = BE



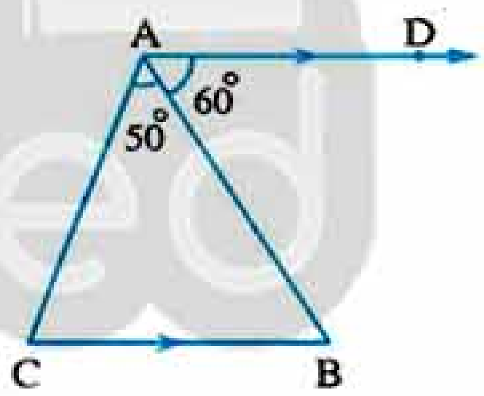
[b] In the opposite figure :

AB = AC , DC > DB
Prove that :
m ($\angle$ ABD) > m ($\angle$ ACD)



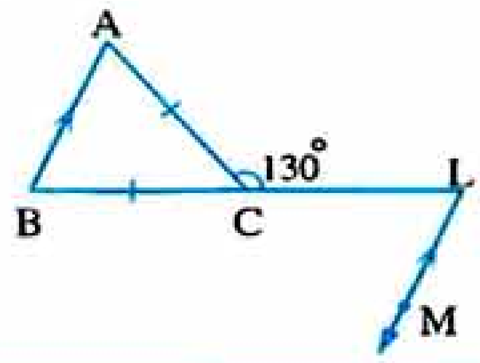
4 **[a] In the opposite figure :**

ABC is a triangle , $\overrightarrow{AD}$ // $\overline{CB}$
, m ($\angle$ DAB) = 60° , m ($\angle$ BAC) = 50°
Prove that : AB > AC



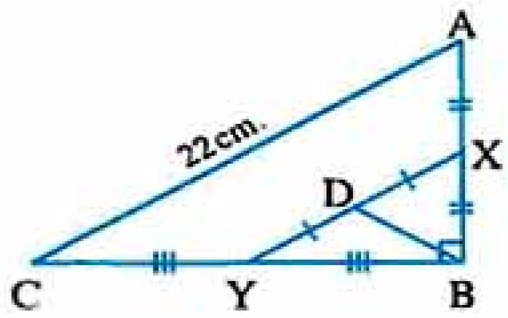
[b] In the opposite figure :

C $\in \overleftrightarrow{LB}$, AC = BC
, m ($\angle$ LCA) = 130°
, $\overleftrightarrow{LM}$ // $\overleftrightarrow{AB}$
Find : m ($\angle$ MLC)



5 **[a] In the opposite figure :**

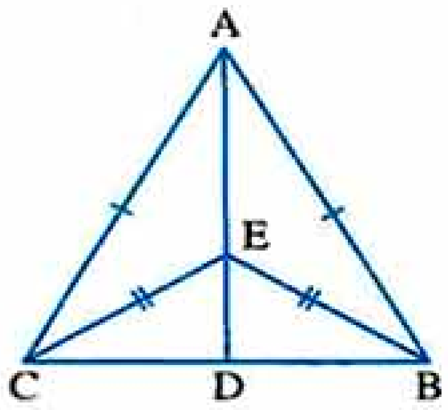
m ($\angle$ ABC) = 90° , X , Y , D
are the midpoints of $\overline{AB}$, $\overline{BC}$, $\overline{XY}$
respectively , if AC = 22 cm.
, **find :** BD



[b] In the opposite figure :

AB = AC , EB = EC

Prove that : BD = CD



12 Suez Governorate

Directorate of Education
Inspection of Mathematics

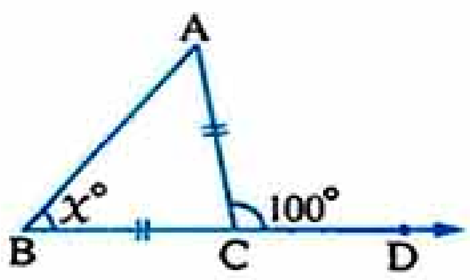
Answer the following questions :

1 Complete :

1. The base angles in an isosceles triangle are ....
2. If the angles of a triangle are congruent , then the triangle is ....
3. In Δ ABC , if m ($\angle$ A) = 70° , m ($\angle$ B) = 50° , then the longest side is ....
4. The point of concurrence of the medians of the triangle divides each median in the ratio of .... : .... from its vertex.
5. In Δ ABC , if m ($\angle$ A) = 30° and m ($\angle$ B) = 90° , then AC = ... BC

2 Choose the correct answer :

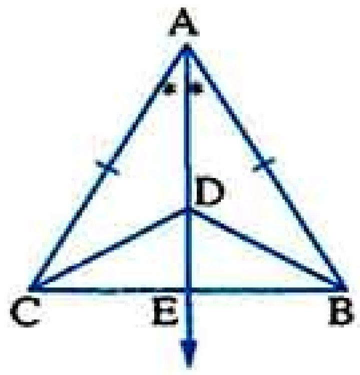
1. The triangle which has three axes of symmetry is ....
   (a) scalene. (b) isosceles. (c) right-angled. (d) equilateral.
2. If the lengths of two sides in an isosceles triangle are 3 cm. and 7 cm. , then the length of the third side equals .... cm.
   (a) 3 (b) 4 (c) 6 (d) 7
3. XYZ is a triangle in which m ($\angle$ Z) = 70° and m ($\angle$ Y) = 60° , then YZ .... XY
   (a) > (b) < (c) = (d) twice
4. **In the opposite figure :**
   CA = CB , m ($\angle$ B) = x° , m ($\angle$ ACD) = 100° where C $\in \overline{BD}$, then x = ...



   (a) 50° (b) 100° (c) 150° (d) 200°
5. In Δ ABC , if AB = AC and $\overline{AD}$ is a median , then $\overline{AD}$ .... $\overline{BC}$
   (a) $\equiv$ (b) $\perp$ (c) $\subset$ (d) //
6. In Δ ABC , if AB = 3 cm. , BC = 5 cm. , then AC $\in$ ....
   (a)]2 , 8[(b)]2 , 7[(c)]2 , 15[(d)]8 , 15[

3 **[a]** ABC is a triangle in which m (∠ A) = 40° , m (∠ B) = 75° Arrange the lengths of sides of the triangle descendingly.

[b] In the opposite figure :

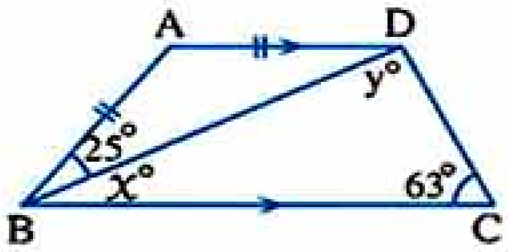


AB = AC , $\overrightarrow{AE}$ bisects ∠ BAC

, $\overline{AE} \cap \overline{BC} = \{E\}$, D ∈ $\overline{AE}$

Prove that : BD = CD

4 **[a] In the opposite figure :**



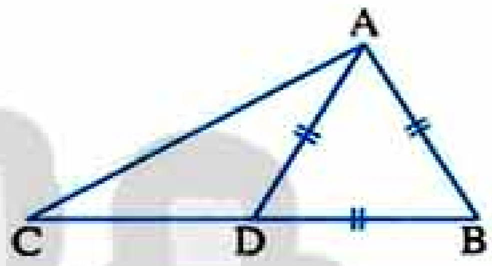
$\overline{AD}$ // $\overline{BC}$, AD = AB

, m (∠ ABD) = 25° , m (∠ C) = 63°

, m (∠ DBC) = x° , m (∠ CDB) = y°

Find the value of each of : x and y

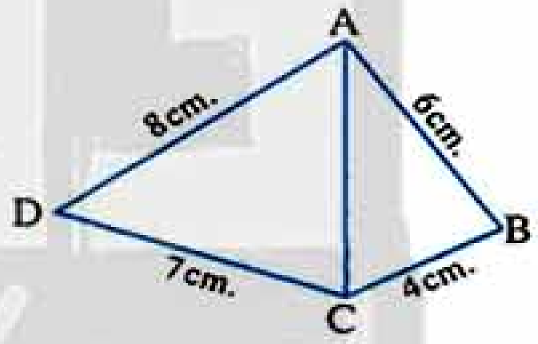
[b] In the opposite figure :



AB = BD = DA

Prove that : BC > AC

5 **[a] In the opposite figure :**



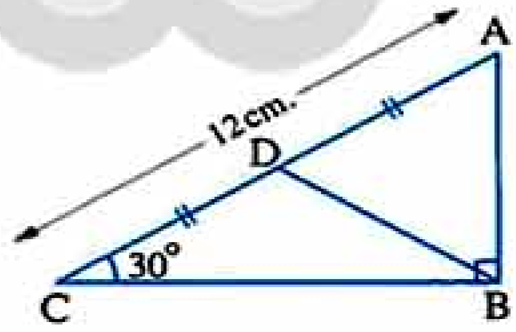
ABCD is a quadrilateral

, AB = 6 cm. , BC = 4 cm.

, CD = 7 cm. , AD = 8 cm.

Prove that : m (∠ BCD) > m (∠ BAD)

[b] In the opposite figure :



ABC is a triangle , m (∠ ABC) = 90°

, D is the midpoint of $\overline{AC}$

, AC = 12 cm. , m (∠ C) = 30°

, then find : The perimeter of Δ ABD

13 El-Beheira Governorate

Damanhur Directorate
Al-Farabi Language School

Answer the following questions :

1 **Complete the following :**

1 The length of the side opposite to the angle of measure 30° in the right-angled triangle equals .... the length of the hypotenuse.

2 If $\overline{AD}$ is a median in Δ ABC , M is the point of intersection of its medians and AM = 12 cm. , then AD = ...

3 The number of axes of symmetry of the isosceles triangle equals ....

4 In a triangle , if two angles are unequal in measure , then the greater angle in measure is opposite to ....

5 If $\overline{AB} \equiv \overline{XY}$ and AB = 5 cm. , then 2 AB – XY = ...

2 Choose the correct answer :

1 The measure of one of the base angles in the isosceles triangle is 65° , then the measure of its vertex angle equals ....°

(a) 65 (b) 50 (c) 130 (d) 55

2 If 4 cm. , $(x + 3)$ cm. and 8 cm. are side lengths of an isosceles triangle , then x = ...

(a) 4 (b) 3 (c) 5 (d) 8

3 If Δ ABC is right-angled at B , AB = 6 cm. , BC = 8 cm. , then the length of the median drawn from B equals .... cm.

(a) 10 (b) 8 (c) 6 (d) 5

4 The diagonals are perpendicular in the ....

(a) trapezium. (b) parallelogram. (c) square. (d) triangle.

5 The point of concurrence of the medians of the triangle divides each median in the ratio of .... from the base.

(a) 1 : 2 (b) 1 : 3 (c) 2 : 1 (d) 3 : 1

6 The acute angle supplements .... angle.

(a) an acute (b) an obtuse (c) a right (d) a reflex

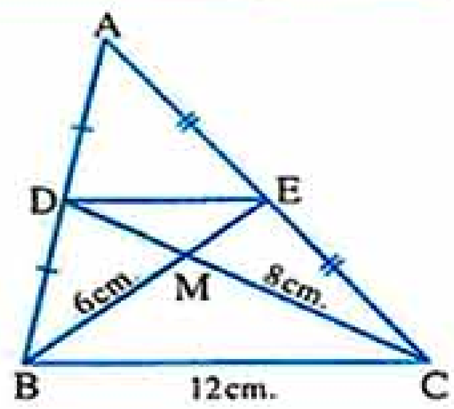
3 [a] In the opposite figure :

$\overline{BE}$, $\overline{CD}$ are medians in Δ ABC

, MB = 6 cm. , MC = 8 cm.

, BC = 12 cm.

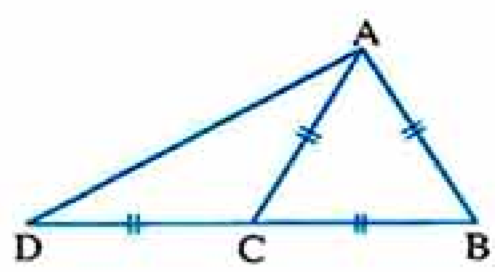
Find : The perimeter of Δ MDE



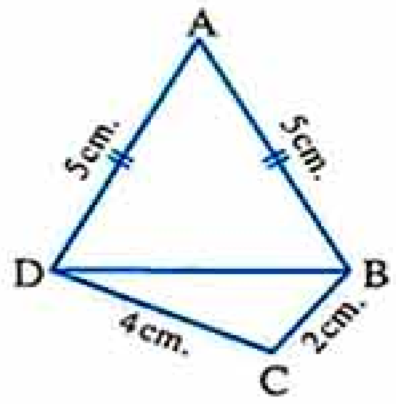
[b] In the opposite figure :

AB = BC = AC = DC

Prove that : m ($\angle$ BAD) = 90°



4 **[a] In the opposite figure :**



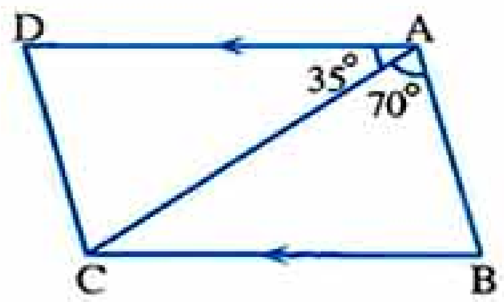
ABCD is a quadrilateral in which AB = AD = 5 cm.

, BC = 2 cm. , DC = 4 cm.

Prove that :

m (∠ ABC) > m (∠ ADC)

[b] In the opposite figure :

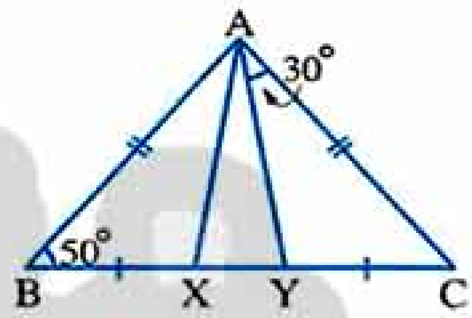


$\overline{AD}$ // $\overline{BC}$, m (∠ BAC) = 70°

and m (∠ DAC) = 35°

Prove that : AC > BC

5 **In the opposite figure :**



ABC is a triangle in which

AB = AC , BX = CY

If m (∠ B) = 50° , m (∠ CAY) = 30°

1 **Prove that :** AYX is an isosceles triangle.

2 **Find :** m (∠ AXY)

14 El-Menia Governorate

El-Menia Directorate of Education
Kafr El-Mansoura Formal Language School

Answer the following questions :

1 **Choose the correct answer :**

1 The triangle in which the measures of two angles of it are 42° and 69° is ....

(a) an isosceles triangle. (b) an equilateral triangle.

(c) a scalene triangle. (d) a right-angled triangle.

2 In Δ ABC which is right-angled at B , if AC = 20 cm. , then the length of the median drawn from B equals ....

(a) 10 cm. (b) 8 cm. (c) 6 cm. (d) 5 cm.

3 In Δ ABC , if m (∠ B) = 130° , then the longest side of it is ....

(a) $\overline{BC}$ (b) $\overline{AC}$ (c) $\overline{AB}$ (d) its median.

4 The two angles are said to be supplementary if the sum of their measures is ....

(a) zero° (b) 90° (c) 180° (d) 360°

5 The lengths which can be lengths of sides of a triangle are ....

(a) (0 , 3 , 5) (b) (3 , 3 , 5) (c) (3 , 3 , 6) (d) (3 , 3 , 7)

6 Δ XYZ is an isosceles triangle in which m ($\angle$ X) = 100° , then m ($\angle$ Y) = ...

(a) 100° (b) 80° (c) 60° (d) 40°

2 Complete :

1 The sum of measures of the accumulative angles at a point is ....°

2 The ray drawn from the midpoint of a side of a triangle parallel to another side .... the third side.

3 If the measure of an angle in an isosceles triangle equals 60° , then the triangle is ....

4 The point of concurrence of the medians of the triangle divides each median in the ratio of .... from the base.

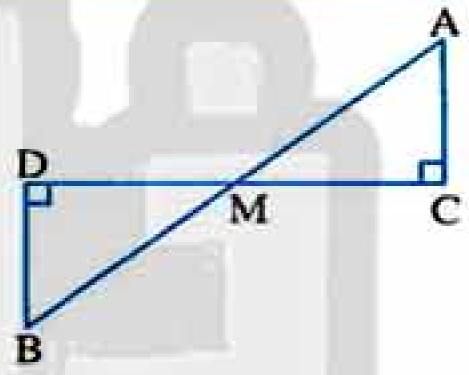
5 In Δ ABC , m ($\angle$ B) = 70° , m ($\angle$ C) = 50° , then AC .... AB

3 [a] In the opposite figure :

$\overline{AB} \cap \overline{CD} = \{M\}$, $\overline{AC} \perp \overline{CD}$

, $\overline{BD} \perp \overline{CD}$

Prove that : AB > CD

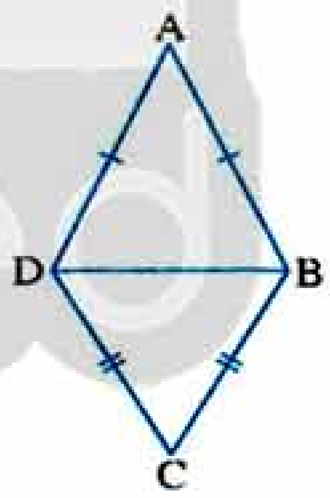


[b] In the opposite figure :

AB = AD , BC = CD

Prove that :

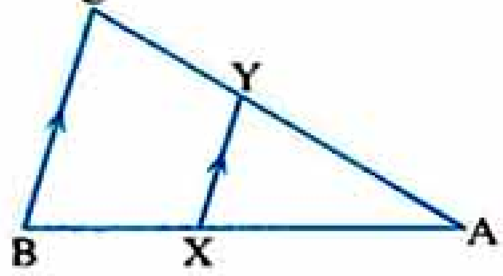
m ($\angle$ ABC) = m ($\angle$ ADC)



4 [a] In the opposite figure :

AB > BC , $\overline{XY} \parallel \overline{BC}$

Prove that : AX > XY



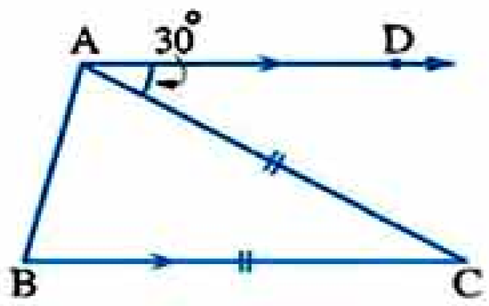
[b] In the opposite figure :

ABC is a triangle in which AC = BC

, $\overrightarrow{AD} \parallel \overline{BC}$, m ($\angle$ DAC) = 30°

Find with proof :

The measures of the angles of Δ ABC



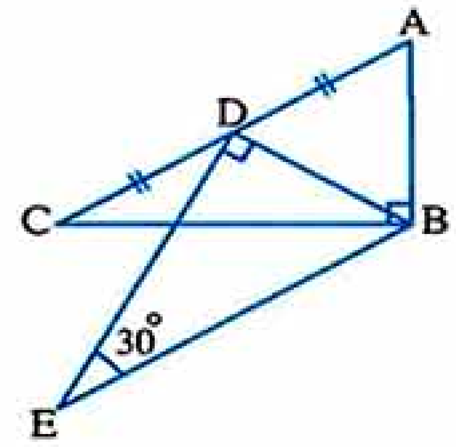
5 **[a] In the opposite figure :**

m (∠ ABC) = m (∠ BDE) = 90°

, m (∠ E) = 30°

, D is the midpoint of $\overline{AC}$

Prove that : AC = BE



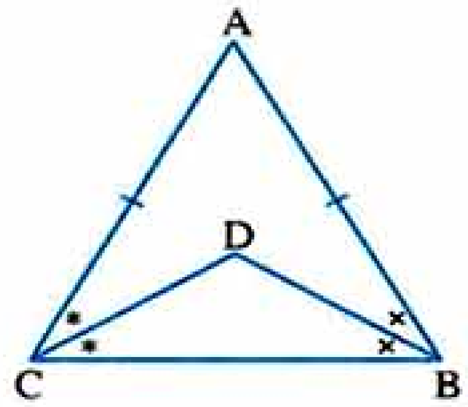
[b] In the opposite figure :

AB = AC , $\overrightarrow{BD}$ bisects ∠ ABC

and $\overrightarrow{CD}$ bisects ∠ ACB

Prove that :

Δ DBC is isosceles.



15 Qena Governorate

Qena Directorate of Education
Math's Supervision

Answer the following questions :

1 Complete each of the following :

1. The number of axes of symmetry of the equilateral triangle equals ....
2. In the triangle ABC , if AC = BC and m (∠ C) = 80° , then m (∠ A) = ...°
3. XYZ is a triangle , m (∠ X) = 60° , m (∠ Y) = 40° , then XZ .... ZY
4. The point of intersection of the medians of the triangle divides each of them with the ratio of .... from the vertex.
5. The perpendicular bisector of a line segment is called ....

2 Choose the correct answer from those given :

1. The lengths 9 cm. , 4 cm. and .... may be the side lengths of an isosceles triangle.

   (a) 9 cm. (b) 13 cm. (c) 5 cm. (d) 4 cm.

2. $\overline{AD}$ is a median of Δ ABC , and M is the point of concurrence of the medians , then AM = ... AD

   (a) $\frac{2}{3}$ (b) $\frac{1}{2}$ (c) $\frac{3}{2}$ (d) 2

3. The measure of the exterior angle of an equilateral triangle equals ....

   (a) 30° (b) 60° (c) 120° (d) 90°

4 In the triangle ABC , if m (∠ B) = 90° , then the greatest side in length is ....

(a) $\overline{AB}$ (b) $\overline{AC}$ (c) $\overline{CB}$ (d) $\overline{XY}$

5 In Δ XYZ , if XY > ZX , then m (∠ Y) .... m (∠ Z)

(a) > (b) < (c) = (d) ≡

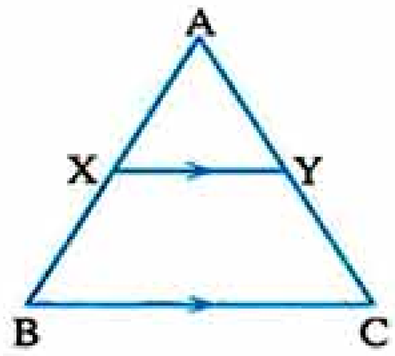
3 [a] **In the opposite figure :**

ABC is a triangle in which AB = AC

, $\overline{XY}$ // $\overline{BC}$

Prove that :

Δ AXY is an isosceles triangle.



[b] In Δ ABC , m (∠ A) = 40° , m (∠ B) = 75° Arrange the lengths of sides of Δ ABC in an ascending order.

4 [a] **In the opposite figure :**

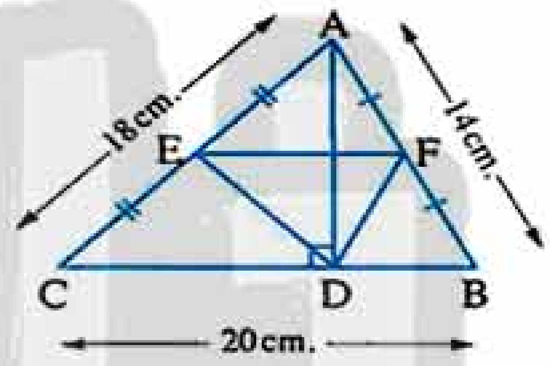
ABC is a triangle in which AB = 14 cm.

, AC = 18 cm. , BC = 20 cm.

, E is the midpoint of $\overline{AC}$

, F is the midpoint of $\overline{AB}$, and $\overline{AD}$ ⊥ $\overline{BC}$

Find : The perimeter of Δ DEF



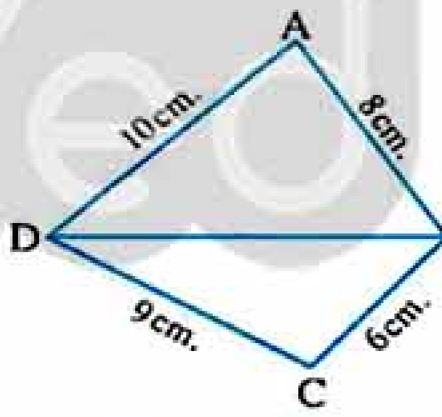
[b] **In the opposite figure :**

ABCD is a quadrilateral in which AB = 8 cm.

, BC = 6 cm. , CD = 9 cm.

and DA = 10 cm.

Prove that : m (∠ ABC) > m (∠ ADC)

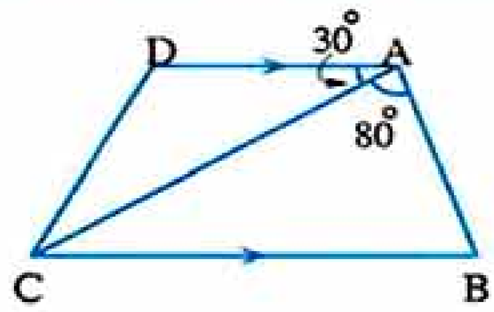


5 [a] **In the opposite figure :**

$\overline{AD}$ // $\overline{BC}$, m (∠ BAC) = 80°

, m (∠ DAC) = 30°

Prove that : BC > AB



[b] **Complete :** In Δ ABC , if AB = 7 cm. , AC = 5 cm. , then .... < BC < ....

Final Examinations of Geometry
2019

Some Schools Examinations on Geometry

1 Cairo Governorate

East Nasr city administration
Heliopolis Language School
Mathematics Department

Answer the following questions :

1 Complete :

(1) The intersection point of the three medians of the triangle divide the median in the ratio .... from the vertex.

(2) In Δ ABC : If CA = CB and m ($\angle$ C) = m ($\angle$ A) , then m ($\angle$ B) = ...°

(3) The bisector of the vertex angle of the isosceles triangle is .... and ....

(4) If the measure of an angle in the isosceles triangle is 100° , then the number of axes of symmetry of Δ ABC is ....

(5) The longest side in the right-angled triangle is ....

2 Choose the correct answer :

(1) In Δ ABC : If m ($\angle$ B) = 90° , then ....
(a) AC > CB (b) AB > AC (c) BC > AC (d) AB = AC

(2) If the lengths of two sides of an isosceles triangle are 3 cm. and 7 cm. , then the length of the third side is ....
(a) 3 (b) 4 (c) 7 (d) 10

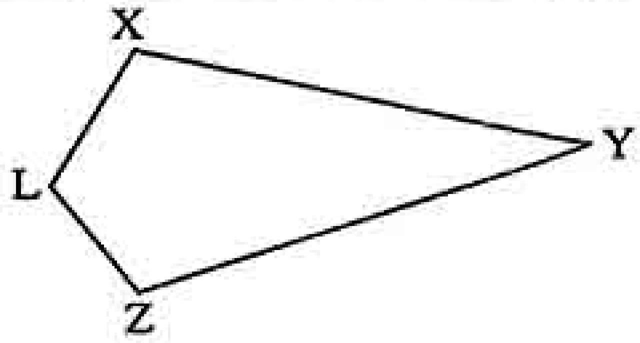
(3) In Δ ABC : If AB = AC and m ($\angle$ A) = 60° , then the number of axes of symmetry of the triangle ABC is ....
(a) 0 (b) 1 (c) 2 (d) 3

(4) Any triangle has .... medians.
(a) 0 (b) 1 (c) 2 (d) 3

(5) If ABCD is a square , then the axes of symmetry of $\overline{AC}$ is ....
(a) $\overleftrightarrow{AD}$ (b) $\overleftrightarrow{BC}$ (c) $\overleftrightarrow{BD}$ (d) $\overleftrightarrow{AB}$

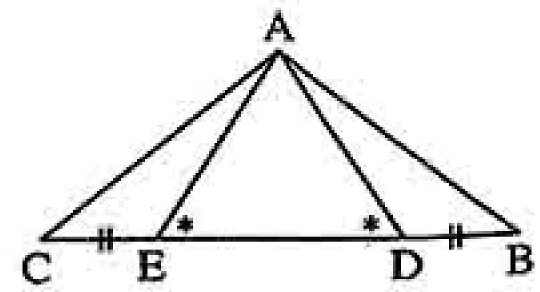
3 [a] In the opposite figure :

XY > XL
and YZ > ZL
Prove that : m ($\angle$ XLZ) > m ($\angle$ XYZ)

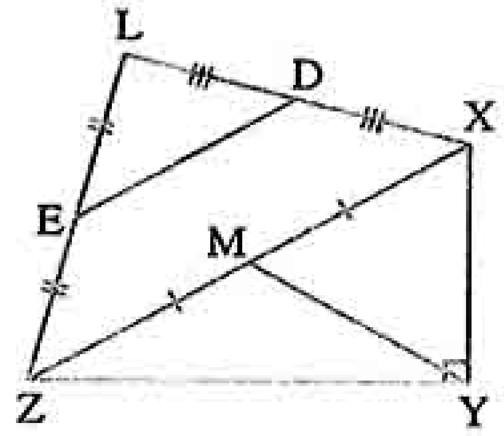


[b] In the opposite figure :t

$\angle$ ADC ≡ $\angle$ AED and BD = CE
, B , D , E and C are collinear.
Prove that : Δ ABC is an isosceles triangle.



4 [a] **In the opposite figure :**



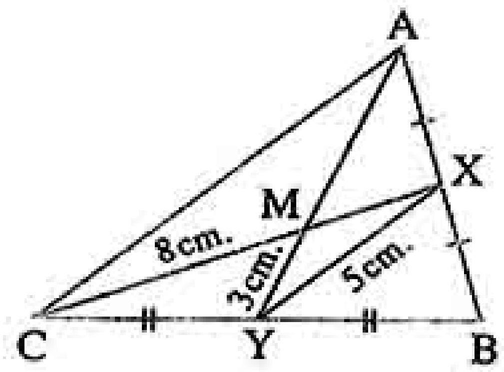
m (∠ XYZ) = 90°

, D is midpoint of $\overline{XL}$

, E is midpoint of $\overline{ZL}$ and M is the midpoint of $\overline{XZ}$

Prove that : DE = YM

[b] **In the opposite figure :**



ABC is a triangle , X is the midpoint of $\overline{AB}$

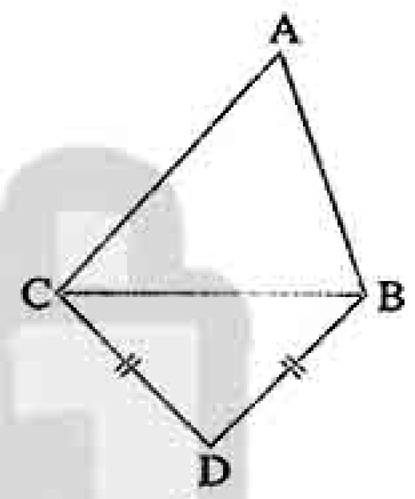
, Y is midpoint of $\overline{BC}$, XY = 5 cm. and $\overline{XC} \cap \overline{AY} = \{M\}$

where CM = 8 cm. , YM = 3 cm.

Find : (1) The perimeter of Δ MXY

(2) The perimeter of Δ MAC

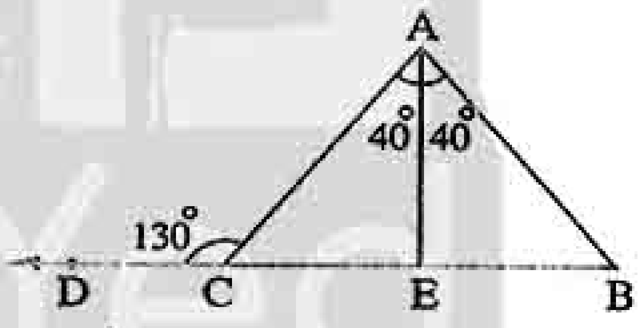
5 [a] **In the opposite figure :**



AC > AB and DB = DC

Prove that : m (∠ ABD) > m (∠ ACD)

[b] **In the opposite figure :**



C ∈ $\overrightarrow{BD}$, m (∠ ACD) = 130°

and m (∠ BAE) = m (∠ CAE) = 40°

Prove that : (1) $\overline{AE} \perp \overline{BC}$

(2) E bisects $\overline{BC}$

2 Cairo Governorate

Maadi Educational Zone
Sakkara Language School

Answer the following questions :

1 **Complete :**

(1) In Δ XYZ , m (∠ X) = 90° , then the longest side is ....

(2) The base angles of the isosceles triangle are ....

(3) ABC is a triangle in which AB = 4 cm. , CB = 7 cm. , then AC ∈]......... , ...[

(4) If A ∈ the axis of symmetry of $\overline{XY}$, then .... = ...

(5) If the measure of an angle in the isosceles triangle equals 60° , then the triangle has .... axes of symmetry.

Geometry

2 Choose the correct answer :

(1) The measure of the exterior angle of equilateral triangle = ...

(a) 90° (b) 120° (c) 45° (d) 60°

(2) If $\overline{AD}$ is a median in Δ ABC and M is the point of intersection of the medians , then AM = ... AD

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) $\frac{3}{2}$ (d) $\frac{1}{2}$

(3) In Δ XYZ , if m (∠ Z) = 70° and m (∠ Y) = 60° , then YZ .... XY

(a) < (b) = (c) > (d) is twice

(4) The numbers 4 , 8 , ... can be lengths of sides of an isosceles triangle.

(a) 4 (b) 8 (c) 12 (d) 3

(5) In Δ ABC , if m (∠ B) = 90° and m (∠ C) = 30° , then AB .... AC

(a) $\frac{1}{3}$ (b) 2 (c) equals (d) $\frac{1}{2}$

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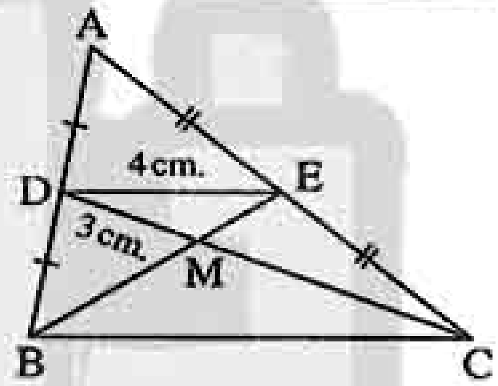
3 [a] In the opposite figure :



D is the midpoint of $\overline{AB}$, E is the midpoint of $\overline{AC}$

, $\overline{CD} \cap \overline{BE} = \{M\}$

If DE = 4 cm. , DM = 3 cm. , BE = 6 cm.

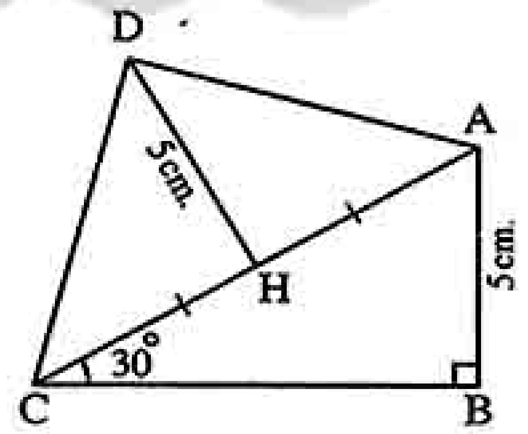
Find : The perimeter of Δ BMC

[b] In Δ ABC , if AB = 5 cm. , BC = 7 cm. and AC = 9 cm.

Arrange the measures of its angles in a descending order.

---

4 [a] In the opposite figure :



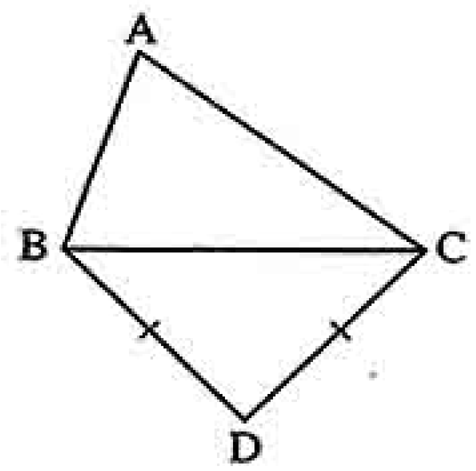
ABC is a right angled triangle at B

, m (∠ ACB) = 30° , AB = 5 cm.

, DH = 5 cm. and H is the midpoint of $\overline{AC}$

Prove that : m (∠ ADC) = 90°

[b] In the opposite figure :



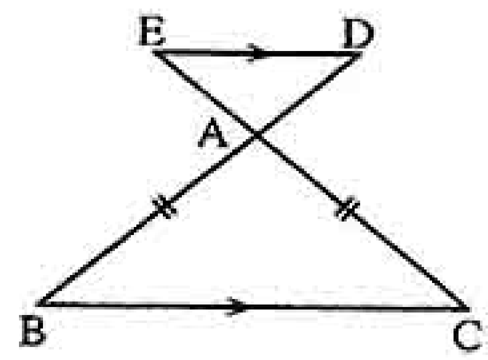
If AC > AB and DC = DB

Prove that : m (∠ ABD) > m (∠ ACD)

5 **[a] In the opposite figure :**

If AB = AC

Prove that : AD = AE



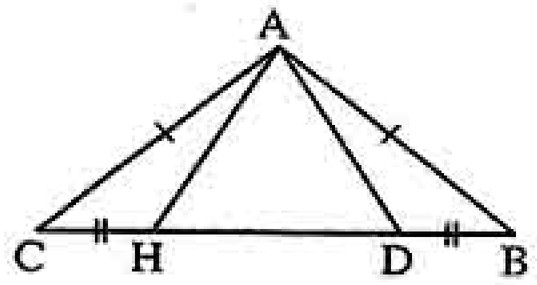
[b] In the opposite figure :

ABC is a triangle in which :

AB = AC , BD = CH

Prove that : ① Δ ADH is an isosceles triangle.

② $\angle$ AHD ≡ $\angle$ ADH



3 Cairo Governorate

El-Sayda Zinab Educational Zone

Answer the following questions :

1 **Choose the suitable answer :**

① The number of axes of symmetry of an equilateral triangle is ....

(a) 0 (b) 1 (c) 2 (d) 3

② An isosceles triangle , one of its base angles has measure 50° , then the measure of the vertex angle = ...

(a) 50° (b) 60° (c) 70° (d) 80°

③ $\overline{AD}$ is a median of triangle ABC , and M is the point of intersection of the medians , then AM = ... AD

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) $\frac{1}{2}$ (d) $\frac{1}{4}$

④ If the lengths of two sides of a triangle are 4 cm. and 8 cm. , then the length of the third side = ... cm.

(a) 3 (b) 4 (c) 8 (d) 12

⑤ In a triangle ABC , if m ($\angle$ A) = 80° and m ($\angle$ C) = 60° , then AB .... BC

(a) < (b) > (c) = (d) ≥

2 **Complete :**

① If XYZ is a right-angled triangle at Y , then the longest side is ....

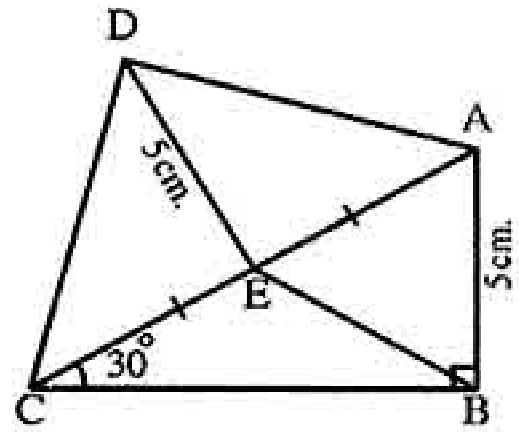
② The sum of measures of any two consecutive angles in the parallelogram = ... °

③ The straight line perpendicular to the midpoint of a line segment is called ....

④ The bisectors of the vertex angle of an isosceles triangle .... and ....

⑤ The measure of the exterior angle of the equilateral triangle = ... °

3 **[a] In the opposite figure :**



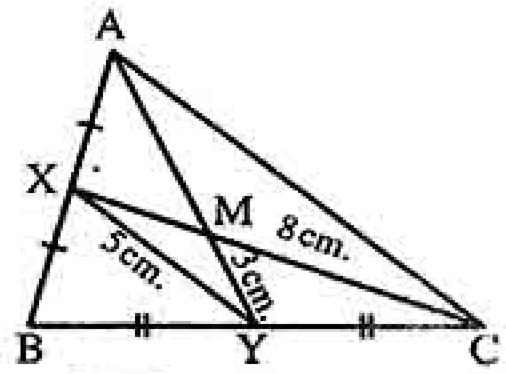
ABC is a right-angled triangle at B

, m (∠ ACB) = 30° , AB = 5 cm.

, E is midpoint of $\overline{AC}$

If DE = 5 cm. **then prove that :** m (∠ ADC) = 90°

[b] In the opposite figure :



ABC is a triangle , X is the midpoint of $\overline{AB}$, Y is the midpoint of $\overline{BC}$

, XY = 5 cm. , $\overline{XC} \cap \overline{AY} = \{M\}$

where : CM = 8 cm. , YM = 3 cm.

Find with proof : The length of each of : (1) $\overline{AM}$ (2) $\overline{MX}$ (3) $\overline{AC}$

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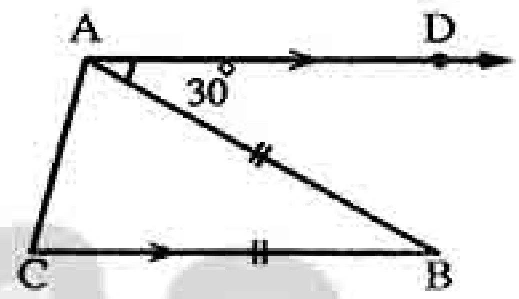
4 **[a] In the opposite figure :**

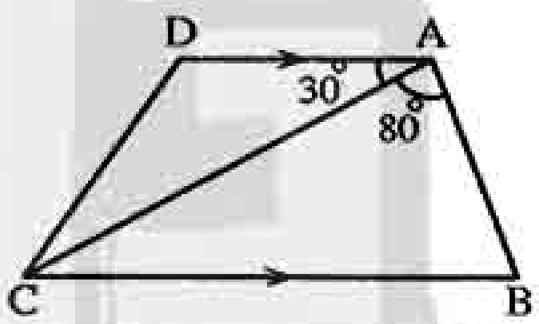


ABC is a triangle in which : AB = BC , $\overrightarrow{AD} // \overline{BC}$

, m (∠ DAB) = 30°

Find : The measures of the angles of Δ ABC

[b] In the opposite figure :



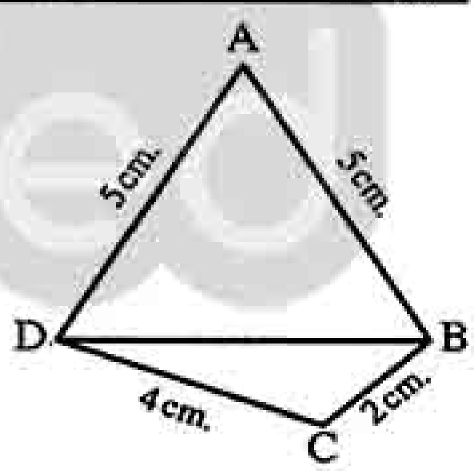
$\overrightarrow{AD} // \overline{BC}$, m (∠ BAC) = 80°

, m (∠ DAC) = 30°

Prove that : BC > AB

---

5 **In the opposite figure :**



ABCD is a quadrilateral in which : AB = AD = 5 cm.

, BC = 2 cm. , DC = 4 cm.

Prove that : m (∠ ABC) > m (∠ ADC)

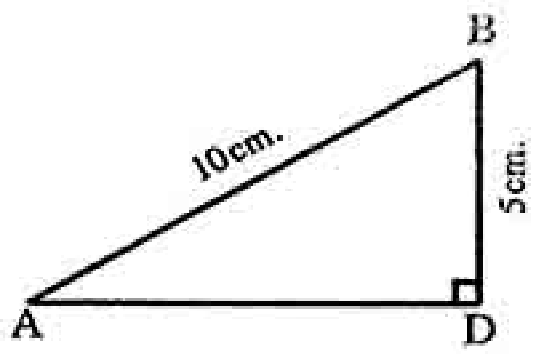
Giza Governorate

Dokki District
Modern Narmer Language School

Answer the following questions :

1 **Choose the correct answer from those given :**

(1) **In the opposite figure :**



Δ ADB , m (∠ ADB) = 90° , BD = 5 cm.

and AB = 10 cm. , then m (∠ A) = ...°

(a) 30 (b) 50 (c) 70 (d) 90

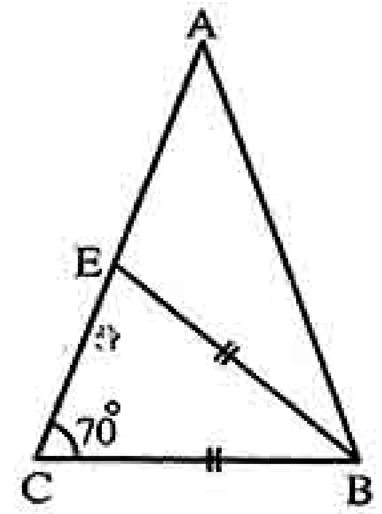
② **In the opposite figure :**

If AB = AC and BE = BC

, then : m (∠ ABE) = ...

(a) 30°   (b) 40°

(c) 70°   (d) 110°



③ **In the opposite figure :**

Δ ABC , AB = BC

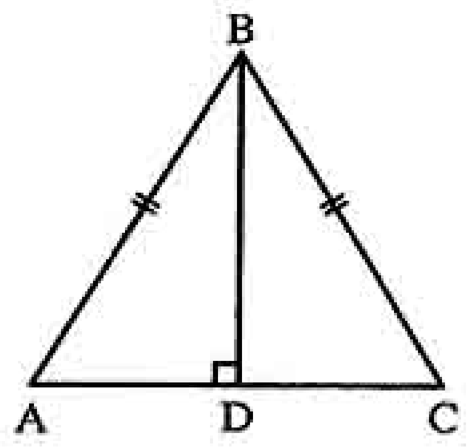
, an altitude is drawn from B to $\overline{AC}$ and intersects $\overline{AC}$ at D

which conclusion is not always true ?

(a) m (∠ ABD) = m (∠ CBD)

(b) m (∠ BDA) = m (∠ BDC)

(c) AD = BD   (d) AD = DC



④ Which set of numbers represents the lengths of the sides of a triangle ?

(a) {5 , 18 , 13}   (b) {6 , 17 , 22}   (c) {16 , 24 , 7}   (d) {26 , 8 , 15}

⑤ The point of concurrency of medians divides each median in the ratio .... from the base.

(a) 1 : 2   (b) 2 : 1   (c) 3 : 1   (d) 2 : 3

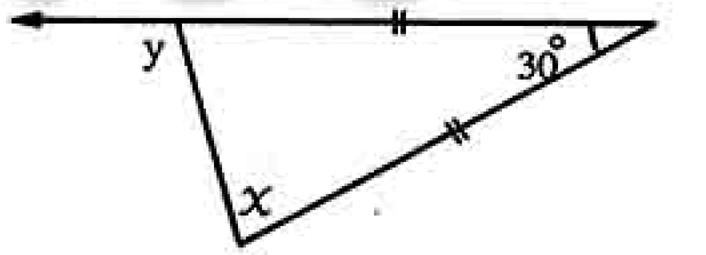
2 Complete :

① The longest side in the right-angled triangle is ....

② If the measure of an angle in the isosceles triangle equals 60° , then the triangle is ....

③ **In the opposite figure :**

x = ...° and y = ... °



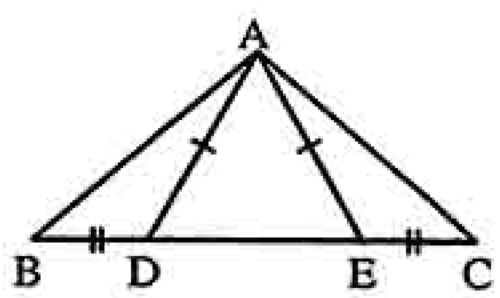
④ If the length of the median drawn from the right vertex of a triangle is 6 cm. , then the length of the hypotenuse is .... cm.

⑤ In Δ ABC , m (∠ A) = 60° , m (∠ B) = 50° , then the longest side is ....

3 [a] In the opposite figure :

AD = AE and BD = CE

Prove that : Δ ABC is an isosceles triangle.



Geometry

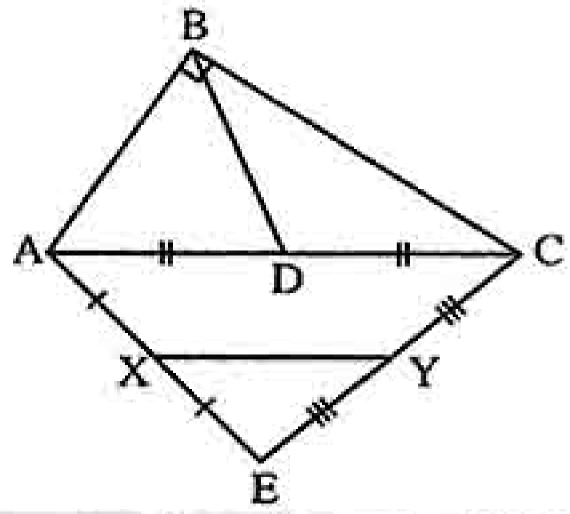
[b] In the opposite figure :

Δ ABC is right-angled at B

, D is the midpoint of $\overline{AC}$

, X and Y are the midpoints of $\overline{AE}$ and $\overline{CE}$ respectively.

Prove that : BD = XY

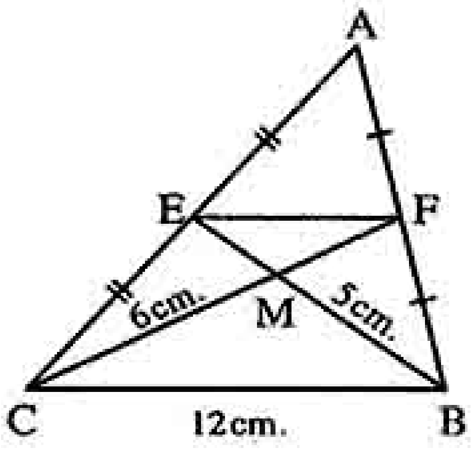


4 **[a] In the opposite figure :**

Δ ABC , F and E are the midpoints of $\overline{AB}$ and $\overline{AC}$ respectively.

If BM = 5 cm. , CM = 6 cm. , BC = 12 cm. ,

then find : The perimeter of Δ MEF



[b] In Δ ABC , m ($\angle$ A) = $3x°$, m ($\angle$ B) = $(4x - 9)°$ and m ($\angle$ C) = $(2x + 9)°$

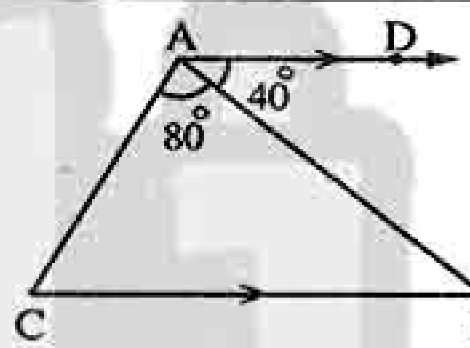
Find the measure of each angle and arrange the sides in a descending order according to their lengths.

5 **[a] In the opposite figure :**

Δ ABC , in which : $\overline{AD}$ // $\overline{BC}$

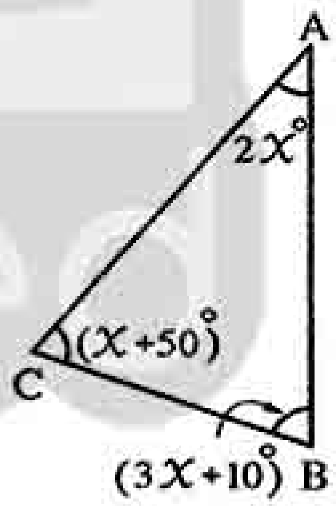
m ($\angle$ DAB) = 40° and m ($\angle$ BAC) = 80°

Prove that : AB > AC



[b] In the opposite figure :

Show with proof , which sides are equal in length.



5 Giza Governorate

Omrania Directorate
El sadat Governmental Language School

Answer the following questions :

1 **Complete each of the following :**

① The point of concurrence of medians of a triangle divides each median in ratio .... : .... from the vertex.

② The longest side in the right-angled triangle is ....

③ The straight line perpendicular to the midpoint of a line segment is called ....

④ The base angles of the isosceles triangle are ....

⑤ In Δ ABC , if AB < BC < AC , then the greatest angle in measure is ....

2 **Choose the correct answer from given ones :**

① The number of axes of symmetry in the scalene triangle is ....

(a) 1 (b) 2 (c) 3 (d) zero

② The measure of the exterior angle of an equilateral triangle is ....

(a) $90°$ (b) $120°$ (c) $60°$ (d) $30°$

③ The numbers 5 , 4 , ... can be lengths of sides of a triangle.

(a) 8 (b) 9 (c) 10 (d) 12

④ In ΔABC , $AB = AC$ and $m(\angle B) = 70°$, then $m(\angle A) =$ ....

(a) $140°$ (b) $70°$ (c) $40°$ (d) $110°$

⑤ ΔABC in which : $m(\angle B) > m(\angle C)$, then AC .... AB

(a) $>$ (b) $<$ (c) $=$ (d) $\le$

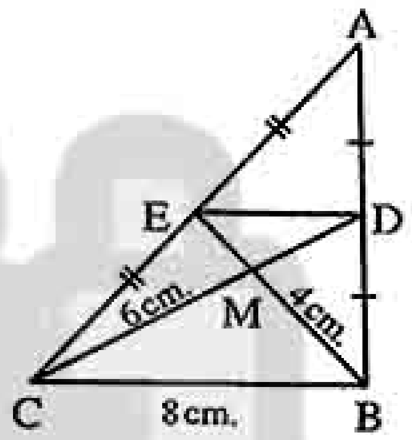
---

3 **[a] In the opposite figure :**

ABC is a triangle in which D , E are midpoints of $\overline{AB}$ and $\overline{AC}$ respectively ,

MC = 6 cm. , MB = 4 cm. and BC = 8 cm.

Find : The perimeter of Δ DME



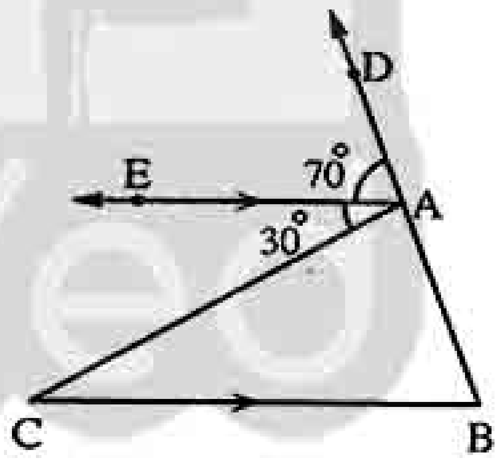
[b] In the opposite figure :

$\overrightarrow{AE} \parallel \overline{BC}$

, $m(\angle DAE) = 70°$

, $m(\angle EAC) = 30°$

Prove that : AC > AB



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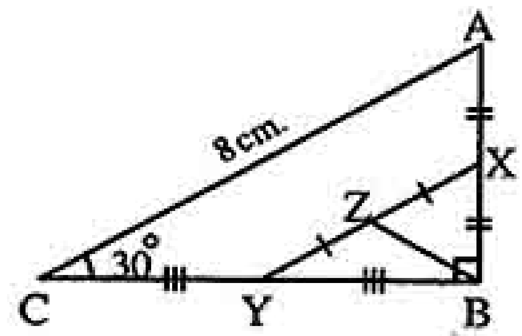
4 **[a] In the opposite figure :**

ABC is a triangle in which : $m(\angle ABC) = 90°$

, $m(\angle C) = 30°$, X , Y and Z are midpoints of $\overline{AB}$, $\overline{BC}$ and $\overline{XY}$ respectively and AC = 8 cm.

Find : The length of each of $\overline{AB}$, $\overline{XY}$, $\overline{BZ}$

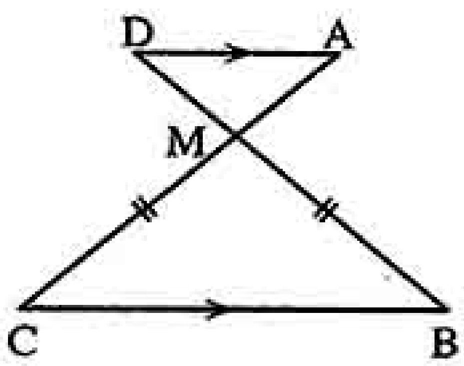


[b] In the opposite figure :

$\overline{AC} \cap \overline{BD} = \{M\}$

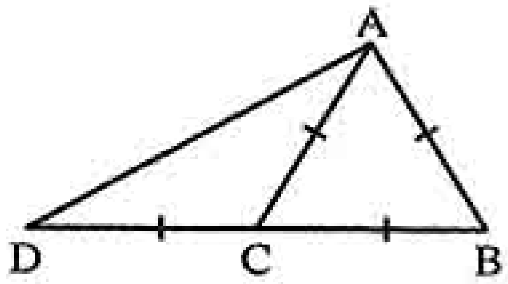
, MB = MC and $\overline{AD} \parallel \overline{BC}$

Prove that : MA = MD



Geometry

5 In the opposite figure :



ABC is an equilateral triangle

, $D \in \overrightarrow{BC}$ such that BC = CD

Prove that : $\overline{BA} \perp \overline{AD}$

6 Alexandria Governorate

Middle Educational Directorate
Math's Supervision

Answer the following questions :

1 Choose the correct answer :

① The isosceles triangle has .... of symmetry.

(a) one axis (b) two axes (c) three axes (d) zero axes

② In Δ ABC , if m ($\angle$ A) = 125° , then the longest side of it is ....

(a) $\overline{AB}$ (b) $\overline{AC}$ (c) $\overline{BC}$ (d) its median

③ If XYZ is an isosceles triangle , m ($\angle$ Y) = 100° , then m ($\angle$ X) = ...

(a) 80° (b) 40° (c) 20° (d) 100°

④ In Δ ABC if m ($\angle$ A) = 30° , m ($\angle$ B) = 90° , then BC = ... AC

(a) $\frac{1}{2}$ (b) $\frac{2}{3}$ (c) $\frac{1}{3}$ (d) 2

⑤ The measure of each exterior angle of equilateral triangle is ....

(a) 180° (b) 360° (c) 60° (d) 120°

2 Complete :

① The point of concurrence divides each median in the ratio .... from the base.

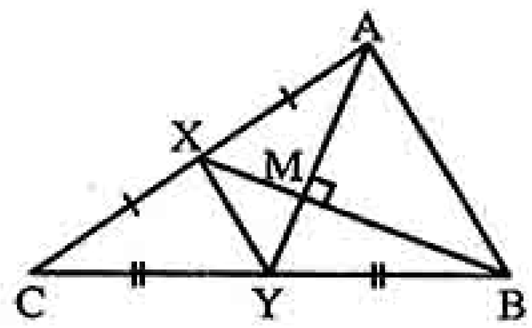
② The longest side in the right angled triangle is ....

③ The sum of measures of the exterior angles of a square is .... °

④ The numbers 8 , 4 , ... can be lengths of sides of an isosceles triangle.

⑤ The axis of symmetry of a line segment is the straight line which is ....

3 [a] In the opposite figure :



$\overline{AY}$ and $\overline{BX}$ are two medians where $\overline{AY} \perp \overline{BX}$

, if AY = 12 cm. and XM = 5 cm.

Find : The area of Δ ABM

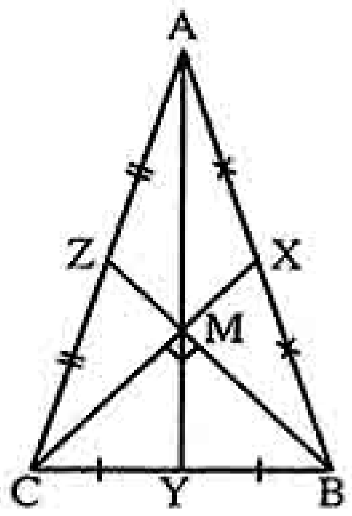
[b] ABC is a triangle in which : m ($\angle$ A) = 6 x° , m ($\angle$ B) = (4 x – 9)° and m ($\angle$ C) = 3 (x – 2)° Arrange the lengths of sides descendingly.

4 **[a] In the opposite figure :**

$\overline{BZ}$ and $\overline{CX}$ are two medians of Δ ABC

, $\overline{CX} \perp \overline{BZ}$

Prove that : AM = BC

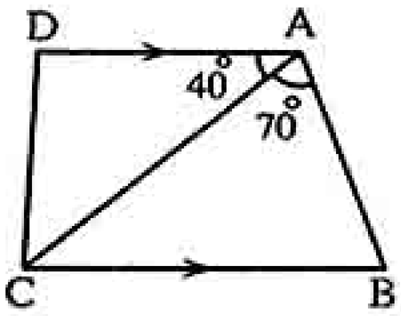


[b] In the opposite figure :

$\overline{AD}$ // $\overline{BC}$, m ($\angle$ DAC) = 40°

, m ($\angle$ BAC) = 70°

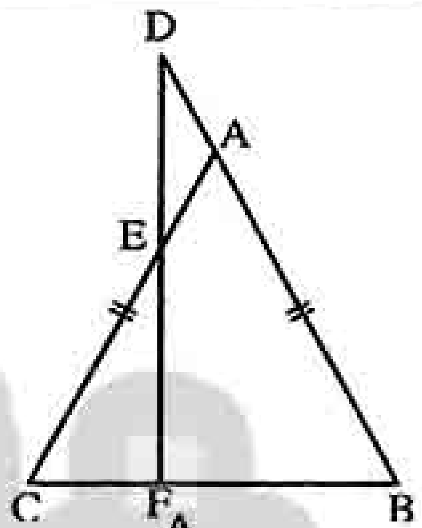
Prove that : BC = AC



5 **[a] In the opposite figure :**

AB = AC

Prove that : EC > EF



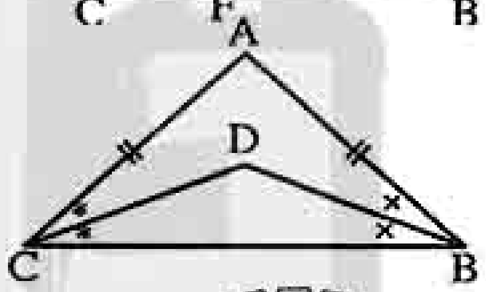
[b] In the opposite figure :

AB = AC

, $\overrightarrow{BD}$ bisects $\angle$ B

, $\overrightarrow{CD}$ bisects $\angle$ C

Prove that : BD = CD



7 Alexandria Governorate

East Educational Zone
Mathematics Directing

Answer the following questions :

1 **Complete the following :**

① If ABCD is a parallelogram and m ($\angle$ A) = 70° , then m ($\angle$ B) = ... °

② The measure of the exterior angle in the equilateral triangle = ...°

③ The length of the median from the vertex of the right angle in the right-angled triangle = ...

④ If AB = AC in Δ ABC and m ($\angle$ B) = 40° , then m ($\angle$ C) = ...°

⑤ In Δ XYZ , if XY < YZ < ZX , then the greatest angle in measure is $\angle$ ....

2 **Choose the correct answer from those given :**

① The diagonals are perpendicular in ....

(a) square and rectangle. (b) rectangle and rhombus.

(c) square and rhombus. (d) parallelogram and rectangle.

② The point of the intersection of the medians in triangle divides each median from the base into the ratio ....

(a) 1 : 2 (b) 2 : 1 (c) 3 : 1 (d) 2 : 3

③ The isosceles triangle has .... axis of symmetry.

(a) 0 (b) 1 (c) 2 (d) 3

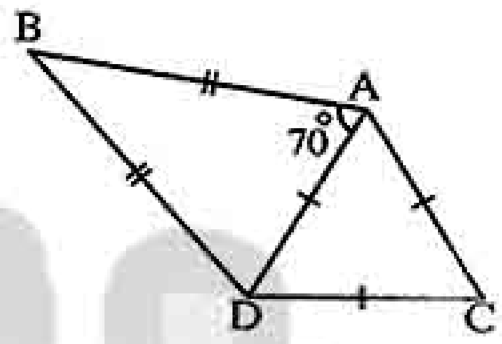
④ If the lengths of two sides in an isosceles triangle 3 cm. and 7 cm. , then the length of the third side = ... cm.

(a) 3 (b) 4 (c) 7 (d) 10

⑤ In ΔABC , if $m(\angle A) < m(\angle B)$, then ....

(a) $AC < BC$ (b) $AC > BC$ (c) $AC = BC$ (d) $\overline{AC} \parallel \overline{BC}$

3 **[a] In the opposite figure :**

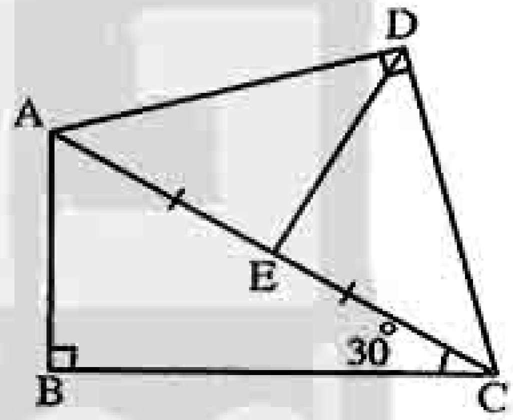


$AB = BD$, $m(\angle BAD) = 70°$

, ΔADC is an equilateral triangle.

Find : $m(\angle BDC)$

[b] In the opposite figure :



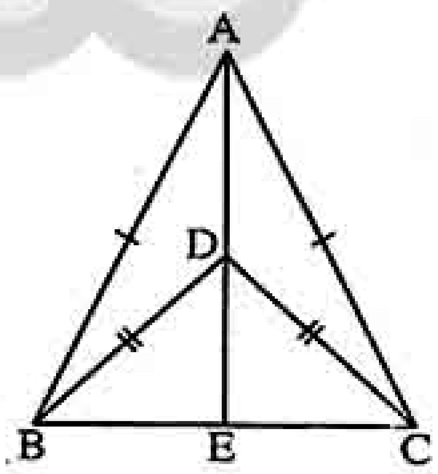
$m(\angle ABC) = m(\angle ADC) = 90°$

, $m(\angle ACB) = 30°$

, E is the midpoint of $\overline{AC}$

Prove that : $AB = ED$

4 **[a] In the opposite figure :**



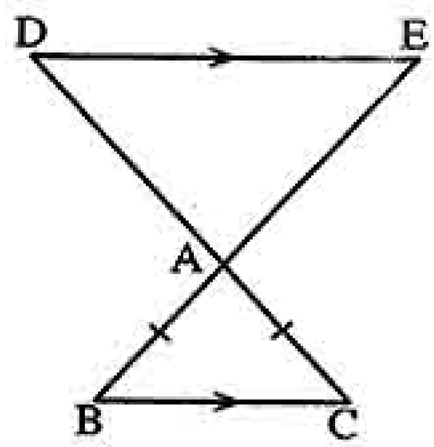
$AB = AC$, $DB = DC$, $D \in \overline{AE}$

Prove that :

① $\overline{AE} \perp \overline{BC}$

② $BE = EC$

[b] In the opposite figure :



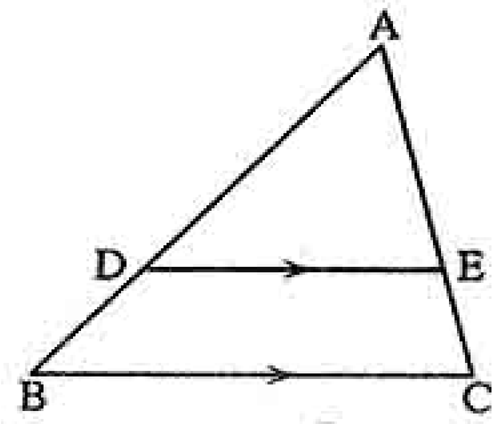
$AB = AC$ and $\overleftrightarrow{DE} \parallel \overline{BC}$

Prove that : $AD = AE$

5 **[a] In the opposite figure :**

AB > AC , $\overline{DE}$ // $\overline{BC}$

Prove that : AD > AE



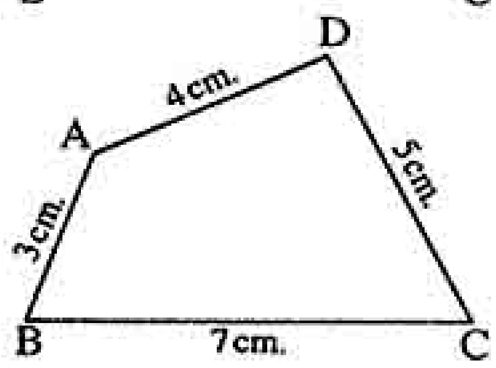
[b] In the opposite figure :

ABCD is a quadrilateral in which :

AB = 3 cm. , BC = 7 cm.

, CD = 5 cm. and DA = 4 cm.

Prove that : m (∠ BAD) > m (∠ BCD)



8 El-Kalyoubia Governorate

Al-Obour Educational Zone
Al-Resala Language School

Answer the following questions :

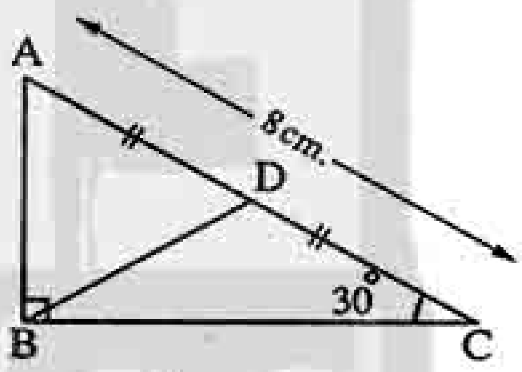
1 **Complete the following :**

① The bisector of the vertex angle of an isosceles triangle bisect the base and ....

② 3 cm. , 8 cm. and .... cm. are three sides of an isosceles triangle.

③ **In the opposite figure :**

The perimeter of Δ ABD = ... cm.



④ The measure of the exterior angle of the equilateral triangle = ...°

⑤ In Δ ABC , m (∠ A) = 100° , then the longest side is ....

2 **Choose the correct answer :**

① In Δ ABC , if m (∠ B) = 90° and m (∠ A) = 30° , then BC = ...

(a) $\frac{1}{2}$ AC (b) 2 AC (c) 2 AB (d) $\frac{1}{2}$ AB

② If A ∈ the axis of symmetry of $\overline{BC}$, then AB = ...

(a) XY (b) XZ (c) AC (d) BC

③ The triangle whose side length are 2 cm. , ($\mathcal{X}$ + 3) cm. and 5 cm. becomes an isosceles triangle when $\mathcal{X}$ = ... cm.

(a) zero (b) 1 (c) 2 (d) 3

④ The number of axis of symmetry of the equilateral triangle = ...

(a) zero (b) 1 (c) 2 (d) 3

⑤ The sum of the lengths of any two sides in the triangle .... the length of the third side.

(a) < (b) ≤ (c) ≥ (d) > (e) =

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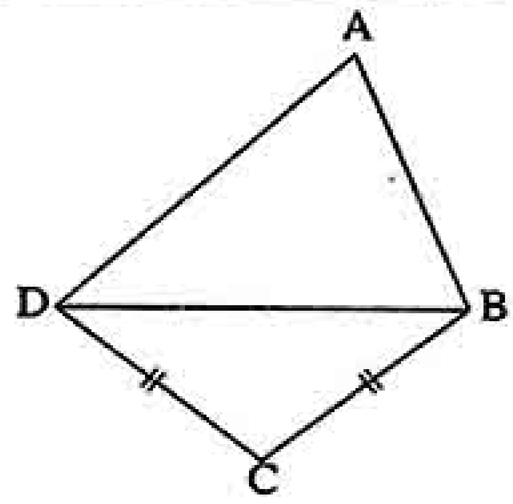
3 [a] **In the opposite figure :**

ABCD is a quadrilateral in which AD > AB and BC = CD

Prove that : m (∠ ABC) > m (∠ ADC)



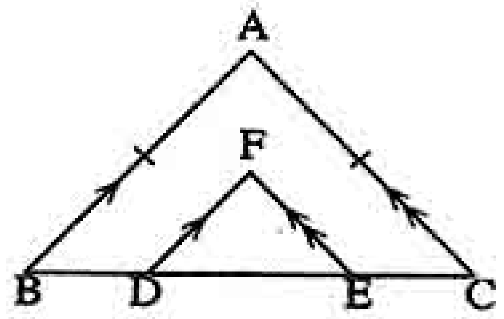
[b] **In the opposite figure :**

$D \in \overline{BC}$, $E \in \overline{BC}$

, $\overline{AB}$ // $\overline{FD}$ and $\overline{AC}$ // $\overline{FE}$

, if AB = AC

Prove that : FDE is an isosceles triangle.



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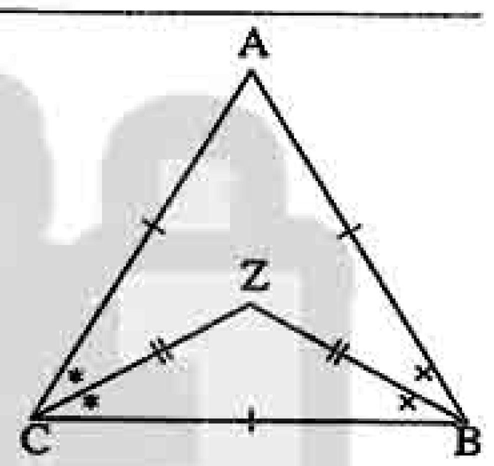
4 [a] **In the opposite figure :**

Δ ABC is an equilateral triangle

, $\overrightarrow{BZ}$ bisects ∠ B

, $\overrightarrow{CZ}$ bisects ∠ C

Find : The measure of the angles in triangle CZB



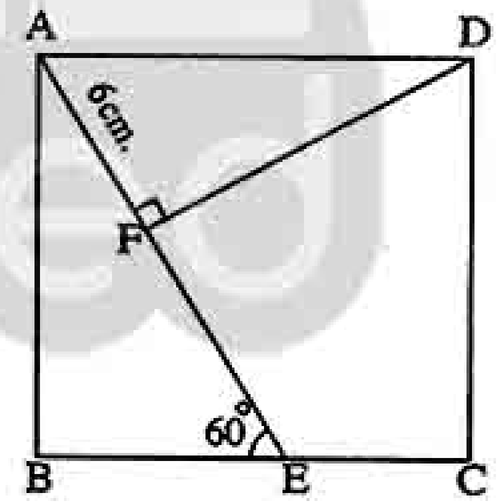
[b] **In the opposite figure :**

ABCD is a square

, m (∠ AEB) = 60°

, AF = 6 cm. , $\overline{DF} \perp \overline{AE}$

Find : The perimeter of the square ABCD



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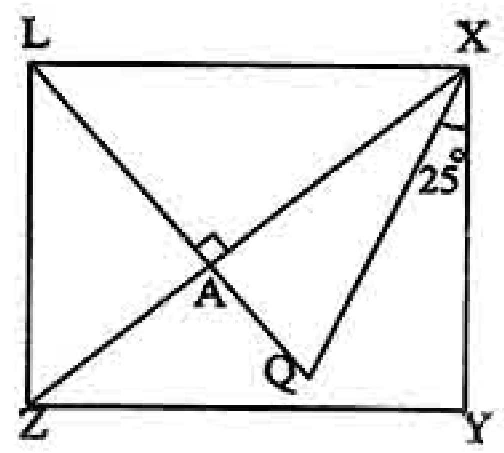
5 [a] **In the opposite figure :**

XYZL is a rectangle in which m (∠ YXQ) = 25°

, $\overline{LQ} \perp \overline{XZ}$

, $\overrightarrow{XQ}$ bisects angle YXZ

Prove that : LQ = XL



[b] In Δ ABC , m (∠ A) = 40° , m (∠ B) = 80°

Arrange the length of the sides of the triangle ABC in a descending order.

9 El-Monofia Governorate

Maths Supervision

Answer the following questions :

1 Complete :

① The perpendicular which is drawn from vertex of an isosceles triangle to its base .... and ....

② The length of the median from the vertex of the right-angled triangle equals ....

③ In Δ ABC , if AB = AC and m ($\angle$ A) = 80° , then m ($\angle$ B) = ...°

④ The measure of the exterior angle of the equilateral triangle = ...°

⑤ In Δ DEF , if DE > DF , then m ($\angle$ F) > ....

2 Choose the correct answer :

① If the length of two sides in an isosceles triangle are 8 cm. and 4 cm. , then the length of the third side is .... cm.

(a) 4 (b) 8 (c) 3 (d) 12

② The number of axes of symmetry in the isosceles triangle = ...

(a) 1 (b) 0 (c) 2 (d) 3

③ $\overline{AD}$ is a median in Δ ABC , M is the point of intersection of the medians , MD = 2 cm. , then AD = ... cm.

(a) 2 (b) 4 (c) 6 (d) 8

④ Δ ABC : m ($\angle$ B) = 125° , then the longest side of it is ....

(a) $\overline{BC}$ (b) $\overline{AC}$ (c) $\overline{AB}$ (d) its median

⑤ In Δ XYZ , if m ($\angle$ Y) = 90° , m ($\angle$ X) = 30° and XZ = 20 cm. , then ZY = ... cm.

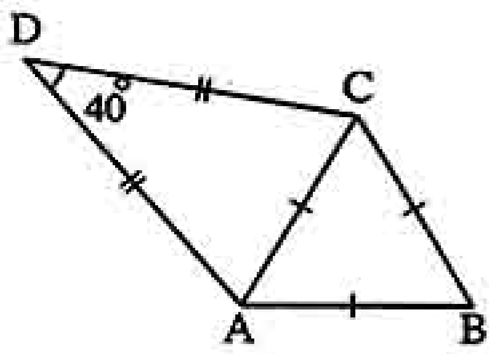
(a) 12 (b) 6 (c) 24 (d) 10

3 [a] In the opposite figure :

m ($\angle$ D) = 40° , DA = DC

and Δ ABC is an equilateral triangle

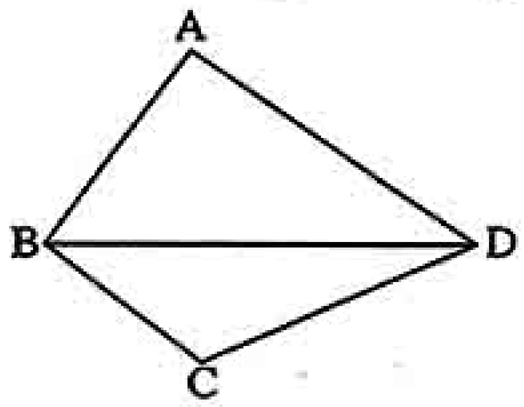
Find : m ($\angle$ DCB)



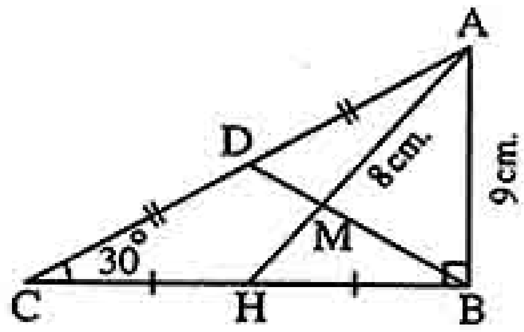
[b] In the opposite figure :

AB < AD and BC < CD

Prove that : m ($\angle$ ABC) > m ($\angle$ ADC)



4 **[a] In the opposite figure :**

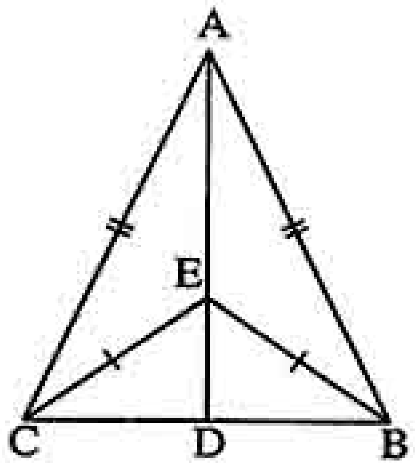


D and H are the midpoints of $\overline{AC}$ and $\overline{CB}$ respectively

m (∠ C) = 30° , m (∠ B) = 90° , AB = 9 cm. , AM = 8 cm.

Find : The length of each of $\overline{BD}$, $\overline{AH}$ and $\overline{MD}$

[b] In the opposite figure :



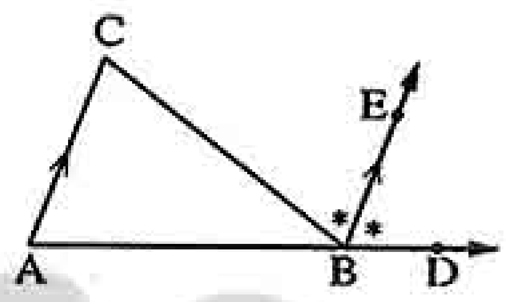
AB = AC and EB = EC

Prove that :

① $\overleftrightarrow{AE}$ is the axis of $\overline{BC}$

② BD = CB

5 **[a] In the opposite figure :**



D ∈ $\overrightarrow{AB}$, $\overrightarrow{BE}$ bisects ∠ CBD

and $\overrightarrow{BE}$ // $\overline{AC}$

Prove that :

Δ ABC is an isosceles triangle,

[b] In Δ ABC : m (∠ A) = 40° and m (∠ B) = 80°

Arrange the lengths of the sides of the triangle ABC descendingly.

10 El-Dakahlia Governorate

Math's Supervision (L.E.S.)

Answer the following questions :

1 **Complete :**

① The number of axes of symmetry of isosceles triangle is ....

② The bisector of the vertex angle of the isosceles triangle ....

③ The medians of the triangle .... at one point.

④ The longest side of the right-angled triangle is the ....

⑤ In Δ ABC , if AB = AC and m (∠ C) = 40° , then m (∠ A) = ... °

2 **Choose the correct answer :**

① Isosceles triangle whose side lengths are 4 cm. , $(x + 3)$ cm. and 8 cm. , then x = ...

(a) 4 (b) 5 (c) 3 (d) 8

② In Δ LMN , if m (∠ M) = 55° and m (∠ N) = 80° , then LM .... MN

(a) < (b) > (c) = (d) twice

③ The measure of the exterior angle of the equilateral triangle = ...°

(a) 30 (b) 60 (c) 90 (d) 120

④ The base angles of the isosceles triangle are ....

(a) alternating (b) corresponding (c) congruent (d) supplementary

⑤ If $\overline{AD}$ is a median of Δ ABC and M is the point of concurrence of the medians , then MD = ... AD

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) $\frac{1}{2}$ (d) $\frac{1}{4}$

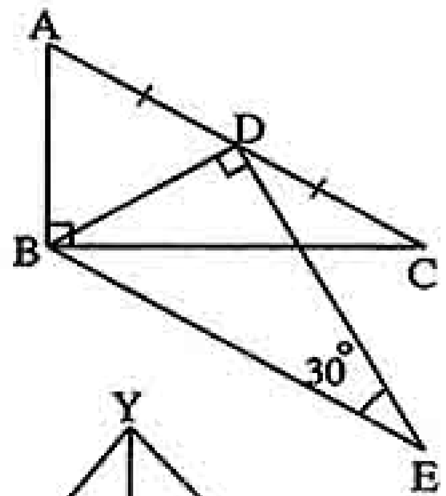
3 **[a] In the opposite figure :**

m ($\angle$ ABC) = m ($\angle$ BDE) = 90°

, m ($\angle$ E) = 30°

, D is the midpoint of $\overline{AC}$

Prove that : AC = BE



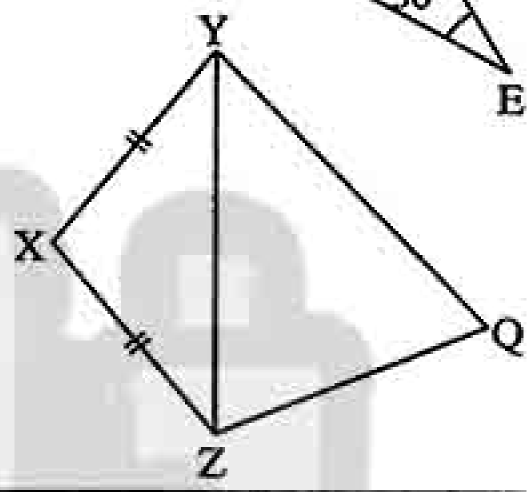
[b] In the opposite figure :

XY = XZ

, QY > QZ

Prove that :

m ($\angle$ XZQ) > m ($\angle$ XYQ)



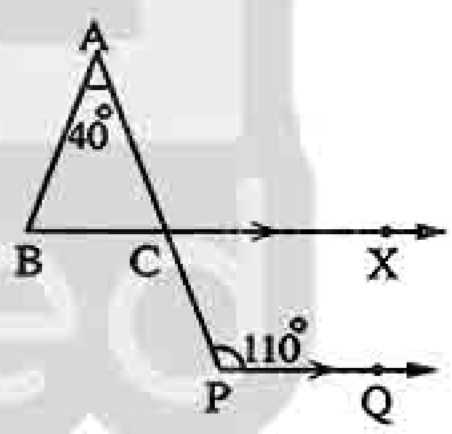
4 **[a] In the opposite figure :**

X $\in \overrightarrow{BC}$, $\overrightarrow{BC}$ // $\overrightarrow{PQ}$

, m ($\angle$ P) = 110°

, m ($\angle$ A) = 40°

Prove that : AB = AC



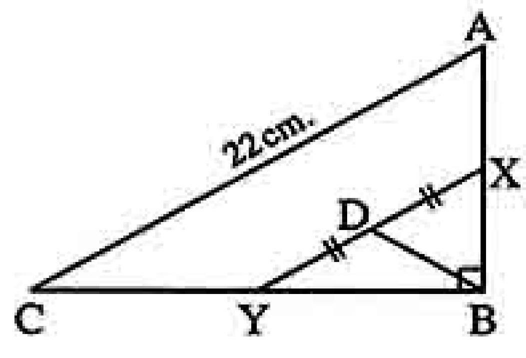
[b] In the opposite figure :

m ($\angle$ ABC) = 90°

X , Y , D are midpoints of $\overline{AB}$, $\overline{BC}$, $\overline{XY}$ respectively.

AC = 22 cm.

Find : BD

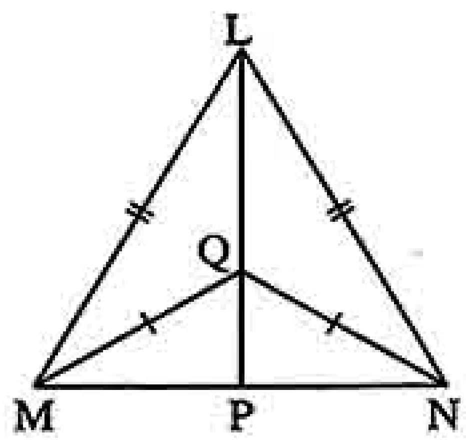


5 **[a] In the opposite figure :**

LM = LN

, QM = QN

Prove that : MP = NP

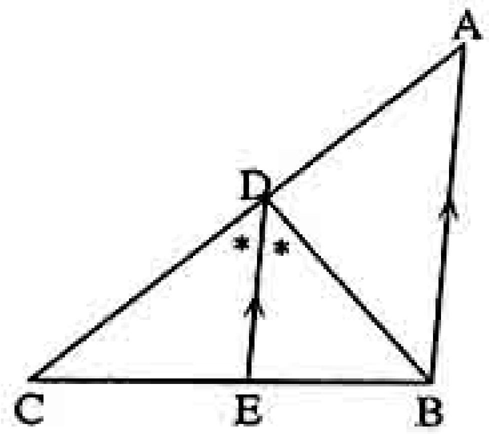


[b] In the opposite figure :

$\overrightarrow{DE}$ bisects $\angle$ BDC and $\overline{DE}$ // $\overline{AB}$

Prove that :

AC > BC



11 Ismailia Governorate

Directorate of Education
Directorate of Math's

Answer the following questions :

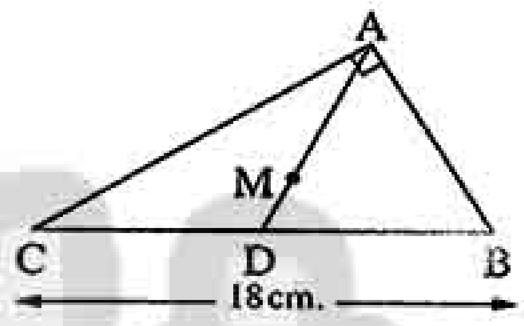
1 Choose the correct answer :

① **In the opposite figure :**

If m ($\angle$ A) = 90° , $\overline{AD}$ is a median ,

M is the point of intersection of its medians

and BC = 18 cm. , then MA = ... cm.



(a) 9 cm. (b) 3 cm. (c) 6 cm. (d) 18 cm.

② In Δ XYZ , if m ($\angle$ Y) < m ($\angle$ Z) , then XY .... XZ

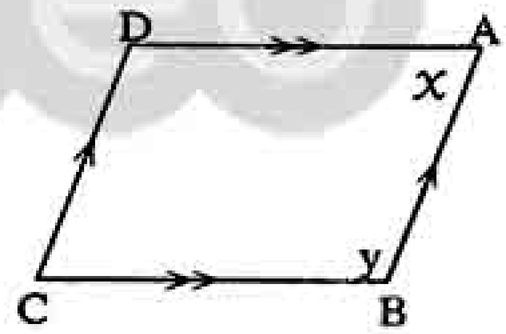
(a) = (b) < (c) > (d) twice

③ If the measures of two angles of a triangle are 65° and 50° , then the triangle is ....

(a) scalene (d) equilateral (c) isosceles (d) right angled

④ If ABCD is a parallelogram , x : y = 1 : 2

, then m ($\angle$ C) = ... °



(a) 60° (b) 120°

(c) 180° (d) 360°

⑤ If 10 cm. , 5 cm. and x cm. are side lengths of an isosceles triangle , then x = ... cm.

(a) 10 (b) 5 (c) 15 (d) 4

2 Complete :

① Number of axes of symmetry of an equilateral triangle = ...

② The perpendicular from the vertex angle of an isosceles triangle bisects each of .... and ....

③ In Δ ABC , if AB = 3 cm. and BC = 5 cm. , then AC $\in$].............. , ...[

④ If ABCD is a square , then m (∠ ACB) = ...°

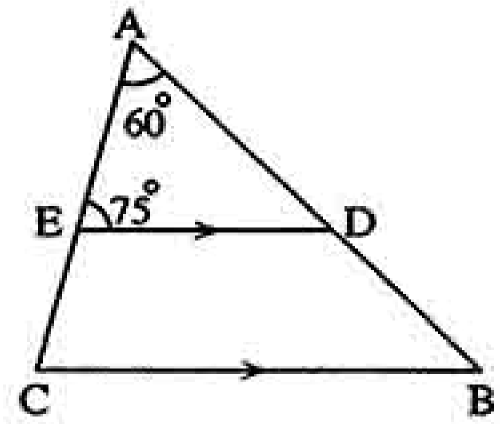
⑤ If A ∈ L where L is the axis of symmetry of $\overline{BC}$, then AB .... AC

3 [a] **In the opposite figure :**

$\overline{ED}$ // $\overline{BC}$

, m (∠ A) = 60° and m (∠ AED) = 75°

Prove that : AB > AC

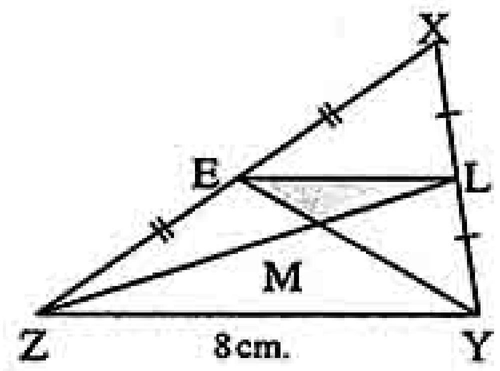


[b] **In the opposite figure :**

Δ XYZ in which : L and E are midpoints of $\overline{XY}$ and $\overline{XZ}$ respectively.

$\overline{YE}$ ∩ $\overline{ZL}$ = {M} , YZ = 8 cm. , YM = 4 cm. and ZL = 9 cm.

Find : The perimeter of Δ EML



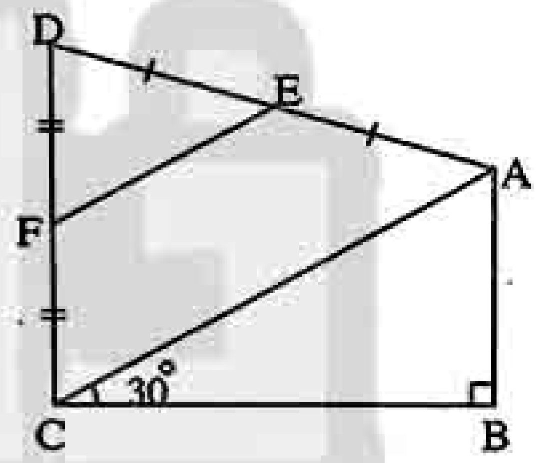
4 [a] **In the opposite figure :**

m (∠ B) = 90° , m (∠ ACB) = 30°

E is the midpoint of $\overline{AD}$

and F is the midpoint of $\overline{CD}$

Prove that : AB = EF

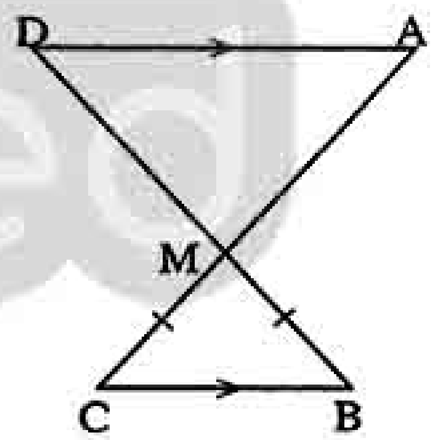


[b] **In the opposite figure :**

If $\overline{AC}$ ∩ $\overline{BD}$ = {M}

, $\overline{AD}$ // $\overline{BC}$ and MB = MC

Prove that : Δ MAD is an isosceles.



5 [a] In Δ ABC : If m (∠ A) = 50° and m (∠ B) = 85°

Find : m (∠ C) , then arrange the lengths of its sides ascendingly.

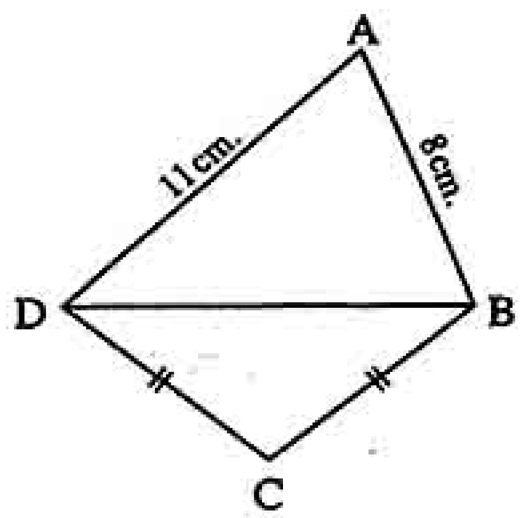
[b] **In the opposite figure :**

ABCD is a quadrilateral

, AD = 11 cm. , AB = 8 cm.

and CB = CD

Prove that : m (∠ ABC) > m (∠ ADC)



12 Damietta Governorate

Damietta Inspection of Mathematic
Official Language Schools

Answer the following questions :

1 Choose the correct answer :

① In △ ABC : m (∠ B) = 80° and m (∠ C) = 50° , then AB = ...

(a) BC (b) AC (c) 2 AC (d) $\frac{1}{2}$ BC

② The lengths 6 cm. , 7 cm. and .... can be lengths of the sides of a triangle.

(a) 15 cm. (b) 13 cm. (c) 18 cm. (d) 11 cm.

③ In △ ABC , if m (∠ A) = 30° and m (∠ B) = 90° , then AC = ...

(a) $\frac{1}{2}$ BC (b) 2 BC (c) 2 AB (d) BC

④ The point of intersection of the medians of the triangle divides each of them with ratio .... from the vertex.

(a) 1 : 2 (b) 3 : 1 (c) 2 : 1 (d) 1 : 3

⑤ In △ ABC , m (∠ A) = 50° and m (∠ B) = 100° then ....

(a) AB > AC (b) AC < AB (c) BC < AC (d) AB = BC

2 Complete :

① The measure of exterior angle of the equilateral triangle = ...°

② If △ ABC ≡ △ XYZ , then ∠ A ≡ ....

③ The longest side in a right-angled triangle is ....

④ If $\overleftrightarrow{XY}$ is an axis of symmetry of $\overline{AB}$, D ∈ $\overleftrightarrow{XY}$, then AD = ...

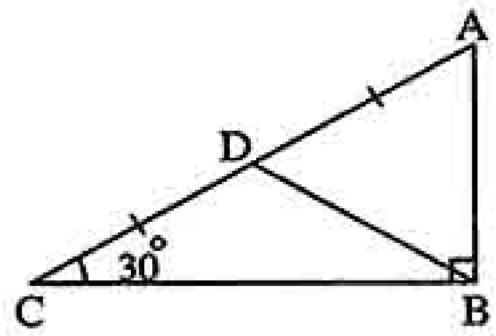
⑤ Square with side length 5 cm. , then its area = ... cm^2.

3 [a] In the opposite figure :

D is a midpoint of $\overline{AC}$

, m (∠ B) = 90° , m (∠ ACB) = 30°

Prove that : △ ABD is an equilateral triangle

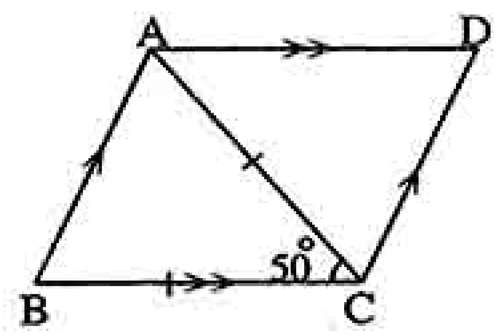


[b] In the opposite figure :

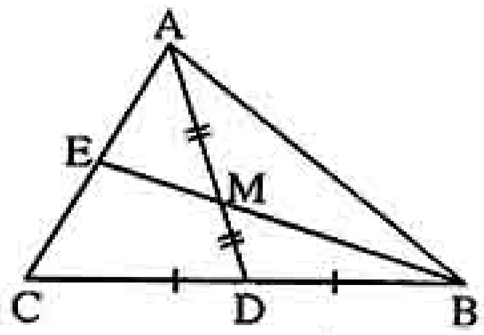
ABCD is a parallelogram

, CA = CB and m (∠ ACB) = 50°

Find with proof : m (∠ D)



4 **[a] In the opposite figure :**

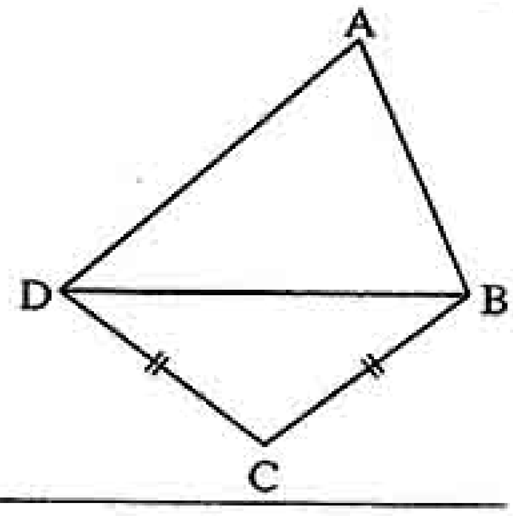


E and D are the midpoints of $\overline{AC}$ and $\overline{CB}$ respectively

If AD = 4.5 cm and BM = 4 cm.

Find : The length of each of $\overline{MD}$ and $\overline{BE}$

[b] In the opposite figure :



ABCD is a quadrilateral in which : AD > AB

and BC = CD

Prove that :

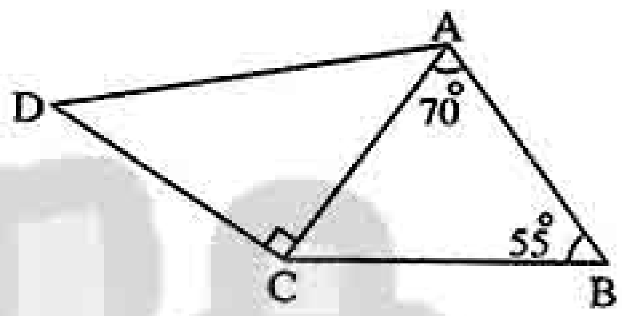
m (∠ ABC) > m (∠ ADC)

---

5 **[a]** ABC is a triangle in which : m (∠ A) = 40° and m (∠ B) = 75°

Arrange the lengths of sides of Δ ABC in ascending order.

[b] In the opposite figure :



m (∠ BAC) = 70° , m (∠ B) = 55°

and m (∠ ACD) = 90°

Prove that : AD > AB

13 El-Behira Governorate

Maths Inspection

Answer the following questions :

1 **Complete the following :**

① If the length of two sides of isosceles triangle are 8 cm. and 4 cm. , then the length of the third side is ....

② The number of axis of symmetry of scalene triangle is ....

③ The length of the median of the right-angled triangle from the vertex of right angle equals .... the length of the hypotenuse.

④ The base angles of the isosceles triangle are .... in measure.

⑤ In Δ ABC , if m (∠ A) = 40° and m (∠ B) = 60° , then the longest side is ....

---

2 **Choose the correct answer :**

① If A lies on the line of symmetry of $\overline{BC}$ then AB .... AC

(a) > (b) < (c) = (d) //

② The measure of the exterior angle of the equilateral triangle = ...

(a) 90° (b) 60° (c) 120° (d) 180°

③ In Δ ABC , if BC > AC , then m (∠ A) .... m (∠ B)

(a) > (b) < (c) = (d) ≥

④ If Δ ABC is a right-angled triangle at B and m (∠ C) = 30° , then AB = ... AC

(a) 2 (b) $\frac{1}{2}$ (c) $\frac{1}{3}$ (d) 3

⑤ The sum of lengths of two sides of a triangle is .... the length of the third side.

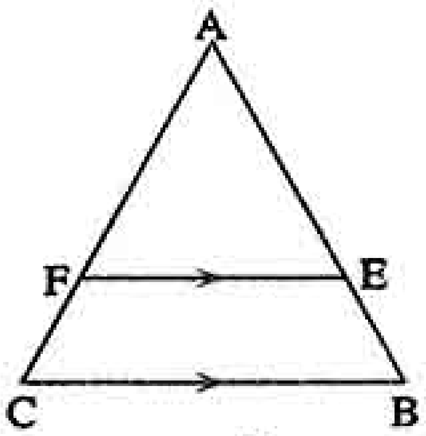
(a) greater than (b) less than (c) equal (d) greater than or equal

---

3 **[a] In the opposite figure :**

AB = AC , $\overline{EF}$ // $\overline{CB}$

Prove that : AE = AF



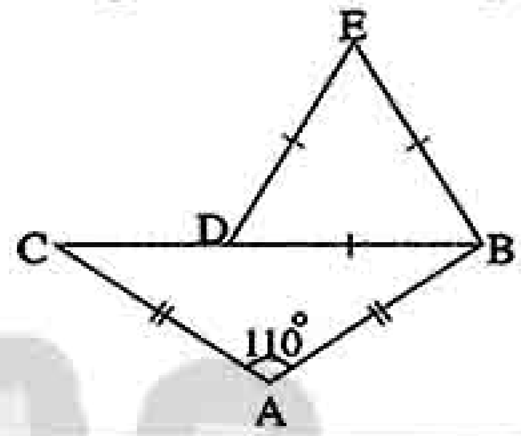
[b] In the opposite figure :

EB = ED = DB

, AB = AC

and m (∠ A) = 110°

Find : m (∠ ABE)



---

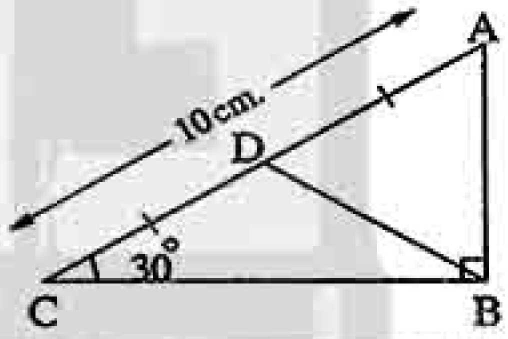
4 **[a]** In Δ ABC , if m (∠ A) = 50° and m (∠ B) = 60°

Arrange the side lengths of Δ ABC ascendingly.

[b] In the opposite figure :

m (∠ ABC) = 90° , m (∠ C) = 30° , AD = DC and AC = 10 cm.

Find : The perimeter of Δ ABD



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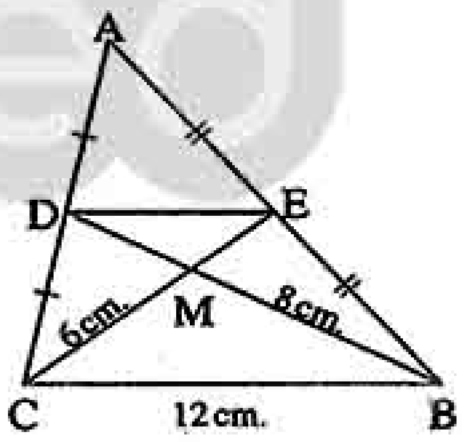
5 **In the opposite figure :**

AE = EB , AD = DC

, MB = 8 cm. , MC = 6 cm.

and BC = 12

Find : The perimeter of Δ MED



14 El-Minia Governorate

El-Minia Directorate of Education
Governmental languages schools

Answer the following questions :

1 **Complete the following : (Calculator is allowed)**

① The number of axes of symmetry in the equilateral triangle equals ....

② If the length of two sides in a triangle are 2 cm. and 7 cm.

, then .... < length of third side < ....

③ The length of median which drawn from the vertex of the right-angle in the right-angled triangle equals ....

④ If the measure of an angle in an isosceles triangle is 60° , then the triangle is ....

⑤ The length of the side opposite to the angle of measure 30° in the right-angled triangle equals ....

2 **Choose the correct answer :**

① XYZ is a triangle in which : m (∠ Z) = 70° and m (∠ Y) = 60° then YZ .... XY

(a) > (b) < (c) = (d) twice

② The numbers which can be lengths of sides of triangle are ....

(a) 0 , 3 , 5 (b) 3 , 3 , 5 (c) 3 , 3 , 6 (d) 3 , 3 , 7

③ The measure of the exterior angle of the equilateral triangle equals ....°

(a) 60 (b) 30 (c) 100 (d) 120

④ If the length of two sides in an isosceles triangle are 8 cm. and 4 cm. , then the length of the third side is .... cm.

(a) 4 (b) 8 (c) 3 (d) 12

⑤ If Δ ABC is a right-angled at B , AB = 6 cm. and BC = 8 cm. , then the length of the median drawn from B is .... cm.

(a) 10 (b) 8 (c) 6 (d) 5

3 [a] In Δ ABC , AB = 7 cm. , BC = 5 cm. and AC = 6 cm.

Arrange its angles measures ascendingly.

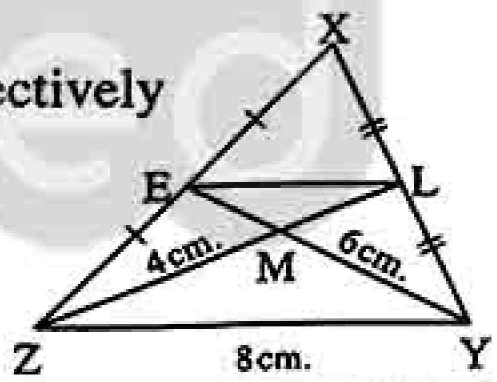
[b] In the opposite figure :

Δ XYZ in which : L and E are the midpoints of $\overline{XY}$ and $\overline{XZ}$ respectively

, $\overline{YE} \cap \overline{ZL} = \{M\}$

, YZ = 8 cm. , YM = 6 cm. , ZM = 4 cm.

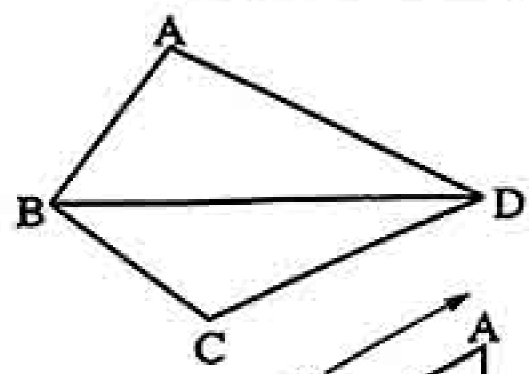
Find : The perimeter of Δ MLE



4 **[a] In the opposite figure :**

AB < AD , BC < CD

Prove that : m (∠ ABC) > m (∠ ADC)



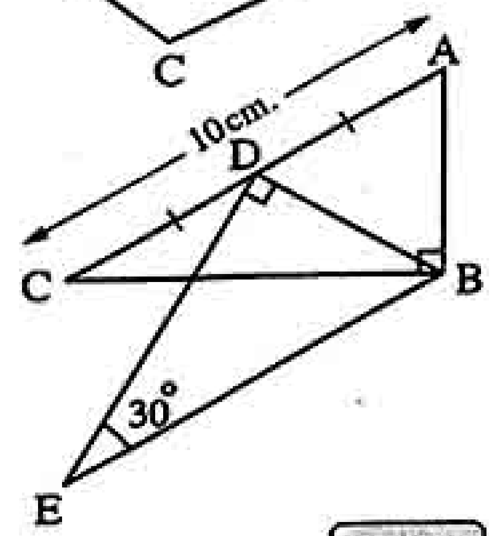
[b] In the opposite figure :

m (∠ ABC) = m (∠ BDE) = 90°

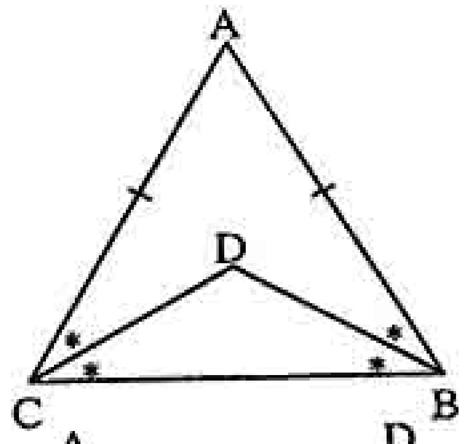
, D is the midpoint of $\overline{AC}$

, m (∠ E) = 30° and AC = 10 cm.

Find : The length of $\overline{BE}$



5 **[a] In the opposite figure :**

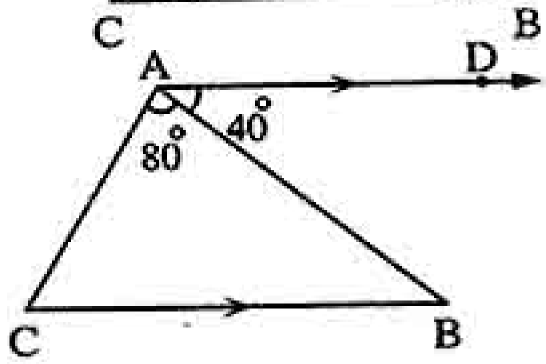


$AB = AC$, $\overrightarrow{BD}$ bisects $\angle B$

and $\overrightarrow{CD}$ bisects $\angle C$

Prove that : ΔDBC is an isosceles triangle.

[b] In the opposite figure :



ΔABC in which : $\overrightarrow{AD} \parallel \overline{CB}$

, $m(\angle DAB) = 40°$ and $m(\angle BAC) = 80°$

Prove that : $AB > AC$

15 South Sinai Governorate

Educational Directorate
Tur Sinai Educational Zone

Answer the following questions :

1 **Choose the correct answer from given answers :**

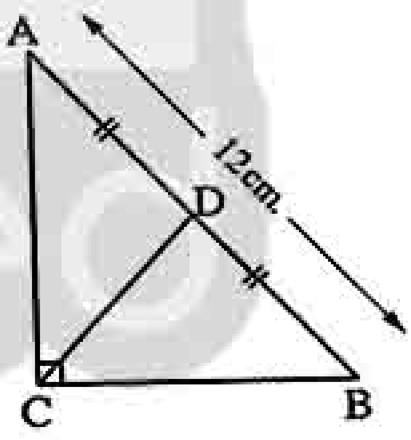
① In isosceles triangle the base angles are ....

(a) complementary. (b) supplementary. (c) adjacent. (d) congruent.

② The sum of the lengths of the two sides of the triangle .... the length of the third side.

(a) double (b) equals (c) greater than (d) less than

③ **In the opposite figure :**



If $AB = 12$ cm.

, then $CD =$ .... cm.

(a) 12 (b) 9

(c) 6 (d) 3

④ The triangle that has one axis of symmetry is .... triangle.

(a) an equilateral (b) an isosceles (c) a scalene (d) a right-angled

⑤ The .... is a parallelogram where one of its angles is right angle.

(a) a rectangle (b) a square (c) a rhombus (d) a trapezium

2 **Complete the following :**

① The point that divides the median of the triangle in the ratio 1 : 2 from the base is the point of intersection of ....

② In ΔABC , if $AB > BC$, then $m(\angle A) < m(\angle$ ....$)$

③ The sum of the measures of accumulative angles at point is ....°

(4) ABC is a triangle in which : m (∠ B) = 130° , then the longest side of its sides is ....

(5) In the right-angled triangle , the length of the side that opposite to the angle of measure 30° = ... the length of the hypotenuse.

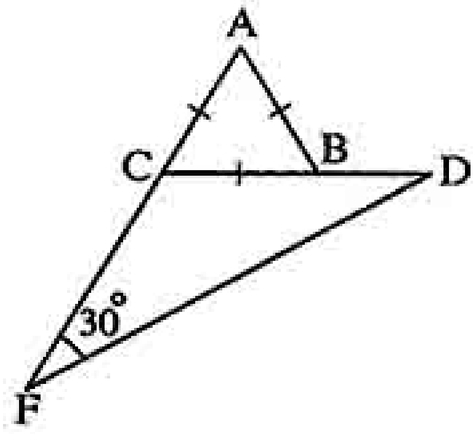
3 **[a] In the opposite figure :**

ABC is an equilateral triangle

, F ∈ $\overrightarrow{AC}$, D ∈ $\overrightarrow{CB}$

, m (∠ DFC) = 30°

Prove that : Δ DCF is an isosceles triangle.



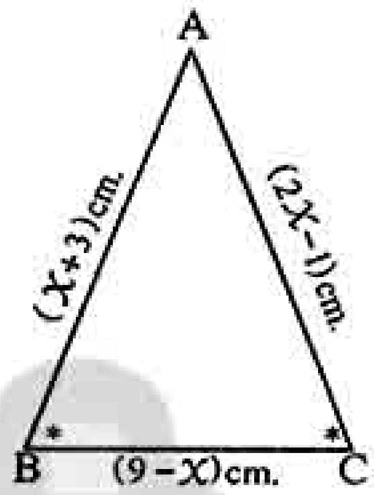
[b] In the opposite figure :

ABC is a triangle in which :

m (∠ B) = m (∠ C)

Find :

The perimeter of Δ ABC

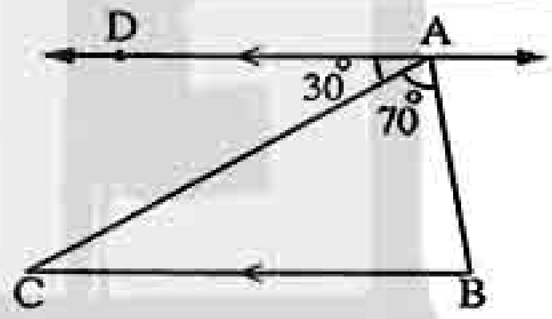


4 **[a] In the opposite figure :**

$\overleftrightarrow{AD}$ // $\overline{BC}$, m (∠ BAC) = 70°

and m (∠ DAC) = 30°

Prove that : AC > BC



[b] ABC is a triangle in which : AB = 7 cm. , BC = 5 cm. and AC = 6 cm.

Arrange the measures of its angles in an ascending order.

5 **[a] In the opposite figure :**

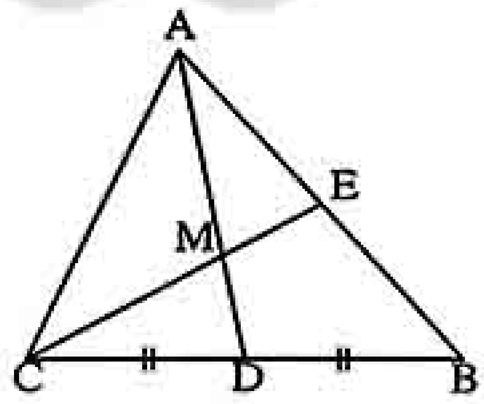
ABC is a triangle

, D is the midpoint of $\overline{BC}$, M ∈ $\overline{AD}$

, where AM = 2 MD

Draw $\overrightarrow{CM}$ cuts $\overline{AB}$ at E , if EC = 12 cm.

, **find :** The length of $\overline{EM}$

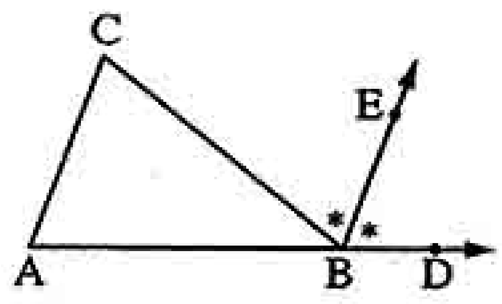


[b] In the opposite figure :

BA = BC

and $\overrightarrow{BE}$ bisects ∠ CBD

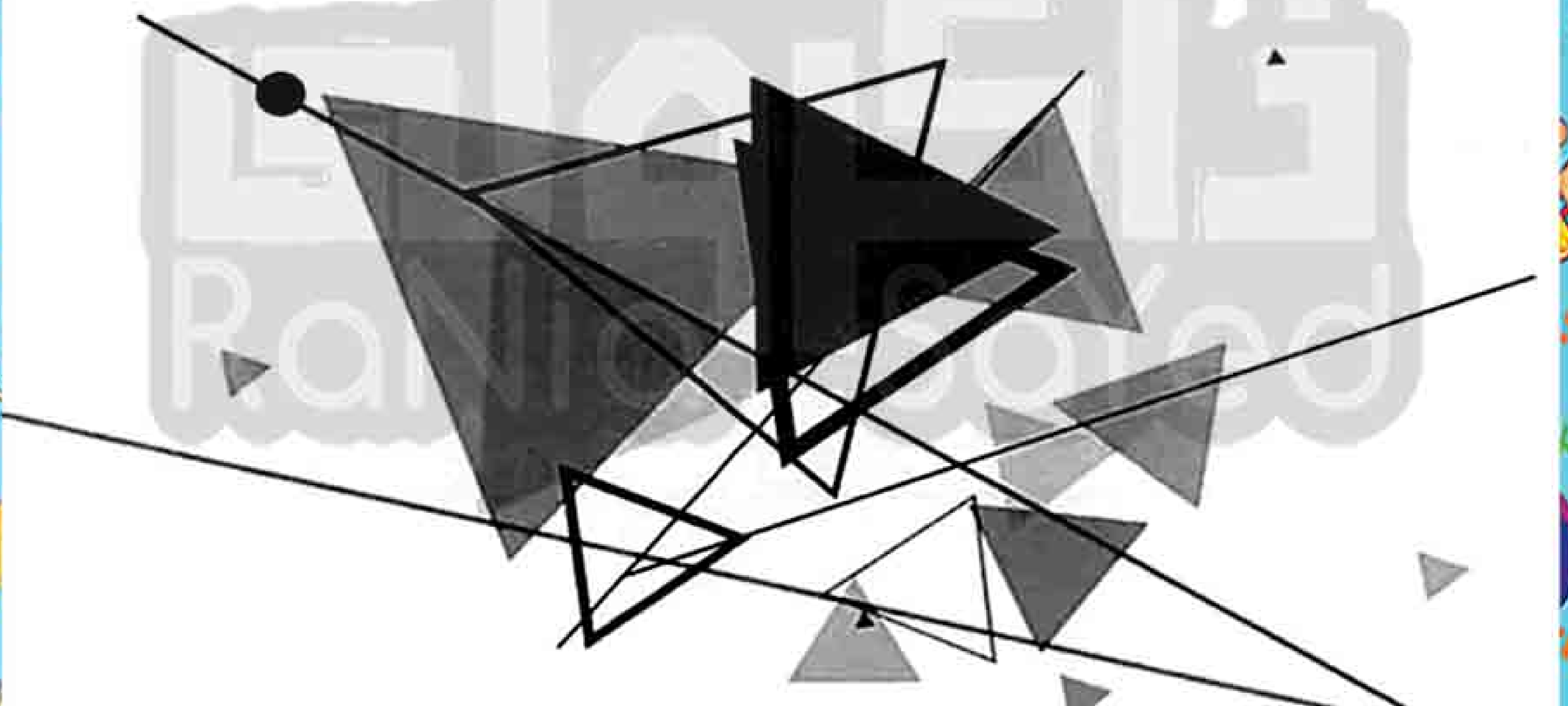
Prove that : $\overrightarrow{BE}$ // $\overline{AC}$



EL-MOASSER

In Mathematics
Guide Answers

For 2nd Prep.
First Term



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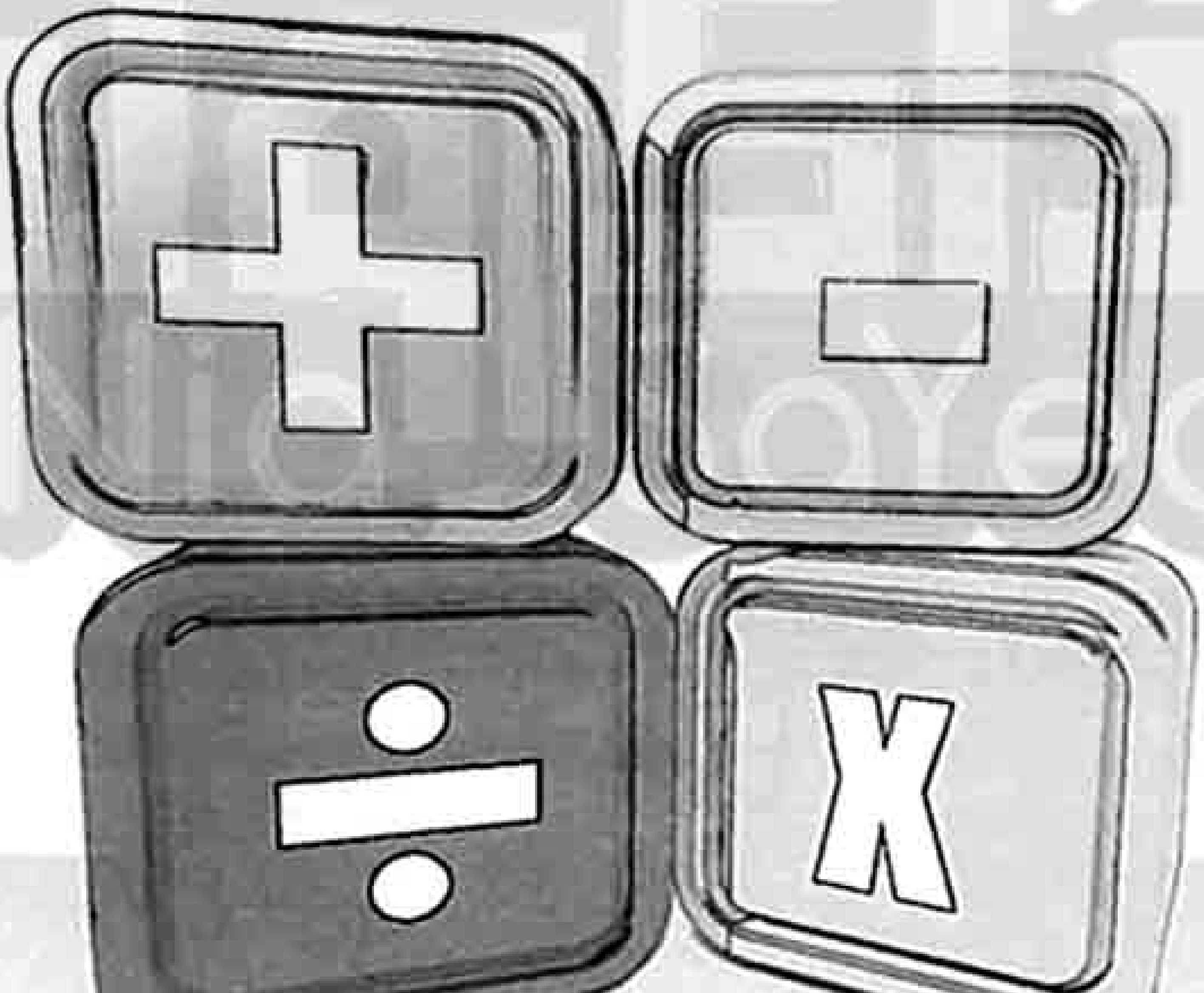
Guide Answers

of Algebra and Statistics Exercises



Answers of revision exercise

1

1) $\frac{1}{5}$ 2) $\frac{3}{10}$ 3) $\frac{1}{4}$ 4) $\frac{3}{4}$ 5) $-\frac{6}{1}$ 6) $\frac{5}{4}$

2

1) b 2) b 3) c
4) a 5) c 6) d
7) d 8) b 9) b

3

1) 13 2) $\frac{1}{2}$ 3) -7
4) zero 5) 1 6) 2
7) 1.5×10^{-4} 8) 4.21×10^{5} 9) 15
10) zero 11) 1

4

1) $\because 5x = 20 - 3 = 17$ $\therefore x = \frac{17}{5}$
2) $\because 7x = 12 - 11 = 1$ $\therefore x = \frac{1}{7}$
3) $\because 3x = 1 - 5 = -4$ $\therefore x = -\frac{4}{3}$
4) $x = 7 - 3 = 4$

5

1) $\because x^2 + 12 = 21$ $\therefore x^2 = 21 - 12$
$\therefore x^2 = 9$ $\therefore x = \pm\sqrt{9}$
$\therefore x = \pm 3$ $\therefore$ The S.S. $= \{3, -3\}$
2) $\because 2x^2 - 1 = -9$ $\therefore 2x^2 = -9 + 1$
$\therefore 2x^2 = -8$ $\therefore x^2 = \frac{-8}{2}$
$\therefore x^2 = -4$ $\therefore x = \pm\sqrt{-4} \notin \mathbb{Q}$
$\therefore$ The S.S. $= \varnothing$
3) $\because |x| = 2$ $\therefore x = \pm 2$
$\therefore$ The S.S. $= \{2, -2\}$
4) $\because \sqrt{x^2} = 4$ $\therefore |x| = 4$
$\therefore x = \pm 4$ $\therefore$ The S.S. $= \{4, -4\}$

Answers of unit one

Answers of Exercise 1

1

Number a	8	125	-27	-1000	$3\frac{3}{8}$	$-\frac{8}{125}$	216	-64
$\sqrt[3]{a}$	2	5	-3	-10	$\frac{3}{2}$	$-\frac{2}{5}$	6	-4

2

1) 6 2) -7 3) $\frac{4}{5}$ 4) $-\frac{2}{3}$
5) 0.1 6) $-\frac{4}{3}$ 7) $2x$ 8) $-3a^2$

3

1) a 2) 64 3) 64 4) 25
5) zero 6) -1 7) 6 8) 1
9) 4 10) zero 11) $-\frac{1}{2}$ 12) 61

4

1) c 2) b 3) a 4) a 5) d
6) c 7) c 8) d 9) a 10) d
11) c 12) d 13) b

5

1) 125 2) $-\frac{1}{64}$
3) $\because \sqrt[3]{x} = -2$ $\therefore x = -8$
4) $\because \sqrt[3]{x} = -1 + 3 = 2$ $\therefore x = 8$
5) -2 6) 4
7) $\because x^3 = 32 - 5 = 27$ $\therefore x = 3$
8) $\because x^3 = 54 \div 2 = 27$ $\therefore x = 3$
9) $\because x^3 = -200 \div \frac{1}{5} = -1000$ $\therefore x = -10$

6

1) $\because x^3 = -27$ $\therefore x = \sqrt[3]{-27} = -3$
$\therefore$ The S.S. $= \{-3\}$
2) $\because 8x^3 = 8 - 7 = 1$ $\therefore x^3 = \frac{1}{8}$
$\therefore x = \sqrt[3]{\frac{1}{8}} = \frac{1}{2}$ $\therefore$ The S.S. $= \{\frac{1}{2}\}$
3) $\because x^3 = \frac{3}{8} - 16 = -\frac{125}{8}$ $\therefore x = \sqrt[3]{-\frac{125}{8}} = -\frac{5}{2}$
$\therefore$ The S.S. $= \{-\frac{5}{2}\}$
4) $\because 2x^3 - x^3 = 3 + 5$ $\therefore x^3 = 8$
$\therefore x = \sqrt[3]{8} = 2$ $\therefore$ The S.S. $= \{2\}$
5) $\because x + 3 = \sqrt[3]{343} = 7$ $\therefore x = 7 - 3 = 4$
$\therefore$ The S.S. $= \{4\}$
6) $\because 3x + 1 = \sqrt[3]{-8} = -2$ $\therefore 3x = -2 - 1 = -3$
$\therefore x = -3 \div 3 = -1$ $\therefore$ The S.S. $= \{-1\}$
7) $\because (2x + 1)^3 = 20 + 7 = 27$ $\therefore 2x + 1 = \sqrt[3]{27} = 3$
$\therefore 2x = 3 - 1 = 2$ $\therefore x = 2 \div 2 = 1$
$\therefore$ The S.S. $= \{1\}$

8 $\because (5x-2)^3 = 18-10 = 8$ $\quad \therefore 5x-2 = \sqrt[3]{8} = 2$

$\therefore 5x = 2+2 = 4$ $\quad \therefore x = \frac{4}{5}$

$\therefore$ The S.S. $= \{\frac{4}{5}\}$

7

1 $\sqrt[3]{2\frac{1}{4} \div \frac{2}{3}} = \sqrt[3]{\frac{9}{4} \times \frac{3}{2}} = \sqrt[3]{\frac{27}{8}} = \frac{3}{2}$

2 $-\sqrt[3]{2^9 \times 3^6} = -\sqrt[3]{(2^3 \times 3^2)^3} = -2^3 \times 3^2 = -8 \times 9 = -72$

3 $\sqrt{\sqrt[3]{729}} = \sqrt{9} = 3$

4 $\sqrt[3]{\sqrt[3]{512}} = \sqrt[3]{8} = 2$

5 $\sqrt{27\sqrt[3]{27}} = \sqrt{27 \times 3} = \sqrt{81} = 9$

8

The edge length of the cube $= \sqrt[3]{15\frac{5}{8}} = \sqrt[3]{\frac{125}{8}} = 2.5$ cm.

9

The edge length of the cube $= \sqrt[3]{216} = 6$ cm.

$\therefore$ Its total area $= 6 \times 6^2 = 216$ cm^2.

10

Let the number be x $\quad \therefore x^3 = 27$

$\therefore x = 3$ $\quad \therefore x^2 = 9$

$\therefore$ The square of the number $= 9$

11

Let the number be x $\quad \therefore \frac{1}{2}x^3 = 32$

$\therefore x^3 = 64$ $\quad \therefore x = 4$

$\therefore$ The number $= 4$

12

The length of the inner edge $= \sqrt[3]{1000} = 10$ cm.

13

The volume of the sphere $= \frac{4}{3}\pi r^3 = \frac{1372}{81}\pi$

$\therefore r^3 = \frac{1372}{81} \times \frac{3}{4} = \frac{343}{27}$

$\therefore r = \sqrt[3]{\frac{343}{27}} = \frac{7}{3}$

$\therefore$ The diameter length of the sphere

$= 2 \times \frac{7}{3} = \frac{14}{3}$ length unit.

14

$\therefore$ The volume of the sphere $= \frac{4}{3}\pi r^3 = 113.04$

$\therefore \frac{4}{3} \times 3.14 \times r^3 = 113.04$

$\therefore r^3 = 27$

$\therefore r = \sqrt[3]{27} = 3$ cm.

$\therefore$ The diameter length of the sphere $= 2 \times 3 = 6$ cm.

15

1 $\because (x^2+6)^3 = 1000$ $\quad \therefore x^2+6 = 10$

$\therefore x^2 = 4$ $\quad \therefore x = \pm 2$

$\therefore$ The S.S. $= \{2, -2\}$

2 $\because (x^3-14)^2 = 169$ $\quad \therefore x^3 - 14 = \pm 13$

$\therefore x^3 = 14 \pm 13$ $\quad \therefore x^3 = 27$ $\quad \therefore x = 3$

or $x^3 = 1$ $\quad \therefore x = 1$

$\therefore$ The S.S. $= \{3, 1\}$

3 Cubing the two sides $\quad \therefore (x-1)^2 = 25$

$\therefore x - 1 = \pm 5$ $\quad \therefore x = 6$ or $x = -4$

$\therefore$ The S.S. $= \{6, -4\}$

4 $\because \sqrt[3]{(x-2)(x-2)^2} = 3$ $\quad \therefore \sqrt[3]{(x-2)^3} = 3$

$\therefore x - 2 = 3$ $\quad \therefore x = 5$

$\therefore$ The S.S. $= \{5\}$

16

Cubing the two sides

$\therefore \sqrt{x} + 19 = 27$ $\quad \therefore \sqrt{x} = 8$

Squaring the two sides

$\therefore x = 64$ $\quad \therefore \sqrt[3]{x} = \sqrt[3]{64} = 4$

17

Let the age of the grandfather be x year

$\therefore$ The age of the man $= \frac{1}{2}x$ year

The age of the grandson (the elder) $= \sqrt{x}$ year

The age of the grandson (the middle) $= \sqrt[3]{x}$ year

The age of the granddaughter $= \frac{\sqrt{x}}{\sqrt[3]{x}}$ year

$\because \sqrt{x} = 2\sqrt[3]{x}$, then cubing the two sides

$\therefore x\sqrt{x} = 8x$, then squaring the two sides

$\therefore x^3 = 64x^2$ $\quad \therefore x = 64$

$\therefore$ The age of the grandfather $= 64$ years

The age of the grandson (the elder) $= \sqrt{64} = 8$ years

The age of the grandson (the middle) $= \sqrt[3]{64} = 4$ years

The age of the granddaughter $= 8 \div 4 = 2$ years

Answers of Exercise 2

1

The rational numbers are No.

1 , 2 , 3 , 4 , 5 , 8 , 9 ,

11 , 13 , 14 , 16 , 17 , 18 , 19

The remained numbers are irrational.

2 1 3.32   2 1.9   3 – 2.1

3

1 $\because \sqrt{4} < \sqrt{5} < \sqrt{9}$ $\therefore 2 < \sqrt{5} < 3$

$\therefore$ The two numbers are 2 , 3

2 $\because \sqrt{9} < \sqrt{12} < \sqrt{16}$ $\therefore 3 < \sqrt{12} < 4$

$\therefore$ The two numbers are 3 and 4

3 $\because \sqrt[3]{8} < \sqrt[3]{10} < \sqrt[3]{27}$ $\therefore 2 < \sqrt[3]{10} < 3$

$\therefore$ The two numbers are 2 and 3

4 $\because \sqrt[3]{-27} < \sqrt[3]{-20} < \sqrt[3]{-8}$ $\therefore -3 < \sqrt[3]{-20} < -2$

$\therefore$ The two numbers are -2 and -3

4

1 $\because \sqrt{1} < \sqrt{2} < \sqrt{4}$ $\therefore 1 < \sqrt{2} < 2$ $\therefore x = 1$

2 $\because \sqrt{64} < \sqrt{80} < \sqrt{81}$ $\therefore 8 < \sqrt{80} < 9$ $\therefore x = 8$

3 $\because \sqrt[3]{1} < \sqrt[3]{5} < \sqrt[3]{8}$ $\therefore 1 < \sqrt[3]{5} < 2$ $\therefore x = 1$

4 $\because \sqrt[3]{27} < \sqrt[3]{50} < \sqrt[3]{64}$ $\therefore 3 < \sqrt[3]{50} < 4$ $\therefore x = 3$

5 $\because \sqrt[3]{-125} < \sqrt[3]{-100} < \sqrt[3]{-64}$

$\therefore -5 < \sqrt[3]{-100} < -4$ $\therefore x = -5$

6 $\because \sqrt{25} < \sqrt{35} < \sqrt{36}$ $\therefore 5 < \sqrt{35} < 6$ $\therefore x = 5$

5

1 $\because \sqrt{16} < \sqrt{20} < \sqrt{25}$ $\therefore 4 < \sqrt{20} < 5$

$\because (4.1)^2 = 16.81 , (4.2)^2 = 17.64 , (4.3)^2 = 18.49 ,$

$(4.4)^2 = 19.36 , (4.5)^2 = 20.25$

$\therefore 4.4 < \sqrt{20} < 4.5$ $\therefore \sqrt{20} \approx 4.4$ or 4.5

Using the calculator $\sqrt{20} \approx 4.47$

2 $\because \sqrt[3]{8} < \sqrt[3]{17} < \sqrt[3]{27}$ $\therefore 2 < \sqrt[3]{17} < 3$

$\because (2.1)^3 = 9.261 , (2.2)^3 = 10.648 , (2.3)^3 = 12.167$

$(2.4)^3 = 13.824 , (2.5)^3 = 15.625 , (2.6)^3 = 17.576$

$\therefore 2.5 < \sqrt[3]{17} < 2.6$

$\sqrt[3]{17} \approx 2.5$ or 2.6

Using the calculator $\sqrt[3]{17} \approx 2.57$

3 $\because \sqrt{4} < \sqrt{5} < \sqrt{9}$ $\therefore 2 < \sqrt{5} < 3$

$\because (2.1)^2 = 4.41 , (2.2)^2 = 4.84 , (2.3)^2 = 5.29$

$\therefore 2.2 < \sqrt{5} < 2.3$ $\therefore 3.2 < \sqrt{5} + 1 < 3.3$

$\therefore \sqrt{5} + 1 \approx 3.2$ or 3.3

Using the calculator $\sqrt{5} + 1 \approx 3.24$

4 $\because \sqrt[3]{8} < \sqrt[3]{9} < \sqrt[3]{27}$ $\therefore 2 < \sqrt[3]{9} < 3$

$\because (2.1)^3 = 9.261$ $\therefore 2 < \sqrt[3]{9} < 2.1$

$\therefore 1 < \sqrt[3]{9} - 1 < 1.1$ $\therefore \sqrt[3]{9} - 1 \approx 1$ or 1.1

Using the calculator $\sqrt[3]{9} - 1 \approx 1.08$

6

1 d   2 b   3 b   4 c

5 c   6 b   7 b   8 d

9 c   10 c   11 d

7

1 $x^2 = \frac{10}{5} = 2$ $\therefore x = \pm\sqrt{2}$ $\therefore x \in \mathbb{Q}'$

2 $x^2 = \frac{9}{4}$ $\therefore x = \pm\sqrt{\frac{9}{4}} = \pm\frac{3}{2}$ $\therefore x \in \mathbb{Q}$

3 $x = \sqrt[3]{125}$ $\therefore x = 5$ $\therefore x \in \mathbb{Q}$

4 $x^3 = \frac{27}{3} = 9$ $\therefore x = \sqrt[3]{9}$ $\therefore x \in \mathbb{Q}'$

5 $x^2 = \frac{10}{0.1} = 100$ $\therefore x = \pm\sqrt{100} = \pm 10$ $\therefore x \in \mathbb{Q}$

6 $x^3 = \frac{-8}{0.001} = -8000$

$\therefore x = \sqrt[3]{-8000} = -20$ $\therefore x \in \mathbb{Q}$

7 $x - 1 = \pm\sqrt{4} = \pm 2$ $\therefore x = 2 + 1 = 3$

or $x = -2 + 1 = -1$ $\therefore x \in \mathbb{Q}$

8 $x - 5 = \sqrt[3]{1} = 1$ $\therefore x = 1 + 5 = 6$ $\therefore x \in \mathbb{Q}$

8

1 $x^2 = 13$ $\therefore x = \pm\sqrt{13}$

$\therefore$ The S.S. $= \{\sqrt{13} , -\sqrt{13}\}$

2 $x^3 = 16$ $\therefore x = \sqrt[3]{16}$ $\therefore$ The S.S. $= \{\sqrt[3]{16}\}$

3 $x^2 = \frac{25}{2} \times \frac{5}{2} = \frac{125}{4}$ $\therefore x = \pm\sqrt{\frac{125}{4}}$

$\therefore$ The S.S. $= \{\sqrt{\frac{125}{4}} , -\sqrt{\frac{125}{4}}\}$

4 $x^3 = -2 \times \frac{4}{5} = -\frac{8}{5}$ $\quad \therefore x = \sqrt[3]{-\frac{8}{5}}$

$\therefore$ The S.S. $= \{\sqrt[3]{-\frac{8}{5}}\}$

5 $125\,x^3 = 27$ $\quad \therefore x^3 = \frac{27}{125}$

$\therefore x = \sqrt[3]{\frac{27}{125}} = \frac{3}{5}$

$\therefore$ The S.S. $= \varnothing$ because $\frac{3}{5} \notin \acute{Q}$

6 $\frac{1}{4}\,x^2 = 64$ $\quad \therefore x^2 = 64 \times 4 = 256$

$\therefore x = \pm\sqrt{256} = \pm 16$

$\therefore$ The S.S. $= \varnothing$ because $16 \notin \acute{Q}$, $-16 \notin \acute{Q}$

7 $\because (x^3 + 5)(x^2 - 3) = 0$

$\therefore x^3 + 5 = 0$ $\quad \therefore x^3 = -5$ $\quad \therefore x = -\sqrt[3]{5}$

or $x^2 - 3 = 0$ $\quad \therefore x^2 = 3$ $\quad \therefore x = \pm\sqrt{3}$

$\therefore$ The S.S. $= \{-\sqrt[3]{5}, \sqrt{3}, -\sqrt{3}\}$

8 $\because (x + \sqrt{7})(x^3 - 6) = 0$

$\therefore x + \sqrt{7} = 0$ $\quad \therefore x = -\sqrt{7}$

or $x^3 - 6 = 0$ $\quad \therefore x^3 = 6$

$\therefore x = \sqrt[3]{6}$ $\quad \therefore$ The S.S. $= \{-\sqrt{7}, \sqrt[3]{6}\}$

9

1 $\because (1.4)^2 = 1.96$, $(1.5)^2 = 2.25$, $(\sqrt{2})^2 = 2$

$\therefore \sqrt{2}$ is included between 1.4 , 1.5

2 $\because (3.31)^2 \simeq 10.96$, $(3.32)^2 \simeq 11.02$, $(\sqrt{11})^2 = 11$

$\therefore \sqrt{11}$ is included between 3.31 , 3.32

3 $\because (1.2)^3 = 1.728$, $(1.3)^3 = 2.197$, $(\sqrt[3]{2})^3 = 2$

$\therefore \sqrt[3]{2}$ is included between 1.2 , 1.3

4 $\because (2.4)^3 = 13.824$, $(2.5)^3 = 15.625$

$(\sqrt[3]{15})^3 = 15$

$\therefore \sqrt[3]{15}$ is included between 2.4 , 2.5

5 $\because (-2.6)^3 = -17.576$, $(-2.5)^3 = -15.625$

$(\sqrt[3]{-17})^3 = -17$

$\therefore \sqrt[3]{-17}$ is included between -2.6 , -2.5

6 $\because 2.7 - 1 = 1.7$, $(1.7)^2 = 2.89$

$2.8 - 1 = 1.8$, $(1.8)^2 = 3.24$

$\sqrt{3} + 1 - 1 = \sqrt{3}$, $(\sqrt{3})^2 = 3$

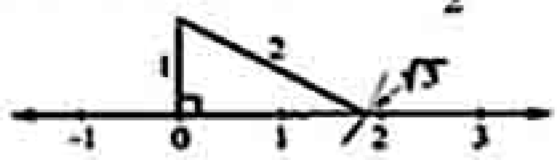
$\therefore \sqrt{3}$ is included between 1.7 , 1.8

$\therefore \sqrt{3} + 1$ is included between 2.7 , 2.8

10

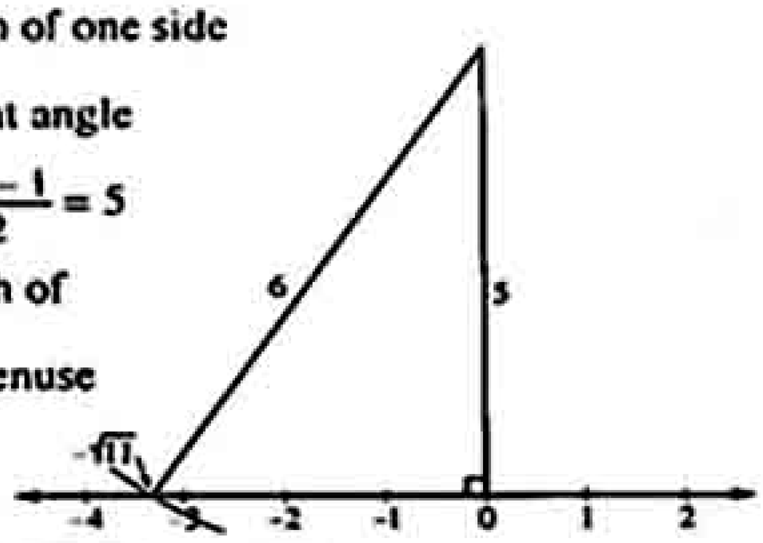
1 The length of one side of the right angle $= \frac{3-1}{2} = 1$

The length of the hypotenuse $= \frac{3+1}{2} = 2$



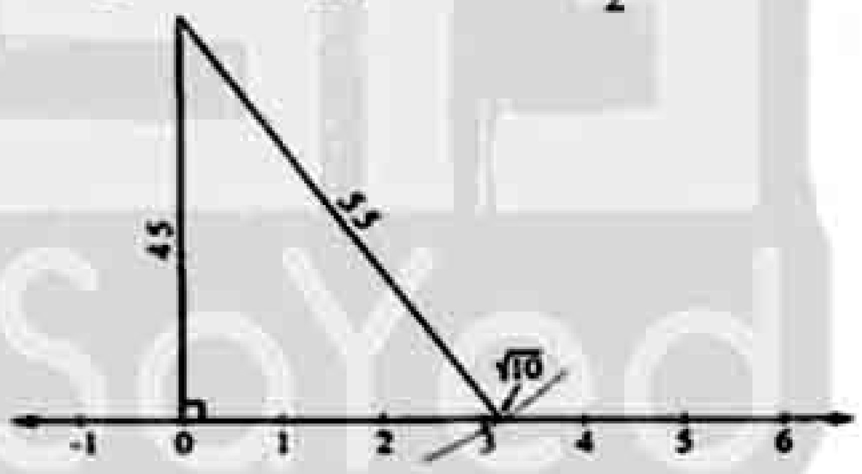
2 The length of one side of the right angle equals $\frac{11-1}{2} = 5$

The length of the hypotenuse $= \frac{11+1}{2} = 6$



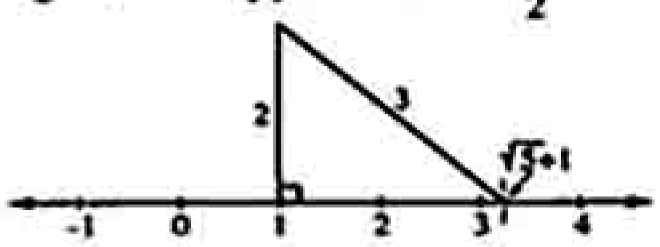
3 The length of one side of the right angle $= \frac{10-1}{2} = 4.5$

The length of the hypotenuse $= \frac{10+1}{2} = 5.5$



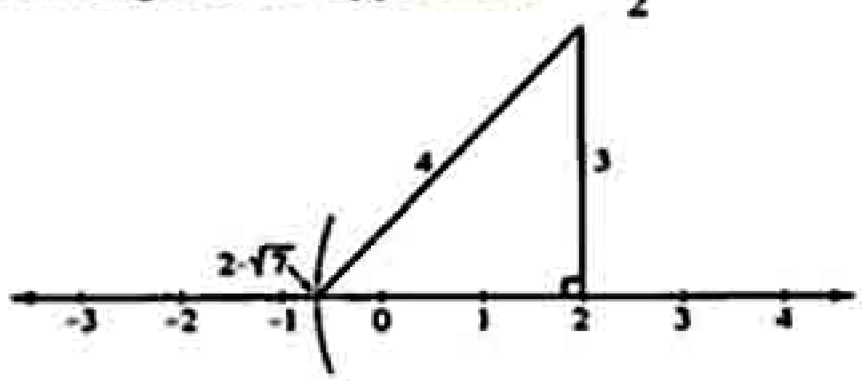
4 The length of one side of the right angle $= \frac{5-1}{2} = 2$

The length of the hypotenuse $= \frac{5+1}{2} = 3$



5 The length of one side of the right angle $= \frac{7-1}{2} = 3$

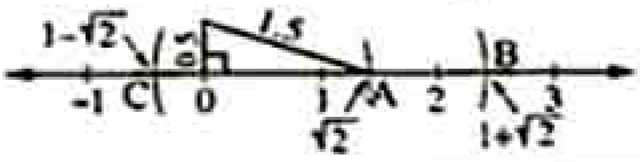
The length of the hypotenuse $= \frac{7+1}{2} = 4$



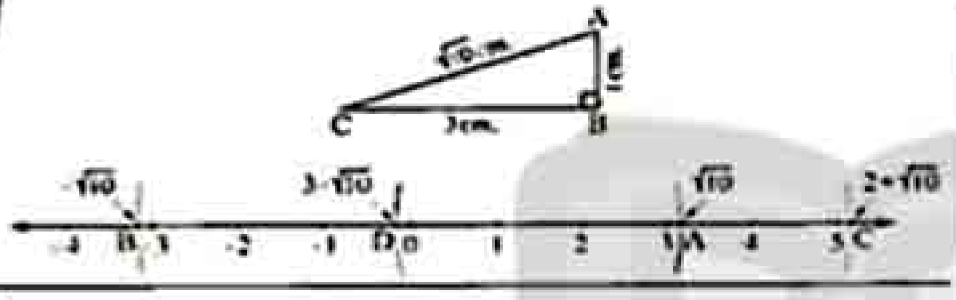
11

The length of one side of the right angle $= \frac{2-1}{2} = 0.5$

The length of the hypotenuse $= \frac{2+1}{2} = 1.5$



12



13

The length of the side of the square $= \sqrt{10}$ cm.

The square of the length of the diagonal

$= (\sqrt{10})^2 + (\sqrt{10})^2 = 10 + 10 = 20$

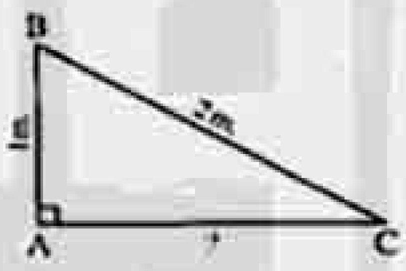
$\therefore$ The length of the diagonal $= \sqrt{20}$ cm.

14

$\because$ The length of the tree = 3 m.

$\therefore$ AB + BC = 3 m.

, $\because$ the length of the left part of the tree = 1 m.



$\therefore$ BC = 3 − 1 = 2 m.

$\because$ In Δ ABC : m ($\angle$ A) = 90°

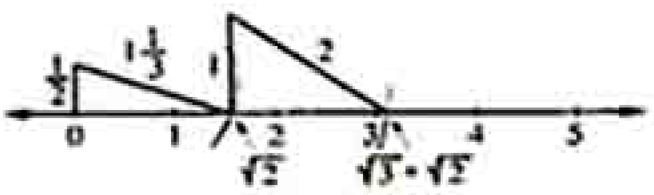
$\therefore (AC)^2 = (BC)^2 - (AB)^2 = 4 - 1 = 3$

$\therefore AC = \sqrt{3}$ m.

$\therefore$ The distance between the base of the tree and the point of touching of its top with the ground $= \sqrt{3}$ m.

15

We represent on the number line the point representing the number $\sqrt{3} + \sqrt{2}$ as shown in the figure :



We find that the point representing the number $\sqrt{3} + \sqrt{2}$ lies between the point representing the number 3 and the point representing the number 4

i.e. $\sqrt{3} + \sqrt{2}$ lies between 3 and 4

Answers of Exercise 3

1

The number	Natural	Integer	Rational	Irrational	Real
-5	✗	✓	✓	✗	✓
$\sqrt{2}$	✗	✗	✗	✓	✓
$1\frac{1}{2}$	✗	✗	✓	✗	✓
$\sqrt[3]{9}$	✗	✗	✗	✓	✓
$\lvert -2 \rvert$	✓	✓	✓	✗	✓
$-\sqrt{4}$	✗	✓	✓	✗	✓
$\frac{5}{2}$	✗	✗	✓	✗	✓
0.3	✗	✗	✓	✗	✓
$\sqrt{-1}$	✗	✗	✗	✗	✗

2

1 $\varnothing$ 2 $\mathbb{R}$ 3 $\varnothing$

4 $\mathbb{R}^{*}$ 5 $\mathbb{Q}$ 6 $\mathbb{Q}'$

3

1 positive 2 negative 3 positive

4 positive 5 negative 6 positive

4

1 > 2 > 3 <

4 < 5 > 6 =

7 > 8 > 9 >

5

1 a 2 b 3 a 4 d

5 a 6 c 7 d

6

1 The ascending order is :

$-\sqrt{11}, -\sqrt{7}, -\sqrt{3}, \sqrt{5}, \sqrt{8}$ and $\sqrt{15}$

2 $\because 0.6 = \sqrt{0.36}, \sqrt[3]{-1} = -1 = -\sqrt{1}$

$\therefore$ The ascending order is :

$-\sqrt{45}, -\sqrt{1}, \sqrt{0.36}, \sqrt{20}$ and $\sqrt{27}$

i.e. $-\sqrt{45}, \sqrt[3]{-1}, 0.6, \sqrt{20}$ and $\sqrt{27}$

7

1. $\because 8 = \sqrt{64}$
   $\therefore$ The descending order is :
   $\sqrt{70}$, $\sqrt{64}$, $\sqrt{62}$ and $-\sqrt{50}$
   i.e. $\sqrt{70}$, 8 , $\sqrt{62}$ and $-\sqrt{50}$
2. $\because 9 = \sqrt{81}$
   $\therefore$ The descending order is :
   $\sqrt{101}$, $\sqrt{81}$, $\sqrt{6}$, $-\sqrt{7}$, $-\sqrt{10}$ and $-\sqrt{50}$
   i.e. $\sqrt{101}$, 9 , $\sqrt{6}$, $-\sqrt{7}$, $-\sqrt{10}$ and $-\sqrt{50}$

8

$\because 2^2 = 4$ $\qquad \therefore 4 > 3 > 2 > \frac{3}{2} > 0$

$\therefore 2 > \sqrt{3} > \sqrt{2} > \sqrt{\frac{3}{2}} > 0$

$\therefore$ The positive irrational numbers are

$\sqrt{3}$, $\sqrt{2}$ and $\sqrt{\frac{3}{2}}$

(There are other solutions)

9

The irrational numbers are

$-\sqrt{5}$, $-\sqrt{3}$ and $-\sqrt{2}$ (There are other solutions)

10

$\because (15)^2 = 225$, $(17)^2 = 289$

Then choosing 4 integers included between 225 , 289

(except 256 because $\sqrt{256} = 16 \in \mathbb{Q}$)

$\therefore 225 < 235 < 245 < 255 < 265 < 289$

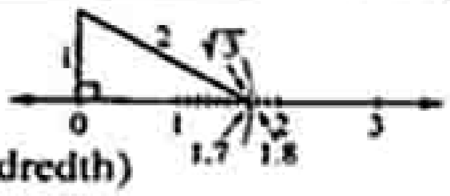
$\therefore 15 < \sqrt{235} < \sqrt{245} < \sqrt{255} < \sqrt{265} < 17$

$\therefore$ The four irrational numbers are

$\sqrt{235}$, $\sqrt{245}$, $\sqrt{255}$ and $\sqrt{265}$

(There are other solutions)

11



Using the calculator

$\sqrt{3} \approx 1.73$ (to the nearest hundredth)

$\therefore 1.7 < \sqrt{3} < 1.8$ for representing $\sqrt{3}$

$\therefore$ The length of the hypotenuse $= \frac{3+1}{2} = 2$

the length of one side of the right angle $= \frac{3-1}{2} = 1$

12

1. $x^2 = 6$ $\qquad \therefore x = \pm\sqrt{6} \approx \pm 2.45$
2. $x^2 = 24 \times \frac{4}{3} = 32$ $\qquad \therefore x = \pm\sqrt{32} \approx \pm 5.66$
3. $\frac{1}{2} x^2 = 5$ $\qquad \therefore x^2 = 5 \times 2 = 10$
   $\therefore x = \pm\sqrt{10}$ $\qquad \therefore x \approx \pm 3.16$
4. $5 x^3 = -1$ $\qquad \therefore x^3 = -\frac{1}{5}$
   $\therefore x = \sqrt[3]{-\frac{1}{5}} \approx -0.58$
5. $\frac{3}{4} x^2 = -13$ $\qquad \therefore x^2 = -13 \times \frac{4}{3} = -\frac{52}{3}$
   (has no solution in $\mathbb{R}$)
6. $\frac{2}{x^3} = 16$ $\qquad \therefore x^3 = \frac{1}{8}$ $\qquad \therefore x = \sqrt[3]{\frac{1}{8}} = \frac{1}{2}$
7. $\because (x^2 - 9)(x^3 - 5) = 0$
   $\therefore x^2 - 9 = 0$ $\qquad \therefore x^2 = 9$
   $\therefore x = \pm\sqrt{9} = \pm 3$
   or $x^3 - 5 = 0$ $\qquad \therefore x^3 = 5$
   $\therefore x = \sqrt[3]{5} \approx 1.71$
8. $\because (2x^3 - 5)(x^2 + 1) = 0$
   $\therefore 2x^3 - 5 = 0$ $\qquad \therefore 2x^3 = 5$
   $\therefore x^3 = \frac{5}{2}$ $\qquad \therefore x = \sqrt[3]{\frac{5}{2}} \approx 1.36$
   or $x^2 + 1 = 0$
   $\therefore x^2 = -1$ (has no solution in $\mathbb{R}$)

13

The side length $= \sqrt{5}$ cm. , $\sqrt{5} \notin \mathbb{Q}$

14

The edge length $= \sqrt[3]{1.728} = \frac{6}{5}$ cm. , $\frac{6}{5} \in \mathbb{Q}$

15

$\because$ The total area of the cube $= 6\,l^2$

$\therefore 13.5 = 6\,l^2$ $\qquad \therefore \frac{13.5}{6} = l^2$

$\therefore l = \sqrt{\frac{13.5}{6}} = 1.5$ cm. , $1.5 \in \mathbb{Q}$

16

The diagonal length $= \sqrt{6^2 + 6^2} = \sqrt{72}$ cm.

17

The side length $= \sqrt{32}$ cm.

$\therefore$ The diagonal length $= \sqrt{(\sqrt{32})^2 + (\sqrt{32})^2}$

$= \sqrt{32 + 32} = \sqrt{64} = 8$ cm.

18

The length of the hypotenuse $= \sqrt{5^2 + 5^2} = \sqrt{50}$ cm.

19

The diagonal length of the rectangle

$= \sqrt{(5)^2 + (7)^2} = \sqrt{74}$ cm.

∵ The area of the square = The area of the rectangle = 5 × 7 = 35 cm.^{2}

∴ The side length of the square = $\sqrt{35}$ cm.

∴ The diagonal length of the square = $\sqrt{35+35}$ = $\sqrt{70}$ cm.

20

Cubing the two sides then squaring them we find that

$(\sqrt[3]{3})^3 = 3$, $3^2 = 9$, $(\sqrt{2})^3 = 2\sqrt{2}$, $(2\sqrt{2})^2 = 8$

∵ $9 > 8$ ∴ $\sqrt[3]{3} > \sqrt{2}$

21

Let the other number = X ∴ $X^2 + 2^2 = 7$

∴ $X^2 = 7 - 4 = 3$ ∴ $X = \pm\sqrt{3}$

∴ The other number is $\sqrt{3}$ or $-\sqrt{3}$

Answers of Exercise 4

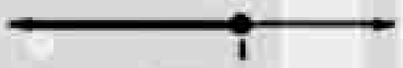
1

2 $\{X : 1 \le X < 3 , X \in \mathbb{R}\}$

3 $]0 , 3]$

4 $]-2 , 3[$, $\{X : -2 < X < 3 , X \in \mathbb{R}\}$

5 $\{X : X \le 1 , X \in \mathbb{R}\}$



6 $]0 , \infty[$, $\{X : X > 0 , X \in \mathbb{R}\}$

7 $]-\infty , 4[$

8 $\{X : X \ge -2 , X \in \mathbb{R}\}$

2

1 c 2 a 3 b 4 c 5 d

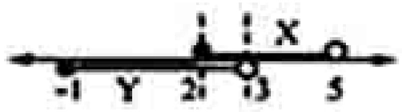
3

1 ∈ 2 ∉ 3 ∈ 4 ∈ 5 ∈
6 ∈ 7 ∈ 8 ∉ 9 ∉ 10 ∉

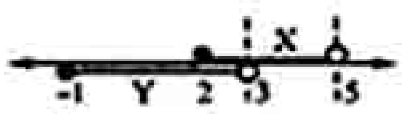
4

1 $[-1 , 5[$

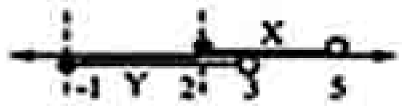
2 $[2 , 3[$



3 $[3 , 5[$



4 $[-1 , 2[$

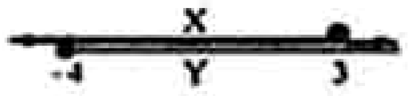


5 $]-\infty , 2[\cup [5 , \infty[$

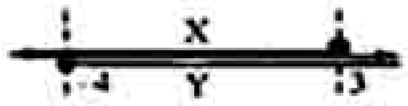
6 $]-\infty , -1[\cup [3 , \infty[$

5

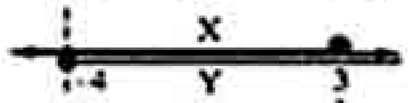
1 $\mathbb{R}$



2 $[-4 , 3]$



3 $]-\infty , -4[$



4 $]3 , \infty[$

5 $]3 , \infty[$

6 $]-\infty , -4[$

6 Use the number line to get the following results :

1 $[-1 , \infty[$ 2 $[3 , 4]$ 3 $[-1 , 3[$
4 $[-1 , 4[- \{3\}$ 5 $\{3 , 4\}$
6 $]4 , \infty[$ 7 $]-\infty , -1[\cup]4 , \infty[$
8 $]-\infty , 3[$

7 Use the number line to get the following results :

1 $[2 , 4]$ 2 $[-1 , 5]$ 3 $]0 , 1[$
4 $]-2 , 3]$ 5 $[3 , 6]$ 6 $[-1 , 2[$
7 $[-3 , 2] - \{0\}$ 8 ∅
9 ∅ 10 $[-2 , 1[\cup]2 , 4]$
11 ∅ 12 $\{-1 , 5\}$

8 Use the number line to get the following results :

1 $[-3 , \infty[$ 2 $[2 , 3[$ 3 $[-4 , 3]$
4 $\mathbb{R}$ 5 $]-\infty , -1[$ 6 $]-\infty , -3[$
7 $]0 , 2]$ 8 $\mathbb{R} - [3 , 4]$

9

1 $[3 , 5]$ 2 $[3 , 5]$ 3 $\{3 , 5\}$ 4 ∅
5 $]3 , 5[$ 6 $]3 , 5[$ 7 ∅
8 $\{3 , 5\}$ 9 $[3 , 5[$ 10 $[3 , 5[$
11 $\{3 , 4\}$ 12 $]-3 , 5]$

10

1 $]1 , 7[$ 2 $]-3 , 0[$ 3 $[3 , 4]$ 4 $]2 , 5[$
5 $\{5\}$ 6 $[3 , 4[$ 7 $\{2 , 7\}$ 8 $\{4\}$
9 $[3 , 4[$ 10 $]0 , 1[$

11

1 b 2 d 3 c
4 b 5 b 6 d

12

1 $[-3, 3]$ 2 $\mathbb{R}$ 3 $]-\infty, -1[$
4 $]-\infty, -3[$ 5 $]-2, 0]$ 6 $[0, 2]$
7 $\{1, 2\}$ 8 $\{0, 1\}$
9 $\{-1, 0, 1, 2\}$ 10 $]0, 5]$
11 $[-3, 0[$

13

1 $[-3, 1[$ 2 $\mathbb{R} -]-3, 1]$ 3 $]-3, 1[$
4 $\mathbb{R} -]-3, 1[$ 5 $[-3, 3[- \{1\}$

14

Let X be the temperature degrees needed to keep the first kind.

, Y be the temperature degrees needed to keep the second kind.

$\therefore X = [-3, 4]$, $Y = [2, 10]$

$\therefore$ The temperature needed to keep the two kinds altogether at the same place $= X \cap Y = [2, 4]$

15

1 d 2 c 3 c 4 c 5 b
6 d 7 c 8 c 9 c

16

$\because X \subset Y \quad \therefore X = X \cap Y = [4, 7]$

$, Y = X \cup Y = [3, 7]$, $Y - X = [3, 4[$

Answers of Exercise 5

1

1 $3\sqrt{3}$ 2 $-2\sqrt{2}$ 3 zero
4 $-\sqrt[3]{7}$ 5 $6\sqrt{5}$ 6 zero

2

1 $3\sqrt{5}$ 2 $3\sqrt{3} - 1$ 3 $8\sqrt{7} - 3\sqrt{2}$
4 $7\sqrt{2} - 2\sqrt[3]{2}$ 5 $\sqrt{2}$
6 $8 \times \frac{1}{2} + 2\sqrt[3]{3} - 4 - 5\sqrt[3]{3} = 4 + 2\sqrt[3]{3} - 4 - 5\sqrt[3]{3}$
$= -3\sqrt[3]{3}$

3

1 3 2 -30 3 $6\sqrt{2}$ 4 1 5 $15\sqrt{3}$
6 $2\sqrt{3} \times \frac{2\sqrt{7}}{7} \times \frac{5\sqrt{7}}{20\sqrt{3}} = 1$

4

1 $2\sqrt{2} + 2\sqrt{5}$ 2 $5\sqrt{2} + 2$
3 $7 + 2\sqrt{7}$ 4 $5\sqrt{3} + 3$
5 $-6\sqrt{5} + 10$ 6 $2 - 7 + 3\sqrt{7} = -5 + 3\sqrt{7}$
7 $-24 - 6\sqrt{3} + 6\sqrt{3} = -24$
8 $3\sqrt{5} - 5 - 2 - 2\sqrt{5} = \sqrt{5} - 7$

5

1 $(\sqrt{2})^2 - (1)^2 = 2 - 1 = 1$
2 $(4)^2 - (3\sqrt{2})^2 = 16 - 18 = -2$
3 $(\sqrt{5})^2 - 2 \times 1 \times \sqrt{5} + (-1)^2 = 5 - 2\sqrt{5} + 1$
$= 6 - 2\sqrt{5}$
4 $(2\sqrt{3})^2 + 2 \times 4 \times 2\sqrt{3} + (4)^2 = 12 + 16\sqrt{3} + 16$
$= 28 + 16\sqrt{3}$
5 $3 + \sqrt{3} - 2 = 1 + \sqrt{3}$
6 $(5)^2 - 2 \times 5 \times \sqrt{3} + (-\sqrt{3})^2 - 28$
$= 25 - 10\sqrt{3} + 3 - 28 = -10\sqrt{3}$

6

1 $\frac{3}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{3\sqrt{3}}{3} = \sqrt{3}$

2 $\frac{10}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{10\sqrt{5}}{5} = 2\sqrt{5}$

3 $-\frac{6}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = -\frac{6\sqrt{3}}{3} = -2\sqrt{3}$

4 $\frac{8}{\sqrt{6}} \times \frac{\sqrt{6}}{\sqrt{6}} = \frac{8\sqrt{6}}{6} = \frac{4\sqrt{6}}{3}$

5 $\frac{2}{3\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{2\sqrt{2}}{6} = \frac{\sqrt{2}}{3}$

6 $\frac{6}{2\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{6\sqrt{3}}{6} = \sqrt{3}$

7 $\frac{25}{2\sqrt{10}} \times \frac{\sqrt{10}}{\sqrt{10}} = \frac{25\sqrt{10}}{20} = \frac{5\sqrt{10}}{4}$

8 $\frac{\sqrt{2} + 3}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{2 + 3\sqrt{2}}{2}$

9 $\frac{\sqrt{5} - 15}{2\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{5 - 15\sqrt{5}}{10} = \frac{1 - 3\sqrt{5}}{2}$

7

1 c 2 d 3 b 4 c 5 a 6 d
7 c 8 b 9 d 10 b 11 d 12 c

8

1 1 , zero 2 $\sqrt{2}-1$ 3 $5\sqrt{3}$
4 1 5 $2 , \sqrt{3}$ 6 $\frac{2}{3}$
7 $4\sqrt{3}$ 8 $3+2\sqrt{2}$ 9 $\pm\sqrt{5}$
10 $8\sqrt{2}$ 11 60 cm^2
12 The additive inverse 13 $\mathbb{R}$

9

1 $\sqrt{5}-2+\sqrt{5}+2=2\sqrt{5}$

2 $\sqrt{5}-2-\sqrt{5}-2=-4$

3 $(\sqrt{5}-2)(\sqrt{5}+2)=5-4=1$

4 $x^2-y^2=(x-y)(x+y)=(-4)(2\sqrt{5})=-8\sqrt{5}$

5 $x^2+2xy+y^2=(x+y)^2=(2\sqrt{5})^2=20$

6 $x^2-2xy+y^2=(x-y)^2=(-4)^2=16$

10

The expression $= a(a-b)^3+b(b-a)^3$
$= a(a-b)^3-b(a-b)^3$
$= (a-b)^3(a-b)=(a-b)^4$
$= (2\sqrt{3})^4=144$

11

$\because x=\sqrt{3+\sqrt{2}}$ $\therefore x^2=3+\sqrt{2}$

$\therefore$ The expression $= x^4-2x^2+1$
$= (x^2-1)^2=(3+\sqrt{2}-1)^2$
$= (2+\sqrt{2})^2=4+4\sqrt{2}+2$
$= 6+4\sqrt{2}$

12

1 $x \simeq 3+2=5$
and using the calculator
$\therefore x \simeq 5.2$
(accepted estimation)
, $y \simeq 1+3=4$
and using the calculator $\therefore y=3.8$ (accepted estimation)

2 $x+y \simeq 5+4=9$
and using the calculator , the expression $\simeq 9.06$
(accepted estimation)

3 $x-y \simeq 5-4=1$
and using the calculator , the expression $\simeq 1.4$
(accepted estimation)

4 $xy \simeq 5\times 4=20$
and using the calculator , the expression equals 20.05
(accepted estimation)

13

1 $x \simeq 4+2=6$
and using the calculator $x \simeq 5.9$ (accepted estimation)
, $y \simeq 4-3=1$
and using the calculator $y \simeq 1.08$ (accepted estimation)

2 $x\times y \simeq 6\times 1=6$
and using the calculator , the expression $\simeq 6.3$
(accepted estimation)

3 $x+y \simeq 6+1=7$
and using the calculator , the expression $\simeq 6.9$
(accepted estimation)

14

The perimeter $= 2(6+\sqrt{5}+6-\sqrt{5})=2\times 12$
$= 24$ cm.

The area $= (6+\sqrt{5})(6-\sqrt{5})=36-5=31$ cm^2.

15

1 $\because$ The area of the small square $= 13$ cm^2.
$\therefore$ The side length of the small square $=\sqrt{13}$ cm.
, $\because$ the side of the chess board consists of 8 small squares.
$\therefore$ The side length of the chess board
$= 8\times\sqrt{13}=8\sqrt{13}$ cm.

2 $\because$ (The diagonal length of the square)^2
$=$ (its side length)^2 + (its side length)^2
«Pythagoras' theorem»
$\therefore$ The diagonal length of the square
$=\sqrt{(8\sqrt{13})^2+(8\sqrt{13})^2}$
$=\sqrt{64\times 13+64\times 13}=\sqrt{1664}$ cm.

16

$(\sqrt{a}-1)\times\frac{\sqrt{a}+1}{4}=1$

$\therefore \frac{a-1}{4}=1$ $\therefore a-1=4$ $\therefore a=5$

17

$x=\sqrt{2}$, $y=\frac{\sqrt{2}}{2}$, $z=\frac{\sqrt{2}}{4}$

$\therefore x^2+2y^2+4z^2=(\sqrt{2})^2+2\times\left(\frac{\sqrt{2}}{2}\right)^2+4\left(\frac{\sqrt{2}}{4}\right)^2$

$=2+2\times\frac{2}{4}+4\times\frac{2}{16}=3\frac{1}{2}$

18

$\frac{1}{2}(2y)=1-\sqrt{2}$ $\therefore y=1-\sqrt{2}$

$\therefore x=-1+\sqrt{2}$

$\therefore xy-2\sqrt{2}=(-1+\sqrt{2})(1-\sqrt{2})-2\sqrt{2}$

$=-1+\sqrt{2}+\sqrt{2}-2-2\sqrt{2}=-3$

Answers of exams on the first part of unit one

Model 1

1

1 c 2 d 3 a 4 b 5 d 6 b

2

1 23 2 5 3 $\hat{Q}$

4 $]-2,7[$ 5 $\sqrt{3}-5$

3 [a] 1 $X\cap Y=[1,3]$

2 $X\cup Y=[-2,5[$

3 $Y-X=]3,5[$

[b] The S.S. $=\{2,-2,\sqrt[3]{7}\}$

4 [a] Prove by yourself. [b] $\sqrt{50}$ cm.

5 [a] Determine by yourself.

[b] 1 2 2 $5+\sqrt{7}$

Model 2

1

1 c 2 c 3 b 4 b 5 a 6 a

2

1 $\sqrt{2}-\sqrt{7}$ 2 $\mathbb{R}^*$ 3 $[-4,0]$

4 -3 5 $\varnothing$

3 [a] $3\sqrt{7}$

[b] 1 $X\cup Y=]-\infty,4[$

2 $X\cap Y=[-2,1[$

3 $\acute{X}=[1,\infty[$

4 [a] 1 The S.S. $=\{\sqrt{20},-\sqrt{20}\}$

2 The S.S. $=\varnothing$

[b] Prove by yourself.

5 [a] 1 $37-20\sqrt{3}$ 2 $5+2\sqrt{5}$

[b] $\sqrt{122}$, $\sqrt{123}$, $\sqrt{124}$ and $\sqrt{125}$

(There are other numbers)

Answers of Exercise 6

1

1 $\sqrt{4\times3}=2\sqrt{3}$ 2 $\sqrt{4\times7}=2\sqrt{7}$

3 $2\sqrt{36\times2}=2\times6\sqrt{2}=12\sqrt{2}$

4 $\frac{2}{5}\sqrt{100\times10}=\frac{2}{5}\times10\sqrt{10}=4\sqrt{10}$

5 $\sqrt{4\times\frac{1}{2}}=\sqrt{2}$ 6 $2\sqrt{\frac{2}{3}\times9}=2\sqrt{6}$

2

1 $5\sqrt{2}+2\sqrt{2}=7\sqrt{2}$

2 $2\sqrt{5}-3\sqrt{5}=-\sqrt{5}$

3 $3\sqrt{2}+2\sqrt{2}-3\sqrt{2}=2\sqrt{2}$

4 $7\sqrt{2}-8\sqrt{2}-3\sqrt{2}+4\sqrt{2}=$ zero

5 $2\times3\sqrt{2}+5\sqrt{2}+\frac{1}{3}\times9\sqrt{2}$

$=6\sqrt{2}+5\sqrt{2}+3\sqrt{2}=14\sqrt{2}$

6 $7\sqrt{2}+5\sqrt{2}-\frac{1}{2}\times10\sqrt{2}-\sqrt{2}$

$=7\sqrt{2}+5\sqrt{2}-5\sqrt{2}-\sqrt{2}=6\sqrt{2}$

7 $3\sqrt{3}+5\times3\sqrt{2}-10\sqrt{3}=15\sqrt{2}-7\sqrt{3}$

3

1 $2\sqrt{5}+4\times2\sqrt{5}-\sqrt{25\times\frac{1}{5}}=2\sqrt{5}+8\sqrt{5}-\sqrt{5}$

$=9\sqrt{5}$

2 $4\sqrt{2}-6\sqrt{2}+3\sqrt{4\times\frac{1}{2}}=4\sqrt{2}-6\sqrt{2}+3\sqrt{2}$

$=\sqrt{2}$

3 $2\sqrt{5}+2\sqrt{9\times\frac{1}{3}}-2\sqrt{3}-\sqrt{25\times\frac{1}{5}}$
$=2\sqrt{5}+2\sqrt{3}-2\sqrt{3}-\sqrt{5}=\sqrt{5}$

4 $\sqrt{3}+\frac{3}{\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}}-\sqrt{12}=\sqrt{3}+\sqrt{3}-2\sqrt{3}=\text{zero}$

5 $3\sqrt{2}-\sqrt{\frac{12}{6}}=3\sqrt{2}-\sqrt{2}=2\sqrt{2}$

6 $5+3\sqrt{2}-\frac{6}{\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}}=5+3\sqrt{2}-3\sqrt{2}=5$

4

1 $10\sqrt{6}$

2 $6\sqrt{36}=6\times 6=36$

3 $2\sqrt{50}=2\times 5\sqrt{2}=10\sqrt{2}$

4 $\sqrt{\frac{2}{7}\times\frac{7}{2}}=\sqrt{1}=1$

5 $3\sqrt{\frac{15}{5}}=3\sqrt{3}$

6 $12\times\sqrt{\frac{2}{3}\times 54}=12\sqrt{36}=12\times 6=72$

5

1 $\sqrt{18}-\sqrt{12}=3\sqrt{2}-2\sqrt{3}$

2 $20+5\sqrt{24}=20+5\times 2\sqrt{6}=20+10\sqrt{6}$

3 $(3\sqrt{5})^2-(\sqrt{7})^2=45-7=38$

4 $(\sqrt{3})^2-2\times\sqrt{3}\times\sqrt{2}+(-\sqrt{2})^2=3-2\sqrt{6}+2$
$=5-2\sqrt{6}$

5 $(\sqrt{3})^2+2\times\sqrt{3}\times\sqrt{5}+(\sqrt{5})^2-2\sqrt{15}$
$=3+2\sqrt{15}+5-2\sqrt{15}=8$

6 $3\sqrt{2}-\frac{12}{\sqrt{6}}\times\frac{\sqrt{6}}{\sqrt{6}}+2\sqrt{6}-3\sqrt{2}$
$=3\sqrt{2}-2\sqrt{6}+2\sqrt{6}-3\sqrt{2}=\text{zero}$

6

1 $\frac{\sqrt{3}}{\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}}=\frac{\sqrt{6}}{2}$

2 $\sqrt{\frac{5}{3}}=\frac{\sqrt{5}}{\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}}=\frac{\sqrt{15}}{3}$

3 $\frac{5\sqrt{3}}{\sqrt{5}}\times\frac{\sqrt{5}}{\sqrt{5}}=\frac{5\sqrt{15}}{5}=\sqrt{15}$

4 $\frac{4\sqrt{3}-\sqrt{2}}{2\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}}=\frac{12-\sqrt{6}}{6}$

7

1 a 2 b 3 c 4 a 5 c
6 a 7 a 8 c 9 b

8

1 $\frac{1}{2}$ 2 $\sqrt{2}$ 3 $\sqrt{3}$ 4 -2
5 $\sqrt{125}$ 6 $\pm\frac{2\sqrt{2}}{3}$ 7 20 , zero

9

1 $x+y=3+\sqrt{5}+1-\sqrt{5}=4$
$x\times y=(3+\sqrt{5})(1-\sqrt{5})=3-2\sqrt{5}-5$
$=-2-2\sqrt{5}$

2 $x+y=\sqrt{3}-\sqrt{2}+\sqrt{3}+\sqrt{2}=2\sqrt{3}$
$x\times y=(\sqrt{3}-\sqrt{2})(\sqrt{3}+\sqrt{2})=3-2=1$

3 $x+y=5-3\sqrt{2}+5-3\sqrt{2}=10-6\sqrt{2}$
$x\times y=(5-3\sqrt{2})(5-3\sqrt{2})$
$=25-30\sqrt{2}+18=43-30\sqrt{2}$

10

$\because x=\frac{\sqrt{2}}{\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}}=\frac{\sqrt{6}}{3}$, $y=\frac{\sqrt{3}}{\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}}=\frac{\sqrt{6}}{2}$

$\therefore 6(x+y)=6\left(\frac{\sqrt{6}}{3}+\frac{\sqrt{6}}{2}\right)=6\times\frac{\sqrt{6}}{3}+6\times\frac{\sqrt{6}}{2}$
$=2\sqrt{6}+3\sqrt{6}=5\sqrt{6}$

11

$\because x=\frac{10}{\sqrt{5}}\times\frac{\sqrt{5}}{\sqrt{5}}=\frac{10\sqrt{5}}{5}=2\sqrt{5}$

, $y=3\sqrt{5}+\sqrt{2}$, $z=2\sqrt{2}+\sqrt{5}$

$\therefore (x-y+z)^2$
$=(2\sqrt{5}-3\sqrt{5}-\sqrt{2}+2\sqrt{2}+\sqrt{5})^2=(\sqrt{2})^2=2$

12 We know that $(x+y)^2=x^2+2xy+y^2$

$\therefore x^2+2xy+y^2=(2\sqrt{5}+\sqrt{2}+2\sqrt{5}-\sqrt{2})^2$
$=(4\sqrt{5})^2=16\times 5=80$

13

$\because x=\sqrt{7}+\frac{1}{2}\times 2\sqrt{3}=\sqrt{7}+\sqrt{3}$

, $y=\frac{1}{3}\times 3\sqrt{7}-\sqrt{3}=\sqrt{7}-\sqrt{3}$

$\therefore x^2y^2=(xy)^2=((\sqrt{7}+\sqrt{3})(\sqrt{7}-\sqrt{3}))^2$
$=(7-3)^2=4^2=16$

14

The perimeter of Δ ABC

$=\sqrt{28}+28\sqrt{\frac{1}{7}}+5\sqrt{7}$

$=\sqrt{4\times7}+4\sqrt{49\times\frac{1}{7}}+5\sqrt{7}$

$=2\sqrt{7}+4\sqrt{7}+5\sqrt{7}=11\sqrt{7}$ cm.

15

1 The area of one square $=\frac{300}{6}=50$ cm^2

$\therefore$ The side length of one square $=\sqrt{50}=5\sqrt{2}$ cm.

$\therefore$ The perimeter of the figure $=14\times5\sqrt{2}$

$=70\sqrt{2}$ cm.

2 The area of one square $=\frac{72}{6}=12$ cm^2

$\therefore$ The side length of one square $=\sqrt{12}=2\sqrt{3}$ cm.

$\therefore$ The perimeter of the figure $=14\times2\sqrt{3}$

$=28\sqrt{3}$ cm.

3 The area of one square $=\frac{40}{5}=8$ cm^2

$\therefore$ The side length of one square $=\sqrt{8}=2\sqrt{2}$ cm.

$\therefore$ The perimeter of the figure $=12\times2\sqrt{2}$

$=24\sqrt{2}$ cm.

16

$a^{x+y}=a^x\times a^y=a^x\div a^{-y}=6\div\sqrt{3}=\frac{6}{\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}}$

$=\frac{6\sqrt{3}}{3}=2\sqrt{3}$

17

1 $\frac{(\sqrt{5})^3\times(\sqrt{5})^5}{(\sqrt{2})^6\times(\sqrt{5})^6}=\frac{(\sqrt{5})^{3+5-6}}{(\sqrt{2})^6}=\frac{5}{8}$

2 $\frac{2\sqrt{2}\times(\sqrt{2})^{-3}\times(\sqrt{3})^{-3}}{(\sqrt{3})^{-3}}=2\times(\sqrt{2})^{-2}$

$=\frac{2}{(\sqrt{2})^2}=\frac{2}{2}=1$

18

$\because 3\sqrt{3}+\sqrt{4\times\frac{1}{2}}+3\sqrt{2}+2\sqrt{3}-5\sqrt{2}$

$=x\sqrt{2}+y\sqrt{3}$

$\therefore 3\sqrt{3}+\sqrt{2}+3\sqrt{2}+2\sqrt{3}-5\sqrt{2}=x\sqrt{2}+y\sqrt{3}$

$\therefore -\sqrt{2}+5\sqrt{3}=x\sqrt{2}+y\sqrt{3}$ $\therefore x=-1$, $y=5$

Answers of Exercise 7

1

1 $\sqrt{5}-\sqrt{3}$ 2 $5+2\sqrt{7}$

3 The number is $\sqrt{5}+\frac{2}{\sqrt{2}}=\sqrt{5}+\sqrt{2}$

$\therefore$ The conjugate number $=\sqrt{5}-\sqrt{2}$

2

1 $\frac{5}{\sqrt{7}-\sqrt{2}}\times\frac{\sqrt{7}+\sqrt{2}}{\sqrt{7}+\sqrt{2}}=\frac{5(\sqrt{7}+\sqrt{2})}{7-2}=\sqrt{7}+\sqrt{2}$

2 $\frac{\sqrt{3}}{2-\sqrt{3}}\times\frac{2+\sqrt{3}}{2+\sqrt{3}}=\frac{\sqrt{3}(2+\sqrt{3})}{4-3}=2\sqrt{3}+3$

3 $\frac{\sqrt{7}+3}{\sqrt{7}-3}\times\frac{\sqrt{7}+3}{\sqrt{7}+3}=\frac{16+6\sqrt{7}}{7-9}=-8-3\sqrt{7}$

3

$\because x=\frac{2}{\sqrt{7}-\sqrt{5}}\times\frac{\sqrt{7}+\sqrt{5}}{\sqrt{7}+\sqrt{5}}=\frac{2(\sqrt{7}+\sqrt{5})}{7-5}$

$=\sqrt{7}+\sqrt{5}$

$\therefore (x+y)^2=(\sqrt{7}+\sqrt{5}+\sqrt{7}-\sqrt{5})^2$

$=(2\sqrt{7})^2=28$

4

$x^2y^2=(xy)^2=\left(\frac{4}{\sqrt{7}-\sqrt{3}}\times\frac{4}{\sqrt{7}+\sqrt{3}}\right)^2$

$=\left(\frac{16}{7-3}\right)^2=4^2=16$

5

L.H.S. $=\frac{4}{x}+2x$

$=\frac{4}{\sqrt{5}+\sqrt{3}}+2(\sqrt{5}+\sqrt{3})$

$=\frac{4(\sqrt{5}-\sqrt{3})}{(\sqrt{5}+\sqrt{3})(\sqrt{5}-\sqrt{3})}+2(\sqrt{5}+\sqrt{3})$

$=\frac{4(\sqrt{5}-\sqrt{3})}{2}+2(\sqrt{5}+\sqrt{3})$

$=2(\sqrt{5}-\sqrt{3})+2(\sqrt{5}+\sqrt{3})$

$=2\sqrt{5}-2\sqrt{3}+2\sqrt{5}+2\sqrt{3}$

$=4\sqrt{5}=$ R.H.S.

6

$\because b = \frac{1}{\sqrt{3}+\sqrt{2}} \times \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}} = \frac{\sqrt{3}-\sqrt{2}}{3-2}$

$= \sqrt{3}-\sqrt{2}$

We know that : $(a-b)(a+b) = a^2 - b^2$

$\therefore (\sqrt{3}+\sqrt{2}-\sqrt{3}+\sqrt{2})(\sqrt{3}+\sqrt{2}+\sqrt{3}-\sqrt{2})$

$= 2\sqrt{2} \times 2\sqrt{3} = 4\sqrt{6}$

7

$\because y = \frac{2}{\sqrt{5}-\sqrt{3}} \times \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}} = \frac{2(\sqrt{5}+\sqrt{3})}{5-3}$

$= \sqrt{5}+\sqrt{3}$

$\therefore x^2 + 2xy + y^2 = (x+y)^2$

$= (\sqrt{5}-\sqrt{3}+\sqrt{5}+\sqrt{3})^2 = (2\sqrt{5})^2 = 20$

8

$\because y = \frac{3}{\sqrt{5}-\sqrt{2}} \times \frac{\sqrt{5}+\sqrt{2}}{\sqrt{5}+\sqrt{2}} = \frac{3(\sqrt{5}+\sqrt{2})}{5-2}$

$= \sqrt{5}+\sqrt{2}$

$, x = \sqrt{5}-\sqrt{2}$

$\therefore$ x and y are two conjugate numbers.

$\therefore x^2 - 2xy + y^2 = (x-y)^2 = (\sqrt{5}-\sqrt{2}-\sqrt{5}-\sqrt{2})^2$

$= (-2\sqrt{2})^2 = 8$

9

$\because x = 3+\sqrt{5}\ , y = \frac{4}{3+\sqrt{5}} \times \frac{3-\sqrt{5}}{3-\sqrt{5}} = \frac{4(3-\sqrt{5})}{9-5}$

$= 3-\sqrt{5}$

$\therefore$ x and y are two conjugate numbers

1) $xy = (3+\sqrt{5})(3-\sqrt{5}) = 9-5 = 4$

2) $x^2 + y^2 = (x+y)^2 - 2xy$

$= (3+\sqrt{5}+3-\sqrt{5})^2 - 2\times 4 = 36 - 8 = 28$

10

$\because x = \frac{2}{\sqrt{5}-\sqrt{3}} \times \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}} = \frac{2(\sqrt{5}+\sqrt{3})}{5-3}$

$= \sqrt{5}+\sqrt{3}$

$, y = \frac{2}{\sqrt{5}+\sqrt{3}} \times \frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}-\sqrt{3}} = \frac{2(\sqrt{5}-\sqrt{3})}{5-3}$

$= \sqrt{5}-\sqrt{3}$

$\therefore x^2 - xy + y^2 = (x-y)^2 + xy$

$= (\sqrt{5}+\sqrt{3}-\sqrt{5}+\sqrt{3})^2 + (\sqrt{5}+\sqrt{3})(\sqrt{5}-\sqrt{3})$

$= (2\sqrt{3})^2 + 2 = 14$

11

$\frac{x+y}{xy-1} = \frac{\sqrt{5}+\sqrt{2}+\sqrt{5}-\sqrt{2}}{(\sqrt{5}+\sqrt{2})(\sqrt{5}-\sqrt{2})-1}$

$= \frac{2\sqrt{5}}{5-2-1} = \frac{2\sqrt{5}}{2} = \sqrt{5}$

12

$\because a = \frac{4}{\sqrt{7}-\sqrt{3}} \times \frac{\sqrt{7}+\sqrt{3}}{\sqrt{7}+\sqrt{3}} = \frac{4(\sqrt{7}+\sqrt{3})}{7-3}$

$= \sqrt{7}+\sqrt{3}$

$, b = \frac{4}{\sqrt{7}+\sqrt{3}} \times \frac{\sqrt{7}-\sqrt{3}}{\sqrt{7}-\sqrt{3}} = \frac{4(\sqrt{7}-\sqrt{3})}{7-3}$

$= \sqrt{7}-\sqrt{3}$

$\therefore \frac{a-b}{ab} = \frac{\sqrt{7}+\sqrt{3}-\sqrt{7}+\sqrt{3}}{(\sqrt{7}+\sqrt{3})(\sqrt{7}-\sqrt{3})} = \frac{2\sqrt{3}}{7-3} = \frac{\sqrt{3}}{2}$

13

$\because x = 2\sqrt{2}-\sqrt{3}\ , y = \frac{5}{2\sqrt{2}-\sqrt{3}}$

$\therefore y = \frac{5}{2\sqrt{2}-\sqrt{3}} \times \frac{2\sqrt{2}+\sqrt{3}}{2\sqrt{2}+\sqrt{3}} = \frac{5(2\sqrt{2}+\sqrt{3})}{8-3}$

$= 2\sqrt{2}+\sqrt{3}$

$\therefore$ x and y are conjugate numbers

$\therefore \frac{x+y}{xy} = \frac{2\sqrt{2}-\sqrt{3}+2\sqrt{2}+\sqrt{3}}{(2\sqrt{2}-\sqrt{3})(2\sqrt{2}+\sqrt{3})}$

$= \frac{4\sqrt{2}}{8-3} = \frac{4\sqrt{2}}{5}$

14

$\because x = \frac{5\sqrt{2}+3\sqrt{5}}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{5\sqrt{10}+15}{5}$

$= \sqrt{10}+3$

$, y = \frac{2\sqrt{5}-3\sqrt{2}}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{2\sqrt{10}-6}{2} = \sqrt{10}-3$

1) $x^2 + y^2 = (x+y)^2 - 2xy$

$= (\sqrt{10}+3+\sqrt{10}-3)^2 - 2(\sqrt{10}+3)(\sqrt{10}-3)$

$= (2\sqrt{10})^2 - 2\times(10-9) = 40 - 2 = 38$

2 $xy = (\sqrt{10}+3)(\sqrt{10}-3) = 10-9 = 1$

$\therefore x^2+y^2 = 38\,xy$

15

$\because x = \frac{1}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}} = \frac{2-\sqrt{3}}{4-3} = 2-\sqrt{3}$

$, y = \frac{12}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{12\sqrt{3}}{3} = 4\sqrt{3}$

$\therefore x^2+y = (2-\sqrt{3})^2 + 4\sqrt{3}$

$= 4-4\sqrt{3}+3+4\sqrt{3} = 7$

16

$\because y = \sqrt{3}-\sqrt{2}$

$, x = \frac{1}{\sqrt{3}-\sqrt{2}} \times \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}} = \sqrt{3}+\sqrt{2}$

$\therefore (x+y)^2 = (\sqrt{3}+\sqrt{2}+\sqrt{3}-\sqrt{2})^2$

$= (2\sqrt{3})^2 = 12$

17

$\because xy = 1$

$\therefore y = \frac{1}{x} = \frac{1}{\sqrt{13}+\sqrt{6}} = \frac{1}{\sqrt{13}+\sqrt{6}} \times \frac{\sqrt{13}-\sqrt{6}}{\sqrt{13}-\sqrt{6}}$

$= \frac{\sqrt{13}-\sqrt{6}}{7}$

$\therefore x^2-49y^2 = (x-7y)(x+7y)$

$= \left(\sqrt{13}+\sqrt{6}-7\left(\frac{\sqrt{13}-\sqrt{6}}{7}\right)\right)$

$\left(\sqrt{13}+\sqrt{6}+7\left(\frac{\sqrt{13}-\sqrt{6}}{7}\right)\right)$

$= (\sqrt{13}+\sqrt{6}-\sqrt{13}+\sqrt{6})(\sqrt{13}+\sqrt{6}+\sqrt{13}-\sqrt{6})$

$= 2\sqrt{6} \times 2\sqrt{13} = 4\sqrt{78}$

18

$\because x = \frac{4(\sqrt{7}+\sqrt{3})}{(\sqrt{7}-\sqrt{3})(\sqrt{7}+\sqrt{3})} = \frac{4(\sqrt{7}+\sqrt{3})}{7-3}$

$= \sqrt{7}+\sqrt{3}$

$, y = \sqrt{7}-\sqrt{3}$

$\therefore x$ and y are two conjugate numbers

$\therefore x^2y^2 = (xy)^2 = [(\sqrt{7}+\sqrt{3})(\sqrt{7}-\sqrt{3})]^2$

$= (7-3)^2 = 4^2 = 16$

19

$\because y = \frac{2}{\sqrt{7}+\sqrt{5}} \times \frac{\sqrt{7}-\sqrt{5}}{\sqrt{7}-\sqrt{5}} = \frac{2(\sqrt{7}-\sqrt{5})}{7-5}$

$= \sqrt{7}-\sqrt{5}$

$\therefore \frac{x+y}{xy} = \frac{\sqrt{7}+\sqrt{5}+\sqrt{7}-\sqrt{5}}{(\sqrt{7}+\sqrt{5})(\sqrt{7}-\sqrt{5})} = \frac{2\sqrt{7}}{7-5} = \sqrt{7}$

20

$\because x = \frac{\sqrt{6}+\sqrt{5}}{\sqrt{6}-\sqrt{5}} \times \frac{\sqrt{6}+\sqrt{5}}{\sqrt{6}+\sqrt{5}} = \frac{11+2\sqrt{30}}{6-5}$

$= 11+2\sqrt{30}$

$, \frac{1}{x} = \frac{1}{11+2\sqrt{30}} \times \frac{11-2\sqrt{30}}{11-2\sqrt{30}} = \frac{11-2\sqrt{30}}{121-120}$

$= 11-2\sqrt{30}$

$\therefore x+\frac{1}{x} = 11+2\sqrt{30}+11-2\sqrt{30} = 22$

21

1 4
2 $3-\sqrt{2}$, 7
3 $\sqrt{3}-\sqrt{2}$
4 $1-\sqrt{7}$
5 $\sqrt{3}-\sqrt{2}$
6 20
7 20
8 $\sqrt{5}+2$
9 $(-1 , 2\sqrt{3})$
10 -1

22

1 $\frac{11}{2\sqrt{5}+3} = \frac{11(2\sqrt{5}-3)}{(2\sqrt{5}+3)(2\sqrt{5}-3)}$

$= \frac{11(2\sqrt{5}-3)}{20-9} = 2\sqrt{5}-3$

$\therefore a = 2 , b = -3$

2 $\frac{3}{2\sqrt{2}-\sqrt{5}} = \frac{3(2\sqrt{2}+\sqrt{5})}{(2\sqrt{2}-\sqrt{5})(2\sqrt{2}+\sqrt{5})}$

$= \frac{3(2\sqrt{2}+\sqrt{5})}{8-5} = 2\sqrt{2}+\sqrt{5}$

$\therefore a = 2 , b = 1$

3 $\frac{7}{\sqrt{8}+1} = \frac{7(\sqrt{8}-1)}{(\sqrt{8}+1)(\sqrt{8}-1)}$

$= \frac{7(\sqrt{8}-1)}{8-1} = \sqrt{8}-1$

$= 2\sqrt{2}-1 = a+b\sqrt{2}$

$\therefore a = -1 , b = 2$

23

1 The expression $= \frac{4(\sqrt{5}-\sqrt{3})+4(\sqrt{5}+\sqrt{3})}{(\sqrt{5}+\sqrt{3})(\sqrt{5}-\sqrt{3})}$

$= \frac{4\sqrt{5}-4\sqrt{3}+4\sqrt{5}+4\sqrt{3}}{5-3}$

$= 4\sqrt{5}$

2 The expression $= \frac{(\sqrt{6}-\sqrt{5})^2-(\sqrt{6}+\sqrt{5})^2}{(\sqrt{6}+\sqrt{5})(\sqrt{6}-\sqrt{5})}$

$= \frac{6+5-2\sqrt{30}-6-5-2\sqrt{30}}{6-5}$

$= -4\sqrt{30}$

3 The expression $= 5\sqrt{3}-5+\frac{10(\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)}$

$= 5\sqrt{3}-5+5(\sqrt{3}+1)$

$= 5\sqrt{3}-5+5\sqrt{3}+5 = 10\sqrt{3}$

24

$BC = \sqrt{28}+2 = 2\sqrt{7}+2 = 2(\sqrt{7}+1)$ cm.

The area of ΔABC

$= \frac{1}{2}\ BC \times AD = \frac{1}{2} \times 2(\sqrt{7}+1)\times(\sqrt{7}-1)$

$= 7-1 = 6\ cm^2$

25

$x y^{-1}+y x^{-1} = \frac{x}{y}+\frac{y}{x} = \frac{x^2+y^2}{xy}$

$= \frac{(\sqrt{5}+1)^2+(\sqrt{5}-1)^2}{(\sqrt{5}+1)(\sqrt{5}-1)}$

$= \frac{6+2\sqrt{5}+6-2\sqrt{5}}{5-1} = \frac{12}{4} = 3$

26

$\because \frac{x^8y^9-y}{(x+y)^5} = \frac{y(x^8y^8-1)}{(x+y)^5}$ (1)

$, x^8y^8 = (xy)^8 = [(\sqrt{7}+\sqrt{6})(\sqrt{7}-\sqrt{6})]^8$

$= 1^8 = 1$

, from (1) : $\therefore \frac{x^8y^9-y}{(x+y)^5} = \frac{(\sqrt{7}-\sqrt{6})(1-1)}{(\sqrt{7}+\sqrt{6}+\sqrt{7}-\sqrt{6})^5}$

$= \frac{\text{zero}}{(2\sqrt{7})^5} = \text{zero}$

Answers of Exercise 8

1

1 $\sqrt[3]{8\times2} = 2\sqrt[3]{2}$ 2 $\sqrt[3]{-27\times2} = -3\sqrt[3]{2}$

3 $2\sqrt[3]{125\times2} = 2\times5\sqrt[3]{2} = 10\sqrt[3]{2}$

4 $\frac{2}{3}\sqrt[3]{-27\times5} = \frac{2}{3}\times-3\sqrt[3]{5} = -2\sqrt[3]{5}$

5 $\sqrt[3]{\frac{1}{3}\times27} = \sqrt[3]{9}$

6 $-2\sqrt[3]{\frac{2}{5}\times125} = -2\sqrt[3]{50}$

2

1 $\sqrt[3]{2\times32} = \sqrt[3]{64} = 4$ 2 $\sqrt[3]{\frac{72}{9}} = \sqrt[3]{8} = 2$

3 $\frac{4}{2}\sqrt[3]{\frac{-54}{-2}} = 2\sqrt[3]{27} = 2\times3 = 6$

4 $\frac{1}{2}\times6\sqrt[3]{10\times100} = 3\sqrt[3]{1000} = 3\times10 = 30$

5 $\sqrt[3]{\frac{2}{5}\times\frac{4}{25}} = \sqrt[3]{\frac{8}{125}} = \frac{2}{5}$

6 $\sqrt[3]{\frac{3}{4}\div\frac{2}{9}} = \sqrt[3]{\frac{3}{4}\times\frac{9}{2}} = \sqrt[3]{\frac{27}{8}} = \frac{3}{2}$

3

1 $2\sqrt[3]{2}-\sqrt[3]{2} = \sqrt[3]{2}$ 2 $5-2\sqrt[3]{3}$

3 $3\sqrt[3]{3}-2\sqrt[3]{3} = \sqrt[3]{3}$

4 $3\sqrt[3]{2}+2\sqrt[3]{2}-5\sqrt[3]{2} = \text{zero}$

5 $2\times3\sqrt[3]{2}-5\sqrt[3]{2}+2\sqrt[3]{2} = 3\sqrt[3]{2}$

6 $2\sqrt[3]{2}-\frac{1}{3}\times3\sqrt[3]{2}-\sqrt[3]{2} = \text{zero}$

7 $2\sqrt[3]{2}+\sqrt[3]{250} = 2\sqrt[3]{2}+5\sqrt[3]{2} = 7\sqrt[3]{2}$

8 $2\sqrt[3]{3}-2\times3\sqrt[3]{\frac{125}{9}} = 2\sqrt[3]{3}-2\sqrt[3]{\frac{125}{9}\times27}$

$= 2\sqrt[3]{3}-2\times5\sqrt[3]{3} = -8\sqrt[3]{3}$

4

1 The left hand side $= 4\sqrt[3]{2}+2\sqrt[3]{2}-2\times3\sqrt[3]{2} = \text{zero}$

= The right hand side.

2 The left hand side $= 3\sqrt[3]{2}\times2\sqrt[3]{2}\div(6\sqrt[3]{4})$

$= 6\sqrt[3]{4}\div6\sqrt[3]{4} = 1$ = The right hand side

5

1 $3\sqrt[3]{3}-2\sqrt[3]{3}-\sqrt[3]{27\times\frac{1}{9}} = 3\sqrt[3]{3}-2\sqrt[3]{3}-\sqrt[3]{3} = \text{zero}$

2) $3\sqrt[3]{2}-4\sqrt[3]{\frac{1}{4}\times 8}+5\times 2\sqrt[3]{2}$

$=3\sqrt[3]{2}-4\sqrt[3]{2}+10\sqrt[3]{2}=9\sqrt[3]{2}$

3) $3\sqrt[3]{4}-2\sqrt[3]{4}-\sqrt[3]{\frac{4}{8}}=3\sqrt[3]{4}-2\sqrt[3]{4}-\frac{1}{2}\sqrt[3]{4}=\frac{1}{2}\sqrt[3]{4}$

4) $\sqrt[3]{3}-\sqrt[3]{24}+\sqrt[3]{27\times\frac{1}{9}}=\sqrt[3]{3}-2\sqrt[3]{3}+\sqrt[3]{3}=$ zero

6

1) $\frac{7}{3}\times 3\sqrt{2}+3\sqrt[3]{2}-7\sqrt{2}+2\sqrt[3]{2}$

$=7\sqrt{2}+3\sqrt[3]{2}-7\sqrt{2}+2\sqrt[3]{2}=5\sqrt[3]{2}$

2) $3\sqrt{3}+\frac{1}{3}\times 3-3\sqrt{9\times\frac{1}{3}}-1$

$=3\sqrt{3}+1-3\sqrt{3}-1=$ zero

3) $-2\sqrt[3]{2}+\frac{14}{\sqrt{7}}\times\frac{\sqrt{7}}{\sqrt{7}}-2\sqrt{7}+3\sqrt[3]{2}$

$=-2\sqrt[3]{2}+2\sqrt{7}-2\sqrt{7}+3\sqrt[3]{2}=\sqrt[3]{2}$

4) $3\sqrt{2}+3\sqrt[3]{2}-\sqrt{\frac{216}{12}}-2\sqrt[3]{2}$

$=3\sqrt{2}+\sqrt[3]{2}-\sqrt{18}=3\sqrt{2}+\sqrt[3]{2}-3\sqrt{2}=\sqrt[3]{2}$

5) $5\sqrt{2}-\frac{1}{2}\times 10\sqrt{2}+\sqrt[3]{125}=5\sqrt{2}-5\sqrt{2}+5=5$

7

$4\sqrt[3]{2}\,(3\sqrt[3]{4}+10\sqrt[3]{4}-\sqrt[3]{\frac{8}{2}})$

$=4\sqrt[3]{2}\,(3\sqrt[3]{4}+10\sqrt[3]{4}-\sqrt[3]{4})$

$=4\sqrt[3]{2}\times 12\sqrt[3]{4}=48\sqrt[3]{8}=96$

8

1) c 2) a 3) c 4) d 5) a

9

1) -2 2) 9 3) 250

4) $\frac{2}{3}\sqrt[3]{7}$ 5) $-\frac{1}{2}$ 6) 1

10

1) $(\sqrt[3]{5}+1-\sqrt[3]{5}+1)^5=2^5=32$

2) $(\sqrt[3]{5}+1+\sqrt[3]{5}-1)^3=(2\sqrt[3]{5})^3=8\times 5=40$

11

$\because x-y=3+\sqrt[3]{6}-3+\sqrt[3]{6}=2\sqrt[3]{6}$

$,\ x+y=3+\sqrt[3]{6}+3-\sqrt[3]{6}=6$

$\therefore\left(\frac{x-y}{x+y}\right)^3=\left(\frac{2\sqrt[3]{6}}{6}\right)^3=\left(\frac{\sqrt[3]{6}}{3}\right)^3=\frac{(\sqrt[3]{6})^3}{3^3}$

$=\frac{6}{27}=\frac{2}{9}$

12

$2\sqrt[3]{4}+2\sqrt[3]{8\times\frac{1}{2}}-4\sqrt[3]{-2}\times\sqrt[3]{-2}+1-\frac{4}{2}$

$=2\sqrt[3]{4}+2\sqrt[3]{4}-4\sqrt[3]{4}+1-2=-1$

13

The edge length of one cube $=\sqrt[3]{24}=2\sqrt[3]{3}$ dm.

$\therefore$ The area of one face of one cube

$=2\sqrt[3]{3}\times 2\sqrt[3]{3}=4\sqrt[3]{9}$ dm^2.

$\therefore$ The area of the using ground

$=5\times 4\sqrt[3]{9}=20\sqrt[3]{9}$ dm^2.

14

L.H.S. $=x^2+y^2=(x+y)^2-2xy$

$=(\sqrt[3]{2}+1+\sqrt[3]{2}-1)^2-2(\sqrt[3]{2}+1)(\sqrt[3]{2}-1)$

$=(2\sqrt[3]{2})^2-2((\sqrt[3]{2})^2-1)$

$=4\sqrt[3]{4}-2\sqrt[3]{4}+2=2\sqrt[3]{4}+2=$ R.H.S.

15

$\frac{2}{\sqrt[3]{2}}=\frac{2\times\sqrt[3]{4}}{\sqrt[3]{2}\times\sqrt[3]{4}}=\frac{2\sqrt[3]{4}}{\sqrt[3]{8}}=\frac{2\sqrt[3]{4}}{2}=\sqrt[3]{4}$

Another solution : $\frac{2}{\sqrt[3]{2}}=\frac{\sqrt[3]{8}}{\sqrt[3]{2}}=\sqrt[3]{\frac{8}{2}}=\sqrt[3]{4}$

Answers of Exercise 9

1

1) 125 2) 96 3) $4l^2$ 4) $6l^2$ 5) $8l^3$

2

$\because$ Area of one face $=\frac{36}{4}=9$ cm^2.

$\therefore$ The edge length of the cube $=\sqrt{9}=3$ cm.

1) Its total area $=6l^2=6\times 3^2=54$ cm^2.

2) Its volume $=l^3=3^3=27$ cm^3.

3

The edge length of the cube $=\frac{12}{4}=3$ cm.

1) Its volume $=l^3=3^3=27$ cm^3.

2) Its lateral area $=4l^2=4\times 3^2=36$ cm^2.

4

The edge length of the cube $= \frac{60}{12} = 5$ cm.

1 Its volume $= l^3 = 5^3 = 125$ cm^3.

2 Its total area $= 6\, l^2 = 6 \times 5^2 = 150$ cm^2.

5

1 d 2 c 3 a 4 d 5 b 6 d 7 a

6

1 The volume of the cuboid $= X \times y \times z$
$= 9 \times 10 \times 5 = 450$ cm^3.

2 Its lateral area $= 2\,(X + y) \times z = 2\,(9 + 10) \times 5$
$= 190$ cm^2.

3 Its total area $= 2\,(Xy + yz + zX)$
$= 2\,(9 \times 10 + 10 \times 5 + 5 \times 9)$
$= 2\,(90 + 50 + 45) = 370$ cm^2.

7

The volume $= X \times y \times z = \sqrt{2} \times \sqrt{3} \times \sqrt{6} = 6$ cm^3.

8

The area of the base $= \sqrt{3}\,(\sqrt{3} - 1) = (3 - \sqrt{3})$ cm^2.

The volume = the area of the base × height
$= (3 - \sqrt{3})\,(3 + \sqrt{3}) = 9 - 3 = 6$ cm^3.

9

$\because$ The lateral area = the perimeter of the base × height

$\therefore$ The height $= \frac{480}{4 \times 10} = 12$ cm.

10

The area of the base $= \frac{\text{volume}}{\text{height}} = \frac{720}{5} = 144$ cm^2.

$\therefore$ The side length of the base $= \sqrt{144} = 12$ cm.

$\therefore$ The total area $= 2\,(Xy + yz + zX)$
$= 2 \times (12 \times 12 + 12 \times 5 + 12 \times 5)$
$= 528$ cm^2.

11

$\because$ The area of the face of the cube $= \frac{294}{6} = 49$ cm^2.

$\therefore$ The edge length $= \sqrt{49} = 7$ cm.

$\therefore$ The volume of the cube $= l^3 = 7 \times 7 \times 7 = 343$ cm^3.

, $\because$ the volume of the cuboid $= X \times y \times z$
$= 7\sqrt{2} \times 5\sqrt{2} \times 5$
$= 350$ cm^3.

$\therefore$ The volume of the cuboid is greater than the volume of the cube

12

The volume of the cuboid $= X \times y \times z = 17 \times 7 \times 4 = 476$ cm^3.

The total area $= 2\,(X + y) \times z + Xy$
$= 2\,(17 + 7) \times 4 + 17 \times 7$
$= 192 + 119 = 311$ cm^2.

13

The circumference of the circle $= 2\,\pi\, r$
$= 2 \times \frac{22}{7} \times 10.5$
$= 66$ cm.

The area of the circle $= \pi\, r^2 = \frac{22}{7} \times (10.5)^2$
$= 346.5$ cm^2.

14

$\because$ The area of the circle $= \pi\, r^2$

$\therefore 154 = \frac{22}{7}\, r^2$ $\therefore r^2 = \frac{154 \times 7}{22} = 49$

$\therefore r = \sqrt{49} = 7$ cm.

The circumference $= 2\,\pi\, r = 2 \times \frac{22}{7} \times 7 = 44$ cm.

The diameter length $= 2 \times 7 = 14$ cm.

15

The area of the circle $= \pi\, r^2$

$\therefore 64\,\pi = \pi\, r^2$ $\therefore r^2 = 64$

$\therefore r = \sqrt{64} = 8$ cm.

The circumference of the circle $= 2\,\pi\, r$

$= 2 \times 3.14 \times 8 \approx 50$ cm.

16

The area of the circle $= 2 \times 12.32 = 24.64$ cm^2.

$\therefore \pi\, r^2 = 24.64$ $\therefore r^2 = 24.64 \times \frac{7}{22} = 7.84$

$\therefore r = \sqrt{7.84} = 2.8$ cm.

$\therefore$ The perimeter of the figure $= \pi\, r + 2\, r$

$= \frac{22}{7} \times 2.8 + 2 \times 2.8 = 14.4$ cm.

17

The area of the shaded part = the area of the great circle − the area of the small circle $= \pi\, r_1^2 - \pi\, r_2^2$

$= \pi \times 25 - \pi \times 9 = 16\,\pi$ cm^2.

18

Let the radius length of the circle $= X$ cm.

$\therefore$ The side length of the square $= 2\,X$ cm.

$\therefore$ The area of the shaded part

$= \frac{\text{the area of the square} - \text{the area of the circle}}{2}$

$= \frac{4x^2 - \pi x^2}{2} = 10\frac{5}{7} = \frac{75}{7}$

$\therefore 4x^2 - \frac{22}{7}x^2 = \frac{75}{7} \times 2$

$\therefore \frac{6}{7}x^2 = \frac{150}{7} \qquad \therefore x^2 = \frac{150}{7} \times \frac{7}{6} = 25$

$\therefore x = \sqrt{25} = 5$ cm.

$\therefore$ The perimeter of the shaded part

$= \frac{1}{2}$ the circumference of the circle

$+ \frac{1}{2}$ the perimeter of the square.

$= \frac{22}{7} \times 5 + 20 = 35\frac{5}{7}$ cm.

19

In the right-angled triangle ABD at A

$\therefore (AB)^2 + (AD)^2 = (BD)^2$ (but AB = AD)

$\therefore 2(AB)^2 = (14)^2 \qquad \therefore (AB)^2 = \frac{196}{2} = 98$

$\therefore AB = \sqrt{98} = 7\sqrt{2}$ cm.

$\therefore$ The area of the shaded part

$= \frac{\text{the area of the circle} - \text{the area of the square}}{4}$

$= \frac{\frac{22}{7} \times (7)^2 - 7\sqrt{2} \times 7\sqrt{2}}{4} = \frac{154 - 98}{4} = 14 \text{ cm}^2.$

The perimeter of the shaded part

$= \frac{1}{4}$ the circumference of the circle

+ side length of the square

$= \frac{1}{4} \times 2 \times \frac{22}{7} \times 7 + 7\sqrt{2} = (11 + 7\sqrt{2})$ cm.

20

The volume of the cylinder $= \pi r^2 h$

$= \frac{22}{7} \times (14)^2 \times 20 = 12320 \text{ cm}^3.$

The total area of the cylinder $= 2\pi rh + 2\pi r^2$

$= 2 \times \frac{22}{7} \times 14 \times 20 + 2 \times \frac{22}{7} \times (14)^2 = 2992 \text{ cm}^2.$

21

$\because$ The volume of the cylinder $= \pi r^2 h$

$\therefore 924 = \frac{22}{7} \times r^2 \times 6$

$\therefore r^2 = \frac{924 \times 7}{6 \times 22} = 49 \qquad \therefore r = 7$ cm.

$\therefore$ The lateral area $= 2\pi rh = 2 \times \frac{22}{7} \times 7 \times 6$

$= 264 \text{ cm}^2.$

22

$\because$ The volume of the cylinder $= \pi r^2 h$

$\therefore 7536 = 3.14 \times r^2 \times 24$

$\therefore r^2 = \frac{7536}{3.14 \times 24} = 100 \qquad \therefore r = 10$ cm.

$\therefore$ The total area $= 2\pi rh + 2\pi r^2$

$= 2 \times 3.14 \times 10 \times 24 + 2 \times 3.14 \times (10)^2 = 2135.2 \text{ cm}^2.$

23

The volume of the cylinder $= \pi r^2 h$

$= \frac{22}{7} \times (7)^2 \times 10 = 1540 \text{ cm}^3.$

The volume of the cube $= l^3 = (11)^3 = 1331 \text{ cm}^3.$

$\therefore$ The volume of the cylinder is greater than the volume of the cube.

24

1. $2\pi rh$, $\pi r^2 h$
2. 2 cm.
3. 20 cm.
4. r cm.
5. r cm.

25

The circumference of the base $= 2\pi r$

$\therefore 44 = 2 \times \frac{22}{7} \times r$

$\therefore r = \frac{44}{2} \times \frac{7}{22} = 7$ cm.

$\therefore$ The volume of the cylinder $= \pi r^2 h$

$= \frac{22}{7} \times (7)^2 \times 25 = 3850 \text{ cm}^3.$

26

The lateral area $= 2\pi rh$

$\therefore 52 = 2 \times \frac{22}{7} \times 4 \times h$

$\therefore h = \frac{52 \times 7}{2 \times 22 \times 4} = \frac{91}{44}$ cm.

$\therefore$ The volume of the cylinder $= \pi r^2 h$

$= \frac{22}{7} \times 4^2 \times \frac{91}{44} = 104 \text{ cm}^3.$

27

$\because$ The volume of the cylinder $= \pi r^2 h$

$\because h = r$

$\therefore$ The volume of the cylinder $= \pi r^3$

$\therefore 72\pi = \pi r^3 \qquad \therefore r^3 = 72 \qquad \therefore r = 2\sqrt[3]{9}$

$\therefore$ The height of the cylinder $= 2\sqrt[3]{9}$ cm.

28

The volume of the tank = the volume of the cuboid $+ \frac{1}{2}$ of the volume of the cylinder

$= 7 \times 7 \times 14 + \frac{1}{2} \times \frac{22}{7} \times (3.5)^2 \times 14$

$= 686 + 269.5 = 955.5 \text{ m}^3$

29

The circumference of the base of the cylinder = BC

$\therefore 2\pi r = 44 \qquad \therefore r = \frac{44 \times 7}{2 \times 22} = 7$ cm.

The height = AB = 10 cm.

The volume $= \pi r^2 h = \frac{22}{7} \times (7)^2 \times 10 = 1540 \text{ cm}^3$

30

The volume of the sphere $= \frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \frac{22}{7} \times (2.1)^3 = 38.808 \text{ cm}^3$

The surface area of the sphere $= 4\pi r^2$

$= 4 \times \frac{22}{7} \times (2.1)^2 = 55.44 \text{ cm}^2$

31

$\because$ The volume of the sphere $= \frac{4}{3}\pi r^3$

$\therefore 4188 = \frac{4}{3} \times 3.141 \times r^3$

$\therefore r^3 = \frac{4188 \times 3}{4 \times 3.141} = 1000 \qquad \therefore r = 10$ cm.

32

The volume of the sphere $= \frac{4}{3}\pi r^3$

$\therefore 562.5\pi = \frac{4}{3}\pi r^3 \qquad \therefore r^3 = \frac{562.5 \times 3}{4} = 421.875$

$\therefore r = \sqrt[3]{421.875} = 7.5$ cm.

$\therefore$ The surface area of the sphere

$= 4\pi r^2 = 4 \times \pi \times (7.5)^2 = 225\pi \text{ cm}^2$

33

1 b 2 a 3 c 4 c 5 b 6 d

34

The volume of the cylinder $= \pi r^2 h$

$= \pi \times (4)^2 \times 18 = 288\pi \text{ cm}^3$

$\because$ The volume of the cylinder = The volume of the sphere.

$\therefore$ The volume of the sphere $= 288\pi \text{ cm}^3$

$\therefore \frac{4}{3}\pi r^3 = 288\pi$

$\therefore r^3 = \frac{288 \times 3}{4} = 216$

$\therefore$ The radius length of the sphere = 6 cm.

35

$\because$ The volume of the cylinder $= \pi r^2 h$

$\therefore 7536 = 3.14 \times r^2 \times 24$

$\therefore r^2 = \frac{7536}{3.14 \times 24} = 100$

$\therefore r = \sqrt{100} = 10$ cm.

, $\because$ the radius length of the sphere = the radius length of the cylinder base

$\therefore$ The volume of the sphere $= \frac{4}{3} \times 3.14 \times (10)^3$

$= 4186\frac{2}{3} \text{ cm}^3$

36

The volume of the cuboid $= 77 \times 24 \times 21 = 38808 \text{ cm}^3$

$\because$ The volume of the cuboid = the volume of the sphere

$\therefore 38808 = \frac{4}{3}\pi r^3$

$\therefore r^3 = \frac{38808 \times 3 \times 7}{4 \times 22} = 9261$

$\therefore r = \sqrt[3]{9261} = 21$ cm.

37

The volume of the sphere $= \frac{4}{3}\pi(3)^3 = 36\pi \text{ cm}^3$

$\because$ The volume of the cylinder = The volume of the sphere

$\therefore$ The volume of the cylinder $= 36\pi \text{ cm}^3$

$\therefore \pi r^2 h = 36\pi \qquad \therefore 9\pi h = 36\pi \qquad \therefore h = 4$ cm.

38

$\because$ The sphere touches the six faces of the cube

$\therefore$ The edge length of the cube = 2 r

$\because$ The volume of the sphere $= \frac{4}{3}\pi r^3$

$\therefore 36\pi = \frac{4}{3}\pi r^3 \qquad \therefore r^3 = \frac{36 \times 3}{4} = 27$

$\therefore r = 3$ cm.

$\therefore$ The edge length of the cube $= 2 \times 3 = 6$ cm.

$\therefore$ The volume of the cube $= \ell^3 = 6^3 = 216 \text{ cm}^3$

39

$\because$ The volume of the sphere = The volume of 8 small spheres

$\therefore \frac{4}{3}\pi r_1^3 = 8 \times \frac{4}{3}\pi r_2^3$

$\therefore (16.8)^3 = 8 r_2^3 \qquad \therefore r_2^3 = \frac{(16.8)^3}{8}$

$\therefore r_2 = \frac{16.8}{2} = 8.4$ cm.

40

The volume of the sphere $= \frac{4}{3} \pi (15)^3 = 4500 \pi$ cm^3.

$\because$ The volume of the cylinder

$= \frac{4}{9}$ The volume of the sphere

$\therefore \pi r^2 h = \frac{4}{9} \times 4500 \pi$ $\quad \therefore r^2 \times 20 = 2000$

$\therefore r^2 = \frac{2000}{20} = 100$ $\quad \therefore r = \sqrt{100} = 10$ cm.

41

$\because$ The sum of lengths of all edges = 52 cm.

, the sum of the 4 heights = 3 × 4 = 12 cm.

$\therefore$ The sum of the remained edges = 52 − 12 = 40 cm.

$\because$ The base is a square

$\therefore$ The side length of the square $= \frac{40}{8} = 5$ cm.

$\therefore$ The volume = 5 × 5 × 3 = 75 cm^3.

42

The volume of the metal = the outer volume

− the inner volume $= \frac{4}{3} \pi r_1^3 - \frac{4}{3} \pi r_2^3$

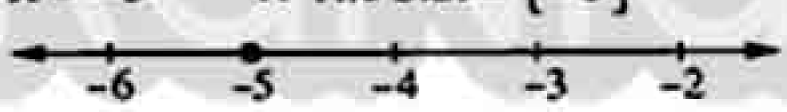
$= \frac{4}{3} \times \pi \left((3.5)^3 - (2.1)^3\right) = \frac{88}{21} \times 33.614 \simeq 140.859$ cm^3.

$\therefore$ The mass of the metal $= 140.859 \times 20 \simeq 2817$ gm.

Answers of Exercise 10

1

1) $\because x = -5$ $\quad \therefore$ The S.S. $= \{-5\}$

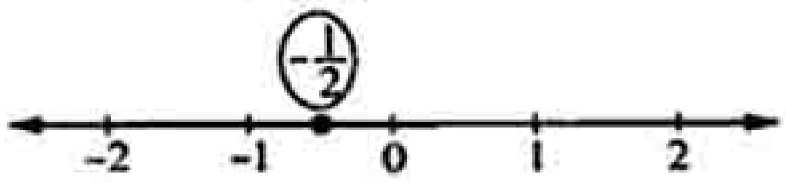


2) $\because 5x = 1 - 6 = -5$ $\quad \therefore x = \frac{-5}{5} = -1$

$\therefore$ The S.S. $= \{-1\}$

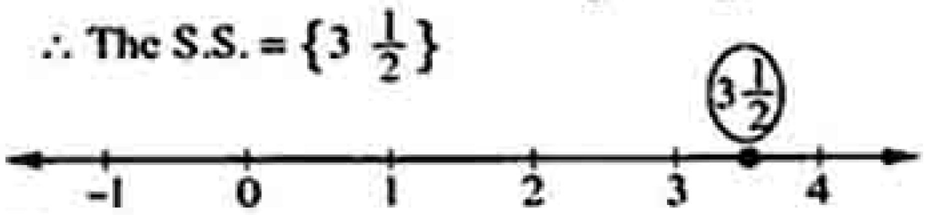
3) $\because 2x = 3 - 4 = -1$ $\quad \therefore x = -\frac{1}{2}$

$\therefore$ The S.S. $= \{-\frac{1}{2}\}$



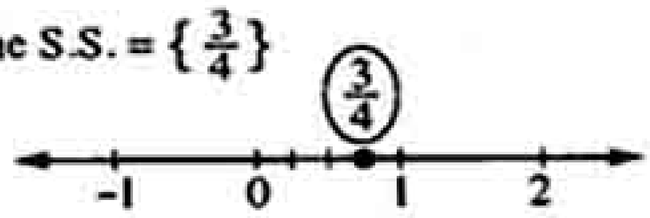
4) $\because 2x = 4 + 3 = 7$ $\quad \therefore x = \frac{7}{2} = 3\frac{1}{2}$

$\therefore$ The S.S. $= \{3\frac{1}{2}\}$



5) $\because 4x - 1 = 2$ $\quad \therefore 4x = 2 + 1 = 3$ $\quad \therefore x = \frac{3}{4}$

$\therefore$ The S.S. $= \{\frac{3}{4}\}$



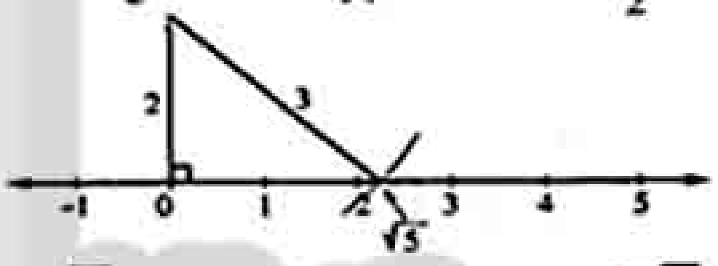
6) $\because \sqrt{5}\, x = 4 + 1 = 5$

$\therefore x = \frac{5}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{5\sqrt{5}}{5} = \sqrt{5}$

$\therefore$ The S.S. $= \{\sqrt{5}\}$

The length of one side of the right angle $= \frac{5-1}{2} = 2$

$\therefore$ The length of the hypotenuse $= \frac{5+1}{2} = 3$

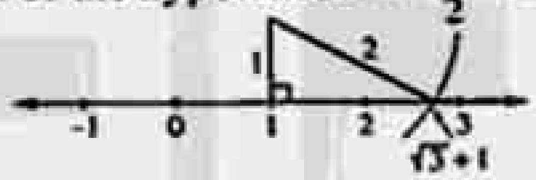


7) $\because x = \sqrt{3} + 1$ $\quad \therefore$ The S.S. $= \{\sqrt{3} + 1\}$

The length of one side of the right angle

$= \frac{3-1}{2} = 1$

The length of the hypotenuse $= \frac{3+1}{2} = 2$



8) $\because 2 - \sqrt{6}\, x = 8$ $\quad \therefore -\sqrt{6}\, x = 8 - 2 = 6$

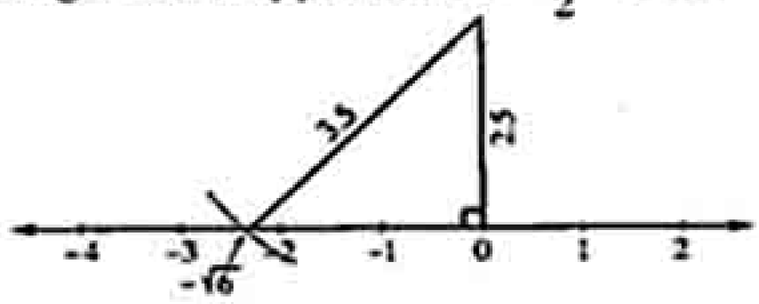
$\therefore x = -\frac{6}{\sqrt{6}} \times \frac{\sqrt{6}}{\sqrt{6}} = \frac{-6\sqrt{6}}{6} = -\sqrt{6}$

$\therefore$ The S.S. $= \{-\sqrt{6}\}$

The length of one side of the right angle $= \frac{6-1}{2}$

$= 2.5$

The length of the hypotenuse $= \frac{6+1}{2} = 3.5$

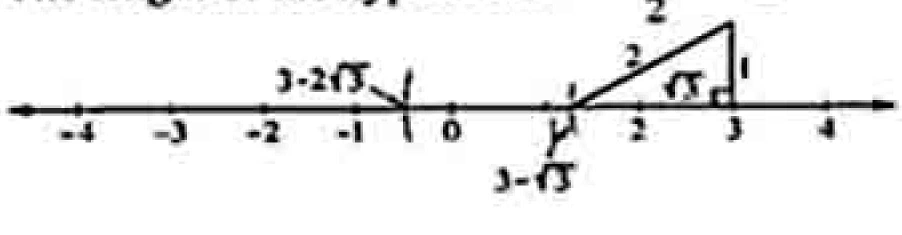


9) $\because x = 3 - 2\sqrt{3}$ $\quad \therefore$ The S.S. $= \{3 - 2\sqrt{3}\}$

The length of one side of the right angle $= \frac{3-1}{2}$

$= 1$

The length of the hypotenuse $= \frac{3+1}{2} = 2$



2

1 a   2 c   3 c   4 d   5 a   6 d

3

1 $\because x > 3$ $\therefore$ The S.S. $=]3 , \infty[$

2 $\because x \leq 2$ $\therefore$ The S.S. $=]-\infty , 2]$

3 $\because x \leq 2$ $\therefore$ The S.S. $=]-\infty , 2]$

4 $\because -x > -2$ $\therefore x < 2$

$\therefore$ The S.S. $=]-\infty , 2[$

5 $\because 2x \geq -2$ $\therefore x \geq -1$

$\therefore$ The S.S. $= [-1 , \infty[$

6 $\because -5x < 5$ $\therefore x > -1$

$\therefore$ The S.S. $=]-1 , \infty[$

7 $\because \frac{1}{2} x \leq 1$ $\therefore x \leq 2$

$\therefore$ The S.S. $=]-\infty , 2]$

8 $\because -2x \leq 4$ $\therefore x \geq -2$

$\therefore$ The S.S. $= [-2 , \infty[$

4

1 $\because 1 < x \leq 4$ $\therefore$ The S.S. $=]1 , 4]$

2 $\because -8 < x < 6$ $\therefore$ The S.S. $=]-8 , 6[$

3 $\because 3 \geq x > -3$ $\therefore$ The S.S. $=]-3 , 3]$

4 $\because -4 < -x \leq -2$ $\therefore 4 > x \geq 2$

$\therefore$ The S.S. $= [2 , 4[$

5 $\because -2 \leq x + 1 \leq 3$ $\therefore -3 \leq x \leq 2$

$\therefore$ The S.S. $= [-3 , 2]$

6 $\because 2 < -x \leq 6$ $\therefore -2 > x \geq -6$

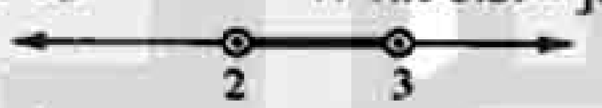
$\therefore$ The S.S. $= [-6 , -2[$

7 $\because -9 \leq 3x \leq 3$ $\therefore -3 \leq x \leq 1$

$\therefore$ The S.S. $= [-3 , 1]$

8 $\because 3 < 2x - 1 < 5$ $\therefore 4 < 2x < 6$

$\therefore 2 < x < 3$ $\therefore$ The S.S. $=]2 , 3[$



9 $\because -1 < \frac{1}{2} x \leq 2$ $\therefore -2 < x \leq 4$

$\therefore$ The S.S. $=]-2 , 4]$

10 By multiplying by 3 $\therefore 0 \leq -2x + 6 < 12$

$\therefore -6 \leq -2x < 6$ $\therefore 3 \geq x > -3$

$\therefore$ The S.S. $=]-3 , 3]$

5 Represent by yourself the S.S. on the number line :

1 $\because 3x - 2x < 4$ $\therefore x < 4$

$\therefore$ The S.S. $=]-\infty , 4[$

2 $\because 7x - 4x \geq 9$ $\therefore 3x \geq 9$ $\therefore x \geq 3$

$\therefore$ The S.S. $= [3 , \infty[$

3 $\because 5x - 2x < 9 + 3$ $\therefore 3x < 12$ $\therefore x < 4$

$\therefore$ The S.S. $=]-\infty , 4[$

4 $\because 7x - 5x \geq -8 + 12$ $\therefore 2x \geq 4$ $\therefore x \geq 2$

$\therefore$ The S.S. $= [2 , \infty[$

5 $\because x + x \leq 3 + 1$ $\therefore 2x \leq 4$ $\therefore x \leq 2$

$\therefore$ The S.S. $=]-\infty , 2]$

6 $\because -x + 2x \geq -3 - 1$ $\therefore x \geq -4$

$\therefore$ The S.S. $= [-4 , \infty[$

6

Represent by yourself the S.S. on the number line :

1. $\because x+3-x \geq 2x-x \geq x-2-x$
   $\therefore 3 \geq x \geq -2$ $\therefore$ The S.S. $= [-2, 3]$
2. $\because -x+x < x+x < 4-x+x$ $\therefore 0 < 2x < 4$
   $\therefore 0 < x < 2$ $\therefore$ The S.S. $=]0, 2[$
3. $\because 4x-4x \leq 5x+2-4x < 4x+3-4x$
   $\therefore 0 \leq x+2 < 3$ $\therefore -2 \leq x < 1$
   $\therefore$ The S.S. $= [-2, 1[$
4. $\because x-1-x < 3x-1-x \leq x+1-x$
   $\therefore -1 < 2x-1 \leq 1$ $\therefore 0 < 2x \leq 2$
   $\therefore 0 < x \leq 1$ $\therefore$ The S.S. $=]0, 1]$
5. $\because 2+2x-2x \leq 3x+3-2x < 5+2x-2x$
   $\therefore 2 \leq x+3 < 5$ $\therefore -1 \leq x < 2$
   $\therefore$ The S.S. $= [-1, 2[$
6. By multiplying by 6
   $\therefore 3x-4 < 6x+6 < 3x+9$
   $\therefore -4 < 3x+6 < 9$
   $\therefore -10 < 3x < 3$ $\therefore -\frac{10}{3} < x < 1$
   $\therefore$ The S.S. $=]-\frac{10}{3}, 1[$

7

1 ≥ 3	2 < 3	3 < -3
4 $\geq -\frac{3}{2}$	5 $\leq 2\sqrt{2}$	6 $]2, 4[$
7 $]-2, 5]$	8 $]2, \infty[$	9 6

8

1 a	2 b	3 c
4 c	5 c	

9

$\because$ The weight of one box = 45 kg.

Let the number of boxes be x

, $\because$ the maximum weight that the lift can carry is 2200 kg.

$\therefore$ The weight of boxes $\leq$ the maximum weight that the lift can carry

$\therefore 45x \leq 2200$ $\therefore x \leq 48\frac{8}{9}$

$\therefore$ The maximum number of boxes can the lift carry in one time is 48 boxes.

10

$\because -4 < -2x < 2$ $\therefore 2 > x > -1$ $\therefore$ The S.S. $=]-1, 2[$

$\because \sqrt{3} \approx 1.7$ $\therefore \sqrt{3} \in]-1, 2[$

11

$\because a+3 \leq x \leq b+3$ $\therefore$ The S.S. $= [a+3, b+3]$

$\therefore [4, 7] = [a+3, b+3]$

$\therefore a+3 = 4$ $\therefore a = 1$

$, b+3 = 7$ $\therefore b = 4$

12

$\because \frac{1}{5} \leq \frac{2x+1}{5} \leq 1$ $\therefore 1 \leq 2x+1 \leq 5$

$\therefore 0 \leq 2x \leq 4$ $\therefore 0 \leq x \leq 2$

$\therefore$ The S.S. $= [0, 2]$ $\therefore m = 0, m+n = 2$ $\therefore n = 2$

13

$\because 5 \leq \frac{2x}{3}+1 \leq 7$ $\therefore 4 \leq \frac{2x}{3} \leq 6$

$\therefore 12 \leq 2x \leq 18$ $\therefore 6 \leq x \leq 9$

$\therefore 4 \leq x-2 \leq 7$ $\therefore$ The smallest value of $x-2$ is 4

14

Multiply both sides by $(\sqrt{3}-\sqrt{5})$

$\therefore x \leq (\sqrt{3}+\sqrt{5})(\sqrt{3}-\sqrt{5})$

"Note that the sign changed because $(\sqrt{3}-\sqrt{5})$ is a negative number because $\sqrt{3} < \sqrt{5}$"

$\therefore x \leq -2$ $\therefore$ The S.S. $=]-\infty, -2]$

Answers of exams on the second part of unit one

Model 1

1

1 a 2 c 3 b 4 d 5 a 6 c

2

1 $\sqrt{3}+\sqrt{2}$ 2 $\sqrt{6}$ 3 $2\sqrt[3]{2}$

4 20 cm. 5 < -4

3 [a] $2\sqrt[3]{2}$

[b] 36π cm^2

4 [a] The S.S. = $]-2 , 3]$ and represent by yourself.

[b] $r = 3$ cm.

5 [a] $\sqrt{2}$

[b] Prove by yourself , $x^2 + 2xy + y^2 = 20$

Model 2

1

1 b 2 b 3 d 4 c 5 a 6 c

2

1 $2\sqrt{2}-\sqrt{5}$ 2 $]-2 , \infty[$ 3 2

4 18 5 3

3 [a] Prove by yourself.

[b] $\frac{\sqrt{3}}{2}$

4 [a] $\sqrt{5}$

[b] 54 cm^2

5 [a] 132 cm^2

[b] The S.S. = $]-\infty , 4[$ and represent by yourself.

Answers of unit two

Answers of Exercise 11

1 (5 , 14) , (2 , 5) , (0 , − 1) , (− 3 , − 10)

2 The ordered pair (− 1 , 3) satisfies the relation.

3
(1) (1 , − 3) , (2 , − 1) , (3 , 1) , (5 , 5)
(2) (0 , 5) , (2 , 6) , (4 , 7) , (6 , 8)
(3) (0 , 2) , (3 , 2) , (5 , 2) , (− 4 , 2)
(4) (2.5 , 7) , (2.5 , 3) , (2.5 , − 7) , (2.5 , 4)
There are other solutions.

4
(1)

x	0	1	2	3
y	1	5	9	13

(2)

x	− 4	− 3	− 2
y	− 5	0	5

(3)

a	1	4	3
b	− 3	0	− 1

(4)

a	2	5	− 1
b	− 1	0	− 2

5 (1) 7 (2) − 9 (3) zero (4) − 1

6 ∵ (3 , 6) satisfies the relation $y = k\,x$
∴ $6 = 3\,k$ ∴ $k = 2$

7 ∵ (3 , 1) satisfies the relation $y - 3\,x = a$
∴ $1 - 3 \times 3 = a$ ∴ $a = -8$

8 ∵ (− 3 , 2) satisfies the relation $3\,x + b\,y = 1$
∴ $3 \times (-3) + b \times 2 = 1$ ∴ $2\,b = 9 + 1$
∴ $2\,b = 10$ ∴ $b = 5$

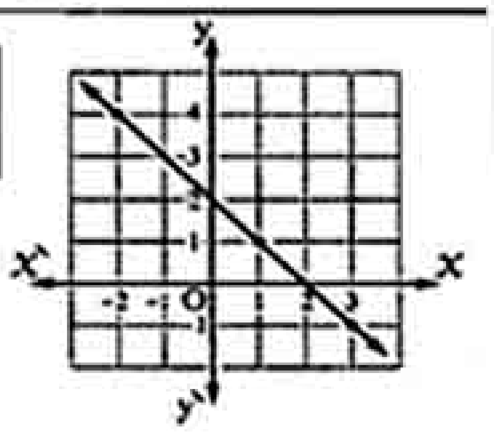
9 ∵ (3 , a) satisfies the relation $y - 2\,x = 4$
∴ $a - 2 \times 3 = 4$ ∴ $a = 10$

10 ∵ (k , 2 k) satisfies the relation $x + y = 15$
∴ $k + 2\,k = 15$ ∴ $3\,k = 15$ ∴ $k = 5$

11 (1) $x = 3$ (2) y = zero

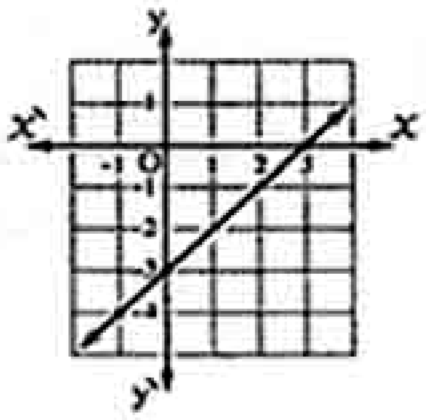
12
(1)

x	1	3	− 2
y	1	− 1	4



(2)

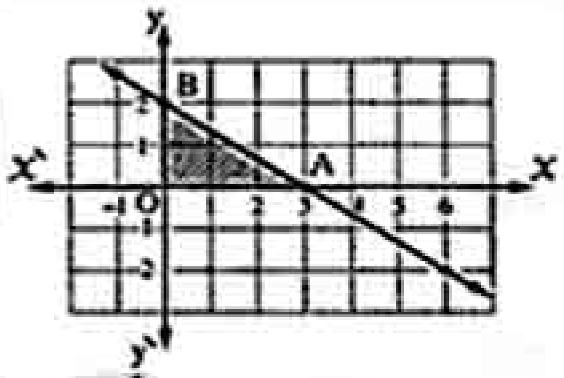
x	0	3	− 1
y	− 3	0	− 4



From (3) to (8) represent the relations graphically by yourself.

13 $y = -\frac{2}{3}x + 2$

x	0	3	6
y	2	0	− 2



From the graph :
The area of Δ OAB $= \frac{1}{2} \times 3 \times 2 = 3$ square units

14 ∵ the straight line intersects x − axis at (3 , b)
∴ b = 0
∴ (3 , 0) satisfies the relation $2\,x - y = a$
∴ $2 \times 3 - 0 = a$ ∴ $a = 6$

15
(1) d (2) b (3) c (4) b
(5) a (6) c (7) c (8) c
(9) c (10) c (11) c (12) d

16 Let the first number be x and the second be y
∴ $2\,x + y = 12$ ∴ $y = 12 - 2\,x$
∵ the two numbers are even natural numbers.
∴ x has the values 0 , 2 , 4 , 6 then we can register the different possibilities to the two numbers in the following table :

x	0	2	4	6
y	12	8	4	0

17 Let the length of the rectangle = x cm. and the width = y cm. ∴ $x > y$
, ∵ the perimeter of the rectangle = 14 cm.
∴ $2\,(x + y) = 14$ ∴ $x + y = 7$
we can record the different possibilities of the length and the width of the rectangle in the opposite table :

x	6	5	4
y	1	2	3

18 Let the number of bills of L.E. 5 be X, then its value $= 5X$

and let the number of bills of L.E. 20 be y then its value $= 20$ y

$\therefore 5X + 20y = 65$ where X and y are natural numbers.

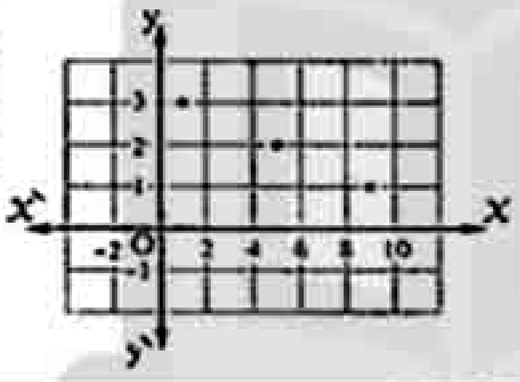
$\therefore X + 4y = 13$ $\qquad \therefore y = \frac{13 - X}{4}$

$\because X \leq 10$, $(13 - X)$ is divisible by 4

i.e. X has the values 9 , 5 and 1

then we can write the different possibilities in the following table :

X	1	5	9
y	3	2	1



19 Let the store sold in this week X computer's table and y chairs.

$\therefore 100X + 50y = 500$

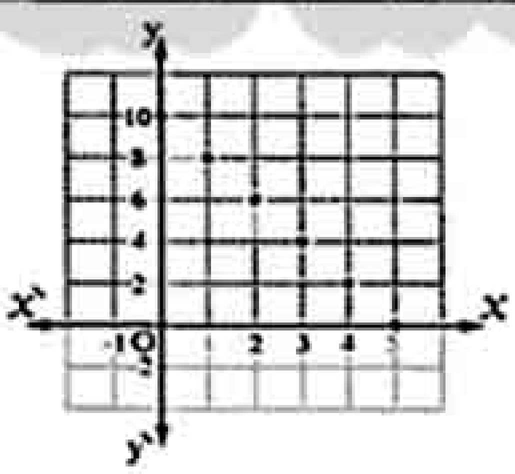
where X and y are natural numbers

$\therefore 2X + y = 10$ $\qquad \therefore y = 10 - 2X$

$\therefore X$ is not more than 5

$\therefore$ We can write the expectations which represent the number of computer's tables and the number of chairs in the following table :

X	0	1	2	3	4	5
y	10	8	6	4	2	0



20 Let the length of any of the two congruent sides in the triangle be X cm. and the length of the third side be y cm.

$\because$ the perimeter of the triangle = 19

$\therefore 2X + y = 19$ $\qquad \therefore y = 19 - 2X$

$\therefore X$ and y are positive integers then X is not more than 9 and from the inequality of the triangle then X has the values 5 , 6 , 7 , 8 and 9

then we can write all the possibilities in the following table :

X	5	6	7	8	9
y	9	7	5	3	1

Answers of Exercise 12

1

figure (1) the slope is positive.

figure (2) the slope is negative.

figure (3) the slope is undefined.

figure (4) the slope equals zero.

2

1 negative 2 zero 3 undefined 4 positive

3

1 zero 2 undefined 3 X-axis 4 $\overleftrightarrow{BC}$ or $\overleftrightarrow{AC}$

4

1 the slope of $\overline{AB} = \frac{4-3}{3-1} = \frac{1}{2}$

2 the slope of $\overline{AB} = \frac{0-2}{5-1} = -\frac{1}{2}$

3 the slope of $\overline{AB} = \frac{5-2}{6-3} = 1$

4 the slope of $\overline{AB} = \frac{-1+1}{4-2} = 0$

5 the slope of $\overline{AB} = \frac{3-3}{2-1} = 0$

6 the slope of $\overline{AB} = \frac{4-2}{5-5} = \frac{2}{\text{zero}}$ undefined

7 the slope of $\overline{AB} = \frac{2+1}{3-3} = \frac{3}{\text{zero}}$ undefined

8 the slope of $\overline{AB} = \frac{1+2}{4-3} = \frac{3}{1} = 3$

9 the slope of $\overline{AB} = \frac{1-3}{2+1} = \frac{-2}{3}$

10 the slope of $\overline{NK} = \frac{-7+2}{-1-4} = \frac{-5}{-5} = 1$

11 the slope of $\overline{EO} = \frac{0+1}{0+3} = \frac{1}{3}$

12 the slope of $\overline{AB} = \frac{-1+9}{-1+6} = \frac{8}{5}$

5

1 Taking the two points (0 , 0) , (1 , 2) which lie on the straight line we find that

the slope $= \frac{2-0}{1-0} = 2$

2 Taking the two points (0 , − 1) , (− 2 , 3) which lie on the straight line we find that

the slope $= \frac{3-(-1)}{-2-0} = \frac{4}{-2} = -2$

6

$\because m(\angle M) = 45°$ $\therefore \Delta MNL$ is an isosceles triangle.

$\therefore ML = LN$

$\because$ the length of $\overline{ML} = 4$ units.

$\therefore$ the length of $\overline{LN} = 4$ units.

$\therefore N = (3 , 6)$

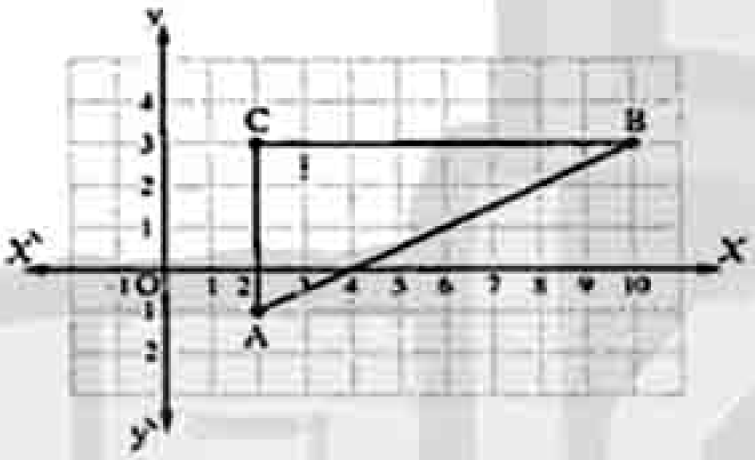
, the slope of $\overline{MN} = \frac{6-2}{3-7} = \frac{4}{-4} = -1$

7

the slope of $\overline{AB} = \frac{3+1}{10-2} = \frac{1}{2}$

the slope of $\overline{BC} = \frac{3-3}{2-10} = \text{zero}$

the slope of $\overline{AC} = \frac{3+1}{2-2} = \frac{4}{0}$ (undefined)



from the graph we find that ΔABC is right-angled.

8

$\because$ the slope of the straight line which passes through the two points $(1 , 3)$ and $(3 , k)$ equals 3

$\therefore \frac{k-3}{3-1} = 3$ $\therefore \frac{k-3}{2} = 3$

$\therefore k - 3 = 6$ $\therefore k = 9$

9

$\because$ the slope of the straight line which passes through the two points $(3 , c)$ and $(5 , -2)$ equals -3

$\therefore \frac{-2-c}{5-3} = -3$ $\therefore \frac{-2-c}{2} = -3$

$\therefore -2 - c = -6$ $\therefore c = 4$

10

$-2 = \frac{2-4}{X-(-1)}$ $\therefore -2 = \frac{-2}{X+1}$

$\therefore X + 1 = 1$ $\therefore X = 0$

11

$\because \frac{-1-y}{3-(-2)} = -0.6$ $\therefore \frac{-1-y}{5} = -0.6$

$\therefore -1 - y = -3$ $\therefore y = 2$

12

$\because$ the straight line is parallel to X-axis

$\therefore$ the slope = zero $\therefore \frac{k-4}{2-3} = \text{zero}$

$\therefore k - 4 = \text{zero}.$ $\therefore k = 4$

13

$\because$ the straight line is parallel to y-axis

$\therefore$ the slope is undefined

$\therefore X_2 - X_1 = \text{zero}$ $\therefore 6 - 2X = 0$

$\therefore -2X = -6$ $\therefore X = 3$

14

$\because$ the straight line is perpendicular to y-axis

$\therefore$ the straight line is parallel to X-axis

$\therefore$ the slope = zero $\therefore y_2 - y_1 = \text{zero}$

$\therefore 3y - 6 = 0$ $\therefore 3y = 6$ $\therefore y = 2$

15

$\because$ the slope of the straight line passing through the two points $(-5 , 11)$ and $(0 , 8) = \frac{8-11}{0-(-5)} = \frac{-3}{5}$ (1)

$\because$ the slope of the straight line passing through the two points $(0 , 8)$ and $(5 , 5) = \frac{5-8}{5-0} = \frac{-3}{5}$ (2)

from (1) and (2) we find that the three points are collinear.

(liying on the straight line whose slope $= -\frac{3}{5}$)

16

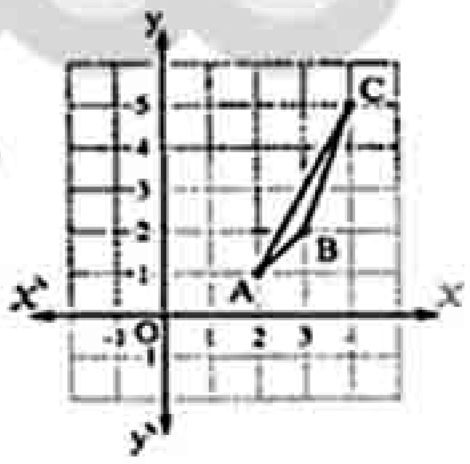
the slope of $\overline{AB} = \frac{2-1}{3-2} = 1$

, the slope of $\overline{BC} = \frac{5-2}{4-3} = 3$

and the slope of $\overline{AC}$

$= \frac{5-1}{4-2} = 2$

we observe that the three points are not collinear.



17

1 $\because$ the slope of $\overline{AB} = \frac{2-1}{2-1} = 1$

$\because$ the slope of $\overline{BC} = \frac{-3-2}{-3-2} = 1$

$\therefore$ the slope of $\overline{AB}$ = the slope of $\overline{BC}$ and the point B is a common point.

$\therefore$ the points A , B and C are collinear.

2 ∵ the slope of $\overline{AB} = \frac{7-(-3)}{-6-4} = \frac{10}{-10} = -1$

∵ the slope of $\overline{BC} = \frac{-4-7}{5-(-6)} = \frac{-11}{11} = -1$

∴ the slope of $\overline{AB}$ = the slope of $\overline{BC}$ and the point B is a common point.

∴ the points A , B and C are collinear.

3 ∵ the slope of $\overline{AB} = \frac{4-12}{2-(-2)} = \frac{-8}{4} = -2$

∵ the slope of $\overline{BC} = \frac{-4-4}{6-2} = \frac{-8}{4} = -2$

∴ the slope of $\overline{AB}$ = the slope of $\overline{BC}$ and the point B is a common point.

∴ the points A , B and C are collinear.

18

1 ∵ the slope of $\overline{AB} = \frac{0-1}{3-2} = \frac{-1}{1} = -1$

∵ the slope of $\overline{BC} = \frac{-1-0}{5-3} = -\frac{1}{2}$

∴ the slope of $\overline{AB} \neq$ the slope of $\overline{BC}$

∴ the points A , B and C are not collinear.

2 ∵ the slope of $\overline{AB} = \frac{1-2}{3-(-1)} = -\frac{1}{4}$

∵ the slope of $\overline{BC} = \frac{2-1}{7-3} = \frac{1}{4}$

∴ the slope of $\overline{AB} \neq$ the slope of $\overline{BC}$

∴ the points A , B and C are not collinear.

3 ∵ the slope of $\overline{AB} = \frac{2-(-3)}{2-0} = \frac{5}{2}$

∵ the slope of $\overline{BC} = \frac{-3-2}{-3-2} = 1$

∴ the slope of $\overline{AB} \neq$ the slope of $\overline{BC}$

∴ the points A , B and C are not collinear.

19

∵ the slope of $\overline{AB} = \frac{5-3}{2+1} = \frac{2}{3}$

∵ the slope of $\overline{BC} = \frac{5-1}{2-8} = -\frac{2}{3}$

∴ the slope of $\overline{AB} \neq$ the slope of $\overline{BC}$

∴ $C \notin \overline{AB}$

20

∵ the slope of the straight line which passes through the two points (4 , 1) and (– 2 , 7)

$= \frac{7-1}{-2-4} = \frac{6}{-6} = -1$

∵ the slope of the straight line which passes through the two points (– 2 , 7) and (3 , y)

$= \frac{y-7}{3-(-2)} = \frac{y-7}{5}$

∵ the three points are collinear.

∴ $\frac{y-7}{5} = -1$ ∴ $y-7=-5$

∴ $y=-5+7$ ∴ $y=2$

21

∵ the slope of the straight line which passes through the two points (3 , – 1) and (x , 1) equals $\frac{2}{3}$

∴ $\frac{1-(-1)}{x-3} = \frac{2}{3}$ ∴ $\frac{2}{x-3} = \frac{2}{3}$

∴ $x-3=3$ ∴ $x=6$

∵ the slope of the straight line which passes through the two points (3 , – 1) and (9 , y) equals $\frac{2}{3}$

∴ $\frac{y-(-1)}{9-3} = \frac{2}{3}$ ∴ $\frac{y+1}{6} = \frac{2}{3}$ ∴ $3y+3=12$

∴ $3y=9$ ∴ $y=3$

Answers of Exercise 13

1

∵ The uniform velocity $= \frac{\text{the covered distance}}{\text{the taken time}}$

$= \frac{180}{3} = 60$ km./hr.

∴ the covered distance = the taken time × the uniform velocity = 60 × 5 = 300 km.

2

∵ The rate of consumption of fuel

$= \frac{\text{the amount of consumpted fuel}}{\text{time}}$

$= \frac{2.47}{3} = \frac{247}{300}$ litre/hr.

∴ the consumpted amount =

The rate of consumption × time

$= \frac{247}{300} \times 10 = 8\frac{7}{30}$ litre

3

1 At the moment of starting the motion, the body is at a distance of 2 metres from the fixed point.

At t = 6 , the body is at a distance of 8 metres.

Taking the two points (0 , 2) and (6 , 8) on the straight line which represents the relation between t and d

$\therefore$ the slope $= \frac{8-2}{6-0} = \frac{6}{6} = 1$

it represents the velocity of the body within a going trip.

2 At the moment of starting the motion , the body is at t distance of 12 metres from the fixed point.

At t = 6 the body is at a distance of 2 metres. Taking the two points (0 , 12) and (6 , 2) on the straight line representing the relation between t and d

$\therefore$ the slope $= \frac{2-12}{6-0} = \frac{-10}{6} = -\frac{5}{3}$

It represents the velocity of the body within the returning back.

3 On starting the motion , the body is at a distance of 8 metres from the fixed point.

At t = 6 the body is at a distance of 8 metres.

$\because$ the straight line representing the relation is horizontal. $\therefore$ The slope = zero

It means that the body is rest.

4

Taking two points on the straight line representing the relation between t and d say (0 , 50) and (4 , 150)

$\therefore$ the uniform velocity = the slope of the straight line $= \frac{150-50}{4-0} = \frac{100}{4} = 25$ km/h.

5

1 Taking two points on the straight line representing the relation between t and d say (0 , 50) and (2 , 200)

$\therefore$ the unifrom velocity = the slope of the straight line $= \frac{200-50}{2-0} = 75$ km/h.

2 from the graph :

The car is at a distance = 275 km. from the point 0 after passing 3 hours from the moment of beginning the motion.

6

1 The velocity within the first 3 hours = the slope of the striaght line $\overline{OB} = \frac{125}{3} = 41\frac{2}{3}$ km/h

The velocity within the next 2 hours = the slope of the straight line $\overline{BC} = \frac{125}{2} = 62\frac{1}{2}$ km/h

2 The average velocity within the all trip $= \frac{\text{total distance}}{\text{total time}} = \frac{250}{5} = 50$ km/h

7

1 The velocity within the first 3 hours = the slope of the striaght line $= \frac{60-20}{3-0} = \frac{40}{3} = 13\frac{1}{3}$ km/h.

2 The velocity within the next 4 hours = the slope of the straight line $= \frac{0-60}{7-3} = \frac{-60}{4} = -15$ km/h.

The negative sign means that the bicycle returns back with velocity 15 km/h.

The total distance = 40 + 60 = 100 km.

8

1 The slope of the straight line $\overline{AB}$

$= \frac{60-20}{4-0} = \frac{40}{4} = 10$

It means the increasing of the capital within the first 4 years with rate equals 10 thousands pounds/year.

The slope of $\overline{BC} = \frac{60-60}{6-4}$ = zero

It means constancy of the capital within the fifth and sixth years.

The slope of $\overline{CD} = \frac{50-60}{8-6} = \frac{-10}{2} = -5$

It means decreasing of the capital within the 7^{th} and 8^{th} years with rate = 5 thousands/year.

2 The starting capital of the company = 20 thousand pounds.

9

1 The slope of $\overline{AB} = \frac{125-50}{8-0} = \frac{75}{8} = 9\frac{3}{8}$

It means that the increase in height goes with respect to the increase in age.

The slope of $\overline{BC} = \frac{175-125}{18-8} = \frac{50}{10} = 5$

It means that the increase in height goes with respect to the increase in age but with a rate less than the rate within the first 8 years.

The slope of $\overline{CD} = \frac{175-175}{22-18} = 0$

It denotes the constancy in height inspite of the increase in age after 18 years.

2 $\because$ the height of the person at age 30 years = 175 cm. and the height of the person at age 8 years = 125 cm.

$\therefore$ the difference = 175 − 125 = 50 cm.

10

1 The greatest capacity of the tank = 70 litre.

2 The tank will be empty after 30 hours.

3 The remained fuel = 35 litre.

4 taking the two points (0 , 70) , (30 , 0) on the straight line representing the relation.

$\therefore$ The rate of consumption of the fuel =

The slope of the straight line $= \frac{70-0}{0-30} = -2\frac{1}{3}$ litre/h.

The negative sign means the rate of consumption.

i.e. the amount of fuel is consumpted with rate $2\frac{1}{3}$ Litre/hr

11

1 100 pages.

2 taking the two points (0 , 100) and (3 , 40) on the straight line representing the relation.

we find that the rate of decreasing the number of pages = the slope of the straight line

$= \frac{40-100}{3-0} = \frac{-60}{3} = -20$ page/h

The negative sign expresses the decreasing in the number of remained pages with rate 20 page/h.

3 $\because$ the remained pages decreases with rate 20 page/h.

$\because$ There are 100 pages.

$\therefore$ The person finishes reading the book after

$\frac{100}{20} = 5$ hours.

12

1 The depth of the well before beginning digging = 5 m.

2 The depth of the well after finishing digging = 40 m.

3 The total time taken in digging = 10 h.

4 The average of digging the well in the first 5 hours

= the slope of the straight line $= \frac{27.5-5}{5-0} = 4.5$ m/h.

5 The average of digging in the last two hours

= the slope of the straight line

$= \frac{40-27.5}{10-8} = 6.25$ m/h.

13

1 The velocity during the going trip

= the slope of the straight line $= \frac{60-0}{3-0} = 20$ km/h.

2 The average velocity during returning back

$= \frac{\text{total distance}}{\text{total time}} = \frac{60}{5} = 12$ km/h.

3 It means that the bicycle stopped within the 6^{th} hour from the beginning.

14

Let the covered distance be d km

The amount of the remained fuel in the tank be y litre.

At the beginning the covered distance = zero

The amount of remained fuel = 40 litre

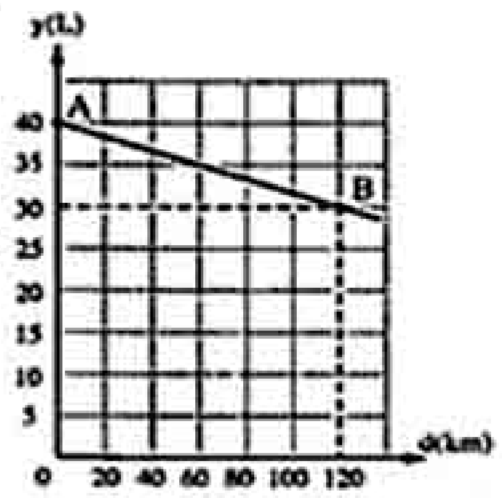
We express this by the point A (0 , 40)

$\because$ After covering distance.

The amount of remained fuel

$= \frac{3}{4} \times 40 = 30$ litre.

we expresses this by the point B (120 , 30)



$\therefore$ the rate of decreasing the amount of fuel

= the slope of $\overline{AB}$

$= \frac{30-40}{120-0} = \frac{-10}{120} = -\frac{1}{12}$

$\therefore$ the amount of fuel decreases with the rate one litre for every 12 km.

$\therefore$ The distance covered by the car when the tank becomes empty $= 12 \times 40 = 480$ km.

15

1 100 km.

2 the train A took 2 hours

the train B took $2\frac{1}{2}$ hours.

3 The average speed $= \frac{\text{total distance}}{\text{total time}}$

with respect to the train A

The average speed $= \frac{100}{2} = 50$ km/h.

with respect to the train B

the average speed $= \frac{100}{2.5} = 40$ km/h.

4 It means that the train A was at rest from half past ten till half past eleven.

16

1 Tortoise

2 The velocity of the tortoise $= \frac{\text{the covered distance}}{\text{the taken time}}$

$= \frac{100}{60} = 1\frac{2}{3}$ metre / minute

3 The average velocity of the rabbit $= \frac{\text{total distance}}{\text{total time}}$

$= \frac{100}{65} = 1\frac{7}{13}$ metre / minute

4 It means that the rabbit was at rest from the tenth minute to 60^{th} minute.

17

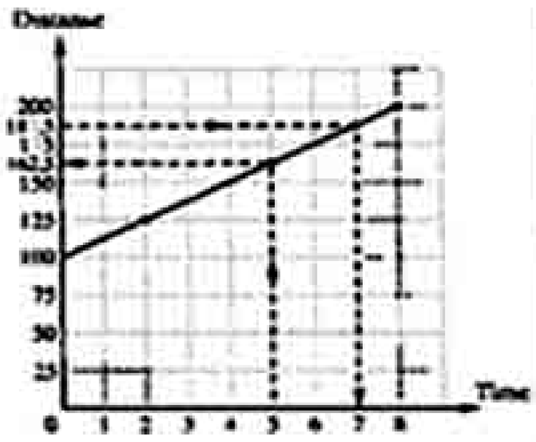
1 the velocity of the bicycle = the slope of the straight line $= \frac{200-125}{8-2} = 12.5$ km/h.

2 300 minutes = 5 hours ,

then the bicycle is at distance = 162.5 km.

3 7 hours.

4 from the graph.
the starting point is far from the fixed point = 100 km.



Answers of exams on unit two

Model 1

1

1 c	2 a	3 c
4 b	5 c	6 c

2

1 (4 , 0)	2 4	3 2
4 undefined	5 zero	

3

[a] Represent by yourself. [b] Prove by yourself.

4

[a] Represent by yourself
, the area of Δ OAB = 3 square units.

[b] $y = \frac{-1}{2}$

5

1 $13\frac{1}{3}$ km./hr. 2 15 km./hr.

3 100 km.

Model 2

1

1 d	2 b	3 d
4 c	5 b	6 b

2

1 zero	2 -1	3 $(\frac{-5}{2}, 0)$
4 undefined	5 collinear	

3

[a] Represent by yourself. [b] $a = -6$, $b =$ zero

4

[a] $a = -2$

[b] Prove by yourself.

5

First : 1 2 m. 2 8 m.

Second : 1

Answers of unit three

Answers of Exercise 14

1

Sets	Tallies	Freq.
25 –	###	5
30 –	### ### ///	13
35 –	### ### ### /	16
40 –	###	5
45 –	/	1
Total		40

Sets	Freq.
25 –	5
30 –	13
35 –	16
40 –	5
45 –	1
Total	40

2

Sets	Tallies	Freq.
30 –	////	4
40 –	###	5
50 –	### //	7
60 –	### ///	8
70 –	### /	6
80 –	////	4
90 –	### /	6
Total		40

Sets	Freq.
30 –	4
40 –	5
50 –	7
60 –	8
70 –	6
80 –	4
90 –	6
Total	40

The set which has the highest frequency is 60 –
The sets which have the lowest frequency are 80 – , 30 –

3

1

Sets	Tallies	Freq.
20 –	///	3
24 –	//	2
28 –	### /	6
32 –	### //	7
36 –	### ### //	12
Total		30

Sets	Freq.
20 –	3
24 –	2
28 –	6
32 –	7
36 –	12
Total	30

2 12 students.

4

Sets	Tallies	Freq.
0 –	//	2
4 –	### //	7
8 –	### ### //	12
12 –	### ### ###	15
16 –	////	4
Total		40

Sets	Freq.
0 –	2
4 –	7
8 –	12
12 –	15
16 –	4
Total	40

The percentage of those who obtained 12 marks at least $= \frac{19}{40} \times 100 = 47.5\ \%$

5

1 The least height = 112 cm. and the greatest height = 199 cm.
The range = 199 – 112 = 87 cm.

2

Sets	Tallies	Freq.
110 –	//	2
120 –	///	3
130 –	///	3
140 –	### /	6
150 –	### ////	9
160 –	### ///	8
170 –	### //	7
180 –	### //	7
190 –	###	5
Total		50

Sets	Freq.
110 –	2
120 –	3
130 –	3
140 –	6
150 –	9
160 –	8
170 –	7
180 –	7
190 –	5
Total	50

6

Sets	Tallies	Freq.
165 –	### ///	8
170 –	### ###	10
175 –	### ### ###	15
180 –	### /	6
185 –	### ###	10
190 –	////	4
195 –	/	1
200 –	/	1
Total		55

Sets	Freq.
165 –	8
170 –	10
175 –	15
180 –	6
185 –	10
190 –	4
195 –	1
200 –	1
Total	55

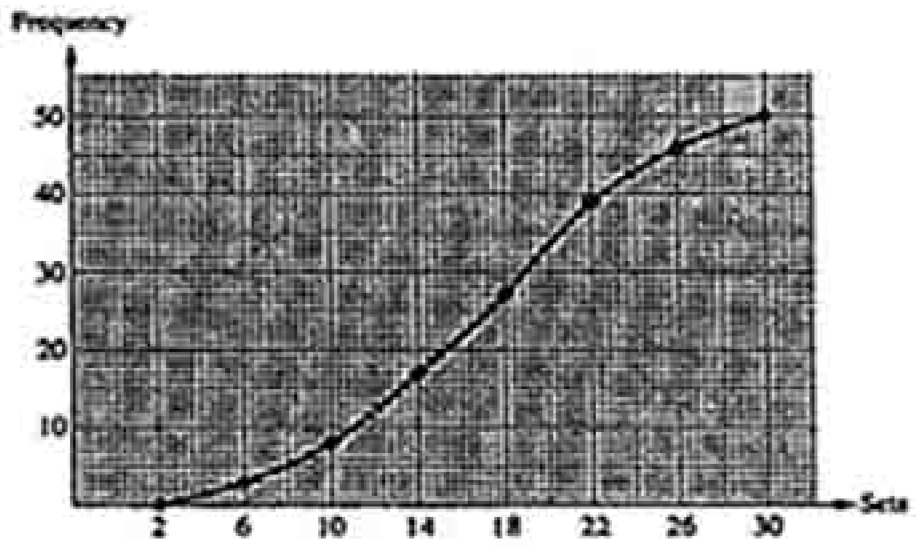
1 39 soldiers. 2 22 soldiers.

Answers of Exercise 15

First : Problems on the ascending cumulative frequency curve.

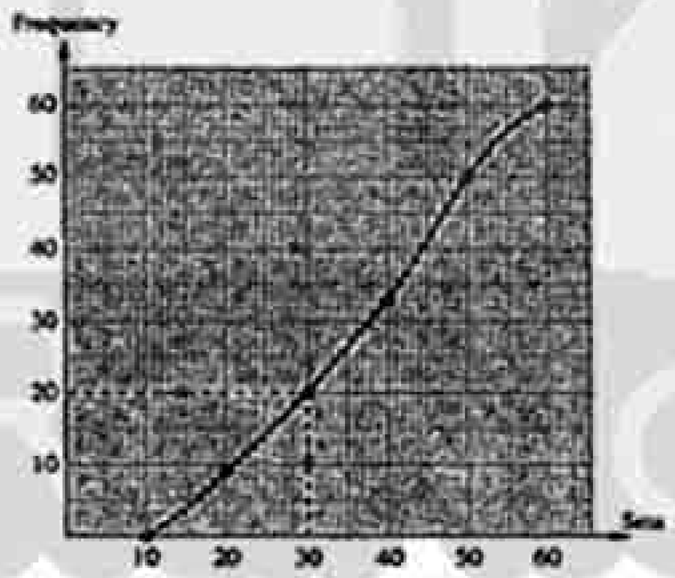
1

The upper boundaries of sets	ascending cumulative frequency
less than 2	0
less than 6	3
less than 10	8
less than 14	17
less than 18	27
less than 22	39
less than 26	46
less than 30	50



2

The upper boundaries of sets	Ascending cumulative frequency
less than 10	0
less than 20	9
less than 30	20
less than 40	33
less than 50	50
less than 60	60



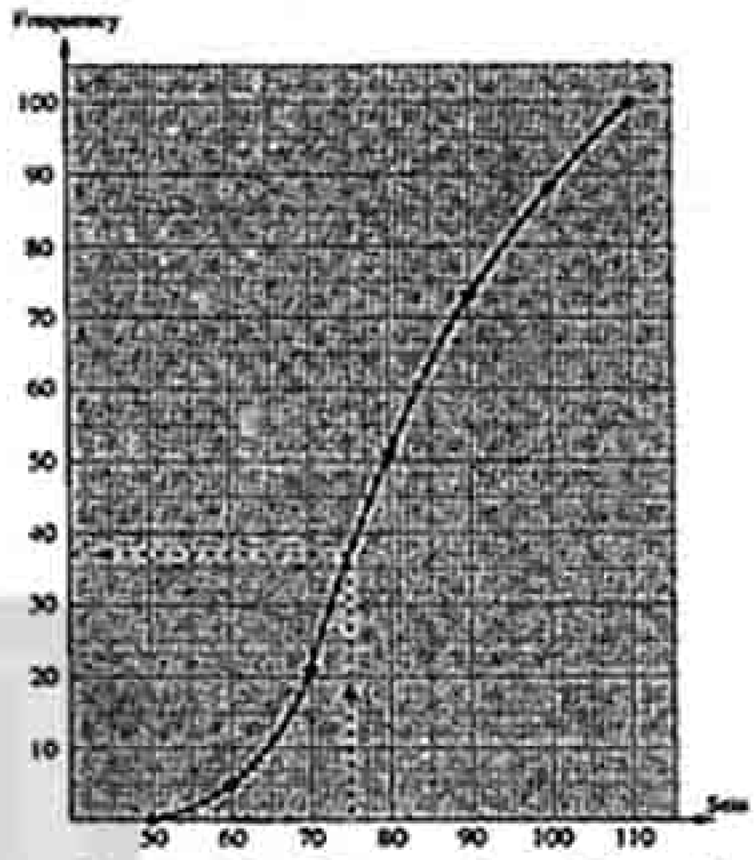
From the graph :

The number of failed pupils = 20 pupils.

3

1

The upper boundaries of sets	Ascending cumulative frequency
less than 50	0
less than 60	5
less than 70	21
less than 80	51
less than 90	73
less than 100	88
less than 110	100



2 **From the graph :** The number of factories which work less than 75 hours = 37 factories.

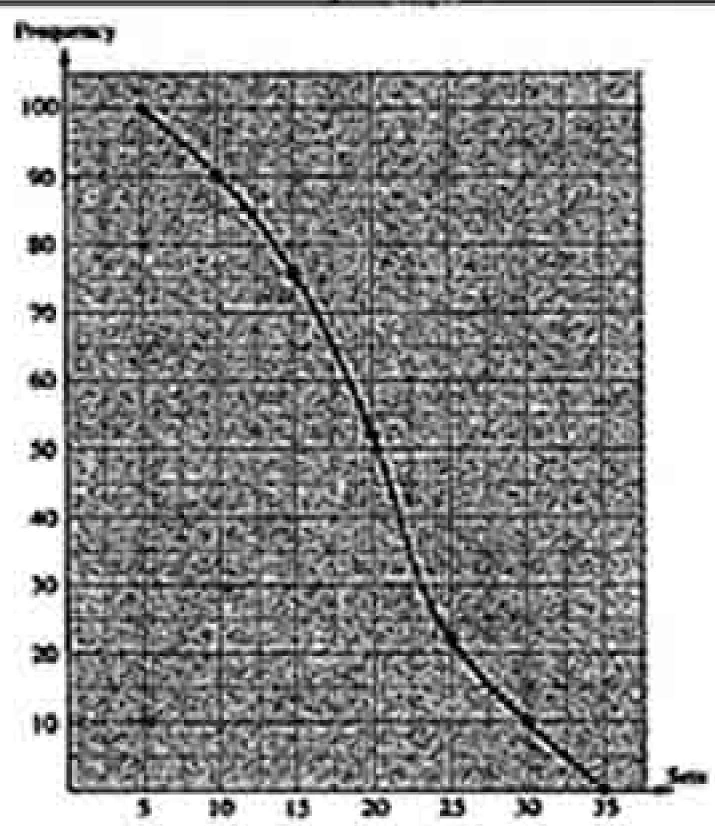
3 The percentage of the number of factories which work less than 75 hours

$= \frac{37}{100} \times 100\ \% = 37\ \%$

Second : Problems on the descending cumulative frequency curve.

4

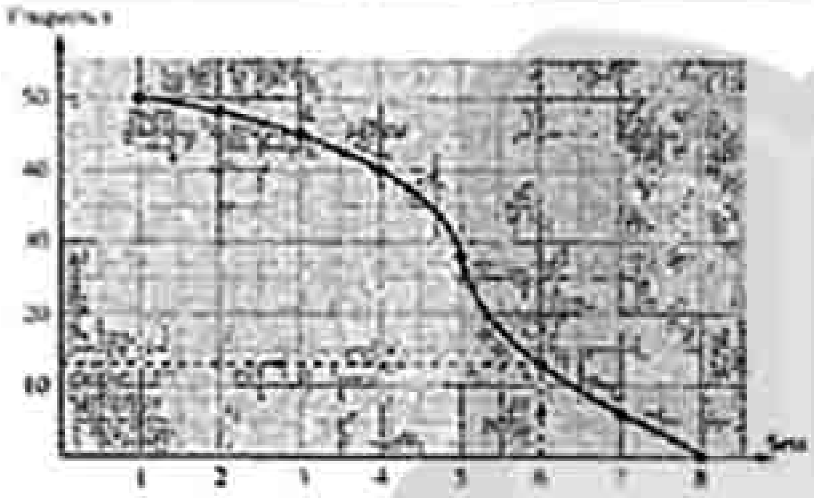
The lower limits of sets	Descending cumulative frequency
5 and more	100
10 and more	90
15 and more	76
20 and more	52
25 and more	22
30 and more	10
35 and more	0



5

1

The lower boundaries of sets	Descending cumulative frequency
1 and more	50
2 and more	48
3 and more	45
4 and more	40
5 and more	28
6 and more	13
7 and more	6
8 and more	0

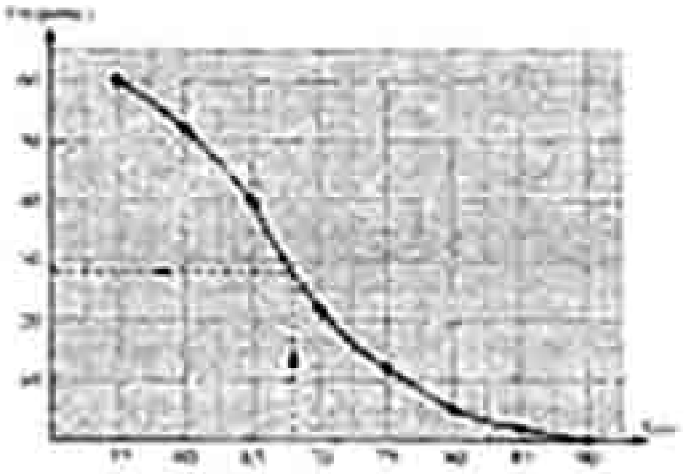


2 From the graph : The number of pupils who study 6 hours and more daily = 13 pupils.

3 The percentage of the number of pupils who study 6 hours and more daily = $\frac{13}{50} \times 100\ \% = 26\ \%$

6 The missed value in the table = 10

The lower limits of sets	Descending cumulative frequency
55 and more	60
60 and more	52
65 and more	40
70 and more	22
75 and more	12
80 and more	5
85 and more	2
90 and more	0



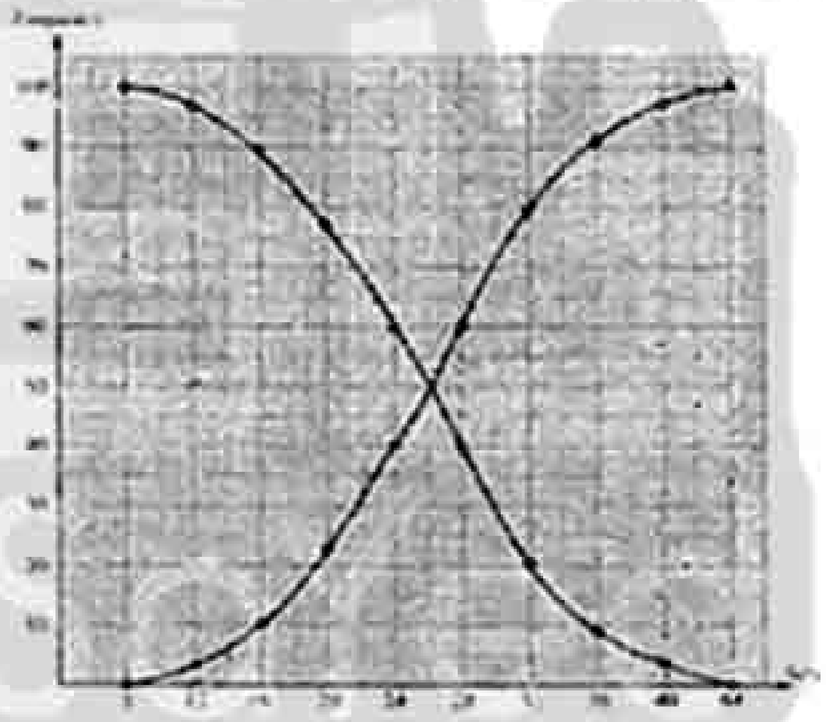
From the graph :

The number of persons whose weights are 68 kg. and more = 28 persons.

Third : Problems on the two curves together.

7

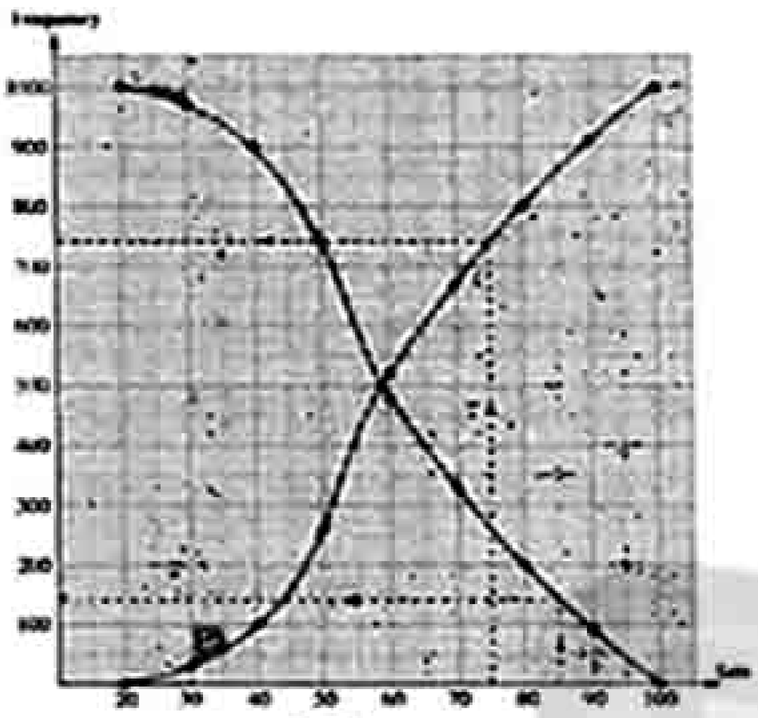
The upper limits of sets	Ascending cumulative frequency	The lower limits of sets	Descending cumulative frequency
less than 8	0	8 and more	100
less than 12	4	12 and more	96
less than 16	11	16 and more	89
less than 20	23	20 and more	77
less than 24	41	24 and more	59
less than 28	61	28 and more	39
less than 32	80	32 and more	20
less than 36	91	36 and more	9
less than 40	97	40 and more	3
less than 44	100	44 and more	0



8

1

The upper boundaries of sets	Ascending cumulative frequency	The lower boundaries of sets	Descending cumulative frequency
less than 20	0	20 and more	1000
less than 30	30	30 and more	970
less than 40	100	40 and more	900
less than 50	260	50 and more	740
less than 60	520	60 and more	480
less than 70	670	70 and more	330
less than 80	800	80 and more	200
less than 90	910	90 and more	90
less than 100	1000	100 and more	0



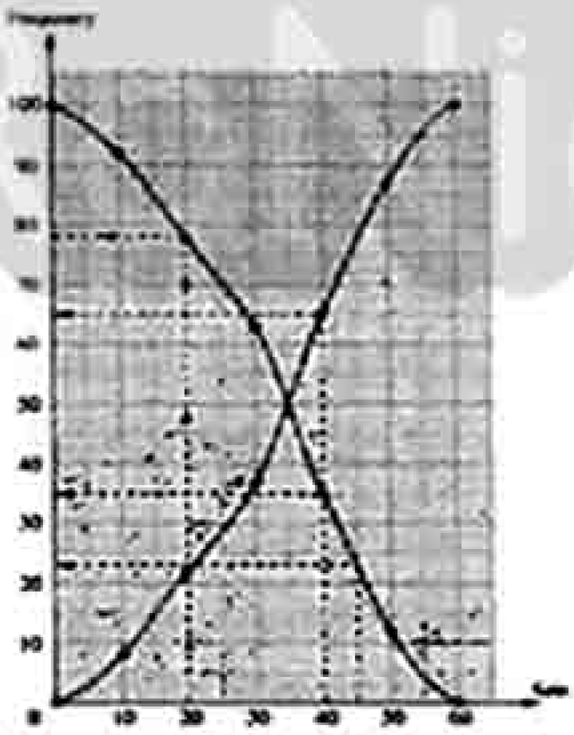
2 740 students. **3** 140 students.

9

1

The upper boundaries of sets	Ascending cumulative frequency	The lower boundaries of sets	Descending cumulative frequency
less than 0	0	0 and more	100
less than 10	8	10 and more	92
less than 20	22	20 and more	78
less than 30	37	30 and more	63
less than 40	65	40 and more	35
less than 50	88	50 and more	12
less than 60	100	60 and more	0

2



3 **From the graph :** The number of students who got less than 40 marks = 65 students and the number of students who got 40 marks or more = 35 students.

4 The number of students who got 20 marks or more = 78 and their percentage = $\frac{78}{100} \times 100\ \% = 78\ \%$

5 The number of students who got 45 marks or more = 23 students and their percentage = $\frac{23}{100} \times 100\ \% = 23\ \%$

10

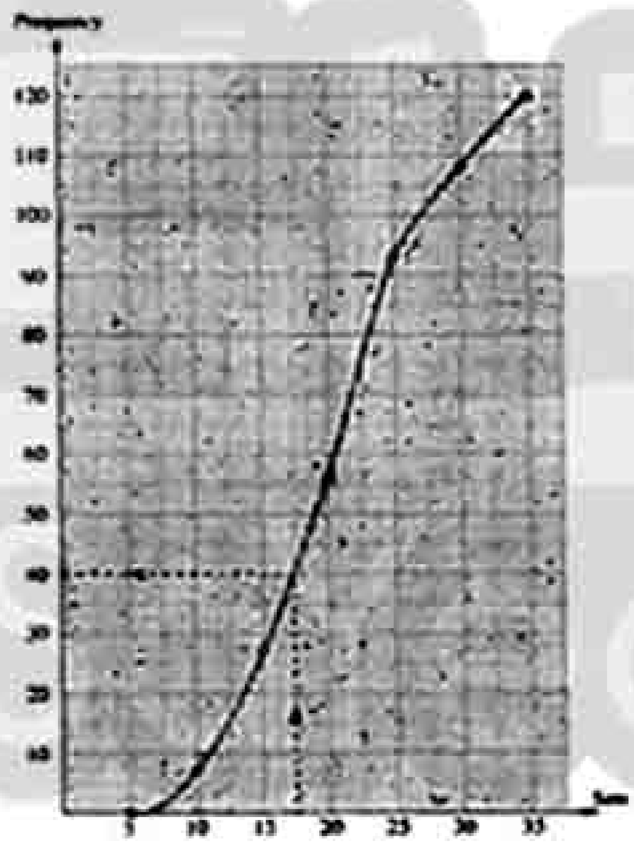
1 The frequency distribution table.

Sets	5 –	10 –	15 –	20 –	25 –	30 –	Total
Frequency	7	20	29	37	15	12	120

2 The ascending cumulative frequency table.

The upper limits of sets	Ascending cumulative frequency
less than 5	0
less than 10	7
less than 15	27
less than 20	56
less than 25	93
less than 30	108
less than 35	120

3



4 **From the graph :** The number of workers whose experience years are less than 17.5 years = 40 workers.

Answers of Exercise 16

1

1 $\frac{\text{The sum of values}}{\text{Number of values}}$ **2** Its lower limit, its upper limit.

3 10 **4** 11 **5** 14 **6** 3940

2

1 c **2** d **3** c **4** b **5** a

3

Sets	Centre of Sets "X"	Frequency "f"	Center of sets × frequency "$X \times f$"
5 –	10	6	60
15 –	20	8	160
25 –	30	4	120
35 –	40	2	80
Total		20	420

$\therefore$ The mean $= \frac{420}{20} = 21$

4 **1**

Sets	"X"	"f"	"$X \times f$"
10 –	15	1	15
20 –	25	2	50
30 –	35	4	140
40 –	45	2	90
50 –	55	1	55
Total		10	350

$\therefore$ The mean of marks of students $= \frac{350}{10} = 35$ marks.

2 The number of failed students = 3 students.

5

Sets	"X"	"f"	"$X \times f$"
16 –	18	10	180
20 –	22	15	330
24 –	26	22	572
28 –	30	25	750
32 –	34	20	680
36 –	38	8	304
Total		100	2816

$\therefore$ The mean $= \frac{2816}{100} = 28.16$

6

Sets	"X"	"f"	"$X \times f$"
15 –	20	2	40
25 –	30	3	90
35 –	40	5	200
45 –	50	8	400
55 –	60	6	360
65 –	70	4	280
75 –	80	2	160
Total		30	1530

$\therefore$ The mean $= \frac{1530}{30} = 51$

7

Sets	"X"	"f"	"$X \times f$"
140 –	142	12	1704
144 –	146	20	2920
148 –	150	38	5700
152 –	154	22	3388
156 –	158	17	2686
160 –	162	11	1782
Total		120	18180

$\therefore$ The mean $= \frac{18180}{120} = 151.5$ cm.

8 **1**

Sets	"X"	"f"	"$X \times f$"
1 –	1.5	2	3
2 –	2.5	3	7.5
3 –	3.5	5	17.5
4 –	4.5	12	54
5 –	5.5	15	82.5
6 –	6.5	7	45.5
7 –	7.5	6	45
Total		50	255

$\therefore$ The mean of the number of hours of study $= \frac{255}{50} = 5.1$ hours.

2 The number of pupils who study less than 4 hours daily = 2 + 3 + 5 = 10 pupils.

9

1 25 – , 10

2

Sets	"X"	"f"	"$X \times f$"
5 –	10	3	30
15 –	20	10	200
25 –	30	12	360
35 –	40	10	400
45 –	50	5	250
Total		40	1240

$\therefore$ The mean $= \frac{1240}{40} = 31$ marks.

3 The number of students whose marks are not less than 35 = 15 students.

10 The missed number is 5

Sets	"X"	"f"	"$X \times f$"
6 –	8	2	16
10 –	12	3	36
14 –	16	5	80
18 –	20	8	160
22 –	24	6	144
26 –	28	4	112
30 –	32	2	64
Total		30	612

$\therefore$ The mean $= \frac{612}{30} = 20.4$ kg.

11

[1] $X = 30$

, $k + 2 = 100 - (10 + 17 + 20 + 32 + 4) = 17$

$\therefore k = 15$

[2]

Sets	"X"	"f"	"$X \times f$"
10 –	15	10	150
20 –	25	17	425
30 –	35	20	700
40 –	45	32	1440
50 –	55	17	935
60 –	65	4	260
Total		100	3910

$\therefore$ The mean $= \frac{3910}{100} = 39.1$

12

[1] $3k + 4k = 50 - (7 + 10 + 8 + 4)$

$\therefore 7k = 21 \qquad \therefore k = \frac{21}{7} = 3$

[2]

Sets	"X"	"f"	"$X \times f$"
30 –	32.5	7	227.5
35 –	37.5	9	337.5
40 –	42.5	12	510
45 –	47.5	10	475
50 –	52.5	8	420
55 –	57.5	4	230
Total		50	2200

$\therefore$ The mean $= \frac{2200}{50} = 44$ kg.

13

[1] $k - 2 = 50 - (4 + 5 + 8 + 7 + 5 + 1)$

$\therefore k - 2 = 20 \qquad \therefore k = 22$

[2]

Sets	"X"	"f"	"$X \times f$"
2 –	4	4	16
6 –	8	5	40
10 –	12	8	96
14 –	16	20	320
18 –	20	7	140
22 –	24	5	120
26 –	28	1	28
Total		50	760

$\therefore$ The mean $= \frac{760}{50} = 15.2$ days.

14 $\because$ The total of marks of the student in 5 months

$= 5 \times 23.8 = 119$ marks.

, let the required mark of the sixth month be X

$\therefore \frac{119 + X}{6} = 24 \qquad \therefore 119 + X = 144$

$\therefore X = 144 - 119 = 25$ marks.

$\therefore$ The mark of the student in the 6th month is 25

15 $\because$ The total of marks of Magdi in 4 exams

$= 4 \times 16 = 64$ marks.

, let the required mark be $X \qquad \therefore \frac{64 + X}{5} = 18$

$\therefore 64 + X = 90 \qquad \therefore X = 90 - 64 \qquad \therefore X = 26$ marks.

$\therefore$ The mark of Magdi in the 5th exam should be 26 marks.

16

[1] $a = \frac{0 + 4}{2} = 2 \quad , b = \frac{90}{6} = 15 \quad , c = \frac{300}{30} = 10$

$\therefore \frac{4 + d}{2} = 6 \qquad \therefore d = 8$

, $e = \frac{16 + 12}{2} = 14$, $f = \frac{16 + 20}{2} = 18$

, $X = 10 \times 18 = 180$

, $y = 1140 - (10 + 90 + 300 + 180) = 560$

, $r = \frac{560}{14} = 40$

, $m = 5 + 15 + 30 + 40 + 10 = 100$

[2] The mean $= \frac{1140}{100} = 11.4$ marks.

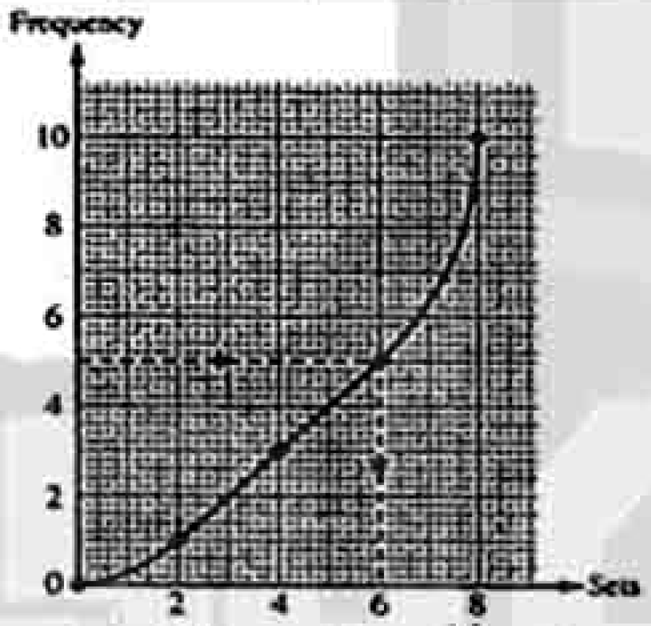
Answers of Exercise 17

1

1) 4 2) 6 3) The third
4) 7 5) 10 6) The median

2

The upper limits of sets	Ascending cumulative frequency
less than 0	0
less than 2	1
less than 4	3
less than 6	5
less than 8	10

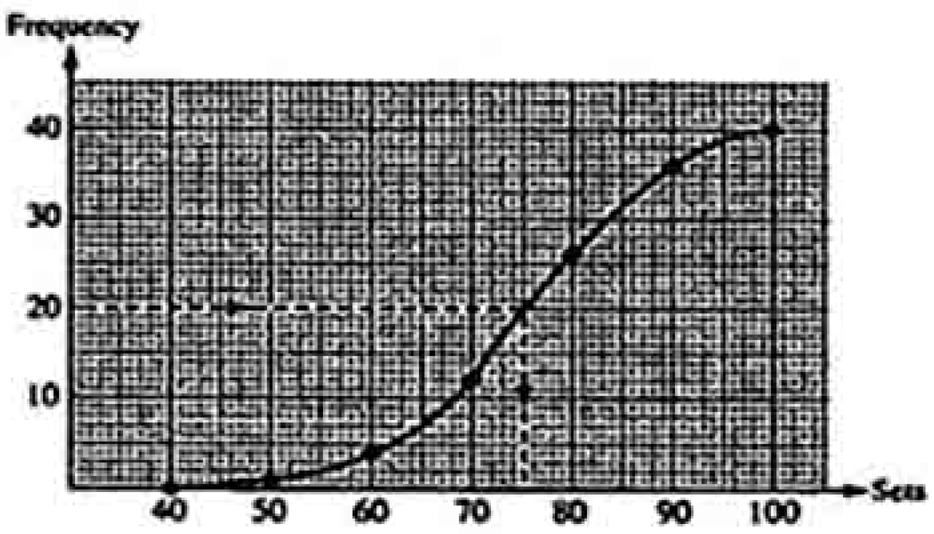


$\because$ The order of the median $= \frac{10}{2} = 5$

$\therefore$ The median $= 6$

3

The upper boundaries of sets	Ascending cumulative frequency
less than 40	0
less than 50	1
less than 60	4
less than 70	12
less than 80	26
less than 90	36
less than 100	40

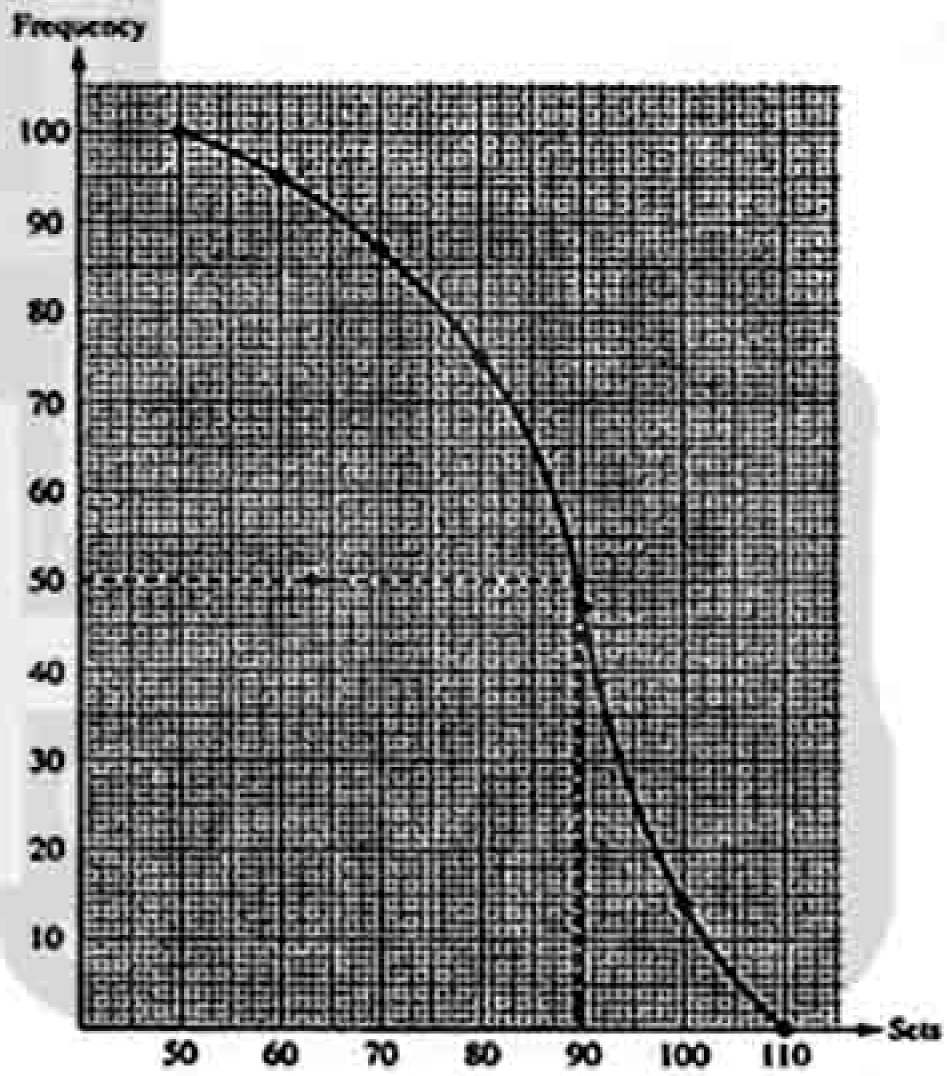


$\because$ The order of the median $= \frac{40}{2} = 20$

$\therefore$ The percentage of intelligence $\simeq 75\%$.

4

The lower boundaries of sets	Descending cumulative frequency
50 and more	100
60 and more	95
70 and more	87
80 and more	75
90 and more	47
100 and more	14
110 and more	0

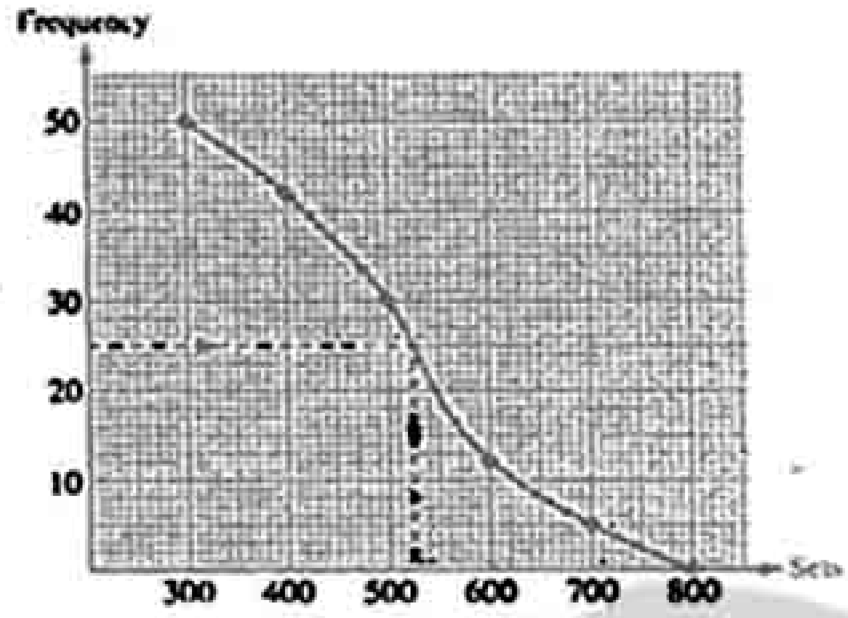


$\because$ The order of the median $= \frac{100}{2} = 50$

$\therefore$ The median of working hours $\simeq 89.5$ hours

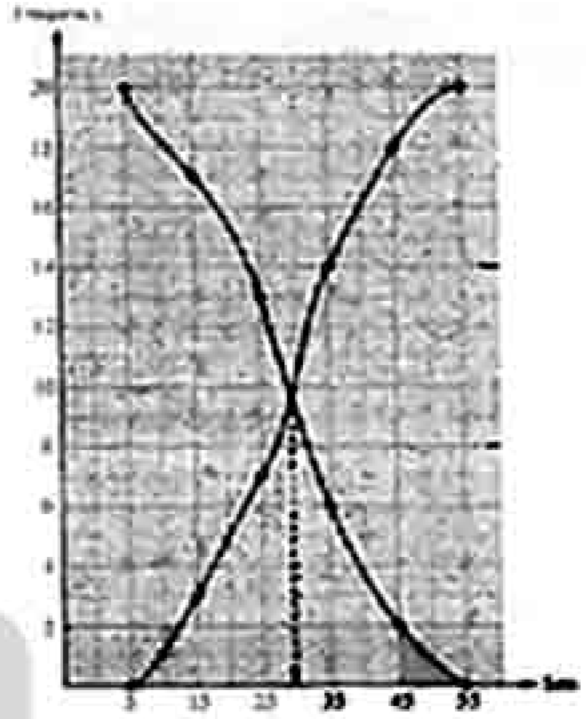
5

The lower boundaries of sets	Descending cumulative frequency
300 and more	50
400 and more	42
500 and more	30
600 and more	12
700 and more	5
800 and more	0



$\because$ The order of the median $= \frac{50}{2} = 25$

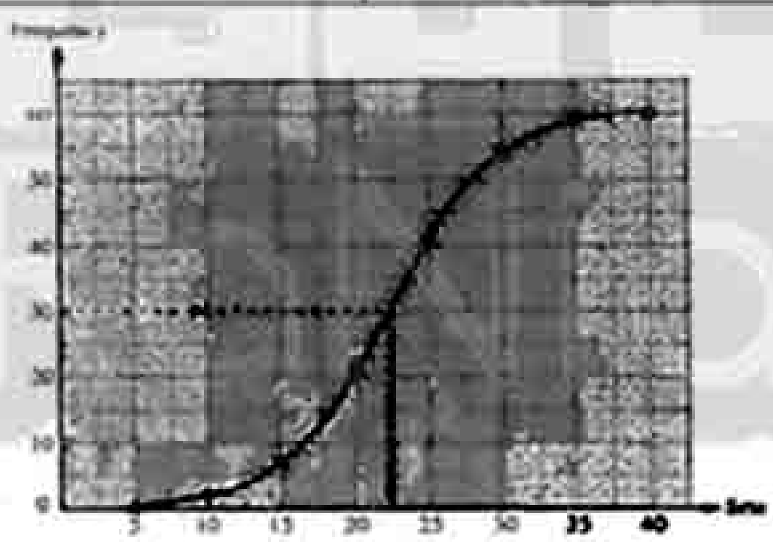
$\therefore$ The median wage = 520 pounds.



From the graph we find that the median ≃ 29 kg.

6

The upper limits of sets	Ascending cumulative frequency
less than 5	0
less than 10	2
less than 15	7
less than 20	21
less than 25	41
less than 30	54
less than 35	59
less than 40	60



$\because$ The order of the median $= \frac{60}{2} = 30$

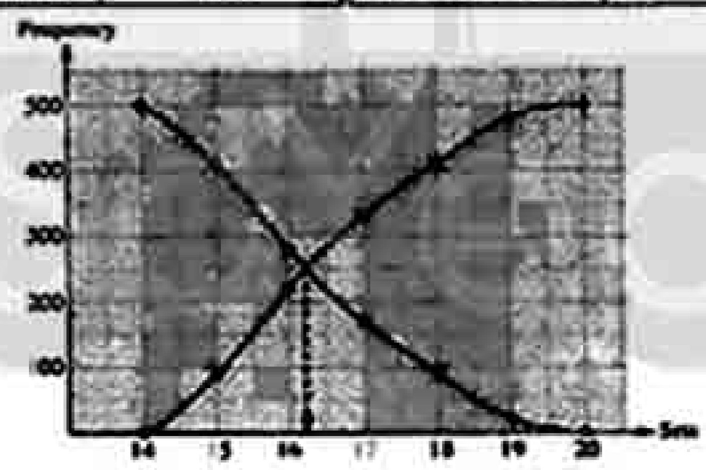
$\therefore$ The median mark ≃ 22 marks.

7

The upper limits of sets	Ascending cumulative frequency	The lower limits of sets	Descending cumulative frequency
less than 5	0	5 and more	20
less than 15	3	15 and more	17
less than 25	7	25 and more	13
less than 35	14	35 and more	6
less than 45	18	45 and more	2
less than 55	20	55 and more	0

8

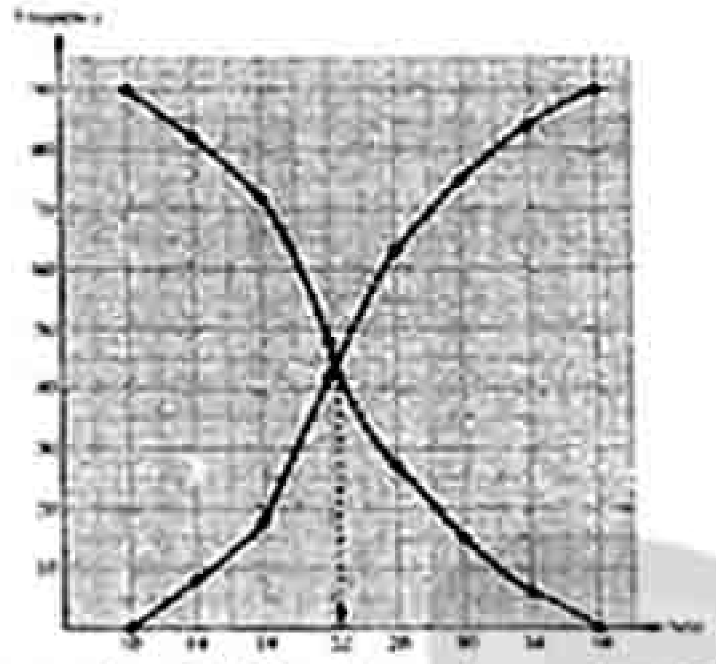
The upper boundaries of sets	Ascending cumulative frequency	The lower boundaries of sets	Descending cumulative frequency
less than 14	0	14 and more	500
less than 15	90	15 and more	410
less than 16	220	16 and more	280
less than 17	330	17 and more	170
less than 18	410	18 and more	90
less than 19	480	19 and more	20
less than 20	500	20 and more	0



$\therefore$ The median age ≃ 16.3 years.

9

The upper limits of sets	Ascending cumulative frequency	The lower limits of sets	Descending cumulative frequency
less than 10	0	10 and more	90
less than 14	8	14 and more	82
less than 18	18	18 and more	72
less than 22	42	22 and more	48
less than 26	63	26 and more	27
less than 30	75	30 and more	15
less than 34	84	34 and more	6
less than 38	90	38 and more	0



From the graph we find that the median mark = 22.5 marks

10

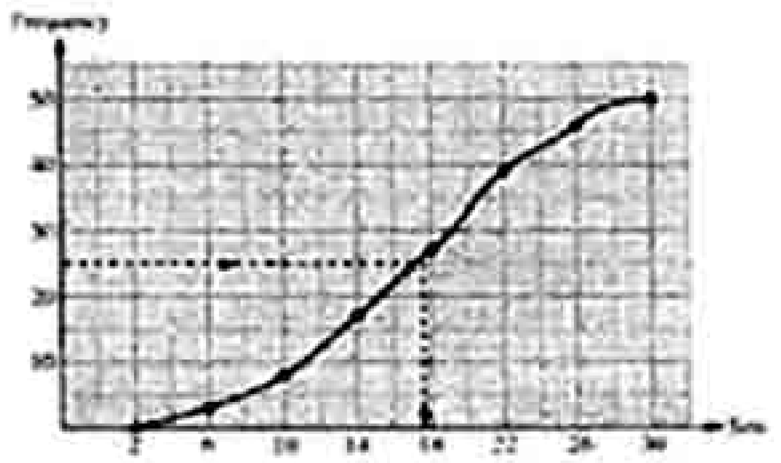
1

Sets	"X"	"f"	"X × f"
2 –	4	3	12
6 –	8	5	40
10 –	12	9	108
14 –	16	10	160
18 –	20	12	240
22 –	24	7	168
26 –	28	4	112
Total		50	840

$\therefore$ The mean $= \frac{840}{50} = 16.8$

2 We form the ascending cumulative frequency table.

The upper limits of sets	Ascending cumulative frequency
less than 2	0
less than 6	3
less than 10	8
less than 14	17
less than 18	27
less than 22	39
less than 26	46
less than 30	50



$\because$ The order of the median $= \frac{50}{2} = 25$

$\therefore$ The median $= 17.6$

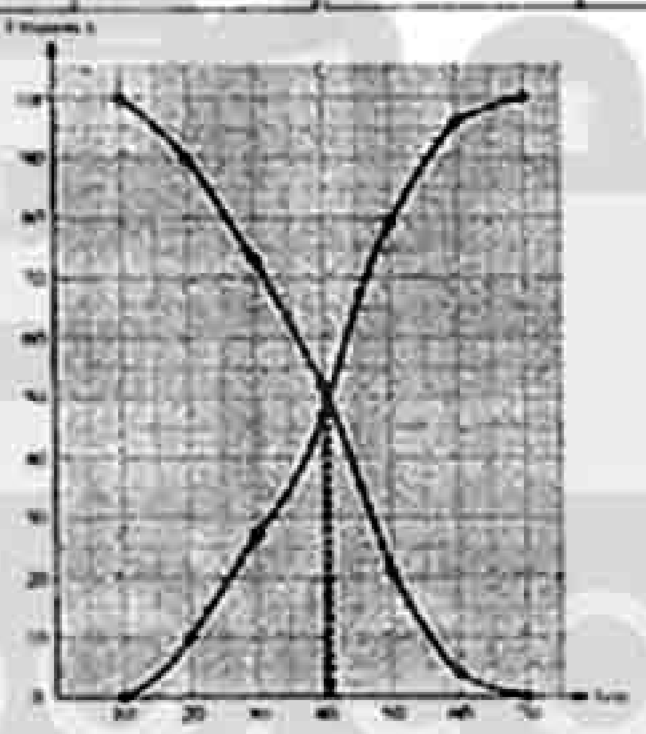
11

1 $X = 30$, $k + 2 = 100 - (10 + 17 + 20 + 32 + 4)$

$\therefore k + 2 = 17$ $\qquad \therefore k = 15$

2

The upper limits of sets	Ascending cumulative frequency	The lower limits of sets	Descending cumulative frequency
less than 10	0	10 and more	100
less than 20	10	20 and more	90
less than 30	27	30 and more	73
less than 40	47	40 and more	53
less than 50	79	50 and more	21
less than 60	96	60 and more	4
less than 70	100	70 and more	0

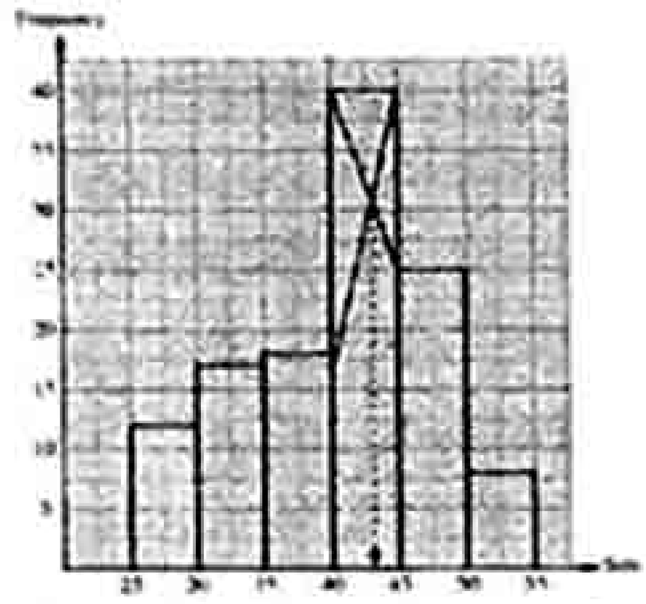


The median = 41

Answers of Exercise 18

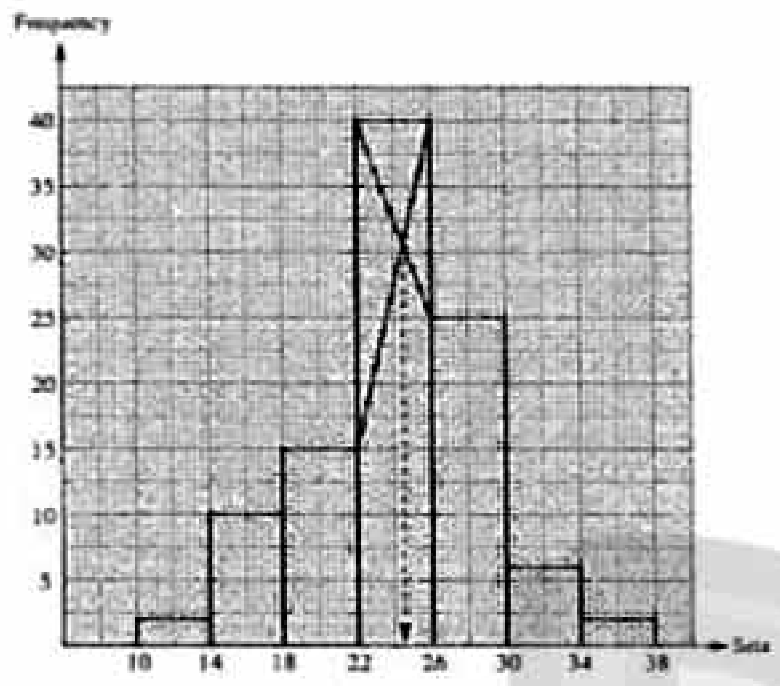
1 1 the most common value in the set. 2 5 3 8 4 3 5 6 6 2

2



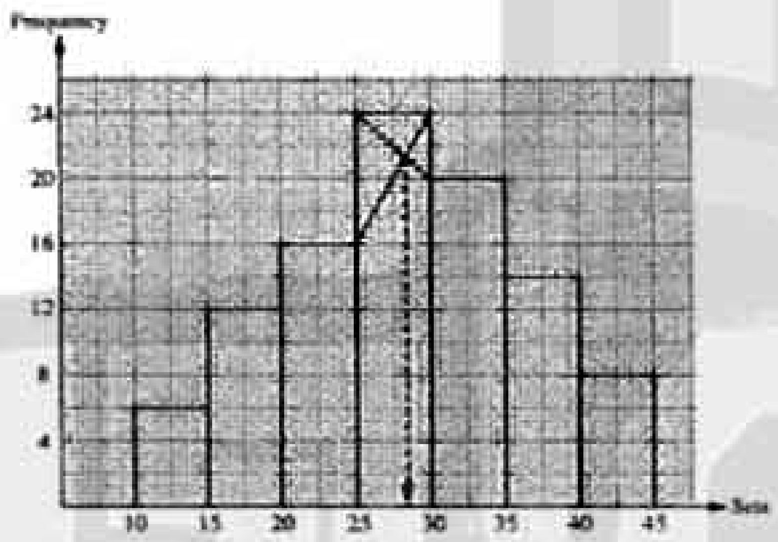
From the graph : The mode age = 43 years.

3



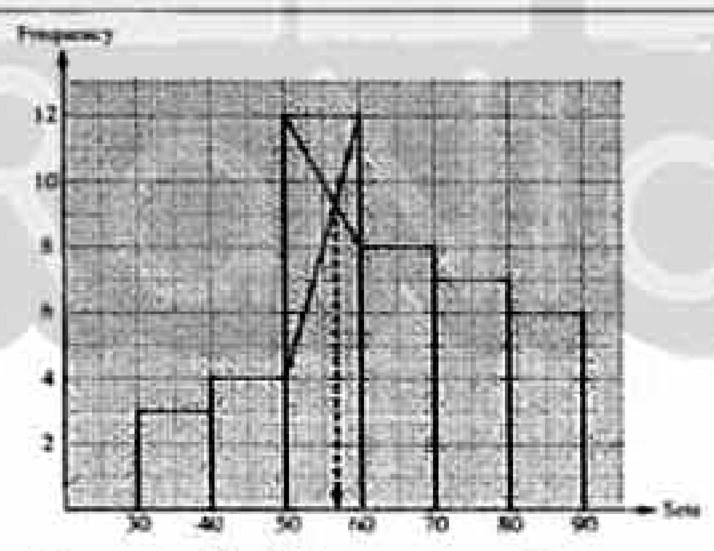
From the graph : the mode mark ≈ 24.5 marks.

4



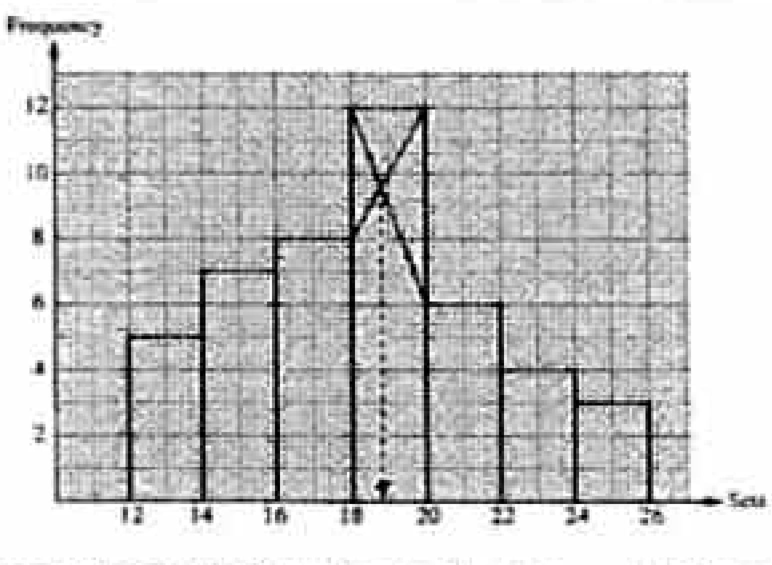
From the graph : The mode wage ≈ 28.5 pounds.

5



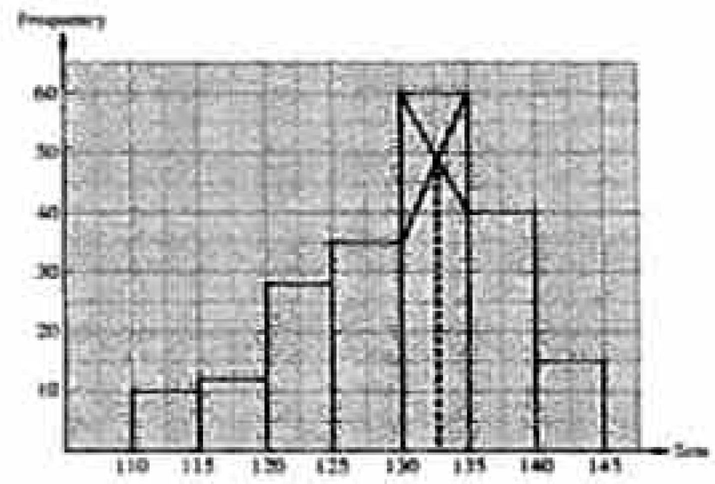
From the graph : The mode = 57

6



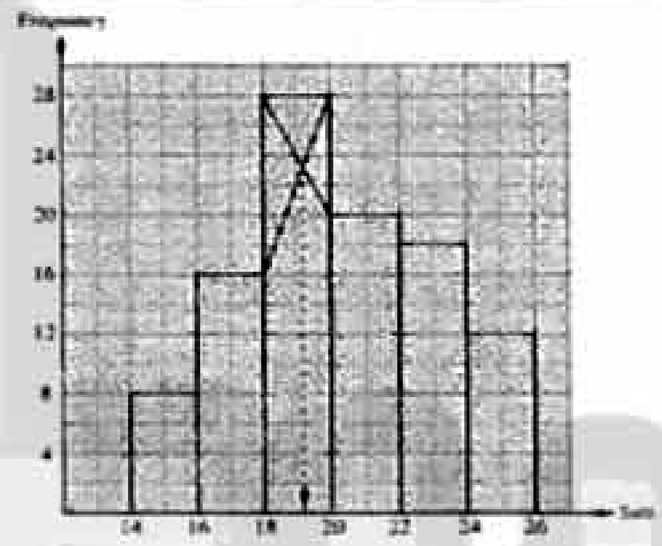
From the graph : the mode age ≈ 18.8 years.

7



From the graph : the mode height ≈ 132.75 cm.

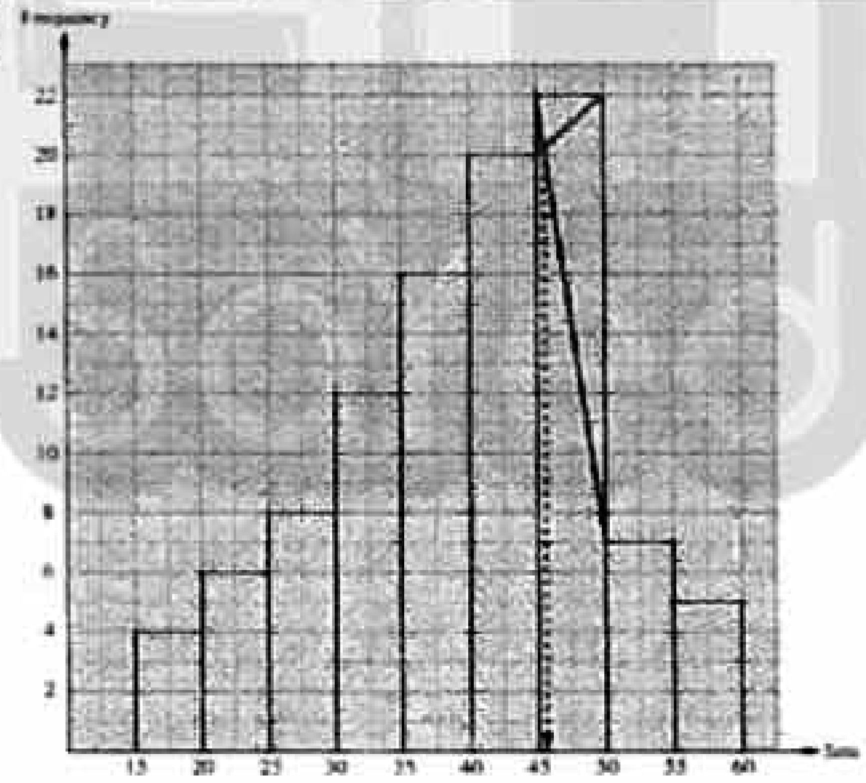
8



From the graph :

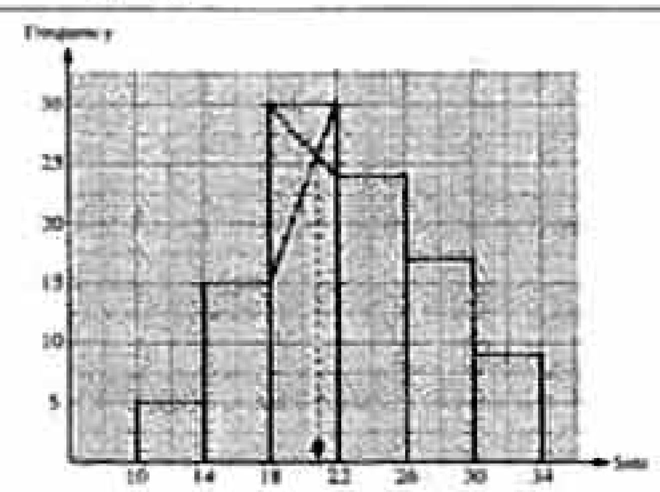
the mode of the amount of milk ≈ 19.2 galoons.

9



From the graph : the mode mark ≈ 45.5 marks.

10



From the graph : the mode weight ≈ 20.8 kg.

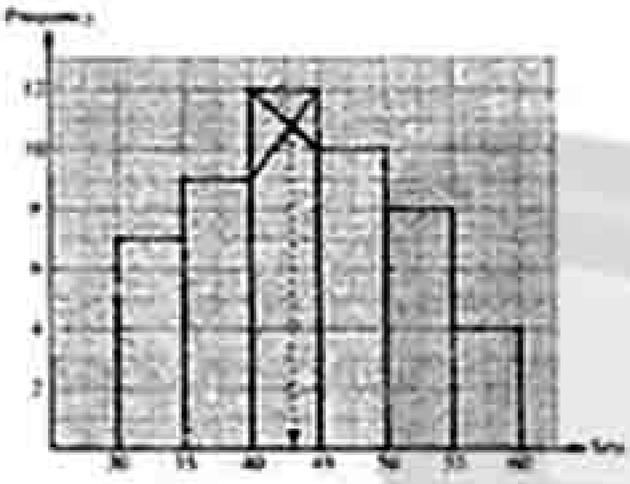
11

1 $\because k + 4 + 3k + 4k + 3k + 1 + 3k - 1 + k + 1 = 50$

$\therefore 15k + 5 = 50 \quad \therefore 15k = 45 \quad \therefore k = 3$

2

Weight in kg.	30 –	35 –	40 –	45 –	50 –	55 –	Total
number of students	7	9	12	10	8	4	50



From the graph : The mode weight = 43 kg.

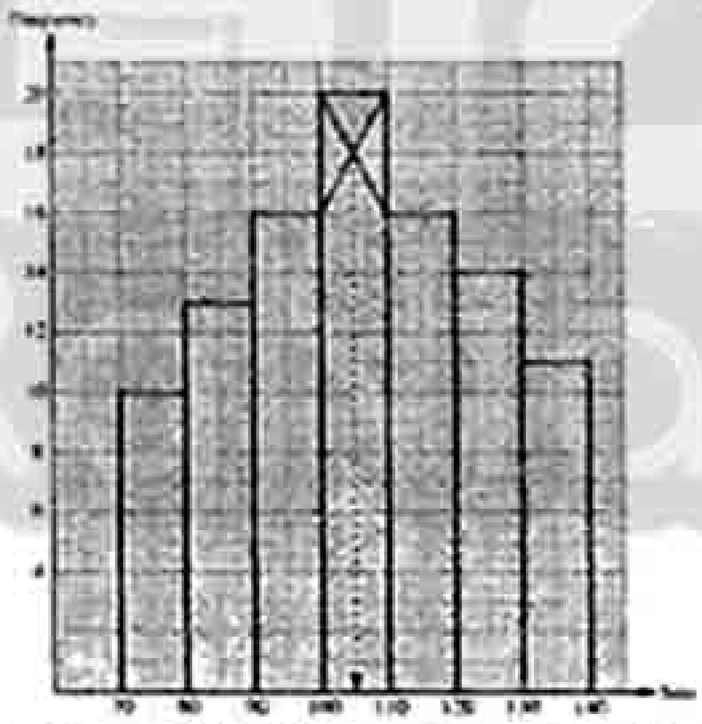
12

1 $X = 110$

$\because k - 4 = 100 - (10 + 13 + 20 + 16 + 14 + 11)$

$\therefore k - 4 = 16 \quad \therefore k = 20$

2



From the graph : The mode wage = 105 pounds.

13

1

Sets	"X"	"f"	"X × f"
35 –	40	15	600
45 –	50	30	1500
55 –	60	23	1380
65 –	70	20	1400
75 –	80	8	640
85 –	90	4	360
Total		100	5880

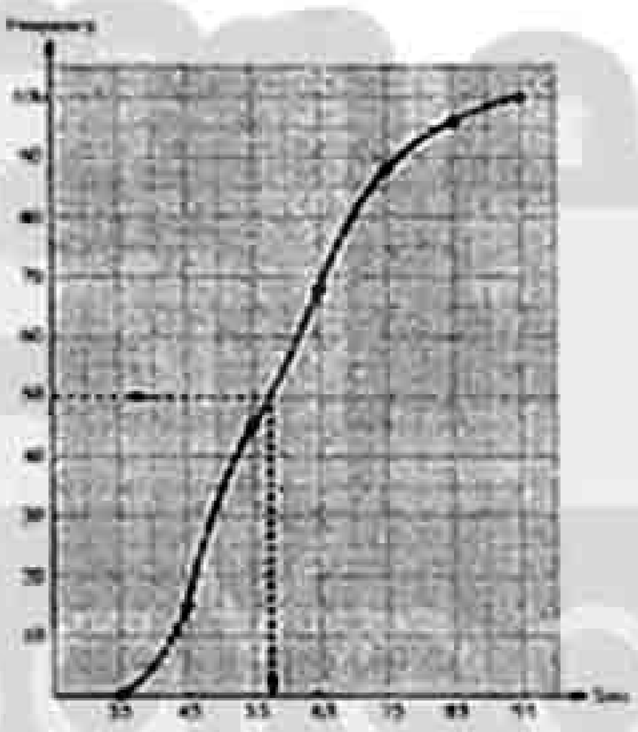
$\therefore$ The mean of working hours $= \frac{5880}{100}$

$= 58.8$ hours.

2 We form the ascending cumulative frequency table as follows :

The upper limits of sets	Ascending cmulative frequency
less than 35	0
less than 45	15
less than 55	45
less than 65	68
less than 75	88
less than 85	96
less than 95	100

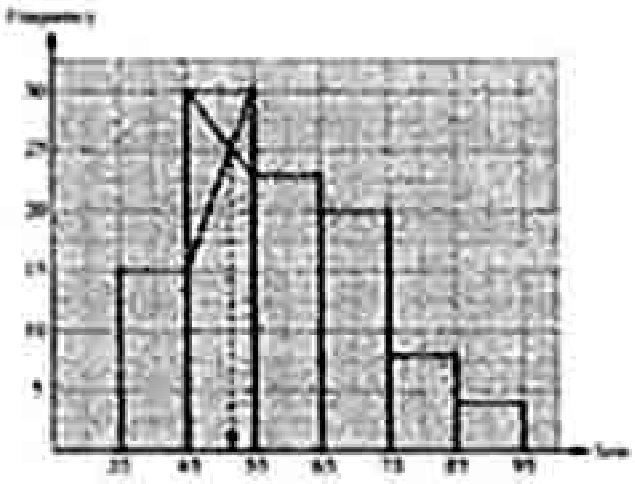
Then we draw the ascending cumulative frequency curve as follows :



$\because$ The order of the median $= \frac{100}{2} = 50$

$\therefore$ The median $\simeq 57.5$ hours.

3 We graph the histogram of the distribution as follows :



From the graph :

we find that the mode $\simeq 52$ hours.

14

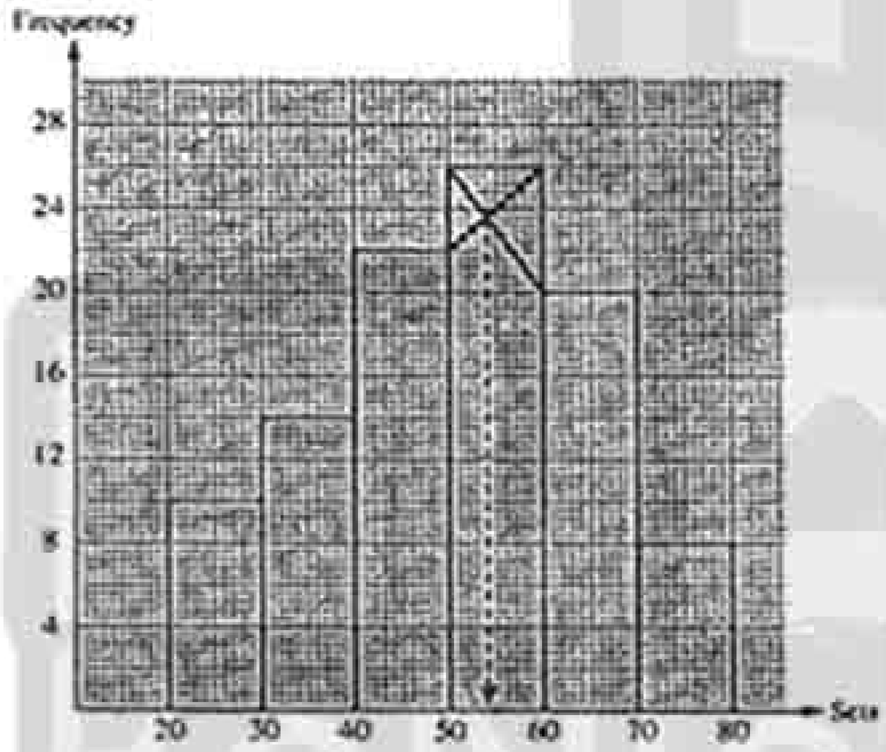
1 $k = 100 - (10 + 22 + 26 + 20 + 8) = 14$

2

Sets	"X"	"f"	"X × f"
20 –	25	10	250
30 –	35	14	490
40 –	45	22	990
50 –	55	26	1430
60 –	65	20	1300
70 –	75	8	600
Total		100	5060

$\therefore$ The mean $= \frac{5060}{100} = 50.6$ pounds.

3



From the graph : The mode value = 54 pounds.

15

1 $\because 3k + 4k = 50 - (7 + 10 + 8 + 4)$

$\therefore 7k = 21 \qquad \therefore k = \frac{21}{7} = 3$

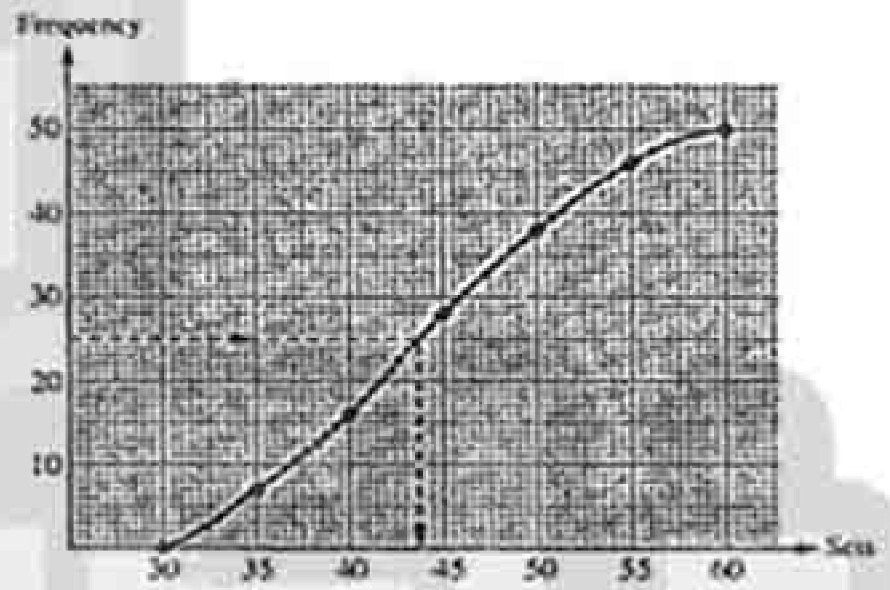
2

Sets	"X"	"f"	"X × f"
30 –	32.5	7	227.5
35 –	37.5	9	337.5
40 –	42.5	12	510
45 –	47.5	10	475
50 –	52.5	8	420
55 –	57.5	4	230
Total		50	2200

$\therefore$ The mean $= \frac{2200}{50} = 44$ kg.

3

The upper limits of sets	Ascending cumulative frequency
less than 30	0
less than 35	7
less than 40	16
less than 45	28
less than 50	38
less than 55	46
less than 60	50



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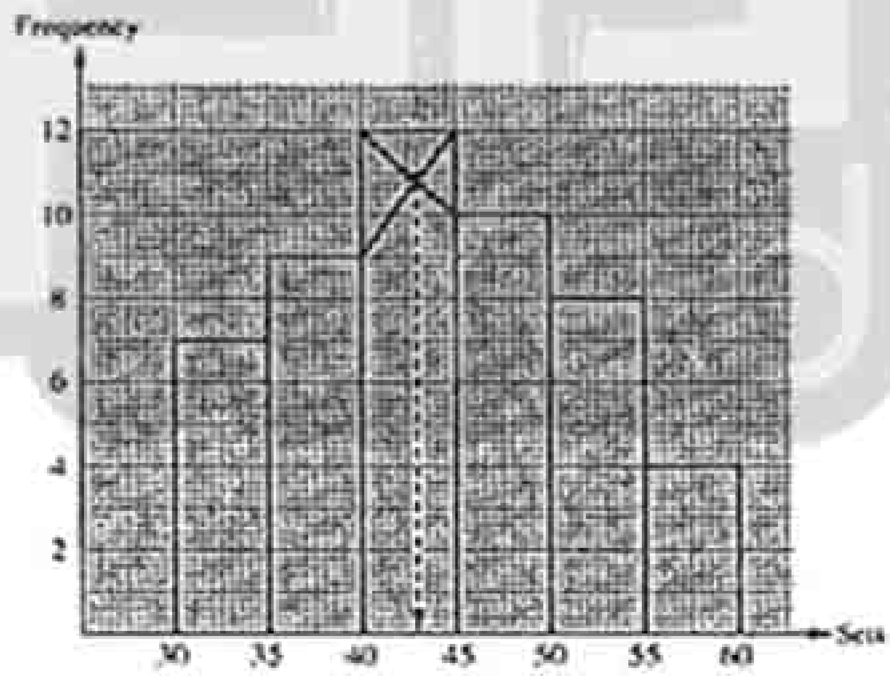


From the graph : The mode weight = 43 kg.

5 The order of the median $= \frac{50}{2} = 25$

$\therefore$ The median $= 43.5$ kg.

Answers of exams on unit three

Model 1

1

1 b 2 c 3 a
4 c 5 d

2

1 9 2 3 3 the median
4 4 5 4

3

1 35 marks
2 3 students

4

1 $k = 8$, $m = 4$
2 The median $\simeq 5.6$

5

The mode $\simeq$ 55 marks.

Model 2

1

1 a 2 b 3 c
4 c 5 d

2

1 the order of the median 2 the mode
3 7140 4 17
5 5

3

The arithmetic mean = 31

4

1 $k = 4$, $m = 3$
2 The median $\simeq 5$

5

Graph by yourself , the mode age $\simeq$ 43 years.

Answers of accumulative basic skills

1

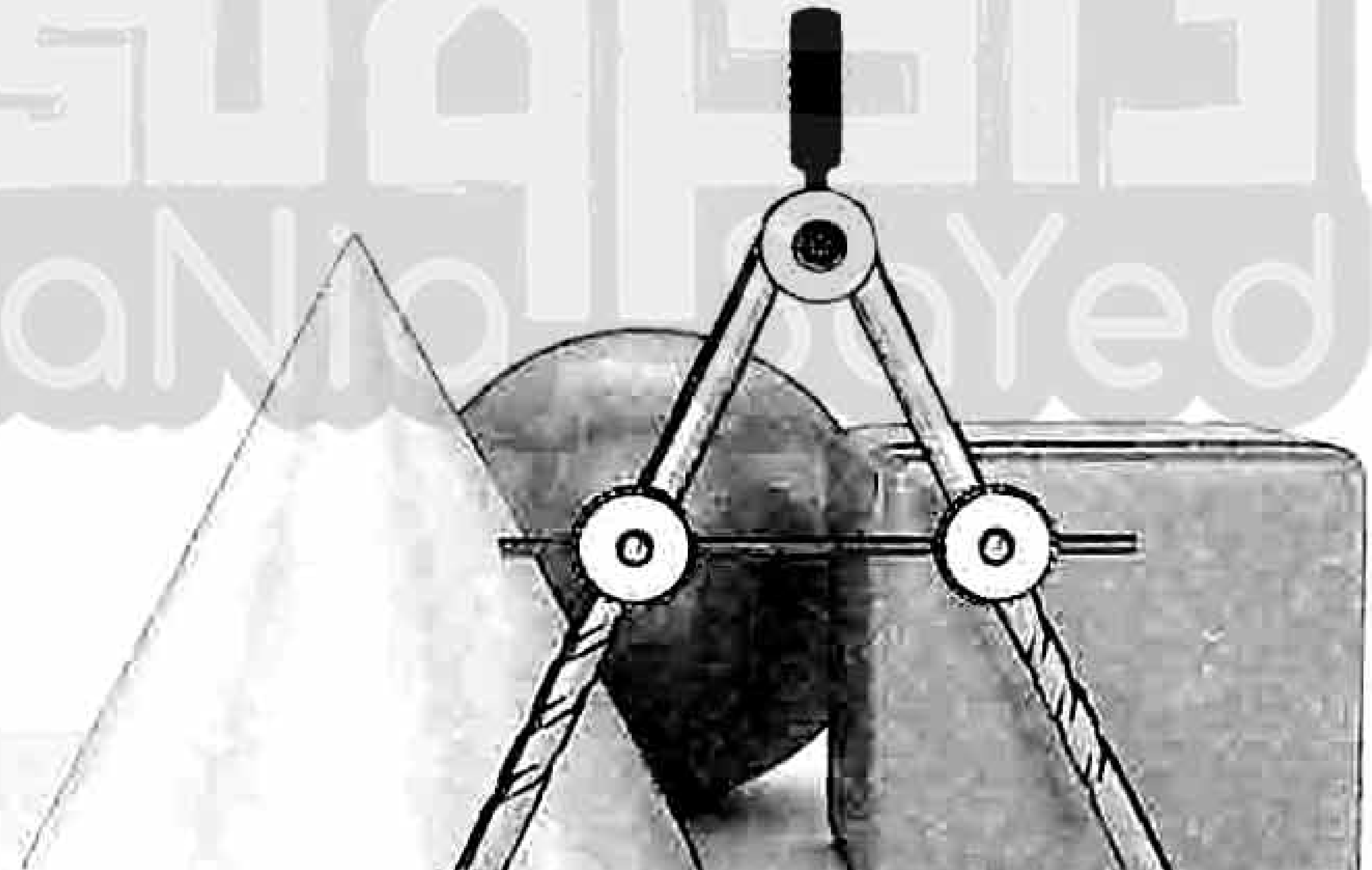
1 6 2 0 3 15
4 154 5 21 6 $\frac{2}{3}$
7 7500 8 4 9 12
10 9 11 27 12 6 , 8 , 2

2

1 c 2 c 3 a
4 a 5 d 6 c
7 d 8 b 9 d
10 d 11 c 12 a

Guide Answers

of Geometry Exercises



Answers of unit four

Answers of Exercise 1

1

1 a median
2 3
3 one point
4 1 : 2
5 2 : 1
6 4

2

1 8 cm. , 5 cm.
2 6 cm. , 4 cm. , $\frac{1}{3}$, $\frac{2}{3}$
3 6 cm. , 3 cm. , 4 cm.
4 5 cm. , 12 cm. , 27 cm.

3

$\because$ $\overline{AD}$, $\overline{BE}$ are two medians in Δ ABC ,
$\overline{AD} \cap \overline{BE} = \{M\}$

$\therefore$ M is the point of concurrence of the medians of Δ ABC

$\therefore$ MD $= \frac{1}{3}$ AD $= \frac{1}{3} \times 6 = 2$ cm. (1)

, ME $= \frac{1}{3}$ BE $= \frac{1}{3} \times 9 = 3$ cm. (2)

$\because$ D is the midpoint of $\overline{BC}$, E is the midpoint of $\overline{AC}$ in Δ ABC

$\therefore$ DE $= \frac{1}{2}$ AB $= \frac{1}{2} \times 9 = 4.5$ cm. (3)

From (1) , (2) and (3) :

$\therefore$ The perimeter of Δ MDE $= 2 + 3 + 4.5 = 9.5$ cm.

(The req.)

4

$\because$ D is the midpoint of $\overline{AB}$
, E is the midpoint of $\overline{AC}$

$\therefore$ BC = 2 DE $\quad \therefore$ BC = 8 cm.

$\because$ M is the intersection point of medians of Δ ABC

$\therefore$ MC = 2 DM $\quad \therefore$ MC = 6 cm.

, BM $= \frac{2}{3}$ BE $\quad \therefore$ BM = 4 cm.

$\therefore$ The perimeter of Δ BMC $= 8 + 6 + 4 = 18$ cm.

(The req.)

5

$\because$ M is the intersection point of the medians of Δ ABC

$\therefore$ XM $= \frac{1}{2}$ MC = 4 cm.

$\therefore$ The perimeter of Δ MXY $= 4 + 5 + 3 = 12$ cm.

(First req.)

, AM = 2 MY = 6 cm.

$\because$ X is the midpoint of $\overline{AB}$,
Y is the midpoint of $\overline{BC}$

Answers of revision exercise

1

1 5 cm. , 3 cm. , 110° , 70°
2 4 cm. , 14 cm.
3 106° , 14 cm.
4 6.3 cm.
5 16 cm. , 77°
6 5 cm. , 60°
7 2 cm. , 3 cm. , 8 cm.
8 16 cm. , 135° , 110°

2

1 $X = 115°$, $y = 65°$, $z = 115°$
2 $X = 80°$, $y = 25°$, $z = 75°$
3 $X = 35°$, $y = 27°$, $z = 35°$
4 $X = 30°$, $y = 30°$, $z = 120°$
5 $X = 138°$, $y = 42°$, $z = 138°$
6 $X = 75°$, $y = 45°$, $z = 30°$

3

1 AB = 35 cm.
2 AB $= \frac{1}{2}$ BD = 7.5 cm.
3 AB = 2 AX = 14 cm.
4 AB $= \frac{1}{3}$ BC $\quad \therefore$ AB $= \frac{1}{3} \times 12 = 4$ cm.
5 AB = 2 BC = 2 AD $\quad \therefore$ AB $= 2 \times 45 = 90$ cm.
6 $\because$ M is the midpoint of $\overline{BD}$, BM = 6 cm.

$\therefore$ BD = 12 cm.

$\therefore$ In Δ DBC which is right-angled at D

$\therefore$ DC $= \sqrt{(BC)^2 - (DB)^2} = \sqrt{225 - 144} = 9$ cm.

$\therefore$ AB = DC = 9 cm.

4

Fig (1) : $X = 44°$, $y = 88°$, $z = 46°$

Fig (2) : $X = 90°$, $y = 45°$, $z = 90°$

Fig (3) : $X = 35°$, $y = 110°$, $z = 70°$

Fig (4) : $X = 60°$, $y = 30°$, $z = 60°$

5

1 E is the midpoint of $\overline{AC}$
2 $\overline{DE}$ // $\overline{BC}$
3 3 cm.
4 90°
5 12 cm.
6 17 cm.
7 2.5 cm.
8 3 cm.
9 3 cm. , 45°

$\therefore AC = 2\,XY = 10$ cm.

$\therefore$ The perimeter of $\Delta MAC = 6 + 8 + 10 = 24$ cm. (Second req.)

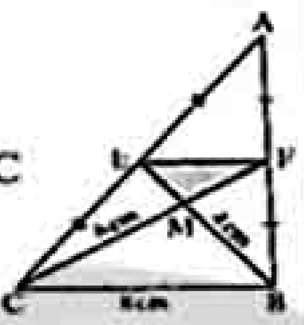
6

$\because$ F is the midpoint of $\overline{AB}$

, E is the midpoint of $\overline{AC}$

$\therefore \overline{BE}$, $\overline{CF}$ are two medians in ΔABC

$\therefore$ M is the intersection point of the medians of ΔABC



$\therefore ME = \frac{1}{2} MB = 2$ cm. (1)

, $MF = \frac{1}{2} MC = 3$ cm. (2)

$\because$ F is the midpoint of $\overline{AB}$, E is the midpoint of $\overline{AC}$

$\therefore FE = \frac{1}{2} BC = 4$ cm. (3)

From (1) , (2) and (3) :

$\therefore$ The perimeter of $\Delta MFE = 2 + 3 + 4 = 9$ cm. (The req.)

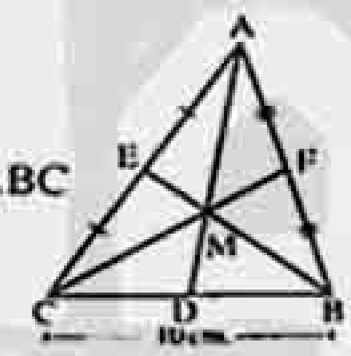
7

$\because$ F is the midpoint of $\overline{AB}$

, E is the midpoint of $\overline{AC}$

$\therefore \overline{BE}$, $\overline{CF}$ are two medians in ΔABC

$\therefore$ M is the intersection point of the medians of ΔABC



$\therefore \overline{AD}$ is a median in ΔABC

$\therefore$ D is the midpoint of $\overline{BC}$

$\therefore BD = \frac{10}{2} = 5$ cm. (First req.)

, $AM = \frac{2}{3} AD = \frac{2}{3} \times 12 = 8$ cm. (Second req.)

8

$\because$ M is the intersection point of medians of ΔABC

$\therefore MF = \frac{1}{2} AM$ (1)

, $MD = \frac{1}{2} MC$ (2)

$\because$ D is the midpoint of $\overline{AB}$,

F is the midpoint of $\overline{BC}$ in ΔABC

$\therefore DF = \frac{1}{2} AC$ (3)

By adding (1) , (2) and (3) :

$\therefore MF + MD + DF = \frac{1}{2} AM + \frac{1}{2} MC + \frac{1}{2} AC$

$\therefore$ The perimeter of ΔMFD

$= \frac{1}{2} (AM + MC + AC)$

$= \frac{1}{2}$ the perimeter of ΔAMC

$= \frac{1}{2} \times 36 = 18$ cm. (The req.)

9

$\because$ E is the midpoint of $\overline{BC}$

, O is the midpoint of $\overline{AC}$

$\therefore \overline{AE}$, $\overline{BO}$ are two medians in ΔABC

$\therefore$ M is the intersection point of the medians of ΔABC

$\therefore \overline{CD}$ is a median in ΔABC

$\therefore AM = \frac{2}{3} AE$, $BM = \frac{2}{3} BO$, $CM = \frac{2}{3} CD$

, $\because AM + BM + CM = 18$

$\therefore \frac{2}{3} AE + \frac{2}{3} BO + \frac{2}{3} CD = 18$

$\therefore \frac{2}{3} (AE + BO + CD) = 18$

$\therefore AE + BO + CD = 18 \times \frac{3}{2} = 27$ cm. (The req.)

10

$\because$ M is the point of concurrence of the medians of ΔABC

$\therefore \overline{CD}$ is a median in ΔABC

$\therefore DM = \frac{1}{2} MC = 3$ cm.

, $\because \Delta AMD$ is a right-angled triangle at M

$\therefore (AM)^2 = (AD)^2 - (DM)^2 = 25 - 9 = 16$

$\therefore AM = 4$ cm.

$\therefore ME = \frac{1}{2} AM = 2$ cm. (The req.)

11

$\because$ ABCD is a parallelogram

$\therefore$ The two diagonals bisect each other

$\therefore$ M is the midpoint of $\overline{AC}$

$\therefore \overline{DM}$ is a median in ΔADC

$\because DE = 2\,EM$

$\therefore$ E is the intersection point of the medians of ΔADC

$\because E \in \overline{FC}$

$\therefore \overline{CF}$ is a median in ΔACD $\therefore AF = FD$ (Q.E.D.)

12

$\because$ The two diagonals of the rectangle bisect each other

$\therefore$ M is the midpoint of $\overline{AC}$

$\therefore \overline{BM}$ is a median in ΔABC

$\because$ E is the midpoint of $\overline{AB}$

$\therefore \overline{CE}$ is a median in ΔABC

$\because \overline{CE} \cap \overline{BM} = \{F\}$

$\therefore$ F is the intersection point of the medians of ΔABC (First req.)

$\therefore BF = \frac{2}{3} BM$ $\therefore 4 = \frac{2}{3} BM$ $\therefore BM = 6$ cm.

$\because$ The two diagonals of the rectangle are equal in length and bisect each other

$\therefore$ AM = BM = 6 cm. (Second req.)

13

$\because$ D is the midpoint of $\overline{BC}$

$\therefore \overline{AD}$ is a median in ΔABC

$\because$ AM = $\frac{2}{3}$ AD

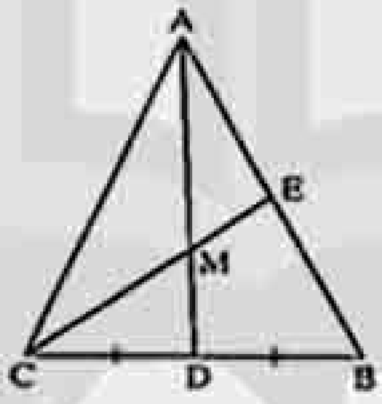
$\therefore$ M is the intersection point of the medians of ΔABC

$\therefore \overline{CF}$ is a median in ΔABC

$\therefore$ F is the midpoint of $\overline{AB}$ $\therefore$ BF = $\frac{1}{2}$ AB

$\because$ AC = AB $\therefore$ BF = $\frac{1}{2}$ AC (Q.E.D.)

14



$\because$ D is the midpoint of $\overline{BC}$

$\therefore \overline{AD}$ is a median in ΔABC

$\because$ AM = 2 MD

$\therefore$ M is the intersection point of the medians of ΔABC

$\because$ M $\in \overline{CE}$

$\therefore \overline{CE}$ is a median in ΔABC

$\therefore$ EM = $\frac{1}{3}$ EC = $\frac{1}{3}$ × 12 = 4 cm. (The req.)

15

$\because$ O is the midpoint of $\overline{AC}$

$\therefore \overline{BO}$ is a median in ΔABC

, $\because$ BO = 3 MO

$\therefore$ M is the intersection point of the medians of ΔABC

$\therefore \overline{AE}$ is a median in ΔABC

$\therefore$ E is the midpoint of $\overline{BC}$

$\therefore$ BE = EC $\therefore x + 3 = 2x - 1$

$\therefore 3 + 1 = 2x - x$ $\therefore x = 4$

$\therefore$ BE = EC = 7 cm.

$\therefore$ BC = 14 cm. (The req.)

16

$\because$ M is the point of concurrence of the medians of ΔABC

$\therefore \overline{CD}$ is a median in ΔABC

$\therefore$ D is the midpoint of $\overline{AB}$

In ΔAMB :

$\because$ D is the midpoint of $\overline{AB}$, E is the midpoint of $\overline{BM}$

$\therefore \overline{MD}$, $\overline{AE}$ are two medians in ΔAMB

$\therefore$ N is the point of concurrence of the medians of ΔAMB

$\therefore$ MN = 2 ND $\therefore x + 3 = 2(x - 1)$

$\therefore x + 3 = 2x - 2$ $\therefore 3 + 2 = 2x - x$

$\therefore x = 5$

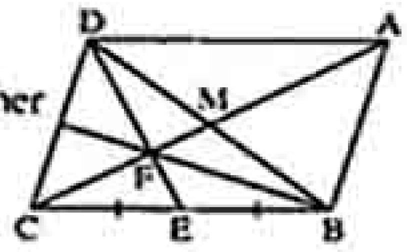
$\therefore$ ND = 5 − 1 = 4 cm. , MN = 5 + 3 = 8 cm.

$\therefore$ MD = ND + MN = 12 cm.

, $\because \overline{CD}$ is a median in ΔABC

$\therefore$ MC = 2 MD = 24 cm. (The req.)

17



$\because$ ABCD is a parallelogram

$\therefore$ The two diagonals bisect each other

$\therefore$ M is the midpoint of $\overline{BD}$

$\therefore \overline{CM}$ is a median in ΔDBC

$\because$ E is the midpoint of $\overline{BC}$

$\therefore \overline{DE}$ is a median in ΔDBC

$\therefore$ F is the intersection point of the medians of ΔDBC

$\therefore \overline{BF}$ bisects $\overline{CD}$ (Q.E.D. 1)

$\therefore$ CF = $\frac{2}{3}$ CM , $\because$ CM = $\frac{1}{2}$ AC

$\therefore$ CF = $\frac{2}{3}$ × $\frac{1}{2}$ AC = $\frac{1}{3}$ AC (Q.E.D. 2)

18

$\because \overline{AD}$ and $\overline{BE}$ are medians in ΔABC

$\therefore$ M is the intersection point of the medians of ΔABC

, $\because$ M $\in \overline{CF}$

$\therefore \overline{CF}$ is a median in ΔABC

$\therefore$ F is the midpoint of $\overline{AB}$

In ΔABM :

$\because$ F is the midpoint of $\overline{AB}$, N is the midpoint of $\overline{BM}$

$\therefore \overline{NF} // \overline{AM}$ $\therefore \overline{NF} // \overline{MD}$ (1)

In ΔBMC :

$\because$ D is the midpoint of $\overline{BC}$, N is the midpoint of $\overline{BM}$

$\therefore \overline{ND} // \overline{CM}$ $\therefore \overline{ND} // \overline{MF}$ (2)

From (1) and (2) :

$\therefore$ The figure FNDM is a parallelogram. (Q.E.D.)

19

$\because$ D is the midpoint of $\overline{BC}$

$\therefore \overline{AD}$ is a median in ΔABC

, $\because$ AM = 2 MD , M $\in \overline{AD}$

$\therefore$ M is the intersection point of the medians of ΔABC

, $\because$ M $\in \overline{BE}$

$\therefore \overline{BE}$ is a median in ΔABC $\therefore$ BM = 2 ME

$\therefore$ BM = 4 cm. $\therefore$ BE = 2 + 4 = 6 cm.

, $\because$ Δ BCE in which :

D is the midpoint of $\overline{BC}$, $\overline{DF} \parallel \overline{BE}$

$\therefore$ F is the midpoint of $\overline{EC}$

$\therefore$ DF = $\frac{1}{2}$ BE = 3 cm. (The req.)

20

$\because$ D is the midpoint of $\overline{BC}$, $\overline{DF} \parallel \overline{AC}$

$\therefore$ F is the midpoint of $\overline{AB}$ $\therefore$ DF = $\frac{1}{2}$ AC

In Δ ABD :

$\because$ E is the midpoint of $\overline{BD}$, F is the midpoint of $\overline{AB}$

$\therefore$ $\overline{AE}$ and $\overline{DF}$ are medians in Δ ABD

$\therefore$ M is the intersection point of the medians of Δ ABD

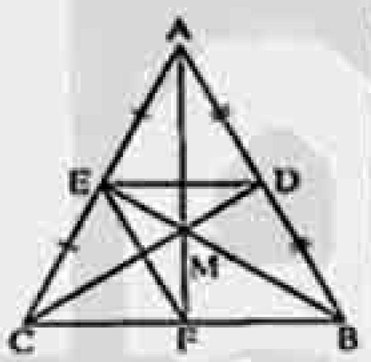
$\therefore$ DM = $\frac{2}{3}$ DF , $\because$ DF = $\frac{1}{2}$ AC

$\therefore$ DM = $\frac{2}{3} \times \frac{1}{2}$ AC = $\frac{1}{3}$ AC (Q.E.D.)

21

$\because$ $\overline{CD}$ and $\overline{BE}$ are two medians in Δ ABC

$\therefore$ M is the intersection point of the medians of Δ ABC



$\therefore$ $\overline{AF}$ is a median in Δ ABC

$\therefore$ F is the midpoint of $\overline{BC}$

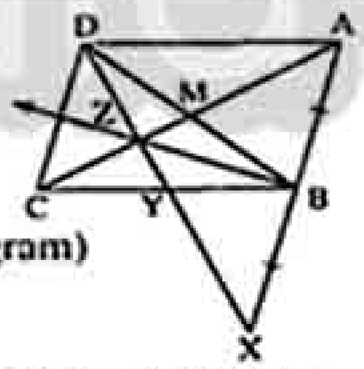
, $\because$ E is the midpoint of $\overline{AC}$

$\therefore$ $\overline{FE} \parallel \overline{AB}$, FE = $\frac{1}{2}$ AB

$\therefore$ $\overline{FE} \parallel \overline{BD}$, FE = BD

$\therefore$ DBFE is a parallelogram (Q.E.D.)

22



From Δ AXD :

$\because$ B is the midpoint of $\overline{AX}$,

$\overline{BC} \parallel \overline{AD}$ (ABCD is a parallelogram)

$\therefore$ BY = $\frac{1}{2}$ AD , $\because$ BC = AD

(two opposite sides in the parallelogram ABCD)

$\therefore$ BY = $\frac{1}{2}$ BC $\therefore$ Y is the midpoint of $\overline{BC}$

$\therefore$ $\overline{DY}$ is a median in Δ DBC (1)

$\because$ M is the midpoint of $\overline{BD}$ (the intersection point of the diagonals of the parallelogram)

$\therefore$ $\overline{CM}$ is a median in Δ DBC (2)

From (1) and (2) :

$\because$ $\overline{CM} \cap \overline{DY} = \{Z\}$

$\therefore$ Z is the intersection point of the medians of Δ DBC

$\therefore$ $\overline{BZ}$ intersects $\overline{DC}$ at the midpoint of $\overline{DC}$ (Q.E.D.)

Answers of Exercise 2

1

1) 3
2) half the length of the hypotenuse
3) right
4) half the length of the hypotenuse
5) twice

2

1) 3
2) 10
3) 8
4) 18 , 9 , $\frac{1}{3}$, 3
5) 5 , 5 , 15
6) 8 , 9 , 10 , 27

3

In Δ ADC :

$\because$ m ($\angle$ D) = 90° , E is the midpoint of $\overline{AC}$

$\therefore$ DE = $\frac{1}{2}$ AC (1)

In Δ ABC :

$\because$ m ($\angle$ B) = 90° , m ($\angle$ ACB) = 30°

$\therefore$ AB = $\frac{1}{2}$ AC (2)

From (1) and (2) :

$\therefore$ AB = DE (Q.E.D.)

4

In Δ LXZ :

$\because$ D is the midpoint of $\overline{LX}$, E is the midpoint of $\overline{LZ}$

$\therefore$ DE = $\frac{1}{2}$ XZ (1)

From Δ XYZ :

$\because$ m ($\angle$ Y) = 90° , M is the midpoint of $\overline{XZ}$

$\therefore$ YM = $\frac{1}{2}$ XZ (2)

From (1) and (2) :

$\therefore$ DE = YM (Q.E.D.)

5

In Δ ACD :

$\because$ E is the midpoint of $\overline{AD}$, F is the midpoint of $\overline{CD}$

$\therefore$ EF = $\frac{1}{2}$ AC $\therefore$ AC = 8 cm.

In Δ ABC :

$\because$ m ($\angle$ B) = 90° , m ($\angle$ ACB) = 30°

$\therefore$ AB = $\frac{1}{2}$ AC = 4 cm. (The req.)

6

In Δ ABC :

$\because$ m ($\angle$ BAC) = 90° , D is the midpoint of $\overline{BC}$

$\therefore$ BC = 2 AD = 2 × 3 = 6 cm.

In Δ CBE :

$\because$ m ($\angle$ CBE) = 90° , m ($\angle$ E) = 30°

$\therefore$ EC = 2 BC = 2 × 6 = 12 cm.

, $\because$ F is the midpoint of $\overline{CE}$

$\therefore$ BF = $\frac{1}{2}$ EC = $\frac{1}{2}$ × 12 = 6 cm. (The req.)

7

In Δ ABC :

$\because$ m ($\angle$ B) = 90° , m ($\angle$ ACB) = 60°

$\therefore$ m ($\angle$ CAB) = 30° $\therefore$ BC = $\frac{1}{2}$ AC

$\because$ DE = BC $\therefore$ DE = $\frac{1}{2}$ AC

$\because$ $\overline{DE}$ is a median in Δ ACD

$\therefore$ m ($\angle$ ADC) = 90° (Q.E.D.)

8

In Δ ABC :

$\because$ m ($\angle$ B) = 90° , m ($\angle$ ACB) = 30°

$\therefore$ AB = $\frac{1}{2}$ AC

, $\because$ AB = DE = 5 cm. $\therefore$ DE = $\frac{1}{2}$ AC

$\because$ $\overline{DE}$ is a median in Δ ACD

$\therefore$ m ($\angle$ ADC) = 90° (Q.E.D.)

9

In Δ ABD :

$\because$ m ($\angle$ A) = 90° , M is the midpoint of $\overline{BD}$

$\therefore$ AM = $\frac{1}{2}$ BD

, $\because$ CM = AM $\therefore$ CM = $\frac{1}{2}$ BD

$\because$ $\overline{CM}$ is a median in Δ DBC

$\therefore$ m ($\angle$ BCD) = 90° (Q.E.D.)

10

In Δ DBC :

$\because$ E is the midpoint of $\overline{BC}$, $\overline{EF}$ // $\overline{BD}$

$\therefore$ EF = $\frac{1}{2}$ BD

, $\because$ AM = EF $\therefore$ AM = $\frac{1}{2}$ BD

$\because$ $\overline{AM}$ is a median in Δ ABD

$\therefore$ m ($\angle$ BAD) = 90° (Q.E.D.)

11

$\because$ $\angle$ ADC is an exterior angle of Δ ABD

$\therefore$ m ($\angle$ ADC) = 33° + 27° = 60°

$\therefore$ In Δ ADC :

m ($\angle$ DAC) = 180° − (60° + 90°) = 30°

$\therefore$ DC = $\frac{1}{2}$ AD

$\therefore$ AD = 8 cm. (The req.)

12

In Δ ABE :

$\because$ m ($\angle$ A) = 30° , m ($\angle$ B) = 90°

$\therefore$ BE = $\frac{1}{2}$ AE $\therefore$ BE = 2 cm. (1)

, m ($\angle$ AEB) = 180° − (30° + 90°) = 60°

, $\because$ E $\in$ $\overline{BC}$

$\therefore$ m ($\angle$ DEC) = 180° − (60° + 60°) = 60°

$\therefore$ In Δ DEC :

m ($\angle$ D) = 180° − (60° + 90°) = 30°

$\therefore$ CE = $\frac{1}{2}$ DE $\therefore$ EC = 5 cm. (2)

Adding (1) and (2) :

$\therefore$ BC = 2 + 5 = 7 cm. (The req.)

13

In Δ ADB :

$\because$ m ($\angle$ ADB) = 90° , AE = EB

$\therefore$ DE = $\frac{1}{2}$ AB

Similarly in Δ ACB :

$\because$ m ($\angle$ ACB) = 90° , AE = EB

$\therefore$ CE = $\frac{1}{2}$ AB $\therefore$ DE = CE

$\therefore$ Δ CED is an isosceles triangle. (Q.E.D.)

14

In Δ LYE :

$\because$ m ($\angle$ YLE) = 90° , m ($\angle$ E) = 30°

$\therefore$ LY = $\frac{1}{2}$ YE = 5 cm.

In Δ ZYX :

$\because$ m ($\angle$ ZYX) = 90° , L is the midpoint of $\overline{ZX}$

$\therefore$ YL = $\frac{1}{2}$ ZX $\therefore$ ZX = 10 cm. (The req.)

15

In Δ ABC :

$\because$ m ($\angle$ ABC) = 90° , m ($\angle$ C) = 30°

$\therefore$ AC = 2 AB = 14 cm.

, $\because$ D is the midpoint of $\overline{AC}$

$\therefore$ BD = $\frac{1}{2}$ AC = 7 cm.

, in Δ DEC :

$\because$ m ($\angle$ DEC) = 90° , m ($\angle$ C) = 30°

$\therefore$ DE = $\frac{1}{2}$ DC , $\because$ DC = $\frac{1}{2}$ AC = 7 cm.

$\therefore$ DE = 3.5 cm. (The req.)

16

In Δ ABC :

$\because$ m ($\angle$ ABC) = 90° , m ($\angle$ C) = 30°

$\therefore$ AB = $\frac{1}{2}$ AC = 4 cm.

$\because$ X is the midpoint of $\overline{AB}$, Y is the midpoint of $\overline{BC}$

$\therefore$ XY = $\frac{1}{2}$ AC = 4 cm.

In Δ XBY :

$\because$ m ($\angle$ XBY) = 90°

, Z is the midpoint of $\overline{XY}$

$\therefore$ BZ = $\frac{1}{2}$ XY = 2 cm. (The req.)

17

In Δ MED :

$\because$ m ($\angle$ MED) = 90° $\therefore$ $(MD)^2 = 3^2 + 4^2 = 25$

$\therefore$ MD = $\sqrt{25}$ = 5 cm.

, $\because$ M is the point of concurrence of the medians of Δ ABC

$\therefore$ AD = 3 MD = 15 cm.

, $\because$ m ($\angle$ BAC) = 90°

, $\overline{AD}$ is a median in Δ ABC

$\therefore$ BC = 2 AD = 30 cm. (The req.)

18

In Δ ABC :

$\because$ m ($\angle$ BAC) = 90°

$\therefore$ $(BC)^2 = (12)^2 + (9)^2 = 225$

$\therefore$ BC = $\sqrt{225}$ = 15 cm.

, $\because$ $\overline{AD}$ is a median in Δ ABC , m ($\angle$ BAC) = 90°

$\therefore$ AD = $\frac{1}{2}$ BC = $7\frac{1}{2}$ cm.

, $\because$ M is the point of concurrence of the medians of Δ ABC

$\therefore$ AM = $\frac{2}{3}$ AD = 5 cm. (The req.)

19

$\because$ ABCD is a parallelogram

$\therefore$ m ($\angle$ C) = m ($\angle$ A) = 60°

$\therefore$ In Δ DEC :

m ($\angle$ EDC) = 180° − (60° + 90°) = 30°

$\therefore$ CE = $\frac{1}{2}$ DC $\therefore$ DC = 8 cm.

$\therefore$ The perimeter of the parallelogram ABCD

= (12 + 8) × 2 = 40 cm. (The req.)

20

$\because$ m ($\angle$ BAD) = 90° , m ($\angle$ BAE) = 30°

$\therefore$ m ($\angle$ DAF) = 60°

From Δ AFD :

$\because$ m ($\angle$ AFD) = 90° , m ($\angle$ DAF) = 60°

$\therefore$ m ($\angle$ ADF) = 30° $\therefore$ AD = 2 AF = 8 cm.

$\therefore$ The area of the square = 64 cm^2. (The req.)

21

In Δ BCE :

$\because$ m ($\angle$ EBC) = 30°

$\therefore$ CE = $\frac{1}{2}$ BE (1)

, $\because$ m ($\angle$ EBC) = 30° $\therefore$ m ($\angle$ ABE) = 60°

$\therefore$ In Δ ABE :

m ($\angle$ EAB) = 30°

, $\because$ m ($\angle$ AEB) = 90°

$\therefore$ BE = $\frac{1}{2}$ AB (2)

From (1) and (2) :

$\therefore$ CE = $\frac{1}{2} \times \frac{1}{2}$ AB = $\frac{1}{4}$ AB (Q.E.D.)

22

In Δ ABC :

$\because$ m ($\angle$ ABC) = 90° , m ($\angle$ A) = 30°

$\therefore$ AC = 2 BC = 16 cm.

, m ($\angle$ C) = 180° − (90° + 30°) = 60°

, in Δ BCD :

$\because$ m ($\angle$ BDC) = 90° , m ($\angle$ C) = 60°

$\therefore$ m ($\angle$ CBD) = 180° − (90° + 60°) = 30°

$\therefore$ CD = $\frac{1}{2}$ BC = 4 cm.

$\therefore$ AD = AC − CD = 16 − 4 = 12 cm. (The req.)

23

In Δ ABC :

$\because$ m ($\angle$ ABC) = 90° , m ($\angle$ C) = 30°

$\therefore$ m ($\angle$ A) = 180° − (90° + 30°) = 60°

$\therefore$ In Δ ABE :

$\because$ m ($\angle$ AEB) = 90° , m ($\angle$ A) = 60°

$\therefore m(\angle ABE) = 30^\circ$ $\therefore AB = 2AE = 8$ cm.

In ΔABC:

$AC = 2AB = 16$ cm.

$\because \overline{BD}$ is a median in ΔABC

$\therefore BD = 8$ cm. , $AD = 8$ cm.

$\therefore$ The perimeter of $\Delta ABD = 8 + 8 + 8 = 24$ cm. (The req.)

24

In ΔABC:

$\because m(\angle B) = 30^\circ$, $m(\angle C) = 90^\circ$

$\therefore AC = \frac{1}{2} AB$

, $\because$ E is the midpoint of $\overline{BC}$

, O is the midpoint of $\overline{AC}$

$\therefore EO = \frac{1}{2} AB$ $\therefore EO = AC$

In ΔDEO:

$\because$ X is the midpoint of $\overline{DE}$

, Y is the midpoint of $\overline{DO}$

$\therefore XY = \frac{1}{2} EO$ $\therefore XY = \frac{1}{2} AC$ (Q.E.D)

25

$\because$ ABCD is a parallelogram $\therefore \overline{AD} \parallel \overline{BC}$

, $\because \overline{XY} \parallel \overline{BC}$ $\therefore \overline{AD} \parallel \overline{XY} \parallel \overline{BC}$

, $\because \overline{AB}$ and $\overline{EF}$ are transversals for them

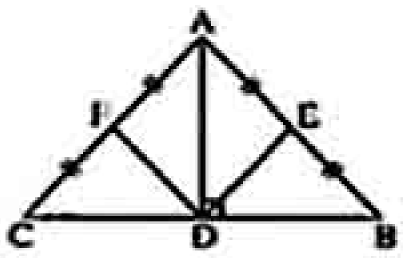
$\therefore EZ = ZF$

In ΔEYF:

$\because m(\angle EYF) = 90^\circ$, Z is the midpoint of $\overline{EF}$

$\therefore YZ = \frac{1}{2} EF$ (Q.E.D.)

26



In ΔADB

$\because m(\angle ADB) = 90^\circ$

, E is the midpoint of $\overline{AB}$

$\therefore DE = \frac{1}{2} AB$

In ΔADC:

$\because m(\angle ADC) = 90^\circ$

, F is the midpoint of $\overline{AC}$ $\therefore DF = \frac{1}{2} AC$

$\therefore DE + DF = \frac{1}{2} AB + \frac{1}{2} AC$ but $AB = AC$ (Given)

$\therefore DE + DF = \frac{1}{2} AB + \frac{1}{2} AB = AB$ (Q.E.D.)

27

In ΔABC:

$\because \overline{EO} \parallel \overline{AC}$, E is the midpoint of $\overline{AB}$

$\therefore$ O is the midpoint of $\overline{BC}$

, $\because BC = 4 + 12 = 16$ cm.

$\therefore BO = \frac{1}{2} BC = 8$ cm.

$\therefore DO = 8 - 4 = 4$ cm.

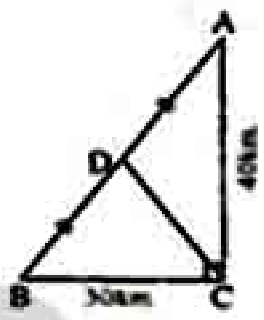
$\therefore BD = DO$

, $\because \overline{EO} \parallel \overline{AC}$, $\overline{AB}$ is a transversal

$\therefore m(\angle BEO) = m(\angle A) = 90^\circ$ (corresponding angles)

$\therefore ED = \frac{1}{2} BO = 4$ cm. (The req.)

28



Let the service station lie at the point D which is the midpoint of $\overline{AB}$

$\therefore$ The road length = the length of $\overline{CD}$

In ΔACB:

$\because m(\angle ACB) = 90^\circ$

$\therefore (AB)^2 = (AC)^2 + (BC)^2 = 1600 + 900 = 2500$

$\therefore AB = 50$ km.

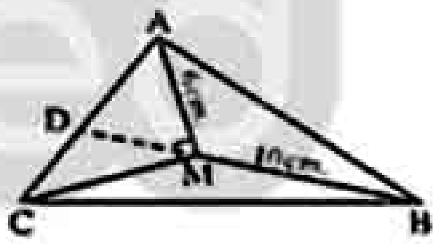
, $\because$ D is the midpoint of $\overline{AB}$

$\therefore CD = \frac{1}{2} AB = \frac{1}{2} \times 50 = 25$ km.

$\therefore$ The length of the road 25 km. (The req.)

29

Constr : Draw $\overrightarrow{BM}$ to intersect $\overline{AC}$ at D



Proof : $\because$ M is the point of concurrence of the medians of ΔABC

$\therefore MD = \frac{1}{2} BM = 5$ cm.

, in ΔAMC : $\because m(\angle AMC) = 90^\circ$

, $\overline{MD}$ is a median

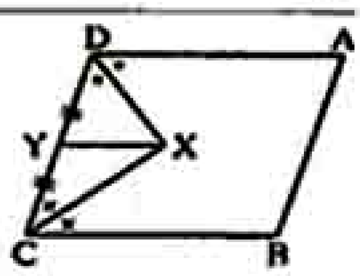
$\therefore MD = \frac{1}{2} AC$ $\therefore AC = 10$ cm. (First req.)

In ΔAMC : $\because m(\angle AMC) = 90^\circ$

$\therefore (MC)^2 = (10)^2 - (6)^2 = 64$

$\therefore MC = \sqrt{64} = 8$ cm. (Second req.)

30



$\because \overline{DA} \parallel \overline{CB}$

, $\overline{DC}$ is a transversal

$\therefore m(\angle ADC) + m(\angle DCB) = 180^\circ$

$\therefore \frac{1}{2} m(\angle ADC) + \frac{1}{2} m(\angle DCB) = 90^\circ$

$\therefore m(\angle XDC) + m(\angle DCX) = 90^\circ$

but the sum of the measures of the interior angles of a triangle XDC = 180°

∴ m (∠ DXC) = 90° , ∵ DY = YC

∴ XY = $\frac{1}{2}$ DC ∴ XY = YC (Q.E.D.)

Answers of Exercise 3

1

1) x = 50° 2) x = 56° 3) y = 63°

4) l = 65° , z = 50° 5) x = 54° , y = 117°

6) x = 69° , y = 111° 7) x = 120°

8) x = 63° , y = 54°

2

1) congruent 2) 60° 3) F

4) 50° 5) 70° 6) C , 50°

3

1) b 2) c 3) b 4) a 5) b

4

In Δ ABC :

∵ AB = AC

∴ m (∠ ABC) = m (∠ ACB)

$= \frac{180° - 40°}{2} = 70°$ (First req.)

∵ m (∠ ABC) = m (∠ ACB) ,

∠ ABD supplements ∠ ABC

, ∠ ACE supplements ∠ ACB

∵ The supplementaries of the congruent angles are congruent

∴ ∠ ABD ≡ ∠ ACE (Second req.)

5

From Δ ABC :

∵ AB = AC ∴ m (∠ B) = m (∠ ACB) = 70°

∴ m (∠ BAC) = 180° − (2 × 70°) = 40°

In Δ ACD :

∵ AC = CD ∴ m (∠ CAD) = m (∠ D)

∵ ∠ ACB is an exterior angle of Δ ACD

∴ m (∠ ACB) = m (∠ CAD) + m (∠ D)

∴ m (∠ CAD) = $\frac{70°}{2}$ = 35°

∴ m (∠ BAD) = m (∠ BAC) + m (∠ CAD)

= 40° + 35° = 75° (The req.)

6

∵ ∠ ACD is an exterior angle of Δ ABC

∴ m (∠ ACD) = 30° + 40° = 70°

From Δ ACD :

∵ AC = AD

∴ m (∠ D) = m (∠ ACD) = 70° (First req.)

∴ m (∠ CAD) = 180° − (70° + 70°) = 40° (Second req.)

7

∵ Δ ACD is an equilateral triangle

∴ m (∠ CAD) = 60° (1)

From Δ ABC :

∵ AB = BC

∴ m (∠ BAC) = m (∠ BCA) = $\frac{180° - 40°}{2}$ = 70° (2)

From (1) and (2) :

∴ m (∠ BAD) = 60° + 70° = 130° (The req.)

8

In Δ ABD :

∵ AB = AD

∴ m (∠ ADB) = m (∠ ABD) = $\frac{180° - 120°}{2}$ = 30° (First req.)

∵ $\overline{AD}$ // $\overline{BC}$, $\overline{DC}$ is a transversal to them

∴ m (∠ C) + m (∠ ADC) = 180°

∴ m (∠ C) = 180° − (65° + 30°) = 85° (Second req.)

9

∵ $\overline{AD}$ // $\overline{BC}$, $\overline{AC}$ is a transversal to them

∴ m (∠ C) = m (∠ DAC) = 30° (alternate angles)

In Δ ABC :

∵ AC = BC

∴ m (∠ CAB) = m (∠ B) = $\frac{180° - 30°}{2}$ = 75° (The req.)

10

∵ Δ DEC is an equilateral triangle

∴ m (∠ ECD) = 60° (1)

From Δ ABC :

∵ AB = AC ∴ m (∠ B) = m (∠ ACB)

, ∵ m (∠ B) + m (∠ ACB) = 180° − 80° = 100°

∴ m (∠ B) = m (∠ ACB) = $\frac{100°}{2}$ = 50° (2)

From (1) and (2) :

∴ m (∠ BCD) = 50° + 60° = 110° (The req.)

11

In ΔABC :

$\because BA = BC$, $m(\angle B) = 80°$

$\therefore m(\angle BAC) = m(\angle BCA) = \frac{180° - 80°}{2} = 50°$

$\therefore m(\angle DAC) = 114° - 50° = 64°$

In ΔADC :

$\because DA = DC$, $m(\angle DAC) = 64°$

$\therefore m(\angle ADC) = 180° - (64° \times 2) = 52°$ (The req.)

12

From ΔABC :

$\because AB = AC$

$\therefore m(\angle B) = m(\angle BCA)$

$\because m(\angle B) + m(\angle BCA) = 180° - 48° = 132°$

$\therefore m(\angle B) = m(\angle BCA) = \frac{132°}{2} = 66°$ (First req.)

$\because \overline{CD}$ bisects $\angle ACB$

$\therefore m(\angle BCD) = \frac{66°}{2} = 33°$ (Second req.)

13

$\because \Delta ABC$ is an equilateral triangle

$\therefore m(\angle ABC) = m(\angle ACB) = 60°$

$\therefore \frac{1}{2} m(\angle ABC) = \frac{1}{2} m(\angle ACB) = 30°$

$\because \overline{BD}$ bisects $\angle ABC$, $\overline{CD}$ bisects $\angle ACB$

$\therefore m(\angle DBC) = m(\angle DCB) = 30°$

$\therefore$ From ΔDBC :

$m(\angle D) = 180° - (2 \times 30°) = 120°$ (The req.)

14

$\because \Delta ABC$ is an equilateral triangle

$\therefore m(\angle ABC) = 60°$ (1)

From ΔDBC :

$\because DB = DC$, $m(\angle D) = 100°$

$\therefore m(\angle DBC) = m(\angle DCB) = \frac{180° - 100°}{2} = 40°$ (2)

From (1) and (2) :

$\therefore m(\angle ABD) = m(\angle ABC) - m(\angle DBC)$

$= 60° - 40° = 20°$ (The req.)

15

$\because \Delta ABC$ is an equilateral triangle

$\therefore m(\angle ACB) = m(\angle B) = m(\angle BAC) = 60°$

$\therefore m(\angle ACD) = 120°$

In ΔACD :

$\because AC = CD$ $\therefore m(\angle CAD) = m(\angle D)$

$\because m(\angle CAD) + m(\angle D) = 180° - 120° = 60°$

$\therefore m(\angle CAD) = \frac{60°}{2} = 30°$

$\therefore m(\angle BAD) = 60° + 30° = 90°$

$\therefore \overline{BA} \perp \overline{AD}$ (Q.E.D.)

16

From ΔABC :

$\because AB = AC$ $\therefore m(\angle B) = m(\angle C)$

$\therefore \Delta\Delta ABD$, ACE in them :

$$\begin{cases} AB = AC \\ m(\angle B) = m(\angle C) \\ BD = EC \end{cases}$$

$\therefore \Delta ABD \equiv \Delta ACE$ then we deduce that $AD = AE$

$\therefore \Delta ADE$ is an isosceles triangle (Q.E.D. 1)

$\therefore m(\angle ADE) = m(\angle AED)$

$\therefore \angle ADE \equiv \angle AED$ (Q.E.D. 2)

17

$\because \Delta\Delta ADE$, BCE in them :

$$\begin{cases} AD = CB \\ AE = EB \\ m(\angle A) = m(\angle B) \end{cases}$$

$\therefore \Delta ADE \equiv \Delta BCE$, then we deduce that $DE = CE$

In ΔDEC :

$\because DE = CE$ $\therefore m(\angle EDC) = m(\angle ECD)$

$\because m(\angle DEC) = 40°$

$\therefore m(\angle EDC) + m(\angle ECD) = 180° - 40° = 140°$

$\therefore m(\angle EDC) = \frac{140°}{2} = 70°$ (The req.)

18

$\because \angle LZX$ is an exterior angle of ΔXYZ

$\therefore m(\angle X) + m(\angle Y) = 130°$

$\because ZX = ZY$ $\therefore m(\angle X) = m(\angle Y)$

$\therefore m(\angle Y) = \frac{130°}{2} = 65°$

$\because \overleftrightarrow{LM} \parallel \overleftrightarrow{XY}$, $\overleftrightarrow{LY}$ is a transversal to them

$\therefore m(\angle MLY) = m(\angle Y) = 65°$ (The req.)

19

$\because \overrightarrow{AE} \parallel \overrightarrow{BC}$ and $\overrightarrow{BD}$ is a transversal to them

$\therefore$ m (ـ ج) = $m(\angle DAE)$ (corresponding angles)

, $\because \overline{AE} \parallel \overline{BC}$, $\overline{AC}$ is a transversal to them.

$\therefore m(\angle C) = m(\angle EAC)$ (alternate angles)

but $m(\angle B) = m(\angle C)$ because $AB = AC$

$\therefore m(\angle DAE) = m(\angle EAC)$

i.e. $\overline{AE}$ bisects $\angle DAC$ (Q.E.D.)

20

$\because B \in \overline{AD}$

$\therefore m(\angle ABC) + m(\angle CBE) + m(\angle EBD) = 180°$ (1)

$\because$ The sum of measures of the angles of the triangle $= 180°$

$\therefore m(\angle ABC) + m(\angle A) + m(\angle C) = 180°$ (2)

From (1) and (2) :

$\therefore m(\angle CBE) + m(\angle EBD) = m(\angle A) + m(\angle C)$

, $\because m(\angle CBE) = m(\angle EBD)$ (Given)

, $m(\angle A) = m(\angle C)$ (because $BA = BC$)

$\therefore m(\angle CBE) = m(\angle C)$ and they are alternate angles

$\therefore \overline{BE} \parallel \overline{AC}$ (Q.E.D.)

21

In Δ DEC :

$\because DE = DC$

$\therefore m(\angle DEC) = m(\angle C) = \frac{180° - 40°}{2} = 70°$

$\because \overline{AD} \parallel \overline{EC}$, $\overline{DE}$ is a transversal to them

$\therefore m(\angle ADE) = m(\angle DEC) = 70°$ (alternate angles)

$\because AD = AE$

$\therefore m(\angle AED) = m(\angle ADE) = 70°$ (First req.)

In Δ AED :

$\because m(\angle EAD) = 180° - (70° + 70°) = 40°$

$\because m(\angle BAD) = m(\angle C) = 70°$

(from properties of the parallelogram)

$\therefore m(\angle BAE) = 70° - 40° = 30°$ (Second req.)

22

In Δ DBC : $\because DB = DC$

$\therefore m(\angle DCB) = m(\angle DBC) = \frac{180° - 140°}{2} = 20°$

, $\because \overline{DE} \parallel \overline{BC}$, $\overline{DC}$ is a transversal.

$\therefore m(\angle EDC) = m(\angle DCB) = 20°$ (alternate angles)

In Δ DCE : $\because DE = EC$

$\therefore m(\angle DCE) = m(\angle EDC) = 20°$

$\therefore m(\angle ACB) = 20° + 20° = 40°$

From Δ ABC :

$m(\angle A) = 180° - (20° + 40°) = 120°$ (The req.)

23

From Δ ABC :

$\because AB = AC$ $\therefore m(\angle B) = m(\angle C)$

$\therefore 2x + 13 = 3x - 17$ $\therefore x = 30°$

$\therefore m(\angle B) = m(\angle C) = 2 \times 30 + 13 = 73°$

$\therefore m(\angle A) = 180° - (73° + 73°) = 34°$ (The req.)

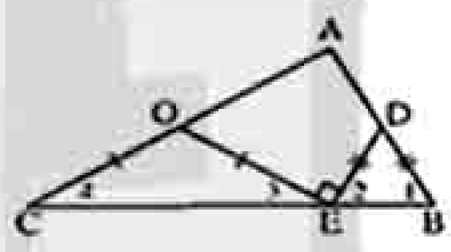
24

1 $x = 60°$, $y = 121°$	2 $x = 45°$, $y = 105°$
3 $x = 50°$, $y = 32°$, $z = 124°$	
4 $x = 75°$, $y = 15°$	5 $x = 25°$, $y = 92°$
6 $y = 110°$, $\ell = 40°$, $z = 70°$	
7 $x = 30°$, $y = 40°$	8 $x = 70°$, $y = 50°$
9 $x = 120°$	10 $x = 40°$
11 $x = 100°$	12 $x = 15°$

25

1 3 cm.	2 5°	3 5°
4 66.5°	5 7°	6 22°

26



In Δ DBE :

$\because DB = DE$

$\therefore m(\angle 1) = m(\angle 2)$

In Δ OEC :

$\because OE = OC$ $\therefore m(\angle 3) = m(\angle 4)$

, $\because E \in \overline{BC}$, $m(\angle DEO) = 90°$

$\therefore m(\angle 2) + m(\angle 3) = 90°$

$\therefore m(\angle 1) + m(\angle 4) = 90°$

In Δ ABC :

$\therefore m(\angle A) = 180° - 90° = 90°$ (The req.)

27

In $\Delta\Delta$ EBD and CBD :

$\left\{ \begin{array}{l} \overline{BD} \text{ is a common side} \\ m(\angle EBD) = m(\angle CBD) \\ m(\angle EDB) = m(\angle CDB) \end{array} \right.$

$\therefore \Delta EBD \equiv \Delta CBD$, then we deduce that :

$BE = BC$, $m(\angle BED) = m(\angle C)$

, $\because BA = BC$ $\therefore BA = BE$

$\therefore m(\angle A) = m(\angle BEA)$

, $\because m(\angle BEA) + m(\angle BED) = 180°$

$\therefore m(\angle A) + m(\angle C) = 180°$ (Q.E.D)

28

$\Delta\Delta$ XYM , MZL in them :

$\begin{cases} XY = MZ \\ YM = LZ \\ m(\angle Y) = m(\angle Z) = 90^\circ \end{cases}$

$\therefore \Delta XYM \equiv \Delta MZL$, then we deduce that

XM = ML , m (∠ XMY) = m (∠ MLZ)

∵ ∠ MLZ complements ∠ LMZ

∴ ∠ XMY complements ∠ LMZ

∴ m (∠ XML) = 90°

∴ From Δ XLM :

∵ MX = ML , m (∠ XML) = 90°

∴ m (∠ MXL) = m (∠ MLX) = $\frac{180^\circ - 90^\circ}{2} = 45^\circ$ (The req.)

29

∵ $\overline{AB} // \overline{CD}$

, $\overline{AC}$ is a transversal to them

∴ m (∠ BAC) + m (∠ ACD) = 180°

(interior angles in the same side of the transversal)

, ∵ $\overline{AE}$ bisects ∠ BAC , $\overline{CE}$ bisects ∠ ACD

∴ m (∠ EAC) + m (∠ ECA) = 90°

From Δ AEC :

∴ m (∠ AEC) = 180° − 90° = 90°

∵ E ∈ $\overline{BD}$

∴ m (∠ BEA) = 180° − (90° + 24°) = 66°

In Δ ABE :

∵ BE = BA

∴ m (∠ BEA) = m (∠ BAE) = 66°

∴ m (∠ ABE) = 180° − 2 × 66° = 48° (The req.)

30

∵ Δ ABD is an equilateral triangle

∴ m (∠ ABD) = 60°

, ∵ Δ CBD is an isosceles triangle where CB = CD

∴ m (∠ CBD) = m (∠ CDB) = $\frac{180^\circ - 50^\circ}{2} = 65^\circ$

∴ m (∠ ABC) = 60° + 65° = 125° (The req.)

31

From Δ BDC :

∵ BD = CD

∴ m (∠ DBC) = m (∠ BCD) (1)

∵ ∠ ADB is an exterior angle of Δ CBD

∴ m (∠ ADB) = m (∠ DBC) + m (∠ BCD)

from (1) : m (∠ ADB) = 2 m (∠ BCD) (2)

In Δ ABD :

∵ AB = AD

∴ m (∠ ABD) = m (∠ ADB)

from (2)

∴ m (∠ ABD) = m (∠ ADB) = 2 m (∠ BCD)

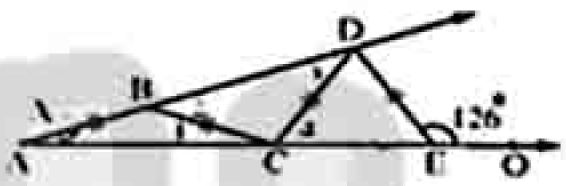
∵ ∠ BAE is an exterior angle of Δ ABD

∴ m (∠ BAE) = m (∠ ABD) + m (∠ ADB)

= 2 m (∠ BCD) + 2 m (∠ BCD)

= 4 m (∠ BCD) (Q.E.D.)

32



In Δ ABC :

∵ BC = BA

∴ m (∠ A) = m (∠ 1) = X

, ∵ ∠ 2 is an exterior angle of Δ ABC

∴ m (∠ 2) = m (∠ A) + m (∠ 1) = $X + X = 2X$

In Δ DBC : ∵ CB = CD

∴ m (∠ 3) = m (∠ 2) = $2X$

, ∵ ∠ 4 is an exterior angle of Δ ACD

∴ m (∠ 4) = m (∠ A) + m (∠ 3) = $X + 2X = 3X$ (1)

, ∵ m (∠ DEC) = 180° − 126° = 54°

In Δ CDE : ∵ DC = DE

∴ m (∠ 4) = m (∠ DEC) = 54° (2)

From (1) and (2) : ∴ $3X = 54^\circ$

∴ $X = \frac{54^\circ}{3} = 18^\circ$ (The req.)

Answers of Exercise 4

1

1 AB = AC	2 YX = YZ	3 XY = XZ
4 AB = AC = BC	5 ML = MN	6 BA = BC
7 ZX = ZY	8 CB = CA	9 AC = AB

2

1 congruent , isosceles	2 equilateral
3 isosceles	4 isosceles
5 equilateral	6 6

3

$\because B \in \overline{DC}$ $\therefore m(\angle ABC) = 180° - 125° = 55°$

In ΔABC : $m(\angle C) = 180° - (55° + 70°) = 55°$

$\therefore m(\angle ABC) = m(\angle C)$

$\therefore AB = AC$

$\therefore \Delta ABC$ is an isosceles triangle. (Q.E.D.)

4

$\because Y \in \overline{ZL}$ $\therefore m(\angle XYZ) = 180° - 120° = 60°$

$, \because XY = XZ$

$\therefore \Delta XYZ$ is an equilateral triangle. (Q.E.D.)

5

$\because B \in \overline{AD}$ $\therefore m(\angle ABC) = 180° - 120° = 60°$

Similarly : $m(\angle ACB) = 60°$

$\therefore m(\angle A) = 180° - (60° + 60°) = 60°$

$\therefore m(\angle A) = m(\angle ABC) = m(\angle ACB)$

$\therefore \Delta ABC$ is an equilateral triangle. (Q.E.D.)

6

$\because \overline{AD} // \overline{BC}$, $\overline{DB}$ is a transversal to them

$\therefore m(\angle DBC) = m(\angle ADB) = 40°$ (alternate angles)

In ΔDBC : $m(\angle C) = 180° - (100° + 40°) = 40°$

$\therefore m(\angle DBC) = m(\angle C)$ $\therefore DB = DC$

$\therefore \Delta DBC$ is an isosceles triangle. (Q.E.D.)

7

$\because \overline{XY} // \overline{AC}$, $\overline{AB}$ is a transversal to them

$\therefore m(\angle A) = m(\angle ABX) = 62°$ (alternate angles)

$\therefore m(\angle ABC) = 180° - (62° + 56°) = 62°$

$\therefore m(\angle ABC) = m(\angle A)$

$\therefore CA = CB$ (Q.E.D.)

8

$\because AB = AC$ $\therefore m(\angle B) = m(\angle C)$ (1)

$, \because \overline{XY} // \overline{BC}$, $\overline{AB}$ is a transversal to them

$\therefore m(\angle AXY) = m(\angle B)$ (corresponding angles) (2)

Similarly $m(\angle AYX) = m(\angle C)$ (3)

From (1) , (2) and (3) :

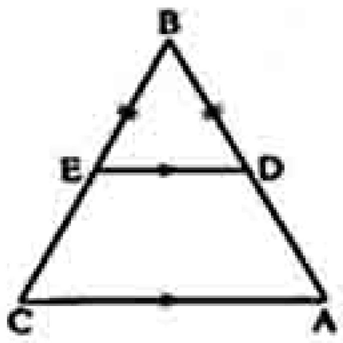
$\therefore m(\angle AXY) = m(\angle AYX)$ $\therefore AX = AY$

$\therefore \Delta AXY$ is an isosceles triangle (Q.E.D. 1)

$\because AB = AC$, $AX = AY$ subtracting

$\therefore XB = YC$ (Q.E.D. 2)

9



From ΔEBD : $\because DB = EB$

$\therefore m(\angle BDE) = m(\angle BED)$ (1)

$\because \overline{DE} // \overline{AC}$, $\overline{AD}$ is a transversal to them

$\therefore m(\angle A) = m(\angle BDE)$ (corresponding angles) (2)

Smilarly $m(\angle C) = m(\angle BED)$ (3)

From (1) , (2) and (3) : $\therefore m(\angle A) = m(\angle C)$

$\therefore AB = BC$ (Q.E.D.)

10

$\because MB = MC$ $\therefore m(\angle B) = m(\angle C)$ (1)

$\because \overline{AD} // \overline{BC}$ and $\overline{AC}$ is a transversal to them

$\therefore m(\angle A) = m(\angle C)$ (alternate angles) (2)

similarly $m(\angle D) = m(\angle B)$ (3)

from (1) , (2) and (3) $\therefore m(\angle A) = m(\angle D)$

$\therefore MA = MD$ (Q.E.D.)

11

$\because B \in \overline{AE}$ $\therefore \angle ABC$ supplements $\angle EBC$

similarly $\angle ACB$ supplements $\angle ACD$

$, \because m(\angle EBC) = m(\angle ACD)$

$\therefore m(\angle ABC) = m(\angle ACB)$

$\therefore AB = AC = 8$ cm.

$\therefore$ The perimeter of $\Delta ABC = 8 + 8 + 10 = 26$ cm.

(The req.)

12

$\because AB = AC$ $\therefore m(\angle B) = m(\angle C)$ (1)

$\because \overline{AB} // \overline{DE}$, $\overline{BE}$ is a transversal to them

$\therefore m(\angle B) = m(\angle DEF)$ (corresponding angles) (2)

similarly $m(\angle C) = m(\angle DFE)$ (3)

from (1) , (2) and (3)

$\therefore m(\angle DEF) = m(\angle DFE)$

$\therefore DE = DF$ (Q.E.D. 1)

In $\Delta\Delta$ ABC , DEF

$\because m(\angle B) = m(\angle DEF)$, $m(\angle C) = m(\angle DFE)$

$\therefore m(\angle BAC) = m(\angle EDF)$ (Q.E.D. 2)

13

$\because \overline{ED} // \overline{BC}$, $\overline{DB}$ is a transversal to them

$\therefore m(\angle EDB) = m(\angle DBC)$ (alternate angles)

but m (∠ EBD) = m (∠ DBC)

∴ m (∠ EDB) = m (∠ EBD) ∴ EB = ED

∴ Δ EBD is an isosceles triangle (Q.E.D.)

14

∵ $\overline{AE}$ // $\overline{BC}$ and $\overline{DB}$ is a transversal to them

∴ m (∠ DAE) = m (∠ B) (corresponding angles)

∵ $\overline{AE}$ // $\overline{BC}$, $\overline{AC}$ is a transversal to them

∴ m (∠ EAC) = m (∠ C) (alternate angles)
but m (∠ DAE) = m (∠ EAC)

∴ m (∠ B) = m (∠ C)

∴ AB = AC (Q.E.D.)

15

∵ m (∠ ABC) = m (∠ ACB) ∴ AB = AC

∴ ΔΔ ADB , AEC

in them $\begin{cases} AB = AC \\ DB = EC \\ m(\angle D) = m(\angle E) = 90^\circ \end{cases}$

∴ Δ ADB ≡ Δ AEC

∴ m (∠ DAB) = m (∠ CAE) (Q.E.D.)

16

In Δ YZX :

∵ YZ = YX ∴ m (∠ Z) = $\frac{180^\circ - 50^\circ}{2} = 65^\circ$

∵ m (∠ ZMX) = 50° + 15° = 65°

∴ In Δ MZX : m (∠ Z) = m (∠ ZMX) ∴ MX = ZX

∴ Δ MZX is an isosceles triangle (Q.E.D.)

17

In Δ ABC : ∵ AB = AC

∴ m (∠ ACB) = m (∠ ABC) = $\frac{180^\circ - 70^\circ}{2} = 55^\circ$

∵ m (∠ MCA) = 25° ∴ m (∠ MCB) = 55° − 25° = 30°

∴ m (∠ MBC) = m (∠ MCB) ∴ MB = MC

∴ Δ MBC is an isosceles triangle. (Q.E.D.)

18

∵ ∠ ADC is an exterior angle of Δ ADB

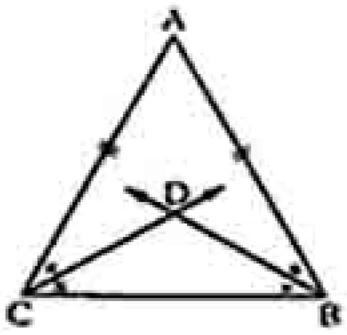
∴ m (∠ ADC) = 40° + 30° = 70°

∵ AD = AC ∴ m (∠ C) = m (∠ ADC) = 70°

∴ In Δ ABC : m (∠ BAC) = 180° − (40° + 70°) = 70°

∴ m (∠ BAC) = m (∠ C) ∴ AB = BC (Q.E.D.)

19



∵ AB = AC

∴ m (∠ ABC) = m (∠ ACB)

∴ $\frac{1}{2}$ m (∠ ABC) = $\frac{1}{2}$ m (∠ ACB)

∵ m (∠ DBC) = $\frac{1}{2}$ m (∠ ABC)
m (∠ DCB) = $\frac{1}{2}$ m (∠ ACB)

∴ m (∠ DBC) = m (∠ DCB) ∴ DB = DC

∴ Δ DBC is an isosceles triangle (Q.E.D.)

20

∵ Δ ABC is an equilateral triangle

∴ m (∠ ACB) = 60°

∵ ∠ ACB is an exterior angle of Δ DCF

∴ m (∠ D) = 60° − 30° = 30°

∴ m (∠ D) = m (∠ F) ∴ CD = CF

∴ Δ DCF is an isosceles triangle (Q.E.D.)

21

∵ DA = DC ∴ m (∠ C) = m (∠ DAC) = 30°

∵ ∠ ADB is an exterior angle of Δ ADC

∴ m (∠ ADB) = 30° + 30° = 60°

∵ DA = DB

∴ Δ ABD is an equilateral triangle (Q.E.D. 1)

∵ m (∠ BAD) = 60° , m (∠ DAC) = 30°

∴ m (∠ BAC) = 90°

∴ Δ ABC is a right-angled triangle (Q.E.D. 2)

22

∵ $\overline{ED}$ // $\overline{AC}$, $\overline{EC}$ is a transversal to them

∴ m (∠ DEC) = m (∠ ACE) (alternate angles)

∵ m (∠ DEC) = m (∠ AEC)

∴ m (∠ ACE) = m (∠ AEC)

∴ AE = AC (1)

, ∵ $\overline{DE}$ // $\overline{AC}$, $\overline{AB}$ is a transversal to them

∴ m (∠ A) = m (∠ BED) = 60°
(corresponding angles) (2)
from (1) and (2)

∴ Δ AEC is an equilateral triangle. (Q.E.D.)

23

In Δ ABC : m (∠ ACB) = 180° − (60° + 90°) = 30°

In Δ ECD : m (∠ ECD) = 180° − (30° + 90°) = 60°

$\because C \in \overline{BD}$

$\therefore m(\angle ACE) = 180° - (30° + 60°) = 90°$

In ΔACE : $m(\angle CAE) = 180° - (90° + 45°) = 45°$

$\therefore m(\angle CAE) = m(\angle CEA) = 45°$ $\therefore CA = CE$

In ΔECD : $\because m(\angle D) = 90°$, $m(\angle CED) = 30°$

$\therefore CE = 2\,CD = 6$ cm. but $AC = CE$

$\therefore AC = 6$ cm. (The req.)

24

In ΔADE : $\because \angle ADE \equiv \angle AED$ $\therefore AD = AE$

$D \in \overline{BC}$, $E \in \overline{BC}$

$\because \angle ADB$ supplements $\angle ADE$,

$\angle AEC$ supplements $\angle AED$

but $m(\angle ADE) = m(\angle AED)$

$\therefore m(\angle ADB) = m(\angle AEC)$

(supplementaries of the congruent angles are congruent)

$\therefore \Delta\Delta$ ADB , AEC in them :

$$\begin{cases} m(\angle ADB) = m(\angle AEC) \\ AD = AE \\ BD = CE \end{cases}$$

$\therefore \Delta ADB \equiv \Delta AEC$

we deduce that $AB = AC$

$\therefore \Delta ABC$ is an isosceles triangle. (Q.E.D.)

25

$\because BY = ZC$ then adding YC to both sides

$\therefore BC = ZY$

$\therefore \Delta\Delta$ ABC , XZY in them :

$$\begin{cases} AB = XZ \\ BC = ZY \\ m(\angle B) = m(\angle Z) \end{cases}$$

$\therefore \Delta ABC \equiv \Delta XZY$

we deduce that $m(\angle ACB) = m(\angle XYZ)$

$\therefore EC = EY$

$\therefore \Delta EYC$ is an isosceles triangle. (Q.E.D.)

26

In ΔBMC :

$\because m(\angle MBC) = m(\angle MCB)$

$\therefore MB = MC$

, $\because m(\angle ABM) = m(\angle MCD)$

(complementaries of equal angles in measure are equal in measure)

$\therefore \Delta\Delta$ ABM , DCM in them :

$$\begin{cases} AB = DC \text{ (two sides in a square)} \\ BM = CM \text{ (proved)} \\ m(\angle ABM) = m(\angle DCM) \text{ (proved)} \end{cases}$$

$\therefore \Delta ABM \equiv \Delta DCM$ we deduce that $AM = DM$

$\therefore \Delta AMD$ is an isosceles triangle (Q.E.D.)

27

In $\Delta\Delta$ ABF , AME : $m(\angle B) = m(\angle AME) = 90°$

$m(\angle BAF) = m(\angle MAE)$ ($\overline{AE}$ bisects $\angle BAC$)

$\therefore m(\angle AFB) = m(\angle E)$ (1)

, $\because \overline{AD} \parallel \overline{BF}$, $\overline{AF}$ is a transversal to them.

$\therefore m(\angle DAE) = m(\angle AFB)$ (alternate angles) (2)

from (1) and (2)

$\therefore m(\angle E) = m(\angle DAE)$

$\therefore DA = DE$ (Q.E.D.)

28

$\because m(\angle EAM) = m(\angle EMA)$ $\therefore EA = EM$

, $\because \overline{AE}$ is a median in ΔABD , $m(\angle BAD) = 90°$

$\therefore AE = \frac{1}{2} BD$ $\therefore EM = \frac{1}{2} BD$ (1)

, $\because$ E is the midpoint of $\overline{BD}$, $\overline{EM} \parallel \overline{BC}$

$\therefore EM = \frac{1}{2} BC$ (2)

from (1) and (2)

$\therefore \frac{1}{2} BD = \frac{1}{2} BC$ $\therefore BD = BC$ (Q.E.D.)

29

$\because m(\angle B) = m(\angle C)$ $\therefore AB = AC$

$\therefore 2X - 1 = X + 3$ $\therefore 2X - X = 3 + 1$ $\therefore X = 4$

$\therefore AB = AC = 2 \times 4 - 1 = 7$ cm. , $BC = 9 - 4 = 5$ cm.

$\therefore$ The perimeter of $\Delta ABC = 7 + 7 + 5 = 19$ cm. (The req.)

30

1 $\because 3X + X + 50° + 30° = 180°$

$\therefore 4X + 80° = 180°$

$\therefore 4X = 180° - 80° = 100°$ $\therefore X = \frac{100°}{4} = 25°$

$\therefore m(\angle A) = 3 \times 25° = 75°$,

$m(\angle B) = 25° + 50° = 75°$

$\therefore m(\angle A) = m(\angle B)$ $\therefore CB = CA$

2 $\because 2z + 3z - 10^\circ + z + 40^\circ = 180^\circ$

$\therefore 6z + 30^\circ = 180^\circ$

$\therefore 6z = 180^\circ - 30^\circ = 150^\circ \quad \therefore z = \frac{150^\circ}{6} = 25^\circ$

$\therefore m(\angle B) = 3 \times 25^\circ - 10^\circ = 65^\circ$

, $m(\angle C) = 25^\circ + 40^\circ = 65^\circ$

$\therefore m(\angle B) = m(\angle C) \quad \therefore AB = AC$

3 $\because \angle DBC$ is an exterior angle of ΔABC

$\therefore 3X = X - 20^\circ + X + 70^\circ$

$\therefore 3X = 2X + 50^\circ \quad \therefore X = 50^\circ$

$\therefore m(\angle A) = 50^\circ - 20^\circ = 30^\circ$,

$m(\angle C) = 50^\circ + 70^\circ = 120^\circ$

$\therefore m(\angle ABC) = 180^\circ - (30^\circ + 120^\circ) = 30^\circ$

$\therefore m(\angle A) = m(\angle ABC) \quad \therefore CB = CA$

31

1 b 2 c 3 b

32

$\because \overline{AB} \parallel \overline{CE}$

$\therefore m(\angle A) = m(\angle ACE) = 40^\circ$

, $\because m(\angle BCE) = 80^\circ$

$\therefore m(\angle BCA) = 80^\circ - 40^\circ = 40^\circ$

$\therefore m(\angle A) = m(\angle BCA) = 40^\circ$

$\therefore BC = BA = 2.4$ km. (The req.)

33

$\because Y \in \overline{XB}$, $m(\angle BYC) = 120^\circ$

$\therefore m(\angle XYZ) = 60^\circ$

$\because XY = YZ$

$\therefore \Delta XYZ$ is an equilateral triangle

$\therefore XY = YZ = XZ$

, $\because BY = ZC = AX$

Adding $\therefore BY + XY = YZ + ZC = AX + XZ$

$\therefore BX = YC = AZ$

$\because X \in \overline{AZ}$

$\therefore m(\angle AXB) = 180^\circ - 60^\circ = 120^\circ$

similarly : $\because Z \in \overline{YC} \quad \therefore m(\angle AZC) = 120^\circ$

$\therefore \Delta\Delta$ AXB , BYC in them :

$\begin{cases} AX = BY \text{ (given)} \\ XB = YC \text{ (proved)} \\ m(\angle AXB) = m(\angle BYC) = 120^\circ \text{ (proved)} \end{cases}$

$\therefore \Delta AXB \equiv \Delta BYC$ we deduce that $AB = BC$

similarly $\Delta AXB \equiv \Delta CZA$ we deduce that

$AB = AC \quad \therefore AB = AC = BC$

$\therefore \Delta ABC$ is an equilateral triangle (Q.E.D.)

Answers of Exercise 5

1

1 An axis of symmetry. 2 3 3 1 4 zero

5 Bisects it and it is perpendicular to the base.

6 Bisects the base and is perpendicular to it.

7 Bisects each of the base and the vertex angle.

8 The straight line perpendicular to it at its middle.

9 at equal distances 10 AC , BC

11 3 12 1 13 3 14 1 15 30°

2

1 35° 2 70° 3 55° 4 2 5 $\overline{AD}$

3

1 30° 2 55° 3 60° 4 4 5 8

6 $4\sqrt{3}$ 7 3 8 1 9 $16\sqrt{3}$

4

$\because BA = BC$, $\overline{BD} \perp \overline{AC}$

$\therefore \overline{BD}$ bisects each of $\angle ABC$, $\overline{AC}$

$\therefore AC = 2AD = 40$ cm.

, $m(\angle DBC) = \frac{1}{2} m(\angle ABC) = 45^\circ$ (1) (First req.)

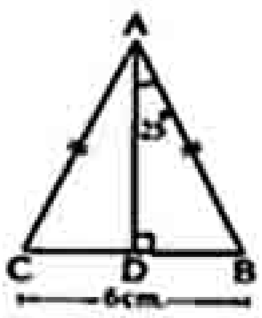
$\because \Delta ABC$ in which $m(\angle B) = 90^\circ$, $BA = BC$

$\therefore m(\angle C) = 45^\circ$ (2)

From (1) and (2) : $\therefore DB = DC$

$\therefore \Delta DBC$ is an isosceles triangle (Second req.)

5



$\because AB = AC$, $\overline{AD} \perp \overline{BC}$

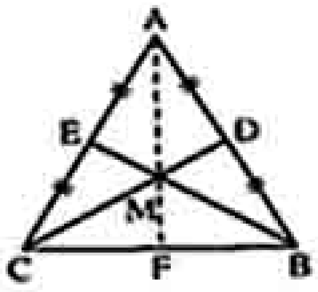
$\therefore \overline{AD}$ bisects each of $\angle BAC$, $\overline{BC}$

$\therefore BD = \frac{1}{2} BC = 3$ cm. (First req.)

$\therefore$ In ΔABD : $m(\angle B)$

$= 180^\circ - (90^\circ + 25^\circ) = 65^\circ$ (Second req.)

6



In ΔABC : $AB = AC$,
M is the point of intersection of its medians
$\therefore \overline{AF}$ is a median of ΔABC
$\therefore \overline{AM} \perp \overline{BC}$ (Q.E.D. 1)
, $\overline{AM}$ bisects $\angle BAC$ (Q.E.D. 2)

7

In ΔABC : $\because AB = AC$, $\overline{AD} \perp \overline{BC}$
$\therefore BC = 2 \times 5 = 10$ cm. (First req.)
In the right-angled triangle ADB at D
$AD = \sqrt{(13)^2 - (5)^2} = 12$ cm.
$\therefore$ The area of $\Delta ABC = \frac{1}{2} \times 10 \times 12 = 60$ cm^2. (Second req.)

8

$\because AB = AC$, $\overline{AD} \perp \overline{BC}$ $\therefore BD = \frac{1}{2} BC = 5$ cm.
, $m(\angle BAC) = 2 \times 30^\circ = 60^\circ$
$\therefore \Delta ABC$ is an equilateral triangle
$\because AB = 10$ cm.
$\because$ In ΔADB which is right-angled at D
$\therefore AD = \sqrt{(10)^2 - (5)^2} = 5\sqrt{3}$ cm. (First req.)
The number of axes of symmetry of $\Delta ABC = 3$ (Second req.)
The area of $\Delta ABC = \frac{1}{2} \times 10 \times 5\sqrt{3} = 25\sqrt{3}$ cm^2. (Third req.)

9

In ΔABC : $\because AB = AC$, $\overline{AE}$ bisects $\angle BAC$
$\therefore BE = \frac{1}{2} BC$ (Q.E.D.1)
, $\overline{AE} \perp \overline{BC}$
$\therefore \overline{AE}$ is the axis of symmetry of $\overline{BC}$, $D \in \overline{AE}$
$\therefore BD = CD$ (Q.E.D.2)

10

$\because C \in \overline{BD}$, $m(\angle ACD) = 130^\circ$
$\therefore m(\angle ACB) = 180^\circ - 130^\circ = 50^\circ$
From ΔABC : $m(\angle B) = 180^\circ - (80^\circ + 50^\circ) = 50^\circ$
$\therefore m(\angle B) = m(\angle ACB)$
$\therefore \Delta ABC$ is an isosceles triangle
, $\overline{AE}$ bisects $\angle BAC$
$\therefore \overline{AE} \perp \overline{BC}$, E is the midpoint of $\overline{BC}$ (Q.E.D)

11

$\because m(\angle ABX) = m(\angle ACY)$
$\therefore m(\angle ABC) = m(\angle ACB)$
(The supplementaries of congruent angles are congruent)
$\therefore AB = AC$
$\because \overline{AD}$ is a median of ΔABC which is isosceles
$\therefore \overline{AD} \perp \overline{BC}$ (Q.E.D.)

12

$\because \overline{AD} // \overline{BC}$, $\overline{DB}$ is a transversal to them
$\therefore m(\angle ADB) = m(\angle DBC)$ (alternate angles)
but $m(\angle ABD) = m(\angle DBC)$
$\therefore m(\angle ADB) = m(\angle ABD)$
$\therefore$ In ΔABD : $AB = AD$ (Q.E.D.1)
, $\because \overline{AE}$ bisects $\angle BAD$ $\therefore \overline{AE} \perp \overline{BD}$ (Q.E.D.2)
, $BE = ED$ (Q.E.D.3)

13

In ΔACD :
$\because$ E is the midpoint of $\overline{AD}$
, $\overline{CE} \perp \overline{AD}$ $\therefore DC = AC$
$\therefore \Delta ACD$ is an isosceles triangle.
, $\because \angle ADC$ is an exterior angle of ΔADB
$\therefore m(\angle ADC) = 20^\circ + 30^\circ = 50^\circ$
From ΔCDE :
$m(\angle DCE) = 180^\circ - (90^\circ + 50^\circ) = 40^\circ$
, $\because \overline{CE}$ bisects $\angle ACD$
$\therefore m(\angle ACE) = m(\angle DCE) = 40^\circ$ (The req.)

14

In ΔADC :
$\because$ E is the midpoint of $\overline{DC}$
, $\overline{AE} \perp \overline{DC}$ $\therefore AD = AC$
$\therefore \Delta ADC$ is an isosceles triangle
$\therefore m(\angle ADC) = m(\angle C) = 70^\circ$
, $\because \angle ADC$ is an exterior angle of ΔABD
$\therefore m(\angle ADC) = m(\angle B) + m(\angle BAD)$
, $\because BD = AC$, $AD = AC$
$\therefore m(\angle B) = m(\angle BAD) = \frac{70^\circ}{2} = 35^\circ$ (The req.)

15

In ΔXYL : $\because XL = XY$, M is the midpoint of $\overline{LY}$
$\therefore \overline{XM}$ is the axis of $\overline{LY}$

similarly in ΔZYL , $\overline{ZM}$ is the axis of $\overline{LY}$

$\therefore$ X , M and Z are on the same straight line. (Q.E.D.)

16

$\because AB = AC \therefore A \in$ the axis of $\overline{BC}$ (1)

, $m(\angle ABC) = m(\angle ACB)$,

$\because m(\angle ABD) = m(\angle ACD)$

by subtracting :

$\therefore m(\angle DBC) = m(\angle DCB)$

$\therefore DB = DC \therefore D \in$ the axis of $\overline{BC}$ (2)

From (1) and (2) :

$\therefore \overline{AD}$ is the axis of $\overline{BC}$ (Q.E.D.)

17

$\because \overline{AD}$ bisects the base of ΔABC which is an isosceles triangle

$\therefore \overline{AD} \perp \overline{BC}$ $\therefore m(\angle ADB) = 90^\circ$

$\because \overline{XY} \parallel \overline{BC}$, $\overline{AD}$ is a transversal to them

$\therefore m(\angle YAD) = m(\angle ADB) = 90^\circ$ (alternate angles)

$\therefore \overline{AD} \perp \overline{XY}$ (Q.E.D.)

18

$\because AB = AC$, $EB = EC$ $\therefore \overline{AE}$ is the axis of $\overline{BC}$

$\therefore BD = DC$ (First req.)

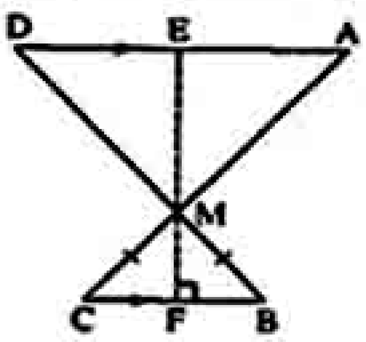
$\therefore DC = 3$ cm.

In ΔADC which is right-angled at D

$\therefore AD = \sqrt{(10)^2 - (3)^2} = \sqrt{100 - 9} = \sqrt{91}$ cm.

(Second req.)

19 Constr. :



Draw $\overline{MF} \perp \overline{BC}$ to meet $\overline{BC}$ at F and $\overline{AD}$ at E

Proof : $\because \overline{AD} \parallel \overline{BC}$, $\overline{AC}$ is a transversal to them

$\therefore m(\angle A) = m(\angle C)$ similarly $m(\angle B) = m(\angle D)$

$\because MB = MC$ $\therefore m(\angle B) = m(\angle C)$

$\therefore m(\angle A) = m(\angle D)$ $\therefore AM = DM$

$\therefore \Delta AMD$ is an isosceles triangle (Q.E.D.1)

In ΔMBC : $\because MB = MC$, $\overline{MF} \perp \overline{BC}$

$\therefore \overline{MF}$ is the axis of symmetry of ΔMBC

$\because \overline{AD} \parallel \overline{BC}$, $\overline{FE}$ is a transversal to them

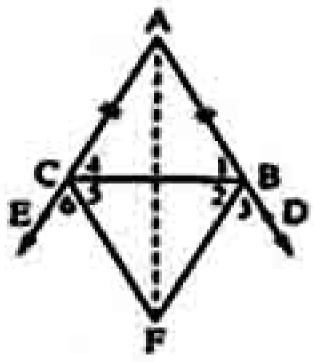
$\therefore m(\angle AEM) = m(\angle BFM) = 90^\circ$

$\therefore \overline{ME} \perp \overline{AD}$

, $\because MA = MD$ $\therefore \overline{ME}$ is the axis of ΔAMD

$\therefore \overline{EF}$ is the axis of symmetry of each of ΔAMD , ΔBMC (Q.E.D.2)

20



$\because AB = AC$

$\therefore m(\angle 1) = m(\angle 4)$ (1)

$\because m(\angle DBC) = 180^\circ - m(\angle 1)$ (2)

, $m(\angle BCE) = 180^\circ - m(\angle 4)$ (3)

From (1) , (2) and (3)

$\therefore m(\angle DBC) = m(\angle BCE)$

$\therefore \frac{1}{2} m(\angle DBC) = \frac{1}{2} m(\angle BCE)$

$\therefore m(\angle 2) = m(\angle 5)$

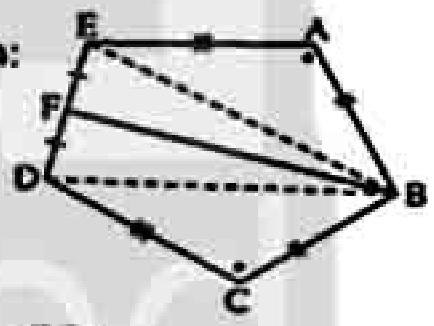
$\therefore FB = FC \therefore \Delta BFC$ is an isosceles triangle (Q.E.D.1)

$\because AB = AC$, $FB = FC$

$\therefore \overline{AF}$ is the axis of $\overline{BC}$ (Q.E.D.2)

21

Constr. : Draw $\overline{BD}$, $\overline{BE}$



Proof: $\Delta\Delta ABE$, CBD in them:

$\begin{cases} m(\angle A) = m(\angle C) \\ AB = CB , \\ AE = CD \end{cases}$

$\therefore \Delta ABE \equiv \Delta CBD$

, then we deduce that: $BE = BD$

, $\because \overline{BF}$ is a median of ΔBED which is isosceles

$\therefore \overline{BF} \perp \overline{DE}$ (Q.E.D.)

22

In ΔABD :

$\because$ E is the midpoint of $\overline{AB}$

, $\overline{DE} \perp \overline{AB}$ $\therefore DA = DB$

$\therefore m(\angle A) = m(\angle ABD)$ (1)

, in ΔDBC :

$\because$ O is the midpoint of $\overline{BC}$

, $\overline{DO} \perp \overline{BC}$ $\therefore DB = DC$

$\therefore m(\angle DBC) = m(\angle C)$ (2)

, $\because m(\angle ABD) + m(\angle DBC) = 130^\circ$ (3)

From (1) , (2) and (3) :

$\therefore m(\angle A) + m(\angle C) = 130^\circ$

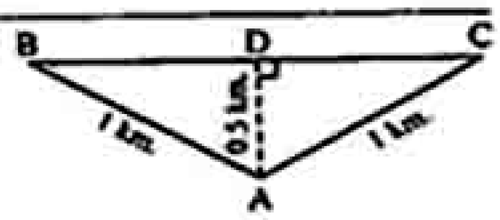
From the quadrilateral ABCD

$\therefore m(\angle ADC) = 360^\circ - (130^\circ + 130^\circ) = 100^\circ$ (The req.)

23

1 c 2 b 3 c 4 b 5 a 6 b

24



In ΔABD :

$\because m(\angle BDA) = 90°$,

$AD = \frac{1}{2} AB$

$\therefore m(\angle B) = 30°$, $m(\angle BAD) = 60°$

In ΔABC :

$\because \overline{AD} \perp \overline{BC}$

$\therefore \overline{AD}$ bisects $\angle BAC$

$\therefore m(\angle BAC) = 60° \times 2 = 120°$

, $\because BA = CA$

$\therefore m(\angle C) = m(\angle B) = 30°$ (First req.)

, $\because \Delta ABD$ is right-angled at D

$\therefore (BD)^2 = (AB)^2 - (AD)^2$

$\therefore (BD)^2 = 1 - \frac{1}{4} = \frac{3}{4}$

$\therefore BD = \sqrt{\frac{3}{4}} = \frac{1}{2}\sqrt{3}$ km.

, $\because \overline{AD} \perp \overline{BC}$

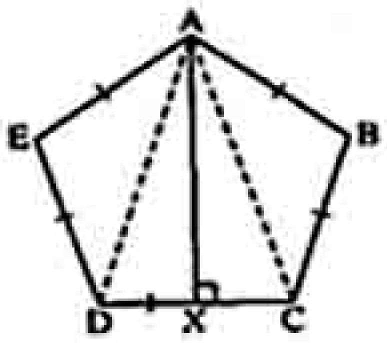
$\therefore$ D is the midpoint of $\overline{BC}$

$\therefore BC = 2 \times \frac{1}{2}\sqrt{3} = \sqrt{3}$ km.

$\therefore$ The distance $= 1 + \sqrt{3} + 1 = (2 + \sqrt{3})$ km.

$\simeq 4$ km. (Second req.)

25



Constr. :

Draw $\overline{AD}$, $\overline{AC}$

Proof :

$\because$ ABCDE is a regular pentagon

$\therefore$ The measure of each interior angle $= 108°$

, $AB = BC = CD = DE = EA$

In ΔABC : $\because AB = BC$, $m(\angle ABC) = 108°$

$\therefore m(\angle BAC) = \frac{180° - 108°}{2} = 36°$

$\therefore \Delta ABC$, ΔAED in them :

$$\begin{cases} AB = AE \\ BC = ED \\ m(\angle ABC) = m(\angle AED) = 108° \end{cases}$$

$\therefore \Delta ABC \equiv \Delta AED$

Then we deduce that : $m(\angle EAD) = m(\angle BAC) = 36°$

, $AC = AD$ $\therefore \Delta ADC$ is an isosceles triangle

, $\because \overline{AX} \perp \overline{CD}$

$\therefore m(\angle CAX) = m(\angle DAX)$

$= \frac{108° - (36° + 36°)}{2} = 18°$ (The req.)

Answers of exams on unit four

Model 1

1

1 b 2 d 3 d
4 c 5 b 6 b

2

1 4

2 half the length of the hypotenuse

3 bisects the vertex angle , perpendicular to the base

4 the angle at this vertex is right

5 65° , 50°

3

[a] The perimeter of $\Delta DEF = 22\frac{1}{2}$ cm.

[b] Prove by yourself.

4

[a] EF = 6 cm.

[b] $m(\angle ABD) = 125°$

5

[a] The perimeter of $\Delta MBC = 26$ cm.

[b] Prove by yourself.

Model 2

1

1 c 2 c 3 b
4 a 5 b 6 b

2

1 perpendicular to the base , bisects it.
2 equal
3 8
4 80° , 130°

3

[a] Prove by yourself.

[b] 1 BD = 10 cm.
2 Prove by yourself

4

[a] BD = 6 cm. , MD = 2 cm.

[b] m (∠ MLY) = 70°

5

[a] The perimeter of the triangle = 19 cm.

[b] Prove by yourself , CD = 3 cm. , AD = $\sqrt{91}$ cm.

Answers of unit five

Answers of Exercise 6

1

1) > 2) > 3) < 4) <

5) < 6) > 7) > , <

2

1) $m(\angle 1) < m(\angle 3)$
2) $m(\angle 4) < m(\angle 2)$
3) $m(\angle 3) < m(\angle 5)$
4) $m(\angle 6) < m(\angle 2)$
5) $m(\angle 1) < m(\angle 3) < m(\angle 5)$
6) $m(\angle 3) < m(\angle 5) < m(\angle 7)$
7) $m(\angle 1) < m(\angle 3) < m(\angle 5) < m(\angle 7)$

3

$\because \overline{AD} \perp \overline{BC}$ from its midpoint

$\therefore \overline{AD}$ is the axis of symmetry of $\overline{BC}$

$\therefore AB = AC$

$\because E \in \overline{AB}$ $\therefore AB > AE$

$\therefore AC > AE$ (Q.E.D.)

4

$\because \overrightarrow{AB} // \overrightarrow{CD}$, $\overrightarrow{BC}$ is a transversal

$\therefore m(\angle BCD) = m(\angle ABC)$ (alternate angles)

$\therefore m(\angle BCD) + m(\angle ACB) > m(\angle ABC)$

$\therefore m(\angle ACD) > m(\angle ABC)$ (1) (Q.E.D.1)

$\because E \in \overrightarrow{CD}$ $\therefore \angle ADE$ is an exterior angle of $\triangle ACD$

$\therefore m(\angle ADE) > m(\angle ACD)$ (2)

From (1) and (2) :

$\therefore m(\angle ADE) > m(\angle ABC)$ (Q.E.D.2)

5

$\because E \in \overrightarrow{CB}$ $\therefore$ The exterior angle ABE of $\triangle ABC$

$\therefore m(\angle ABE) > m(\angle A)$ (1)

$\triangle\triangle$ ABM , CDM in them :

$$\begin{cases} AM = MC \\ MB = MD \\ m(\angle AMB) = m(\angle DMC) \quad \text{(V.O.A)} \end{cases}$$

$\therefore \triangle ABM \equiv \triangle CDM$, then we deduce that

$m(\angle A)$ $m(\angle ACD)$ and from (1) :

$\therefore m(\angle ABE) > m(\angle ACD)$ (Q.E.D.)

6

$\because$ The figure is a parallelogram

$\therefore AD = BC$, $AB = CD$

$\because DX < BY$ $\therefore AX > CY$

$\therefore AX + AB > CY + CD$ (Q.E.D.)

7

$\because D \in \overline{AB}$ $\therefore \angle ADC$ is an exterior angle of $\triangle DBC$

$\therefore m(\angle ADC) > m(\angle B)$

But $m(\angle ADC) = m(\angle ACD)$

because $\triangle ADC$ in which $AD = AC$

$\therefore m(\angle ACD) > m(\angle B)$

$\therefore m(\angle ACD) + m(\angle DCB) > m(\angle B)$

$\therefore m(\angle ACB) > m(\angle B)$ (Q.E.D.)

8

In $\triangle AXY$: $\because m(\angle AXY) = m(\angle AYX)$

$\therefore AX = AY$ (1)

$\because AC > AB$ $\therefore AY + YC > AX + XB$ (2)

From (1) and (2) :

$\therefore YC > XB$ (Q.E.D.)

9

$\because \angle ADC$ is an exterior angle of $\triangle DBC$

$\therefore m(\angle ADC) > m(\angle B)$

But $m(\angle B) = m(\angle ACB)$

(because $AB = AC$ in $\triangle ABC$)

$\therefore m(\angle ADC) > m(\angle ACB)$ (Q.E.D.)

10

$\because m(\angle ACB) > m(\angle ABC)$

$\therefore$ The supplement

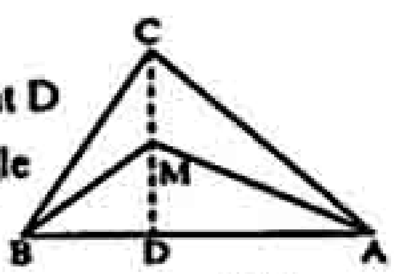
of $\angle ABC >$ the supplement of $\angle ACB$

$\therefore m(\angle ABD) > m(\angle ACE)$

$\therefore \frac{1}{2} m(\angle ABD) > \frac{1}{2} m(\angle ACE)$

i.e. $m(\angle ABX) > m(\angle ACY)$ (Q.E.D.)

11



Const: Draw $\overline{CM}$ to intersect $\overline{BA}$ at D

Proof: $\because \angle AMD$ is an exterior angle of $\triangle AMC$

$\therefore m(\angle AMD) > m(\angle ACM)$ (1)

$\because \angle BMD$ is an exterior angle of $\triangle CMB$

$\therefore m(\angle BMD) > m(\angle BCM)$ (2)

Adding (1) and (2)

$\therefore$ m ($\angle$ AMD) + m ($\angle$ BMD) > m ($\angle$ ACM) + m ($\angle$ BCM)

$\therefore$ m ($\angle$ AMB) > m ($\angle$ C) (Q.E.D.)

12

$\because$ m ($\angle$ B) > m ($\angle$ C)

$\therefore$ m ($\angle$B) + $\frac{1}{2}$m ($\angle$ BAC) > m ($\angle$ C) + $\frac{1}{2}$m ($\angle$ BAC)

$\therefore$ m ($\angle$ B) + m ($\angle$ BAD) > m ($\angle$C) + m ($\angle$ CAD)

but m ($\angle$ B) + m ($\angle$ BAD) = m ($\angle$ CDA)

(an exterior angle of ΔABD),

m ($\angle$ C) + m ($\angle$ CAD) = m ($\angle$ BDA)

(an exterior angle of Δ ACD)

$\therefore$ m ($\angle$ ADC) > m ($\angle$ ADB)

$\because$ Their sum = 180° $\therefore$ m ($\angle$ ADC) > $\frac{180°}{2}$

i.e. m ($\angle$ ADC) > 90°

i.e. $\angle$ ADC is an obtuse angle. (Q.E.D.)

13

$\because$ AC = AD $\therefore$ m ($\angle$ D) = m ($\angle$ ACD)

$\because$ m ($\angle$ ACB) > m ($\angle$ ABC)

$\therefore$ m ($\angle$ ACB) + m ($\angle$ ACD) > m ($\angle$ ABC) + m ($\angle$ D)

$\therefore$ m ($\angle$ BCD) > m ($\angle$ B) + m ($\angle$ D)

but the sum of measures of the interior angles of ΔBCD = 180°

$\therefore$ m ($\angle$ BCD) > $\frac{180°}{2}$ i.e. m ($\angle$ BCD) > 90°

i.e. $\angle$ BCD is an obtuse angle. (Q.E.D.)

Answers of Exercise 7

1

1 The angle of the greater measure

2 $\angle$A 3 m ($\angle$ D)

4 m ($\angle$ A) < m ($\angle$ B) < m ($\angle$ C)

2

1 > , > , < 2 < , < , > 3 > , > , >

3

1 $\because$ $\overline{BC}$ is the longest side

$\therefore$ $\angle$ A is the greatest angle in measure

$\because$ $\overline{AC}$ is the shortest side

$\therefore$ $\angle$ B is the smallest angle in measure

$\therefore$ The ascending order of measures of the angles is : m ($\angle$ B) , m ($\angle$ C) and m ($\angle$ A)

2 $\because$ $\overline{BC}$ is the longest side

$\therefore$ $\angle$ A is the greatest angle in measure

$\because$ $\overline{AB}$ is the shortest side

$\therefore$ $\angle$ C is the smallest angle in measure

$\therefore$The ascending order of the measures of the angles is : m ($\angle$ C) , m ($\angle$ B) and m ($\angle$ A)

4

In ΔABC : $\because$ AC > AB

$\therefore$ m ($\angle$ ABC) > m ($\angle$ ACB) (1)

In ΔBDC : $\because$ DB = DC

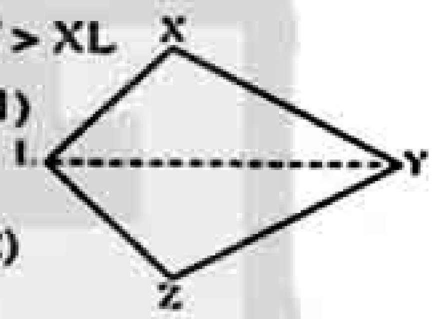
$\therefore$ m ($\angle$ DBC) = m ($\angle$ DCB) (2)

Adding (1) and (2) :

$\therefore$ m ($\angle$ ABC) + m ($\angle$ DBC) > m ($\angle$ ACB) + m ($\angle$ DCB)

$\therefore$ m ($\angle$ ABD) > m ($\angle$ ACD) (Q.E.D.)

5



Construction : Draw $\overline{YL}$

Proof : In ΔXYL $\because$ XY > XL

$\therefore$ m ($\angle$ XLY) > m ($\angle$ XYL) (1)

In ΔZYL : $\because$ YZ > ZL

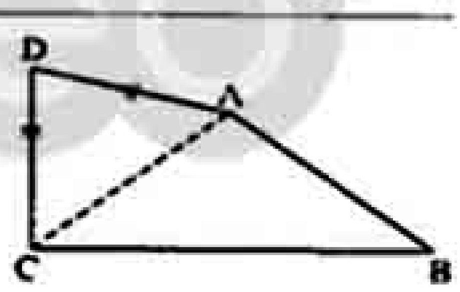
$\therefore$ m ($\angle$ ZLY) > m ($\angle$ ZYL) (2)

Adding (1) and (2) :

$\therefore$ m ($\angle$ XLY) + m ($\angle$ ZLY) > m ($\angle$ XYL) + m ($\angle$ ZYL)

$\therefore$ m ($\angle$ XLZ) > m ($\angle$ XYZ) (Q.E.D.)

6



Construction : Draw $\overline{AC}$

Proof : In ΔABC

$\because$ BC > AB

$\therefore$ m ($\angle$ BAC) > m ($\angle$ ACB) (1)

In ΔDAC : $\because$ DA = DC

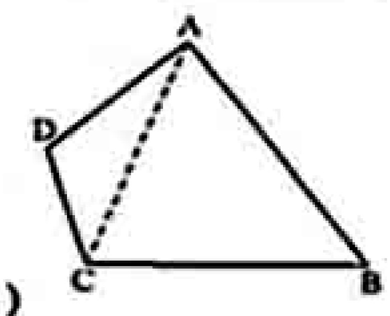
$\therefore$ m ($\angle$ DAC) = m ($\angle$ DCA) (2)

Adding (1) and (2) :

$\therefore$ m ($\angle$ BAC) + m ($\angle$ DAC) > m ($\angle$ ACB) + m ($\angle$ DCA)

$\therefore$ m ($\angle$ BAD) > m ($\angle$ BCD) (Q.E.D.)

7



Construction : Draw $\overline{AC}$

Proof : In ΔABC

$\because$ AB > BC

$\therefore$ m ($\angle$ ACB) > m ($\angle$ BAC) (1)

In ΔADC : $\because AD > DC$

$\therefore m(\angle ACD) > m(\angle CAD)$ (2)

Adding (1) and (2) :

$\therefore m(\angle BCD) > m(\angle BAD)$ (Q.E.D.)

8

In ΔMBC : $\because MC > MB$

$\therefore m(\angle MBC) > m(\angle MCB)$

$\because m(\angle MBC) = \frac{1}{2} m(\angle ABC)$

, $m(\angle MCB) = \frac{1}{2} m(\angle ACB)$

$\therefore \frac{1}{2} m(\angle ABC) > \frac{1}{2} m(\angle ACB)$

$\therefore m(\angle ABC) > m(\angle ACB)$ (Q.E.D.)

9

In ΔDBC : $\because DB > DC$

$\therefore m(\angle DCB) > m(\angle DBC)$

In ΔABC : $\because AB = AC$

$\therefore m(\angle ACB) = m(\angle ABC)$

$\therefore m(\angle ACB) - m(\angle DCB) < m(\angle ABC) - m(\angle DBC)$

$\therefore m(\angle ACD) < m(\angle ABD)$

i.e. $m(\angle ABD) > m(\angle ACD)$ (Q.E.D.)

10

In ΔABC : $\because AB > AC$ $\therefore m(\angle C) > m(\angle B)$ (1)

$\because \overline{XY} \parallel \overline{BC}$ and $\overline{AC}$ is a transversal

$\therefore m(\angle AYX) = m(\angle C)$ (Corresponding angles) (2)

Similarly : $\because \overline{XY} \parallel \overline{BC}$, $\overline{AB}$ is a transversal

$\therefore m(\angle AXY) = m(\angle B)$ (3)

From (1) , (2) and (3) :

$\therefore m(\angle AYX) > m(\angle AXY)$ (Q.E.D.)

11

$\because AB > AC$ $\therefore m(\angle C) > m(\angle B)$

But $m(\angle C) = m(\angle AED)$ (corresponding angles)

, $m(\angle B) = m(\angle ADE)$ (corresponding angles)

$\therefore m(\angle AED) > m(\angle ADE)$

In ΔADE :

$\because m(\angle A) = 90^\circ$

$\therefore m(\angle AED) + m(\angle ADE) = 90^\circ$

, $\because m(\angle AED) > m(\angle ADE)$

$\therefore m(\angle AED) > \frac{90^\circ}{2}$

$\therefore m(\angle AED) > 45^\circ$ (Q.E.D.)

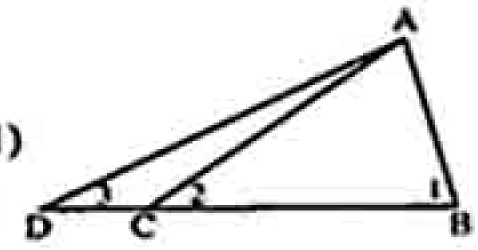
12

$\because AB > AC$, $BD = CE$ Subtracting $\therefore AD > AE$

$\therefore$ In ΔADE : $\because AD > AE$

$\therefore m(\angle AED) > m(\angle ADE)$ (Q.E.D.)

13



In ΔABC : $\because AC > AB$

$\because m(\angle 1) > m(\angle 2)$ (1)

But $\angle 2$ is an exterior angle of ΔACD

$\because m(\angle 2) > m(\angle 3)$ (2)

From (1) and (2) : $\therefore m(\angle 1) > m(\angle 3)$

$\therefore m(\angle ABD) > m(\angle D)$ (Q.E.D.)

14

$\because \Delta ABC$ is an equilateral triangle

$\therefore m(\angle ABC) = m(\angle ACB) = 60^\circ$

$\because m(\angle EBC) < m(\angle ECB)$ Subtracting

$\therefore m(\angle ABC) - m(\angle EBC) > m(\angle ACB) - m(\angle ECB)$

$\therefore m(\angle ABE) > m(\angle ACE)$ (1) (Q.E.D.1)

$\because m(\angle A) = m(\angle B)$

$\therefore m(\angle A) = m(\angle ABE) + m(\angle EBC)$

$\therefore m(\angle A) > m(\angle ABE)$ and from (1) :

$\therefore m(\angle A) > m(\angle ABE) > m(\angle ACE)$ (Q.E.D.2)

15

In ΔXBC : $\because XC > XB$

$\therefore m(\angle XBC) > m(\angle XCB)$

$\because ABCD$ is a rectangle

i.e. $m(\angle ABC) = m(\angle DCB) = 90^\circ$

$\therefore 90^\circ - m(\angle XBC) < 90^\circ - m(\angle XCB)$

$\therefore m(\angle ABX) < m(\angle XCD)$ (Q.E.D.)

16

In ΔADE : $\because AD = 5$ cm. , $AE = 3$ cm. $\therefore AD > AE$

$\therefore m(\angle AED) > m(\angle ADE)$

From the equilateral triangle ABC we find that

$m(\angle A) = 60^\circ$

$\because m(\angle AED) + m(\angle ADE) = 180^\circ - 60^\circ = 120^\circ$

$\therefore m(\angle AED) > 60^\circ$ (Q.E.D.)

17

In ΔABC : $\because AB > AC$ $\therefore m(\angle ACB) > m(\angle ABC)$

$\because 180^\circ - m(\angle ACB) < 180^\circ - m(\angle ABC)$

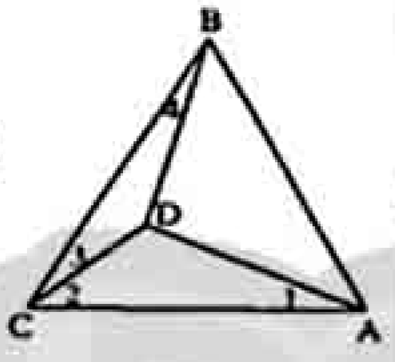
$\because D \in \overline{AB}$, $E \in \overline{AC}$ $\therefore m(\angle BCE) < m(\angle DBC)$

$\because \overline{BF}$ bisects $\angle DBC$, $\overline{CF}$ bisects $\angle BCE$

$\therefore m(\angle BCF) < m(\angle FBC)$

$\therefore m(\angle FBC) > m(\angle BCF)$ (Q.E.D.)

18



In ΔDBC : $\because DB > DC$

$\therefore m(\angle 3) > m(\angle 4)$ (1)

In ΔDAC : $\because DA > DC$

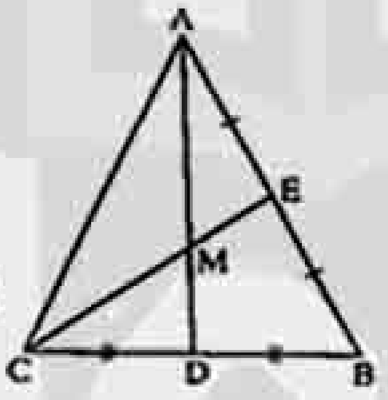
$\therefore m(\angle 2) > m(\angle 1)$ (2)

From (1) and (2) and adding :

$\therefore m(\angle 3) + m(\angle 2) > m(\angle 4) + m(\angle 1)$

$\therefore m(\angle ACB) > m(\angle DBC) + m(\angle DAC)$ (Q.E.D.)

19



$\because \overline{AD}$, $\overline{CE}$ are two medians of ΔABC intersecting at M

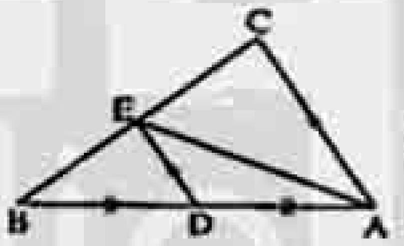
$\because AM = 2MD$, $MC = 2ME$

$\because MD > ME$ $\therefore AM > MC$

, thus in ΔAMC

$\therefore m(\angle CAM) < m(\angle MCA)$ (Q.E.D.)

20



In ΔABC : $\because$ D is the midpoint of $\overline{AB}$, $\overline{DE} \parallel \overline{AC}$

$\therefore DE = \frac{1}{2} AC$

, $\because AD = \frac{1}{2} AB$, $\because AB > AC$

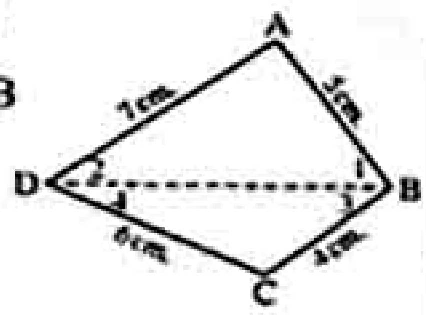
$\therefore \frac{1}{2} AB > \frac{1}{2} AC$

$\therefore AD > DE$ $\therefore m(\angle AED) > m(\angle DAE)$

But $m(\angle AED) = m(\angle CAE)$ (alternate angles)

$\therefore m(\angle CAE) > m(\angle DAE)$ (Q.E.D.)

21



First construction : Draw $\overline{BD}$

Proof : In ΔABD : $\because AD > AB$

$\therefore m(\angle 1) > m(\angle 2)$ (1)

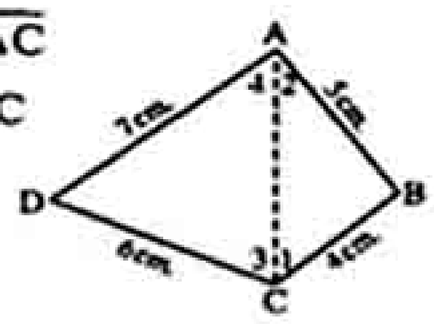
In ΔCBD : $\because CD > CB$

$\therefore m(\angle 3) > m(\angle 4)$ (2)

Adding (1) and (2) :

$\therefore m(\angle 1) + m(\angle 3) > m(\angle 2) + m(\angle 4)$

$\therefore m(\angle ABC) > m(\angle ADC)$ (Q.E.D.1)



Second construction : Draw $\overline{AC}$

Proof : In ΔABC : $\because BA > BC$

$\therefore m(\angle 1) > m(\angle 2)$ (3)

In ΔADC : $\because AD > DC$

$\therefore m(\angle 3) > m(\angle 4)$ (4)

Adding (3) and (4) :

$\therefore m(\angle 1) + m(\angle 3) > m(\angle 2) + m(\angle 4)$

$\because m(\angle BCD) > m(\angle BAD)$ (Q.E.D.2)

$\because$ The sum of measure of the interior angles of the quadrilateral = 360°

and from the two preceding requirements

$\therefore m(\angle B) + m(\angle C) > \frac{360°}{2}$

$\therefore m(\angle B) + m(\angle C) > 180°$ (Q.E.D.3)

22

$\because \overline{AE}$ is a median in ΔABD , $m(\angle A) = 90°$

$\therefore AE = \frac{1}{2} BD$

, $\because$ E is the midpoint of $\overline{BD}$, $\overline{EX} \parallel \overline{AC}$

$\therefore EX = \frac{1}{2} DC$

, $\because AE > EX$

$\therefore \frac{1}{2} BD > \frac{1}{2} DC$

$\therefore BD > DC$

$\therefore m(\angle C) > m(\angle DBC)$ (Q.E.D.)

23

In ΔABM : $\because AM > BM$ $\therefore m(\angle ABM) > m(\angle A)$ (1)

$\because AM = CM$ $\therefore$ In ΔCBM : $MC > MB$

$\therefore m(\angle MBC) > m(\angle C)$ (2)

Adding (1) and (2) :

$\therefore m(\angle ABM) + m(\angle MBC) > m(\angle A) + m(\angle C)$

$\therefore m(\angle ABC) > m(\angle A) + m(\angle C)$

$\therefore \angle ABC$ is an obtuse angle. (Q.E.D)

24

In ΔABD : $\because m(\angle B) = 90° - m(\angle BAD)$ (1)

From ΔACD: $m(\angle C) = 90° - m(\angle CAD)$ (2)

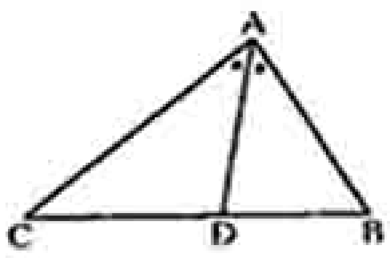
From ΔABC : $AC > AB$ $\therefore m(\angle B) > m(\angle C)$ (3)

From (1) , (2) and (3) :

$\therefore 90° - m(\angle BAD) > 90° - m(\angle CAD)$

$\therefore m(\angle BAD) < m(\angle CAD)$ (Q.E.D.)

25



In ΔABC : $\because AC > AB$

$\therefore m(\angle B) > m(\angle C)$

$\because m(\angle BAD) = m(\angle DAC)$

($\overline{AD}$ bisects $\angle A$)

$\therefore m(\angle B) + m(\angle BAD) > m(\angle C) + m(\angle DAC)$

$\because \angle ADC$ is an exterior angle of ΔABD

$\therefore m(\angle ADC) = m(\angle B) + m(\angle BAD)$

$\because \angle ADB$ is an exterior angle of ΔADC

$\therefore m(\angle ADB) = m(\angle C) + m(\angle DAC)$

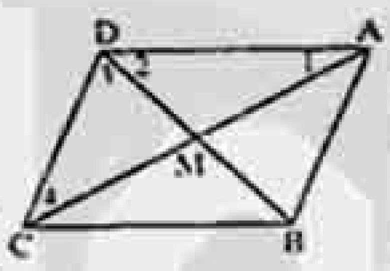
$\therefore m(\angle ADC) > m(\angle ADB)$

$\because m(\angle ADC) + m(\angle ADB) = 180°$

$\therefore m(\angle ADC) > \frac{180°}{2}$ i.e. $m(\angle ADC) > 90°$

i.e. $\angle ADC$ is an obtuse angle. (Q.E.D.)

26



Let $\overline{CA} \cap \overline{DB} = \{M\}$

$\because$ The two diagonals in the parallelogram bisects each other

, $\because AC > BD$ $\therefore MA > MD$,

$MC > MD$,

From ΔAMD : $\because AM > MD$

$\therefore m(\angle 2) > m(\angle 1)$ (1)

From ΔDMC : $\because MC > MD$

$\therefore m(\angle 3) > m(\angle 4)$ (2)

Adding (1) and (2) :

$\therefore m(\angle 2) + m(\angle 3) > m(\angle 1) + m(\angle 4)$

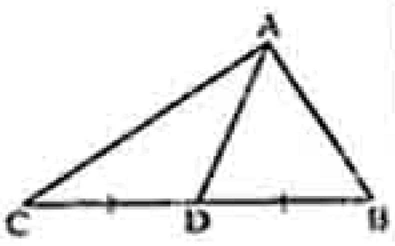
$\therefore m(\angle D) > m(\angle 1) + m(\angle 4)$

$\therefore$ In ΔADC :

$\because m(\angle D) > m(\angle CAD) + m(\angle ACD)$

$\therefore \angle D$ is an obtuse angle. (Q.E.D.)

27



$\because$ The perimeter of ΔACD

$= CD + DA + AC$

The perimeter of ΔABD

$= BD + DA + AB$

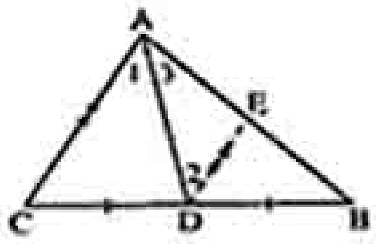
$\because$ The perimeter of $\Delta ACD >$ The perimeter of ΔABD

$\therefore CD + DA + AC > BD + DA + AB$

But $CD = BD$

$\therefore AC > AB$ $\therefore m(\angle B) > m(\angle C)$ (Q.E.D.)

28



Construction : Draw $\overline{DE} \parallel \overline{AC}$ to intersect $\overline{AB}$ at E

Proof : In ΔABC : $\because \overline{DE} \parallel \overline{AC}$,

D is the midpoint of $\overline{BC}$

$\therefore$ E is the midpoint of $\overline{BA}$

$\therefore AE = \frac{1}{2}AB$, $DE = \frac{1}{2}AC$

, $\because AB > AC$ $\therefore AE > DE$

$\because m(\angle 2) > m(\angle 3)$ (1)

$\because \overline{DE} \parallel \overline{AC}$, $\overline{AD}$ is a transversal to them

$\therefore m(\angle 1) = m(\angle 2)$ (Alternate angles)

From (1) : $\therefore m(\angle 1) > m(\angle 3)$

$\therefore m(\angle BAD) < m(\angle CAD)$ (Q.E.D.)

Answers of Exercise 8

1

1. A side greater in length than that opposite to the other angle , greater in measure than the measure of the angle opposite to the other side.
2. The shortest side.
3. The hypotenuse.
4. The length of the line segment drawn from the given point perpendicular to the given straight line.
5. $\overline{AB}$
6. $\overline{AC}$
7. $\overline{BC}$

2

1. c
2. a
3. d

3

1. > , > , <
2. > , > , >
3. > , > , > , >
4. > , < , < , >

YZ < XY < XZ

AC > AB > BC

$\because \overline{AE} \parallel \overline{BC}$, $\overline{AC}$ is a transversal

$\therefore m(\angle C) = m(\angle EAC) = 30°$ (alternate angles) (1)

, $\because \overline{AE} \parallel \overline{BC}$, $\overline{AB}$ is a transversal

$\therefore m(\angle B) = m(\angle DAE) = 70°$ (corresponding angles) (2)

From (1) and (2) : $\therefore m(\angle B) > m(\angle C)$

$\therefore AC > AB$ (Q.E.D.)

7

$\because C \in \overrightarrow{AE}$ $\therefore m(\angle ACB) = 180° - 120° = 60°$

$\because B \in \overrightarrow{CD}$ $\therefore m(\angle ABC) = 180° - 110° = 70°$

$\therefore m(\angle A) = 180° - (60° + 70°) = 50°$

$\therefore m(\angle ACB) > m(\angle A)$ $\therefore AB > BC$ (Q.E.D.)

8

In ΔABC : $\because AB = AC$

$\therefore m(\angle ACB) = m(\angle B) = 65°$

$\therefore m(\angle DCB) = 65° + 20° = 85°$

In ΔDBC : $\therefore m(\angle D) = 180° - (65° + 85°) = 30°$

$\therefore$ In ΔDAC : $m(\angle D) > m(\angle ACD)$

$\therefore AC > AD$ but $AB = AC$

$\therefore AB > AD$ (Q.E.D.)

9

In ΔDBC : $\because DB = DC$

$\therefore m(\angle B) = m(\angle DCB) = \frac{180° - 100°}{2} = 40°$

$\because \overrightarrow{CD}$ bisects $\angle ACB$ $\therefore m(\angle ACD) = 40°$

$\because D \in \overline{AB}$

$\therefore m(\angle ADC) = 180° - 100° = 80°$

$\therefore$ In ΔADC : $m(\angle A) = 180° - (40° + 80°) = 60°$

$\therefore m(\angle ADC) > m(\angle A)$

$\therefore AC > DC$ but $DC = DB$

$\therefore AC > DB$ (Q.E.D.)

10

$\because \overline{AD} // \overline{BC}$, $\overline{AC}$ is a transversal

$\therefore m(\angle ACB) = m(\angle DAC) = 30°$ (alternate angles)

In ΔABC : $\because m(\angle BAC) > m(\angle ACB)$

$\therefore BC > AB$ (Q.E.D.)

11

In ΔACM : $\because m(\angle C) = 90°$ $\therefore AM > CM$ (1)

In ΔBDM : $\because m(\angle D) = 90°$ $\therefore BM > DM$ (2)

Adding (1) and (2) : $\therefore AM + MB > CM + MD$

$\therefore AB > CD$ (Q.E.D.)

12

In ΔABC : $\because AB = AC$

$\therefore m(\angle ABC) = m(\angle ACB)$

$\because m(\angle ABM) < m(\angle ACM)$

$\therefore m(\angle ABC) - m(\angle ABM) > m(\angle ACB) - m(\angle ACM)$

$\therefore m(\angle MBC) > m(\angle MCB)$

From ΔMBC : $\therefore MC > MB$ (Q.E.D.)

13

In ΔABC : $\because \angle B$ is an obtuse angle

$\therefore m(\angle B) > m(\angle C)$ (1)

$\because \overline{DE} // \overline{BC}$, $\overleftrightarrow{DB}$ is a transversal

$\therefore m(\angle ADE) = m(\angle B)$ (corresponding angles) (2)

$\because \overline{DE} // \overline{BC}$, $\overleftrightarrow{EC}$ is a transversal

$\therefore m(\angle AED) = m(\angle C)$ (corresponding angles) (3)

From (1) , (2) and (3) :

$\therefore m(\angle ADE) > m(\angle AED)$

$\therefore AE > AD$ (Q.E.D.)

14

In ΔABC : $\because AB > AC$ $\therefore m(\angle C) > m(\angle B)$ (1)

$\because \overline{DE} // \overline{BC}$ and $\overleftrightarrow{DC}$ is a transversal

$\therefore m(\angle D) = m(\angle C)$ (alternate angles) (2)

$\therefore \overline{DE} // \overline{BC}$, $\overleftrightarrow{BE}$ is a transversal

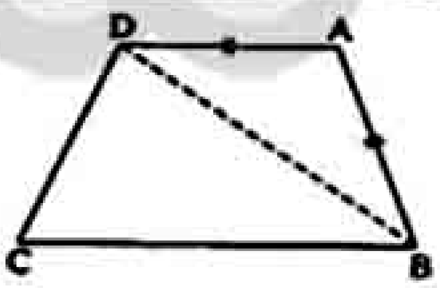
$\therefore m(\angle E) = m(\angle B)$ (alternate angles) (3)

From (1) , (2) and (3) :

$\therefore m(\angle D) > m(\angle E)$ and from ΔADE

$\therefore AE > AD$ (Q.E.D.)

15



Const. : Draw $\overline{BD}$

Proof : In ΔADB

$\because AD = AB$

$\therefore m(\angle ADB) = m(\angle ABD)$

$, \because m(\angle ADC) > m(\angle ABC)$

$\therefore m(\angle ADC) - m(\angle ADB) > m(\angle ABC) - m(\angle ABD)$

$\therefore m(\angle BDC) > m(\angle DBC)$

$\therefore$ In ΔBDC : $BC > CD$ (Q.E.D.)

16

In ΔABC : $\because AB > AC$

$\therefore m(\angle ABC) < m(\angle ACB)$

$, \because B \in \overline{AD}$, $C \in \overline{AE}$

$\therefore 180° - m(\angle ABC) > 180° - m(\angle ACB)$

$\therefore m(\angle CBD) > m(\angle BCE)$

$\because \overline{BF}$ bisects $\angle DBC$, $\overline{CF}$ bisects $\angle BCE$

$\therefore m(\angle FBC) > m(\angle BCF)$ (Q.E.D.1)

$\therefore CF > BF$ (Q.E.D.2)

17

In ΔABD : $\because BD = AD$

$\therefore m(\angle BAD) = m(\angle B)$

$\therefore m(\angle BAD) + m(\angle DAC) > m(\angle B)$

$\therefore m(\angle BAC) > m(\angle B)$ $\therefore BC > AC$ (Q.E.D.)

18

In ΔDBC : $\because m(\angle B) > m(\angle DCB)$

$\therefore DC > DB$ but $DB = AD$ $\therefore DC > AD$

$\therefore$ In ΔADC : $m(\angle A) > m(\angle ACD)$ (Q.E.D.1)

$\because m(\angle BDC) = 180° - (70° + 50°) = 60°$

$\because \angle BCD$ is an exterior angle of ΔADC

$\therefore m(\angle BDC) = m(\angle A) + m(\angle ACD) = 60°$

$\because m(\angle A) > m(\angle ACD)$ $\therefore m(\angle ACD) < 30°$

$\therefore m(\angle ACD) + m(\angle DCB) < 30° + 50°$

$\therefore m(\angle ACB) < 80°$

$\therefore \angle ACB$ is an acute angle (Q.E.D.2)

19

In ΔAFB : $\because FA = FB$

$\therefore m(\angle FBA) = m(\angle FAB) = 50°$ (1)

$\because \angle AFD$ is an exterior angle of ΔAFB

$\therefore m(\angle AFD) = 50° + 50° = 100°$ (2)

$\therefore$ In ΔAFD

$\because FA = FD$ $\therefore m(\angle FDA) = \frac{180° - 100°}{2} = 40°$

From (1) and (2) : $\therefore$ In ΔABD

$m(\angle ABD) > m(\angle ADB)$

$\therefore AD > AB$ (Q.E.D.1)

In ΔABD , $\because \overline{AF}$ is a median , $AF = \frac{1}{2}BD$

$\therefore m(\angle DAB) = 90°$

$\therefore \overline{BC}$ is a hypotenuse of ΔBAC

$\therefore BC > AC$ (Q.E.D.2)

20

$\because \angle ADB$ is an exterior angle of ΔADC

$\therefore m(\angle ADB) > m(\angle C)$

, $\because m(\angle C) = m(\angle B)$, ($AB = AC$ in ΔABC)

$\therefore m(\angle ADB) > m(\angle B)$

And from ΔABD : $AB > AD$ (Q.E.D.)

21

$\Delta\Delta$ ABD , AED in them

$m(\angle B) = m(\angle AED)$, $m(\angle BAD) = m(\angle DAE)$

$\therefore m(\angle ADB) = m(\angle ADE)$

$\therefore \Delta\Delta$ ABD , AED

In them $\begin{cases} m(\angle BAD) = m(\angle EAD) \\ m(\angle ADB) = m(\angle ADE) \\ \overline{AD} \text{ is a common side} \end{cases}$

$\therefore \Delta ABD \equiv \Delta AED$ then we deduce that

$BD = DE$ (Q.E.D.1)

$\therefore$ In ΔDEC

$\because m(\angle DEC) = 90°$ $\therefore DC > DE$

$\because DE = DB$ $\therefore DC > DB$ (Q.E.D.2)

22

$\because m(\angle ADC) = 180° - 110° = 70°$

$\therefore \Delta ACD$ in which $m(\angle ADC) > m(\angle C)$

$\therefore AC > AD$ (1)

$\because \Delta ADB$ is an obtuse-angled at D

$\therefore AB > AD$ (2)

By adding (1) and (2) : $\therefore AB + AC > 2\,AD$ (Q.E.D.)

23

In ΔABC , $\because m(\angle B) = 90°$

$\therefore$ The hypotenuse $\overline{AC}$ is the longest side

$\therefore AB < AC$, $BC < AC$

By adding : $\therefore AB + BC < 2\,AC$ (Q.E.D.)

24

$\because \overline{AD} \parallel \overline{CE}$, $\overline{AC}$ is a transversal

$\therefore m(\angle DAC) = m(\angle ACE)$

$\therefore m(\angle BCE) > m(\angle DAC)$

$\therefore m(\angle BCE) > m(\angle BAD)$, ($\overline{AD}$ bisects $\angle BAC$) (1)

$\because \overline{AD} \parallel \overline{CE}$ and $\overline{BE}$ is a transversal

$\therefore m(\angle BAD) = m(\angle E)$ (corresponding angles) (2)

From (1) and (2) :

$\therefore m(\angle BCE) > m(\angle E)$ and from ΔBCE

$\therefore BE > BC$ (Q.E.D.)

25

$\because \Delta XYM$ is right-angled at Y

$\therefore \angle XMY$ is an acute angle

$\therefore \angle XMZ$ is an obtuse angle

$\because \Delta XMZ$ is an obtuse-angled at M

$\therefore XZ > XM$ (Q.E.D.)

26

In ΔABC : $\because AB > BC$

$\therefore m(\angle C) > m(\angle A)$ (1)

, $\because m(\angle ABC) = 90°$

$\therefore \angle A$ complements $\angle C$ (2)

, in ΔABD : $\because m(\angle ADB) = 90°$

$\therefore \angle A$ complements $\angle ABD$ (3)

From (2) and (3) :

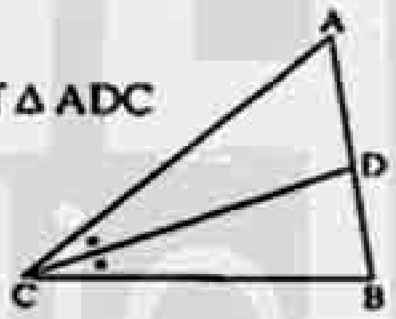
$\therefore m(\angle C) = m(\angle ABD)$

, from (1) :

$\therefore m(\angle ABD) > m(\angle A)$

$\therefore$ In ΔABD : $AD > BD$ (Q.E.D.)

27



$\because \angle BDC$ is an exterior angle of ΔADC

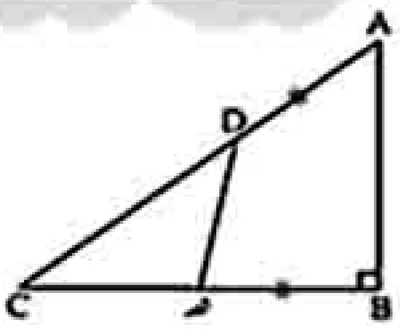
$\therefore m(\angle BDC) > m(\angle ACD)$

$\because m(\angle BCD) = m(\angle ACD)$

$\therefore m(\angle BDC) > m(\angle BCD)$

In ΔDBC : $\therefore BC > BD$ (Q.E.D.)

28



In ΔABC : $\because m(\angle B) = 90°$

$\therefore AC > BC$

$\because AD = BE$

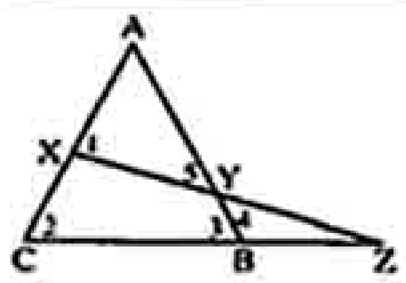
$\therefore AC - AD > BC - BE$

$\therefore DC > EC$

$\therefore$ In ΔDEC : $\because DC > EC$

$\therefore m(\angle CED) > m(\angle CDE)$ (Q.E.D.)

29



$\because \angle 1$ is an exterior angle of ΔXZC

$\therefore m(\angle 1) > m(\angle 2)$

But $m(\angle 3) = m(\angle 2)$, ($AB = AC$ in ΔABC)

$\therefore m(\angle 1) > m(\angle 3)$

But $\angle 3$ is an exterior angle of ΔYZB

$\therefore m(\angle 3) > m(\angle 4)$ $\therefore m(\angle 1) > m(\angle 4)$

, $\because m(\angle 4) = m(\angle 5)$ (V.O.A.)

$\therefore m(\angle 1) > m(\angle 5)$

and from ΔAYX $\therefore AY > AX$ (Q.E.D.)

30

$\because m(\angle A) + m(\angle B) + m(\angle C) = 180°$

$\therefore 5X + 2° + 6X - 10° + X + 20° = 180°$

$\therefore 12X + 12° = 180°$ $\therefore 12X = 180° - 12° = 168°$

$\therefore X = \frac{168°}{12°} = 14°$ $\therefore m(\angle A) = 5 \times 14° + 2° = 72°$

, $m(\angle B) = 6 \times 14° - 10° = 74°$

$m(\angle C) = 14° + 20° = 34°$

$\therefore AB < BC < AC$ (The req.)

31

In ΔABC : $\because AB < AC$

$\therefore m(\angle ACB) < m(\angle ABC)$

$\therefore \frac{1}{2} m(\angle ACB) < \frac{1}{2} m(\angle ABC)$

$\therefore m(\angle MCB) < m(\angle MBC)$ (1)

And from ΔMBC : $MB < MC$ (2)

$\because \overline{XY} \parallel \overline{CB}$, $\overline{XB}$ is a transversal

$\therefore m(\angle X) = m(\angle MBC)$ (alternate angles) (3)

$\because \overline{XY} \parallel \overline{BC}$, $\overline{CY}$ is a transversal

$\therefore m(\angle Y) = m(\angle MCB)$ (alternate angles)

$\because$ In ΔXMY from (1) , (2) and (3) :

$\because m(\angle Y) < m(\angle X)$ $\therefore XM < MY$ (4)

By adding (2) and (4) : $\therefore MB + MX < MC + MY$

$\therefore BX < CY$ (Q.E.D.)

Answers of Exercise 9

1

1. $\because 3 + 4 < 9$ $\therefore$ lengths are not suitable
2. $\because 5 + 7 > 8$ $\therefore$ lengths are suitable
3. $\because 4 + 6 = 10$ $\therefore$ lengths are not suitable
4. $\because 6 + 8 > 13$ $\therefore$ lengths are suitable
5. $\because 3 + 4 > 5$ $\therefore$ lengths are suitable
6. $\because 9 + 9 < 19$ $\therefore$ lengths are not suitable

2

Let the length of the third side be l

1 $\because 9-6<l<9+6 \quad \therefore 3<l<15$

$\therefore l \in]3 , 15[$

2 $\because 3-3<l<3+3 \quad \therefore 0<l<6$

$\therefore l \in]0 , 6[$

3 $\because 3.2-2.9<l<3.2+2.9$

$\therefore 0.3<l<6.1 \quad \therefore l \in]0.3 , 6.1[$

4 $\because 7.3-5.7<l<7.3+5.7$

$\therefore 1.6<l<13 \quad \therefore l \in]1.6 , 13[$

3

1 b 2 b 3 c 4 d 5 a

6 b 7 d 8 b 9 a

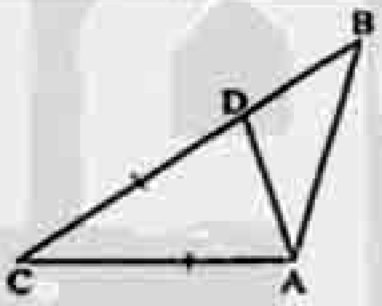
4

In Δ XLY : XL + LY > XY (The triangle inequality)

But XL = LZ $\quad \therefore$ LZ + LY > XY

$\therefore$ YZ > XY (Q.E.D.)

5



In Δ ABC :

$\because$ CA + AB > BC

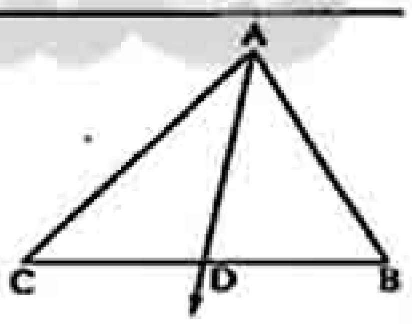
(triangle inequality)

$\therefore$ CA + AB > BD + DC

But CA = DC

$\therefore$ AB > BD (Q.E.D.)

6



From Δ ABD :

AD + DB > AB

(triangle inequality) (1)

From Δ ADC : AD + DC > AC

(Triangle inequality) (2)

Adding (1) and (2) :

$\therefore$ BD + DC + 2 AD > AB + AC (Q.E.D.)

7

From Δ ABM : MA + MB > AB

(Triangle inequality) (1)

From Δ BMC : MB + MC > BC

(Triangle inequality) (2)

From Δ AMC : MA + MC > AC (Triangle inequality) (3)

Adding (1) , (2) and (3) :

$\therefore$ 2 MA + 2 MB + 2 MC > AB + BC + AC

$\therefore$ MA + MB + MC > $\frac{1}{2}$ the perimeter of Δ ABC

(Q.E.D.)

8

From Δ AEZ : AE + AZ > EZ (Triangle inequality) (1)

From Δ EBF :

EB + BF > EF (Triangle inequality) (2)

From Δ ZFC : ZC + CF > ZF (Triangle inequality) (3)

Adding (1) , (2) and (3) :

$\therefore$ AB + AC + BC > EZ + EF + ZF

$\therefore$ The perimeter of Δ ABC > The perimeter of Δ EFZ

(Q.E.D.)

9

In Δ DAC : DA + DC > AC (1)

In Δ DBC : DB + DC > BC (2)

In Δ DBA : DB + DA > AB (3)

Adding (1) , (2) and (3) :

$\therefore$ 2 (DA + DB + DC) > AC + BC + AB

$\therefore$ AC + BC + AB < 2 (DA + DB + DC)

$\therefore$ The perimeter of Δ ABC < 2 (DA + DB + DC)

(Q.E.D.)

10

Assuming that ABC is a triangle

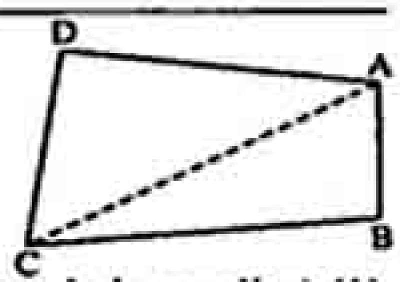
$\because$ AB < AC + BC adding AB to both sides

$\therefore$ 2 AB < AC + BC + AB

$\therefore$ AB < $\frac{1}{2}$ the perimeter of Δ ABC

$\therefore$ The length of any side in the triangle is less than the half of the perimeter of the triangle (Q.E.D.)

11



Construction : Draw $\overline{AC}$

Proof : From Δ ABC

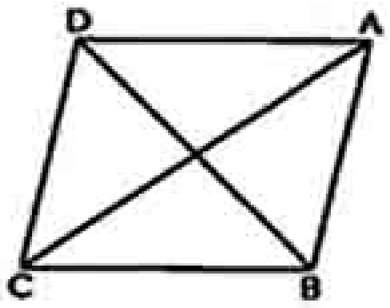
AB + BC > AC (Triangle inequality) (1)

From Δ ADC :

AC + CD > AD (Triangle inequality) (2)

From (1) and (2) : $\therefore$ AB + BC + CD > AD (Q.E.D.)

12



Let ABCD be a quadrilateral

In Δ ABC : AB + BC > AC (1)

In Δ BCD : BC + CD > BD (2)

In Δ ACD : AD + CD > AC (3)

In Δ ABD : AB + AD > BD (4)

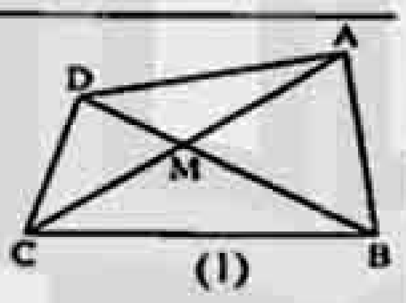
Adding (1) , (2) , (3) , and (4) :

$\because$ 2 AB + 2 BC + 2 CD + 2 AD > 2 AC + 2 BD

$\therefore$ AB + BC + CD + AD > AC + BD

$\therefore$ The sum of lengths of the two diagonals in any convex quadrilateral is less than the perimeter of the quadrilateral (Q.E.D.)

13



Let ABCD be a quadrilateral ,

$\overline{AC} \cap \overline{BD} = \{M\}$

From Δ ABM : AB < MA + MB (1)

From Δ BMC : BC < MB + MC (2)

From Δ CMD : CD < MC + MD (3)

From Δ AMD : AD < MA + MD (4)

Adding (1) , (2) , (3) and (4)

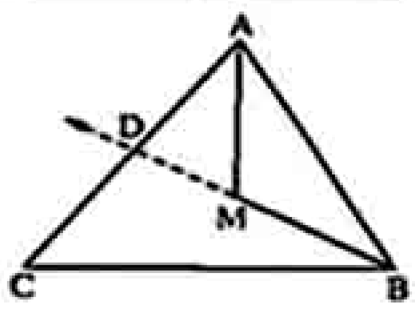
$\therefore$ AB + BC + CD + AD < 2 MA + 2 MC + 2 MB + 2 MD

$\therefore$ AB + BC + CD + DA < 2 (MA + MC) + 2 (MB + MD)

$\therefore$ AB + BC + CD + DA < 2 (AC + BD)

$\therefore$ The perimeter of the quadrilateral ABCD < twice the sum of lengths of the two diagonals. (Q.E.D.)

14



Construction :

Draw $\overline{BM}$ to cut $\overline{AC}$ at D

Proof :

In Δ BDC :

BC + DC > BD (Triangle inequality)

$\therefore$ BC + DC > BM + MD (1)

$\because$ In Δ AMD : AD + MD > AM (Triangle inequality)

$\therefore$ AD > AM − MD (2)

Adding (1) , (2) :

$\therefore$ BC + AD + DC > BM + MD + AM − MD

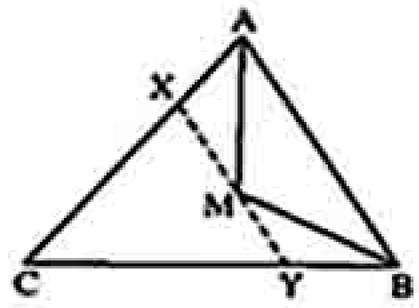
$\therefore$ BC + AC > BM + AM

$\therefore$ AM + MB < BC + AC (Q.E.D.)

Another solution :

Construction :

Draw $\overline{XY}$ Passing through the point M where



$X \in \overline{AC}$, $Y \in \overline{BC}$

Proof : In Δ CXY

CY + CX > XM + MY adding BY and AX to both sides

$\therefore$ CY + BY + CX + AX > XM + AX + MY + BY

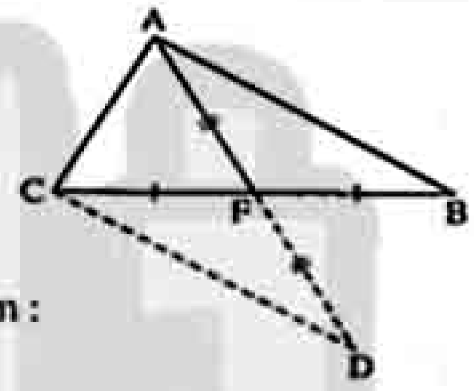
$\therefore$ BC + AC > XM + AX + MY + BY

$\because$ XM + AX > AM , MY + BY > MB

$\therefore$ BC + AC > AM + MB

$\therefore$ AM + MB < AC + BC (Q.E.D.)

15



Construction :

Extend $\overline{AF}$ as its length to D then draw $\overline{CD}$

Proof : $\Delta\Delta$ AFB , DFC in them :

$\begin{cases} AF = DF \text{ const.} \\ BF = FC \text{ (given)} \\ m(\angle AFB) = m(\angle DFC) \text{ (V.O.A.)} \end{cases}$

$\therefore$ The two triangles are congruent

then we deduce that AB = DC

but in Δ ACD we find that

AC + CD > AD (triangle inequality)

$\therefore$ AC + AB > AD

$\because$ AD = 2 AF

$\therefore$ AC + AB > 2 AF (1) (Q.E.D.1)

From Δ ABC : $\because$ AB + AC > BC

i.e. AB + AC > 2 BF (2)

Adding (1) and (2) : 2 AB + 2 AC > 2 AF + 2 BF

Dividing by 2 : $\therefore$ AB + AC > AF + BF (Q.E.D.2)

Answers of exams on unit five

Model 1

1

1 b 2 a 3 a
4 a 5 c 6 b

2

1 greater in measure than the angle opposite to the other side.
2 the hypotenuse 3 $\angle C$
4 < 5 2 , 8

3

[a] The order is : AB , AC , BC
[b] Prove by yourself.

4

[a] Prove by yourself.
[b] Prove by yourself.

5

[a] Prove by yourself.
[b] Prove by yourself.

Model 2

1

1 c 2 c 3 a
4 d 5 d 6 a

2

1 a side greater in length than that opposite to the other angle.
2 $\overline{BC}$ 3 BC , AC
4 $\overline{BC}$ 5 <

3

[a] Prove by yourself.
[b] Prove by yourself.

4

[a] The order is : m ($\angle$ A) , m ($\angle$ B) and m ($\angle$ C)
[b] Prove by yourself.

5

[a] Prove by yourself.
[b] Prove by yourself.

Answers of accumulative basic skills

1

1 $2\sqrt{10}$ 2 2 : 3 3 5
4 150° 5 18 6 54
7 $\frac{1}{2}$ P – y 8 $5\sqrt{3}$ 9 108°
10 60 11 9 12 19

2

1 (b) 2 (c) 3 (a)
4 (c) 5 (a) 6 (c)
7 (d) 8 (a) 9 (c)
10 (b) 11 (d) 12 (c)

Guide Answers

of The Notebook

- Quizzes.
- Final Examinations.

Answers of the quizzes on Algebra and Statistics

Quiz (1)

1

1 d 2 b 3 c

2

[a] 1 $\{3\}$ 2 $\{1\}$

[b] 20 cm.

Quiz (2)

1

1 c 2 c 3 a

2

[a] Prove by yourself.

[b] Represent by yourself.

Quiz (3)

1

1 $\varnothing$ 2 $\dot{Q}$ 3 -125

2

[a] 1 $X \simeq 2.4$ or $X \simeq -2.4$

2 $X \simeq 4.7$

[b] $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$ (There are other numbers)

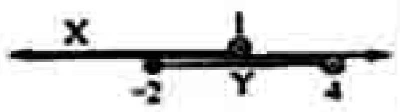
Quiz (4)

1

1 c 2 b 3 b

2

[a] 1 $X \cup Y =]-\infty , 4[$



2 $X \cap Y = [-2 , 1[$

3 $X - Y =]-\infty , -2[$

4 $\grave{X} = [1 , \infty[$

[b] $\{-\sqrt{32} , \sqrt{32}\}$

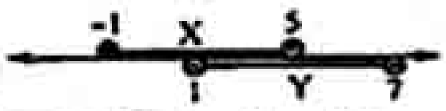
Quiz (5)

1

1 b 2 d 3 b

2

[a] 1 $\sqrt{3} - 3$ 2 $-2 + \sqrt{3}$

[b]


1 $X \cap Y =]1 , 5[$

2 $X \cup Y = [-1 , 7[$

3 $X - Y = [-1 , 1]$

Quiz (6)

1

1 c 2 c 3 d

2

[a] 0

[b] $\{-\sqrt{3} , \sqrt{3}\}$

Quiz (7)

1

1 $\sqrt{3} - \sqrt{2}$ 2 2 3 $]1 , 5[$

2

[a] 12

[b]


$\mathbb{R} - [1 , 2]$

Quiz (8)

1

1 b 2 d 3 c

2

[a] $\frac{1}{2}$

[b] $\sqrt{2} - \sqrt[3]{9}$

Quiz (9)

1

1 a 2 c 3 c

2

[a] 18π cm^2

[b] 20

Quiz 10

1

1 c   2 d   3 c

2

[a] $[-2, 1[$

[b] $\sqrt{5}$

Quiz 11

1

1 c   2 b   3 a

2

[a] $(0, -2)$, $(3, 0)$, $(6, 2)$

, there are other solutions , represent by yourself.

[b] $a = -3$, $b = 0$

Quiz 12

1

1 -125 2 $\frac{1}{3}$ 3 undefined

2

[a] Represent by yourself

, the area of $\Delta AOB = 4$ square units

[b] Prove by yourself.

Quiz 13

1

1 c   2 b   3 c

2

[a] 14 cm.

[b] 1 40 litre   2 $\frac{1}{3}$ litre/min.

3 After 2 hr.

Quiz 14

1

1 c   2 b   3 a

2

Answer by yourself.

Quiz 15

1

1 b   2 c   3 c

2

1 20 workers

2 Draw by yourself.

Quiz 16

1

1 3   2 $]-\infty, -3[$ 3 1230

2

[a] 1 $k = 22$

2 The arithmetic mean = 50.6

[b] $\sqrt{7}$

Quiz 17

1

1 c   2 d   3 c

2

15 approximately

Quiz 18

1

1 2   2 $2\sqrt[3]{4}$ 3 0

2

[a] 1 $x = 30$, $k = 5$

2 24.5 approximately.

[b] Prove by yourself , 1

Answers of school book model examinations on algebra and statistics

Model 1

1

1 $\{-1\}$ 2 20 3 $[-2, 2]$

4 24 5 $\sqrt{3}-\sqrt{2}$

2

1 d 2 c 3 c

4 c 5 a 6 b

3

[a] $\sqrt{2\times 9}+\sqrt[3]{2\times 27}-3\sqrt{2}-\frac{1}{2}\sqrt[3]{2\times 8}$

$=3\sqrt{2}+3\sqrt[3]{2}-3\sqrt{2}-\frac{1}{2}\times 2\sqrt[3]{2}$

$=2\sqrt[3]{2}$

[b] $\because X=\frac{3}{\sqrt{5}-\sqrt{2}}\times\frac{\sqrt{5}+\sqrt{2}}{\sqrt{5}+\sqrt{2}}=\frac{3(\sqrt{5}+\sqrt{2})}{5-2}$

$=\sqrt{5}+\sqrt{2}$

$\therefore$ X and y are two conjugate numbers.

4

[a] $\because$ The area of the square $=\frac{1}{2}d^2$

$\therefore \frac{1}{2}d^2=1089$

$\therefore d^2=2178$

$\therefore d=\sqrt{2178}=33\sqrt{2}$ cm.

[b] $\because 6\times\frac{3X+1}{6}<6(X+1)<6\times\frac{X+4}{2}$

$\therefore 3X+1<6X+6<3X+12$

$\therefore 3X-3X+1<6X-3X+6<3X-3X+12$

$\therefore 1<3X+6<12$ $\therefore 1-6<3X<12-6$

$\therefore -5<3X<6$ $\therefore -\frac{5}{3}<X<2$

$\therefore$ The S.S. $=]-\frac{5}{3}, 2[$

5

[a] The volume of the cylinder $=\pi r^2 h$

$=(4\sqrt{2})^2\times 9\times\pi$

$=288\pi$ cm³

, $\because$ the volume of the cylinder

= the volume of the sphere

$\therefore$ The volume of the sphere $=288\pi$ cm³

$\therefore \frac{4}{3}\pi r^3=288\pi$

$\therefore r^3=288\times\frac{3}{4}=216$

$\therefore r=\sqrt[3]{216}=6$ cm.

[b]

Sets	X	f	$X\times f$
5 –	10	7	70
15 –	20	10	200
25 –	30	12	360
35 –	40	13	520
45 –	50	8	400
Total		50	1550

$\therefore$ The mean $=\frac{1550}{50}=31$

Model 2

1

1 $\sqrt{3}+\sqrt{5}$ 2 6 3 $3+\sqrt{10}$

4 3 5 $]3, 4]$

2

1 b 2 a 3 b

4 c 5 c 6 d

3

[a] $\frac{\sqrt{3}(\sqrt{5}+\sqrt{3})+\sqrt{5}(\sqrt{5}-\sqrt{3})}{(\sqrt{5}-\sqrt{3})(\sqrt{5}+\sqrt{3})}$

$=\frac{\sqrt{15}+3+5-\sqrt{15}}{5-3}=\frac{8}{2}=4$

[b] The left hand side

$=\sqrt[3]{2\times 64}+\sqrt[3]{2\times 8}-2\sqrt[3]{2\times 27}$

$=4\sqrt[3]{2}+2\sqrt[3]{2}-2\times 3\sqrt[3]{2}=6\sqrt[3]{2}-6\sqrt[3]{2}=0$

= the right hand side

4

[a] $\because -2-7 < 3X+7-7 \le 10-7$

$\therefore -9 < 3X \le 3$

$\therefore -3 < X \le 1$

$\therefore$ The S.S. $=]-3 , 1]$

[b] $\because X = \sqrt{2+\sqrt{3}}$ $\therefore X^2 = 2+\sqrt{3}$

$\therefore X^4 = (2+\sqrt{3})^2 = 4+4\sqrt{3}+3 = 7+4\sqrt{3}$

$\therefore X^4 - 2X^2 + 1 = 7+4\sqrt{3}-4-2\sqrt{3}+1 = 4+2\sqrt{3}$

5

[a] 20

[b]

Sets	X	f	$X \times f$
5 –	10	4	40
15 –	20	5	100
25 –	30	6	180
35 –	40	3	120
45 –	50	2	100
Total		20	540

$\therefore$ The mean $= \frac{540}{20} = 27$

Answers of model for the merge students

1

1 $\sqrt{3}-\sqrt{2}$ 2 $3\sqrt{6}$ 3 3

4 5 5 $\varnothing$

2

1 a 2 b 3 a 4 a 5 a

3

1 $\{5 , -5\}$ 2 $[0 , 2]$ 3 7

4 irrational

5

4

1 ✔ 2 ✔ 3 ✔ 4 ✘ 5 ✔

5

[a] The centre $= \frac{8+4}{2} = 6$

[b]

Sets	The centre of the set « X »	Frequency « f »	$X \times f$
5 –	10	7	$10 \times 7 = 70$
15 –	20	10	$20 \times 10 = 200$
25 –	30	12	$30 \times 12 = 360$
35 –	40	13	$40 \times 13 = 520$
45 –	50	8	$50 \times 8 = 400$
Total		50	1550

$\therefore$ The arithmetic mean $= \frac{\text{The sum of } (X \times f)}{\text{The sum of } (f)}$

$= \frac{1550}{50} = 31$

Answers of schools examinations on algebra and statistics

1 Cairo

1 ① (d) ② (b) ③ (d) ④ (a) ⑤ (a) ⑥ (d)

2 ① $[1, 3]$ ② 4 ③ 6 ④ $]2, 3[$ ⑤ 0

3

[a] $a^2 - ab + b^2 = (a - b)^2 + ab$

$= \left(\sqrt{3}+\sqrt{2}-\sqrt{3}+\sqrt{2}\right)^2 + \left(\sqrt{3}+\sqrt{2}\right)\left(\sqrt{3}-\sqrt{2}\right)$

$= \left(2\sqrt{2}\right)^2 + 1 = 8 + 1 = 9$

[b] ① $\because 5x - 3 < 2x + 9 \qquad \therefore 3x < 12$

$\therefore x < 4$

$\therefore$ The S.S. $=]-\infty, 4[$

② $\because 1 \le 3 - 2x < 5 \qquad \therefore -2 \le -2x < 2$

$\therefore -1 < x \le 1$

$\therefore$ The S.S. $=]-1, 1]$

4

[a]

① $M \cap J = [2, 3[$ ② $M - J = [3, \infty[$

[b] $\dfrac{\sqrt{3}}{\sqrt{5}-\sqrt{3}} + \dfrac{\sqrt{5}}{\sqrt{5}+\sqrt{3}} = \dfrac{\sqrt{3}\left(\sqrt{5}+\sqrt{3}\right)+\sqrt{5}\left(\sqrt{5}-\sqrt{3}\right)}{\left(\sqrt{5}-\sqrt{3}\right)\left(\sqrt{5}+\sqrt{3}\right)}$

$= \dfrac{\sqrt{15}+3+5-\sqrt{15}}{5-3} = \dfrac{8}{2} = 4$

5

[a] $2\sqrt{9 \times 2} + \sqrt{25 \times 2} + \frac{1}{3}\sqrt{81 \times 2}$

$= 6\sqrt{2} + 5\sqrt{2} + 3\sqrt{2} = 14\sqrt{2}$

[b]

Sets	x	f	$x \times f$
5 –	10	4	40
15 –	20	5	100
25 –	30	6	180
35 –	40	3	120
45 –	50	2	100
Total		20	540

$\therefore$ The mean $= \frac{540}{20} = 27$

2 Cairo

1 ① (a) ② (d) ③ (b) ④ (a) ⑤ (b) ⑥ (c)

2 ① $\{2, 7\}$ ② 2 ③ $\mathbb{R}_-$ ④ $\frac{2}{3}$ ⑤ $\{\sqrt{12}, -\sqrt{12}\}$

3

[a] $2\sqrt{4 \times 2} + \sqrt{25 \times 2} - \sqrt{16 \times 2}$

$= 4\sqrt{2} + 5\sqrt{2} - 4\sqrt{2} = 5\sqrt{2}$

[b] $\because 3x - 4 \le 5 \qquad \therefore 3x \le 9$

$\therefore x \le 3 \qquad \therefore$ The S.S. $=]-\infty, 3]$

4

[a] $\because x = \dfrac{2}{\sqrt{7}-\sqrt{5}} \times \dfrac{\sqrt{7}+\sqrt{5}}{\sqrt{7}+\sqrt{5}}$

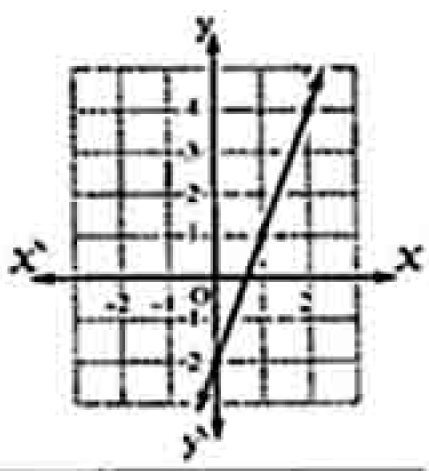
$= \dfrac{2\left(\sqrt{7}+\sqrt{5}\right)}{7-5} = \sqrt{7}+\sqrt{5}$

$\therefore (x + y)^2 = \left(\sqrt{7}+\sqrt{5}+\sqrt{7}-\sqrt{5}\right)^2$

$= \left(2\sqrt{7}\right)^2 = 28$

[b] $y = 3x - 2$

x	0	1	2
y	−2	1	4



5

[a] $\because$ The volume $= \frac{4}{3}\pi r^3 \qquad \therefore \frac{500}{3}\pi = \frac{4}{3}\pi r^3$

$\therefore r^3 = \frac{500}{3} \times \frac{3}{4} = 125 \qquad \therefore r = 5$ cm.

[b]

Sets	x	f	$x \times f$
5 –	10	7	70
15 –	20	10	200
25 –	30	12	360
35 –	40	13	520
45 –	50	8	400
Total		50	1550

$\therefore$ The mean $= \frac{1550}{50} = 31$

3 Cairo

1 1 (d) 2 (b) 3 (a) 4 (a) 5 (b) 6 (b)

2 1 -27 2 2 3 9 4 $[3, 5]$ 5 7

3

[a] $\because$ The volume $= \frac{4}{3}\pi r^3$ $\therefore 562.5\,\pi = \frac{4}{3}\pi r^3$

$\therefore r^3 = \frac{562.5 \times 3}{4} = 421.875$ $\therefore r = 7.5$ cm.

$\therefore$ The area $= 4 \times \pi \times (7.5)^2 = 225\,\pi$ cm^2.

[b] $\because X = \frac{4}{\sqrt{7}+\sqrt{3}} \times \frac{\sqrt{7}-\sqrt{3}}{\sqrt{7}-\sqrt{3}} = \frac{4\left(\sqrt{7}-\sqrt{3}\right)}{7-3}$

$= \sqrt{7}-\sqrt{3}$

$\therefore X^2 - 2Xy + y^2 = (X-y)^2$

$= \left(\sqrt{7}-\sqrt{3}-\sqrt{7}-\sqrt{3}\right)^2$

$= \left(-2\sqrt{3}\right)^2 = 12$

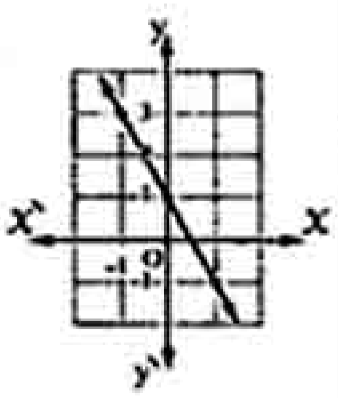
4

[a] $\because -1 < 3X + 5 \le 14$ $\therefore -6 < 3X \le 9$

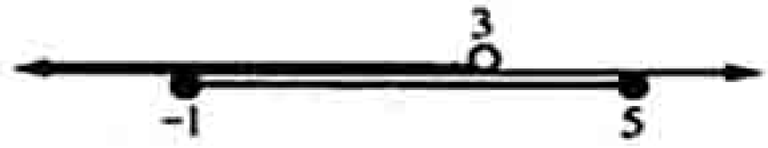
$\therefore -2 < X \le 3$ $\therefore$ The S.S. $=]-2, 3]$

[b] $2X + y = 1$

x	-1	0	1
y	3	1	-1



[c]



1 $A \cap B = [-1, 3[$ 2 $A - B =]-\infty, -1[$

5

[a] The slope of $\overleftrightarrow{AB} = \frac{5-3}{2+1} = \frac{2}{3}$

, $\because$ the slope of $\overleftrightarrow{BC} = \frac{1-5}{8-2} = \frac{-2}{3}$

$\therefore$ The slope of $\overleftrightarrow{AB} \neq$ the slope of $\overleftrightarrow{BC}$ $\therefore C \notin \overleftrightarrow{AB}$

[b]

Sets	x	f	$x \times f$
5 –	10	7	70
15 –	20	10	200
25 –	30	12	360
35 –	40	13	520
45 –	50	8	400
Total		50	1550

$\therefore$ The mean $= \frac{1550}{50} = 31$

4 Giza

1 1 8 2 $[-3, 4[$ 3 3 4 2.5 5 3

2 1 (b) 2 (a) 3 (c) 4 (d) 5 (b) 6 (c)

3

[a]

1 $\acute{X} =]-\infty, -2[\cup]4, \infty[$

2 $X \cap Y =]1, 4]$

3 $X - Y = [-2, 1]$

[b] $\because 2X + 1 < 7$ $\therefore 2X < 6$

$\therefore X < 3$ $\therefore$ The S.S. $=]-\infty, 3[$

4

[a] $2\sqrt{2 \times 9} + \sqrt{2 \times 25} - \sqrt{2 \times 81}$

$= 6\sqrt{2} + 5\sqrt{2} - 9\sqrt{2} = 2\sqrt{2}$

[b] $\because y = \frac{4}{3+\sqrt{5}} \times \frac{3-\sqrt{5}}{3-\sqrt{5}} = \frac{4\left(3-\sqrt{5}\right)}{9-5} = 3-\sqrt{5}$

, $X = 3+\sqrt{5}$ $\therefore X, y$ are conjugate numbers.

$\therefore X^2 - 2Xy + y^2 = (X-y)^2$

$= \left(3+\sqrt{5}-3+\sqrt{5}\right)^2 = \left(2\sqrt{5}\right)^2 = 20$

5

[a] $\because$ The volume of the cuboid = $77 \times 24 \times 21$

$= 38808\ cm^3$

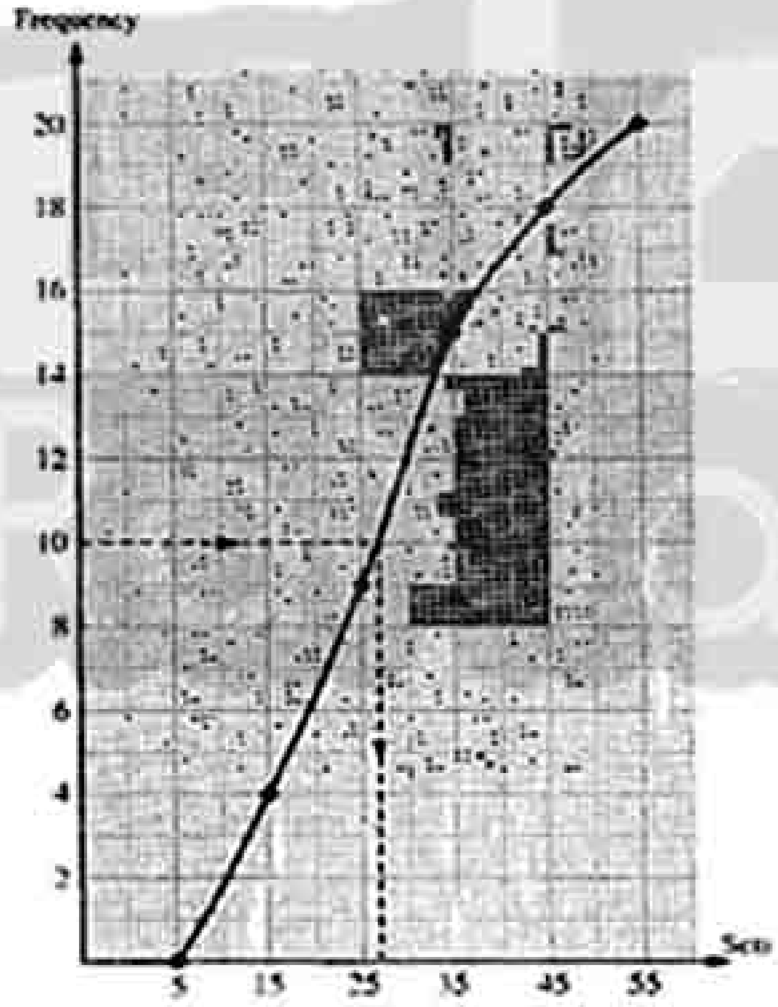
$\therefore$ The volume of the sphere = $38808\ cm^3$

$\therefore \frac{4}{3} \pi r^3 = 38808$

$\therefore r^3 = \frac{38808 \times 7 \times 3}{22 \times 4} = 9261$ $\therefore r = 21$ cm.

[b]

The upper boundaries of sets	Ascending cumulative frequency
less than 5	0
less than 15	4
less than 25	9
less than 35	15
less than 45	18
less than 55	20



$\therefore$ The order of the median = $\frac{20}{2} = 10$

$\therefore$ The median = 27

5 Giza

1 [1] (c) [2] (d) [3] (d) [4] (d) [5] (b) [6] (a)

2 [1] 4 [2] $2X$ [3] $1 : 2$ [4] 0 [5] $]2, 3[$

3

[a] $\because y = \frac{1}{X} = \frac{1}{\sqrt{7}-\sqrt{6}} \times \frac{\sqrt{7}+\sqrt{6}}{\sqrt{7}+\sqrt{6}}$

$= \frac{\sqrt{7}+\sqrt{6}}{7-6} = \sqrt{7}+\sqrt{6}$

$\therefore (X+y)^2 = \left(\sqrt{7}-\sqrt{6}+\sqrt{7}+\sqrt{6}\right)^2$

$= \left(2\sqrt{7}\right)^2 = 28$

[b] $\because -15 \le 2X - 3 \le 5$ $\therefore -12 \le 2X \le 8$

$\therefore -6 \le X \le 4$ $\therefore$ The S.S. = $[-6, 4]$

[c] $\sqrt[3]{27 \times 2} + 4\sqrt[3]{8 \times \frac{1}{4}} + 5\sqrt[3]{8 \times 2}$

$= 3\sqrt[3]{2} + 4\sqrt[3]{2} + 10\sqrt[3]{2} = 17\sqrt[3]{2}$

4

[a]

[1] $X \cap Y =]1, 5]$ [2] $X \cup Y =]-\infty, 9[$

[3] $X - Y =]-\infty, 1]$

[b] The slope = $\frac{5-4}{4-2} = \frac{1}{2}$

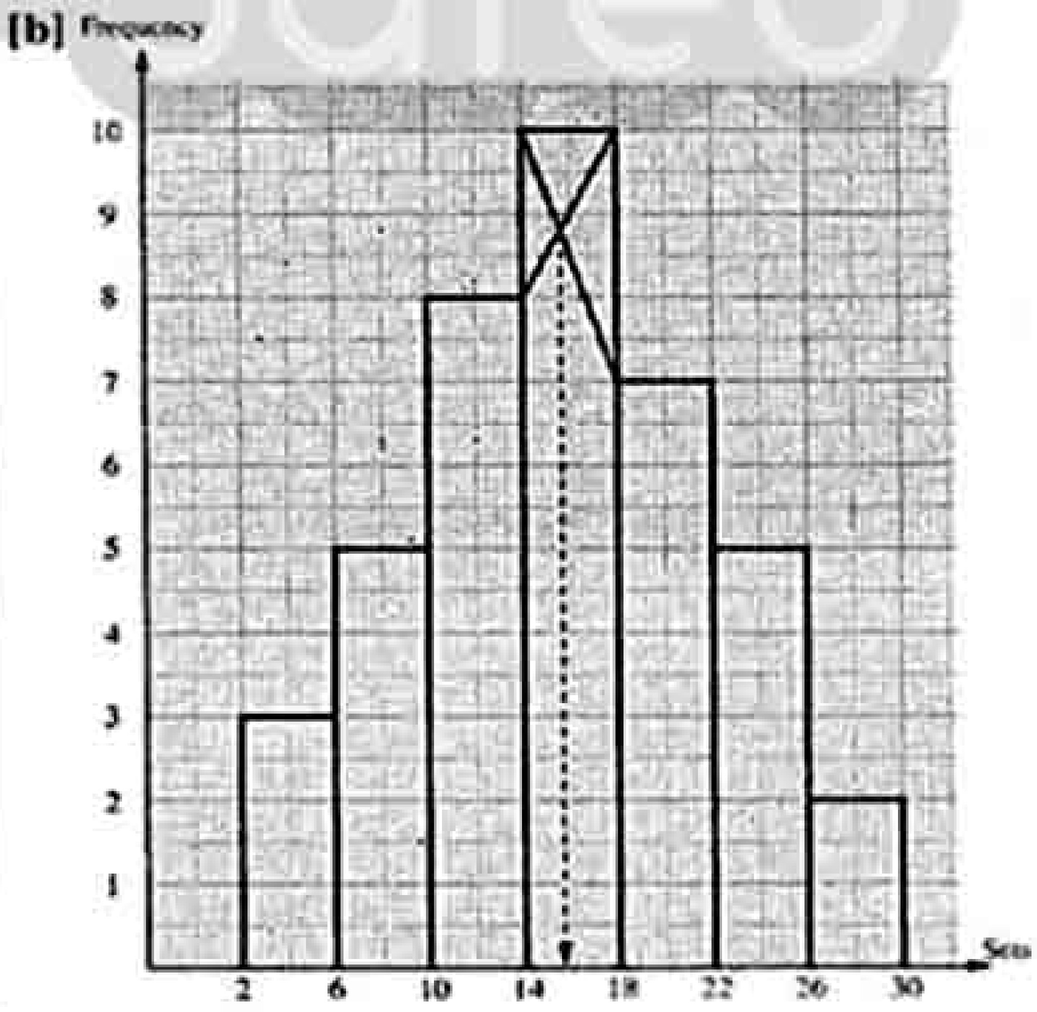
5

[a] $\because 125X^3 - 7 = 20$ $\therefore 125X^3 = 27$

$\therefore X^3 = \frac{27}{125}$ $\therefore X = \frac{3}{5}$

$\therefore$ The S.S. = $\left\{\frac{3}{5}\right\}$

[b]



$\therefore$ The mode = 16

6 Alexandria

1
1. (a) 2. (a) 3. (c)
4. (b) 5. (d) 6. (c)

2
1. zero 2. 5 3. $[-2, 2]$
4. 3 5. 24 cm.

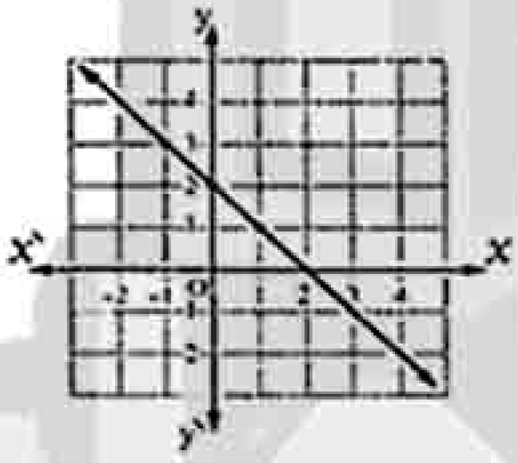
3

[a] $\sqrt{9\times 2}+\sqrt[3]{27\times 2}-3\sqrt{2}-\frac{1}{2}\sqrt[3]{8\times 2}$

$=3\sqrt{2}+3\sqrt[3]{2}-3\sqrt{2}-\sqrt[3]{2}=2\sqrt[3]{2}$

[b] $y=2-X$

X	-1	0	1
y	3	2	1



4

[a] $\because -2<3X+7\leq 10$ $\quad\therefore -9<3X\leq 3$

$\therefore -3<X\leq 1$ $\quad\therefore$ The S.S. $=]-3, 1]$

[b] $\dfrac{\sqrt{3}(\sqrt{5}+\sqrt{3})+\sqrt{5}(\sqrt{5}-\sqrt{3})}{(\sqrt{5}-\sqrt{3})(\sqrt{5}+\sqrt{3})}$

$=\dfrac{\sqrt{15}+3+5-\sqrt{15}}{5-3}=\dfrac{8}{2}=4$

5

[a] $\because (\sqrt{3})^{X}=(2\sqrt{2})^2-(\sqrt{5})^2=8-5=3$

$\therefore (\sqrt{3})^{X}=(\sqrt{3})^2$ $\quad\therefore X=2$

[b]

Sets	X	f	$X\times f$
5 –	10	7	70
15 –	20	10	200
25 –	30	12	360
35 –	40	13	520
45 –	50	8	400
Total		50	1550

$\therefore$ The mean $=\dfrac{1550}{50}=31$

7 Alexandria

1. (c) 2. (c) 3. (b)
4. (b) 5. (d) 6. (d)

1. $-\sqrt[3]{5}$ 2. $\mathbb{R}^*$ 3. $\sqrt{a}-\sqrt{b}, 2\sqrt{a}$
4. 2 5. 0

3

[a] 1. $\sqrt{16\times 2}-\sqrt{25\times 2}+2\sqrt{4\times\frac{1}{2}}$

$=4\sqrt{2}-5\sqrt{2}+2\sqrt{2}=\sqrt{2}$

2. $\sqrt[3]{8\times 2}-\frac{1}{3}\sqrt[3]{27\times 2}=2\sqrt[3]{2}-\sqrt[3]{2}=\sqrt[3]{2}$

[b] $\because y=\dfrac{2}{X}=\dfrac{2}{\sqrt{7}+\sqrt{5}}\times\dfrac{\sqrt{7}-\sqrt{5}}{\sqrt{7}-\sqrt{5}}$

$=\dfrac{2(\sqrt{7}-\sqrt{5})}{7-5}=\sqrt{7}-\sqrt{5}$

$\therefore \dfrac{X+y}{Xy}=\dfrac{\sqrt{7}+\sqrt{5}+\sqrt{7}-\sqrt{5}}{(\sqrt{7}+\sqrt{5})(\sqrt{7}-\sqrt{5})}=\dfrac{2\sqrt{7}}{7-5}=\sqrt{7}$

4

[a] $\because -1\leq 3-2X<5$ $\quad\therefore -4\leq -2X<2$

$\therefore 2\geq X>-1$ $\quad\therefore$ The S.S. $=]-1, 2]$

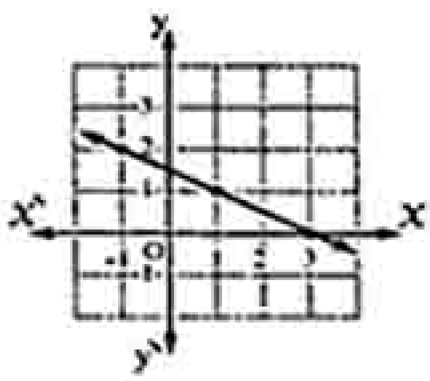
[b] $\because$ The volume $=\pi r^2 h$

$\therefore \pi r^2\times r=72\pi$ $\quad\therefore r^3=72$

$\therefore r=2\sqrt[3]{9}$ $\quad\therefore h=2\sqrt[3]{9}$ cm.

[c] $X+2y=3$

X	-1	1	3
y	2	1	0



5

[a] $\because$ The slope of $\overleftrightarrow{AB}=\dfrac{5-3}{2+1}=\dfrac{2}{3}$

, the slope of $\overleftrightarrow{BC}=\dfrac{1-5}{8-2}=\dfrac{-2}{3}$

$\therefore$ The slope of $\overleftrightarrow{AB}\neq$ the slope of $\overleftrightarrow{BC}$

$\therefore C\notin\overleftrightarrow{AB}$

[b]

Sets	x	f	$x \times f$
8 –	10	4	40
12 –	14	10	140
16 –	18	16	288
20 –	22	12	264
24 –	26	8	208
Total		50	940

$\therefore$ The mean $= \frac{940}{50} = 18.8$

8 El-Kalyoubia

1 (1) (b) (2) (c) (3) (c) (4) (a) (5) (a) (6) (a)

2 (1) $\frac{1}{4}$ (2) 0 (3) $\frac{3}{4}$ (4) -5 (5) $\sqrt{3}-\sqrt{2}$

3

[a] $\because x^3 - 1000 = 0$ $\therefore x^3 = 1000$

$\therefore x = \sqrt[3]{1000} = 10$

[b] $\because$ The area $= \pi r^2$ $\therefore \pi r^2 = 3\pi$

$\therefore r^2 = 3$ $\therefore r = \sqrt{3}$ cm.

$\therefore$ The circumference $= 2\pi r = 2\sqrt{3}\,\pi$ cm.

4

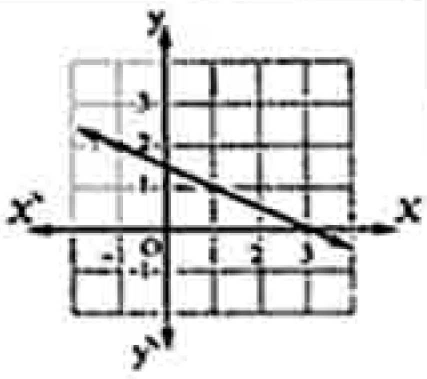
[a] $[2 , 3[$

[b] $(\sqrt{2}+5)(3+\sqrt{2}) = 3\sqrt{2} + 17 + 5\sqrt{2} = 17 + 8\sqrt{2}$

5

[a] $x + 2y = 3$

x	–1	1	3
y	2	1	0



[b]

Sets	x	f	$x \times f$
5 –	10	4	40
15 –	20	5	100
25 –	30	6	180
35 –	40	3	120
45 –	50	2	100
Total		20	540

$\therefore$ The mean $= \frac{540}{20} = 27$

9 El-Gharbia

1 (1) (d) (2) (c) (3) (c) (4) (d) (5) (b) (6) (b)

2 (1) 20 (2) $\sqrt{3}-\sqrt{2}$ (3) $]3 , 4]$ (4) 4 (5) $\frac{-4}{3}$

3

[a] (1) $\because y = \frac{2}{\sqrt{7}+\sqrt{5}} \times \frac{\sqrt{7}-\sqrt{5}}{\sqrt{7}-\sqrt{5}} = \frac{2(\sqrt{7}-\sqrt{5})}{7-5}$

$= \sqrt{7}-\sqrt{5}$

$, x = \sqrt{7}+\sqrt{5}$

$\therefore x , y$ are conjugate numbers.

(2) $xy = (\sqrt{7}+\sqrt{5})(\sqrt{7}-\sqrt{5}) = 7 - 5 = 2$

$, (x+y)^2 = (\sqrt{7}+\sqrt{5}+\sqrt{7}-\sqrt{5})^2$

$= (2\sqrt{7})^2 = 28$

[b] $\sqrt{4 \times 3} + \sqrt[3]{27 \times 2} - \sqrt{3} - \sqrt[3]{8 \times 2}$

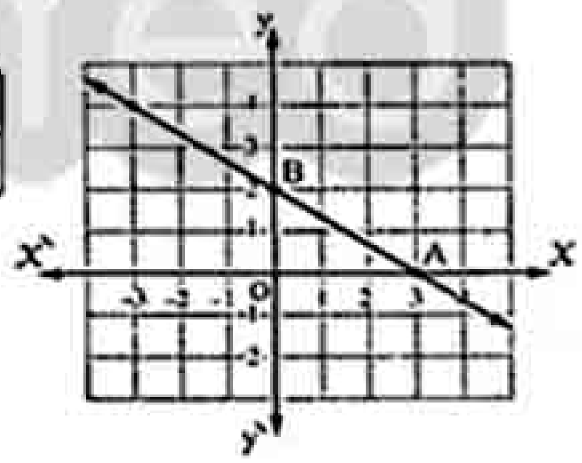
$= 2\sqrt{3} + 3\sqrt[3]{2} - \sqrt{3} - 2\sqrt[3]{2}$

$= \sqrt{3} + \sqrt[3]{2}$

4

[a] $2x + 3y = 6$

x	–3	0	3
y	4	2	0



From the graph :

The area of Δ OAB $= \frac{1}{2} \times 3 \times 2 = 3$ square units.

[b] $\because 8x^3 + 7 = 8$ $\therefore 8x^3 = 1$

$\therefore x^3 = \frac{1}{8}$ $\therefore x = \frac{1}{2}$

$\therefore$ The S.S. $= \{\frac{1}{2}\}$

5

[a] $\because 2x - 1 \geq 5$ $\therefore 2x \geq 6$

$\therefore x \geq 3$ $\therefore$ The S.S. $= [3 , \infty[$

[b]

Sets	x	f	$x \times f$
5 –	10	4	40
15 –	20	5	100
25 –	30	6	180
35 –	40	3	120
45 –	50	2	100
Total		20	540

$\therefore$ The mean $= \frac{540}{20} = 27$

10 El-Dakahlia

1 1 (d) 2 (d) 3 (b)
4 (a) 5 (c) 6 (c)

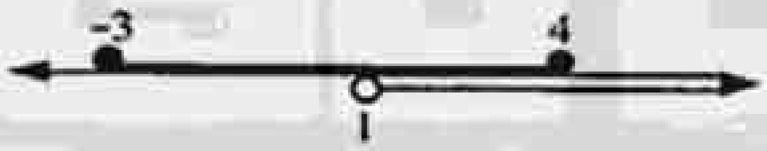
2 1 $]-3 , 7[$ 2 $\varnothing$ 3 3
4 0 5 $\frac{3}{2}$ cm.

3

[a] $\sqrt{9 \times 2} + \sqrt[3]{27 \times 2} - 3\sqrt{2} - \frac{1}{2}\sqrt[3]{8 \times 2}$

$= 3\sqrt{2} + 3\sqrt[3]{2} - 3\sqrt{2} - \sqrt[3]{2} = 2\sqrt[3]{2}$

[b]



1 $X \cap Y =]1 , 4]$

2 $X - Y = [-3 , 1]$

4

[a] $\because -7 \le -3x + 1 < 13$ $\therefore -8 \le -3x < 12$

$\therefore -4 < x \le \frac{8}{3}$ $\therefore$ The S.S. $=]-4 , \frac{8}{3}]$

[b] 1 $\because y = \frac{1}{\sqrt{6}+\sqrt{5}} \times \frac{\sqrt{6}-\sqrt{5}}{\sqrt{6}-\sqrt{5}} = \sqrt{6}-\sqrt{5}$

, $x = \sqrt{6}+\sqrt{5}$

$\therefore x , y$ are conjugate numbers.

2 $(x-y)^2 = \left(\sqrt{6}+\sqrt{5}-\sqrt{6}+\sqrt{5}\right)^2$

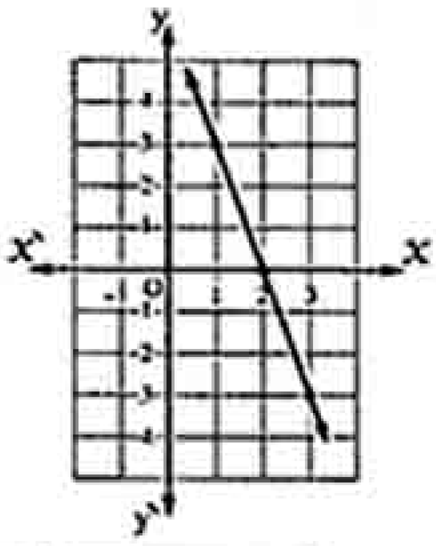
$= \left(2\sqrt{5}\right)^2 = 20$

5

[a] $y + 3x = 6$

x	1	2	3
y	3	0	–3

The slope $= \frac{-3-0}{3-2} = -3$



[b]

Sets	x	f	$x \times f$
10 –	15	5	75
20 –	25	15	375
30 –	35	20	700
40 –	45	25	1125
50 –	55	10	550
Total		75	2825

$\therefore$ The mean $= \frac{2825}{75} = \frac{113}{3}$

11 Ismailia

1 1 (b) 2 (c) 3 (a)
4 (b) 5 (d) 6 (b)

2 1 12 2 $\sqrt{3}+\sqrt{2}$ 3 11
4 0 5 3

3

[a] $\because 8 \le 3x + 2 \le 17$ $\therefore 6 \le 3x \le 15$

$\therefore 2 \le x \le 5$ $\therefore$ The S.S. $= [2 , 5]$

[b] $\sqrt{36 \times 2} + 3\sqrt{9 \times 2} - \sqrt{4 \times \frac{1}{2}}$

$= 6\sqrt{2} + 9\sqrt{2} - \sqrt{2} = 14\sqrt{2}$

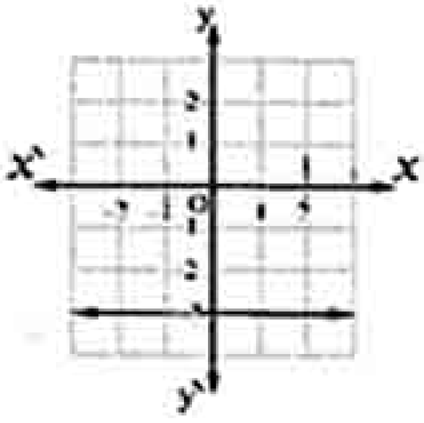
4

[a] $\because$ The volume $= \pi r^2 h$

$\therefore 1540 = \frac{22}{7} \times r^2 \times 10$ $\therefore r^2 = \frac{1540 \times 7}{22 \times 10} = 49$

$\therefore r = 7$ cm. $\therefore d = 14$ cm.

[b]



5

[a]

1 $X \cap Y = [-1, 3]$ 2 $X \cup Y =]-4, \infty[$

3 $\acute{X} =]-\infty, -1[$

[b]

Sets	x	f	$x \times f$
10 –	15	8	120
20 –	25	12	300
30 –	35	14	490
40 –	45	9	405
50 –	55	7	385
Total		50	1700

$\therefore$ The mean $= \frac{1700}{50} = 34$

12 Damietta

1 1 (b) 2 (d) 3 (b)
4 (c) 5 (b) 6 (a)

2 1 – 2 2 5 3 the median
4 4 5 6

3

[a] $\because y = \frac{3}{x} = \frac{3}{\sqrt{5}+\sqrt{2}} \times \frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}-\sqrt{2}} = \frac{3(\sqrt{5}-\sqrt{2})}{5-2}$

$= \sqrt{5}-\sqrt{2}$

$\therefore \frac{x+y}{xy} = \frac{\sqrt{5}+\sqrt{2}+\sqrt{5}-\sqrt{2}}{(\sqrt{5}+\sqrt{2})(\sqrt{5}-\sqrt{2})} = \frac{2\sqrt{5}}{3}$

[b] $\because -3 \le 4x - 7 \le 5$ $\therefore 4 \le 4x \le 12$

$\therefore 1 \le x \le 3$ $\therefore$ The S.S. $= [1, 3]$

[c] $\because$ The volume $= \pi r^2 h$ $\therefore 72\pi = \pi \times r^2 \times 8$

$\therefore r^2 = \frac{72}{8} = 9$ $\therefore r = 3$ cm.

4

[a] $\sqrt{25 \times 2} + \sqrt[3]{27 \times 2} - 5\sqrt{4 \times \frac{1}{2}} - \sqrt[3]{8 \times 2}$

$= 5\sqrt{2} + 3\sqrt[3]{2} - 5\sqrt{2} - 2\sqrt[3]{2} = \sqrt[3]{2}$

[b]

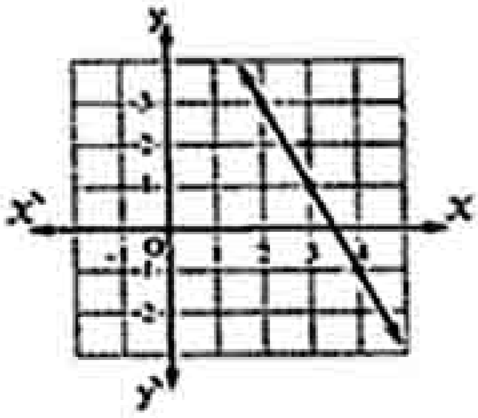
1 $X \cup Y = [-1, \infty[$ 2 $X \cap Y = [2, 5[$

3 $X - Y = [-1, 2[$

5

[a] $2x + y = 7$

x	2	3	4
y	3	1	–1



[b]

Sets	x	f	$x \times f$
5 –	10	4	40
15 –	20	5	100
25 –	30	6	180
35 –	40	3	120
45 –	50	2	100
Total		20	540

$\therefore$ The mean $= \frac{540}{20} = 27$

13 Kafr El-Sheikh

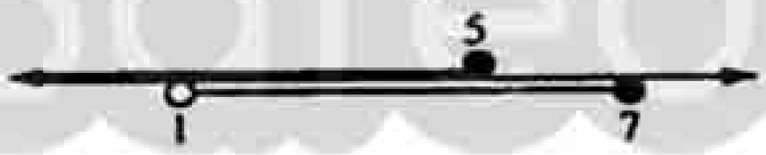
1 1 (c) 2 (b) 3 (a)
4 (d) 5 (c) 6 (b)

2 1 1 2 2 3 8.5
4 6 5 fourth

3

[a] The volume $= \pi r^2 h = \frac{22}{7} \times 5^2 \times 7 = 550$ cm^3

[b]



1 $X \cap Y =]1, 5]$ 2 $X \cup Y =]-\infty, 7]$

3 $Y - X =]5, 7]$

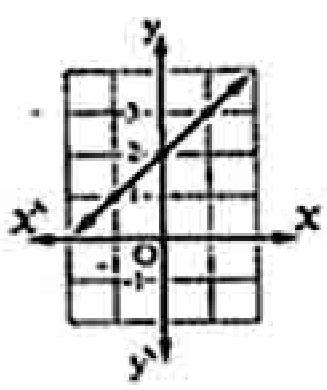
[c] $\because 8x^3 + 7 = 8$ $\therefore 8x^3 = 1$ $\therefore x^3 = \frac{1}{8}$

$\therefore x = \frac{1}{2}$ $\therefore$ The S.S. $= \{\frac{1}{2}\}$

4

[a] $y = x + 2$

x	–1	0	1
y	1	2	3



$\because (-4, a)$ satisfies the relation

$\therefore a = -4 + 2 = -2$

[b] $\sqrt{9\times2}+\sqrt{25\times2}-2\sqrt{4\times2}$

$=3\sqrt{2}+5\sqrt{2}-4\sqrt{2}=4\sqrt{2}$

[c] $\because -8<3X+1\le4$ $\quad\therefore -9<3X\le3$

$\therefore -3<X\le1$ $\quad\therefore$ The S.S. $=]-3,1]$

5

[a] $\because y=\frac{1}{\sqrt{3}+\sqrt{2}}\times\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}}$

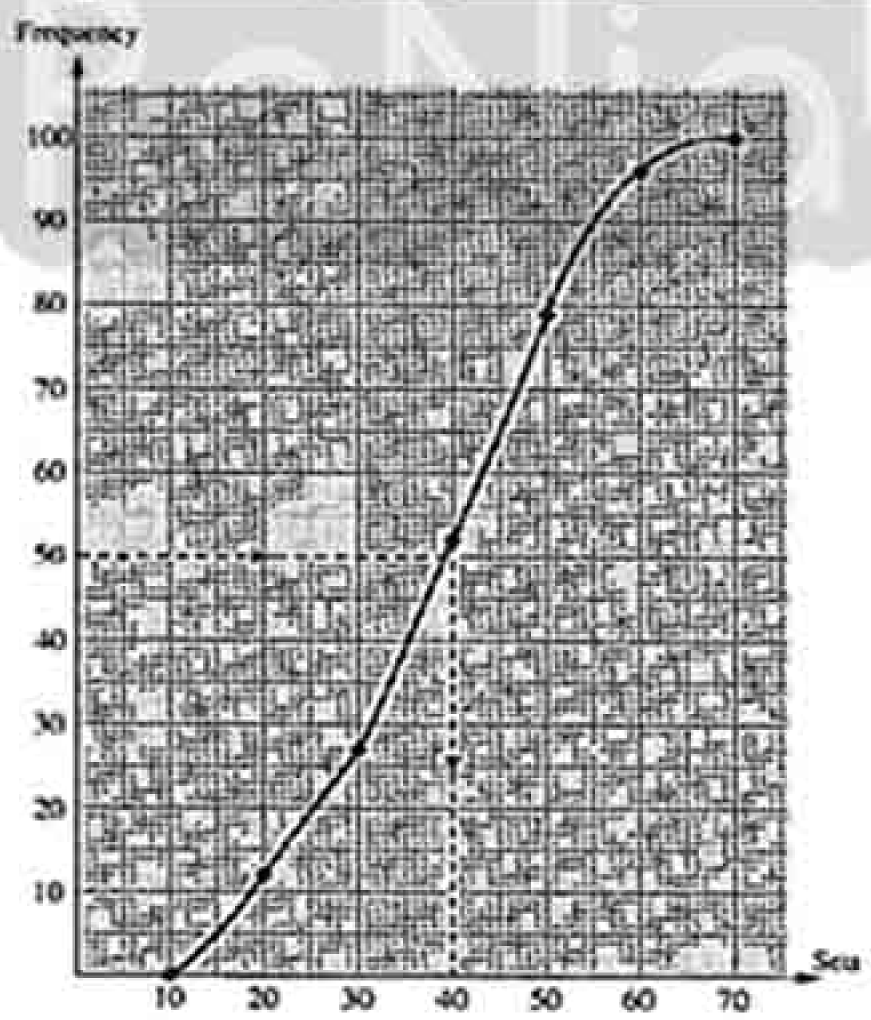
$=\sqrt{3}-\sqrt{2}$

$\therefore \frac{X+y}{Xy}=\frac{\sqrt{3}+\sqrt{2}+\sqrt{3}-\sqrt{2}}{(\sqrt{3}+\sqrt{2})(\sqrt{3}-\sqrt{2})}=2\sqrt{3}$

[b] 1 k = 13

2

The upper limits of sets	Ascending cumulative frequency
less than 10	0
less than 20	12
less than 30	27
less than 40	52
less than 50	79
less than 60	96
less than 70	100



$\because$ The order of the median $=\frac{100}{2}=50$

$\therefore$ The median $\simeq 40$

14 Souhag

1 1 (c) 2 (b) 3 (d) 4 (c) 5 (c) 6 (c)

2 1 $]1,5[$ 2 $\{0,1,-1\}$ 3 $6X$ 4 5 5 8

3

[a] $y=2X+1$

[b] $\because -2<3X+7\le10$ $\quad\therefore -9<3X\le3$

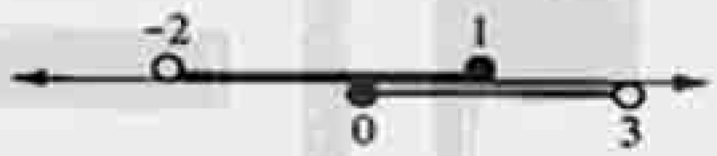
$\therefore -3<X\le1$ $\quad\therefore$ The S.S. $=]-3,1]$

4

[a] $\because y=\frac{1}{\sqrt{3}+\sqrt{2}}\times\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}}=\sqrt{3}-\sqrt{2}$

$\therefore \frac{X+y}{Xy}=\frac{\sqrt{3}+\sqrt{2}+\sqrt{3}-\sqrt{2}}{(\sqrt{3}+\sqrt{2})(\sqrt{3}-\sqrt{2})}=2\sqrt{3}$

[b]



1 $X\cap Y=[0,1]$ 2 $X\cup Y=]-2,3[$

3 $X-Y=]-2,0[$

5

[a] 1 $\sqrt{25\times2}+\sqrt{9\times2}-\sqrt{16\times2}$

$=5\sqrt{2}+3\sqrt{2}-4\sqrt{2}=4\sqrt{2}$

2 $\sqrt[3]{27\times2}+4\sqrt[3]{8\times\frac{1}{4}}+5\sqrt[3]{8\times2}$

$=3\sqrt[3]{2}+4\sqrt[3]{2}+10\sqrt[3]{2}=17\sqrt[3]{2}$

[b]

Sets	X	f	$X\times f$
5 –	10	4	40
15 –	20	5	100
25 –	30	6	180
35 –	40	3	120
45 –	50	2	100
Total		20	540

$\therefore$ The mean $=\frac{540}{20}=27$

15 Luxor

1 (1) (c) (2) (a) (3) (c) (4) (a) (5) (c) (6) (c)

2 (1) $]2 , 7[$ (2) 5 (3) 4 (4) undefined (5) 6

3

[a] $\sqrt{9 \times 3} - \sqrt{4 \times 3} + \sqrt{100 \times 3}$

$= 3\sqrt{3} - 2\sqrt{3} + 10\sqrt{3} = 11\sqrt{3}$

[b] $a^2 + 2ab + b^2 = (a + b)^2$

$= \left(\sqrt{5} + \sqrt{3} + \sqrt{5} - \sqrt{3}\right)^2 = \left(2\sqrt{5}\right)^2 = 20$

4

[a] $\because 2x + 1 \le 7$ $\therefore 2x \le 6$ $\therefore x \le 3$

$\therefore$ The S.S. $=]-\infty , 3]$

[b] The volume $= \frac{4}{3} \pi r^3 = \frac{4}{3} \times \frac{22}{7} \times (2.1)^3$

$= 38.808 \text{ cm}^3$.

5

[a] The slope of $\overline{AB} = \frac{3 + 1}{10 - 2} = \frac{1}{2}$

The slope of $\overline{BC} = \frac{3 - 3}{2 - 10} = 0$

[b]

Sets	x	f	$x \times f$
5 –	10	4	40
15 –	20	5	100
25 –	30	6	180
35 –	40	3	120
45 –	50	2	100
Total		20	540

$\therefore$ The mean $= \frac{540}{20} = 27$

Answers of the quizzes on Geometry

Quiz 1

1

1 one point 2 2 : 1 3 9

2

[a] The perimeter of Δ MBC = 26 cm.

[b] Prove by yourself.

Quiz 2

1

1 half the length of the hypotenuse

2 8

3 half the length of the hypotenuse

2

[a] BD = 4.5 cm. , BM = 3 cm. , AB = 4.5 cm.

[b] Prove by yourself.

Quiz 3

1

1 120° 2 35° 3 right

2

[a] Prove by yourself. [b] Prove by yourself.

Quiz 4

1

1 an equilateral triangle 2 AC

3 $\frac{1}{2}$

2

[a] Prove by yourself.

[b] Prove by yourself.

Quiz 5

1

1 bisects the base and is perpendicular to it

2 $\frac{1}{3}$ 3 equidistant

2

[a] AD = 6 cm.

[b] m ($\angle$ MLY) = 70°

Quiz 6

1

1 the measure of any interior angle of the triangle except its adjacent angle.

2 $\frac{2}{3}$ 3 >

2

[a] Prove by yourself.

[b] Prove by yourself.

Quiz 7

1

1 to the angle of the greater measure

2 axis of symmetry

3 B , A , C

2

[a] Prove by yourself.

[b] Prove by yourself.

Quiz 8

1

1 The hypotenuse 2 $\overline{AB}$

3 45°

2

[a] Prove by yourself.

[b] The perimeter of the figure ADME = 13 cm.

Quiz 9

1

1 c 2 b 3 d

2

[a] Prove by yourself.

[b] Prove by yourself.

Answers of school book model examinations on geometry

Model 1

1

1. The hypotenuse 2. 5 cm. , 9 cm.
3. a side greater in length than that opposite to the other angle.
4. The angle at this vertex is right 5. equilateral

2

1 c	2 a	3 b
4 b	5 a	6 d

3

[a] >

[b] $\because$ Δ DBC is equilateral triangle

$\therefore m(\angle DBC) = 60°$

In Δ ABC : AB = AC

$\therefore m(\angle ABC) = m(\angle ACB) = \frac{180° - 50°}{2} = 65°$

$\therefore m(\angle ABD) = 60° + 65° = 125°$ (The req.)

[c] $\because$ $\overleftrightarrow{AD} \parallel \overleftrightarrow{BC}$, $\overleftrightarrow{AC}$ is a transversal

$\therefore m(\angle ACB) = m(\angle DAC) = 50°$ (alternate angles)

In Δ ABC : $m(\angle B) = 180° - (70° + 50°) = 60°$

$\therefore m(\angle BAC) > m(\angle B)$

$\therefore$ BC > AC (Q. E. D.)

4

[a] Theoretical

[b] In Δ ABC : $\because$ AB = AC

$\therefore m(\angle ABC) = m(\angle ACB)$

$\therefore \frac{1}{2} m(\angle ABC) = \frac{1}{2} m(\angle ACB)$

$\therefore m(\angle DBC) = m(\angle DCB)$

$\therefore$ Δ DBC is an isosceles triangle (Q.E.D)

5

[a] $\because$ $\overline{AC}$ is the longest side.

$\therefore$ $\angle$ B is the greatest angle in measure

, $\because$ $\overline{AB}$ is the shortest side.

$\therefore$ $\angle$ C is the smallest angle in measure

$\therefore$ The descending order of measures of the angles is $m(\angle B)$, $m(\angle A)$ and $m(\angle C)$ (Q. E. D.)

[b] In Δ ABC : $\because$ AB > BC

$\therefore m(\angle ACB) > m(\angle BAC)$ (1)

, $\because$ $\overline{XY} \parallel \overline{BC}$, $\overline{AC}$ is a transversal

$\therefore m(\angle XYA) = m(\angle ACB)$

(corresponding angles) (2)

From (1) and (2) : $\therefore m(\angle XYA) > m(\angle BAC)$

$\therefore$ AX > XY (Q.E.D.)

Model 2

1

1 d	2 a	3 b
4 b	5 d	6 d

2

1. an isosceles triangle. 2. less than
3. XY 4. $\frac{1}{2}$ 5. perpendicular.

3

[a] $\because$ $\overline{AB}$ is the longest side.

$\therefore$ $\angle$ C is the greatest angle in measure

, $\because$ $\overline{CB}$ is the shortest side

$\therefore$ $\angle$ A is the smallest angle in measure

$\therefore$ The ascending order of measure of the angles is $m(\angle A)$, $m(\angle B)$ and $m(\angle C)$ (Q.E.D.)

[b] In Δ ABC : $\because m(\angle B) = 90°$

, $\because$ D is the midpoint of $\overline{AC}$

, $\because$ E is the midpoint of $\overline{BC}$

$\therefore$ $\overline{BD}$, $\overline{AE}$ are two medians in Δ ABC

$\therefore$ M is the intersection point of the medians of Δ ABC

$\therefore BD = \frac{1}{2} AC = \frac{1}{2} \times 9 = 4.5$ cm.

, $BM = \frac{2}{3} BD = \frac{2}{3} \times 4.5 = 3$ cm.

, $\because m(\angle C) = 30°$

$\therefore AB = \frac{1}{2} AC = \frac{1}{2} \times 9 = 4.5$ cm. (The req.)

4

[a] In Δ ABC : $\because$ D is the midpoint of $\overline{AC}$

$\therefore$ $\overline{BD}$ is a median

, $\because m(\angle ABC) = 90°$ $\therefore BD = \frac{1}{2} AC$ (1)

, In Δ BDE : $\because m(\angle BDE) = 90°$, $m(\angle E) = 30°$

$\therefore BD = \frac{1}{2} BE$ (2)

From (1) and (2) : $\therefore$ AC = BE (Q. E .D.)

[b] $\because \overline{AD} \parallel \overline{BC}$, $\overrightarrow{AC}$ is a transversal

$\therefore m(\angle ACB) = m(\angle CAD) = 30°$ (Alternate angles)

In ΔABC : $m(\angle B) = 180° - (70° + 30°) = 80°$

$\therefore m(\angle B) > m(\angle BAC)$

$\therefore AC > BC$ (Q. E. D.)

5

[a] a side greater in length than that opposite to the other angle.

[b] $\because \overrightarrow{AB}$ bisects $\angle YAZ$

$\therefore m(\angle YAB) = m(\angle BAZ)$ (1)

, $\because \overline{AB} \parallel \overline{XY}$, $\overline{AY}$ is a transversal

$\therefore m(\angle BAY) = m(\angle AYX)$ (Alternate angles)(2)

, $\because \overline{AB} \parallel \overline{XY}$, $\overline{ZX}$ is a transversal

$\therefore m(\angle X) = m(\angle BAZ)$ (Corresponding angles) (3)

From (1) , (2) and (3) : $m(\angle AYX) = m(\angle X)$

$\therefore m(\angle AYX) + m(\angle AYZ) > m(\angle X)$

$\therefore m(\angle ZYX) > m(\angle X)$ $\therefore XZ > YZ$ (Q.E.D.)

Answers of model for the merge students

1

1. 1 : 2
2. half the length of the hypotenuse
3. congruent
4. >
5. bisects it , perpendicular to the base.

2

1 b 2 a 3 d 4 a 5 a

3

$\because m(\angle B) = 90°$, $m(\angle C) = 30°$

$\therefore AB = \frac{1}{2} \times AC$

$\therefore AC = 10$ cm.

4

[a] AC , AB , BC

[b] 1 40° 2 $\overline{AB}$

5

1 ✔ 2 ✘ 3 ✘ 4 ✔ 5 ✔

Answers of schools examinations on geometry

1 Cairo

1

1 (c) 2 (c) 3 (c)
4 (c) 5 (b) 6 (b)

2

1 3 2 congruent 3 the hypotenuse
4 bisects the base and is perpendicular to it
5 6

3

[a] In Δ ABC :
$\because$ m ($\angle$ A) + m ($\angle$ B) + m ($\angle$ C) = 180°
$\therefore$ 6 X + (4 X − 9) + 3 (X − 2) = 180°
$\therefore$ 13 X = 195° $\therefore$ X = 15°
$\therefore$ m ($\angle$ A) = 90° , m ($\angle$ B) = 51° , m ($\angle$ C) = 39°
$\therefore$ m ($\angle$ C) < m ($\angle$ B) < m ($\angle$ A)
$\therefore$ AB < AC < BC (The req.)

[b] In Δ ABC : $\because$ m ($\angle$ B) = 90° , m ($\angle$ C) = 30°
$\therefore$ AB = $\frac{1}{2}$ AC = 5 cm.
, $\because$ $\overline{BD}$ is a median $\therefore$ BD = $\frac{1}{2}$ AC = 5 cm.
, $\because$ AD = 5 cm.
$\therefore$ The perimeter of Δ ABD = 5 + 5 + 5 = 15 cm. (The req.)

4

[a] In Δ MBC : $\because$ MB = MC
$\therefore$ m ($\angle$ B) = m ($\angle$ C) (1)
, $\because$ $\overline{AD}$ // $\overline{BC}$, $\overline{AC}$ is a transversal.
$\therefore$ m ($\angle$ A) = m ($\angle$ C) (2)
, $\because$ $\overline{AD}$ // $\overline{BC}$, $\overline{BD}$ is a transversal.
$\therefore$ m ($\angle$ D) = m ($\angle$ B) (3)
From (1) , (2) and (3) :
$\therefore$ m ($\angle$ A) = m ($\angle$ D) $\therefore$ MA = MD
$\therefore$ Δ MAD is isosceles. (Q.E.D.)

[b] In Δ ABC :
$\because$ m ($\angle$ BAC) + m ($\angle$ B) + m ($\angle$ ACB) = 180°
$\therefore$ m ($\angle$ ACB) = 180° − (55° + 70°) = 55°
$\therefore$ m ($\angle$ B) = m ($\angle$ ACB) $\therefore$ AB = AC (1)
In Δ ACD : $\because$ m ($\angle$ ACD) = 90°
$\therefore$ $\overline{AD}$ is the hypotenuse $\therefore$ AD > AC (2)
From (1) and (2) :
$\therefore$ AD > AB (Q.E.D.)

5

[a] $\because$ Δ ABC is equilateral.
$\therefore$ m ($\angle$ ACB) = 60°
In Δ ACD : $\because$ DC = DA , m ($\angle$ D) = 40°
$\therefore$ m ($\angle$ DCA) = m ($\angle$ DAC) = $\frac{180° - 40°}{2}$ = 70°
$\therefore$ m ($\angle$ DCB) = 70° + 60° = 130° (The req.)

[b] In Δ ABD : $\because$ AD > AB
$\therefore$ m ($\angle$ ABD) > m ($\angle$ ADB) (1)
In Δ BCD : $\because$ CD > BC
$\therefore$ m ($\angle$ DBC) > m ($\angle$ BDC) (2)
Adding (1) , (2) :
$\therefore$ m ($\angle$ ABC) > m ($\angle$ ADC) (Q.E.D.)

2 Cairo

1

1 it , the base
2 120° 3 congruent 4 2 , 10
5 the hypotenuse

2

1 (a) 2 (a) 3 (c)
4 (c) 5 (c) 6 (b)

3

[a] In Δ ABC : $\because$ AB = AC
$\therefore$ m ($\angle$ B) = m ($\angle$ C) = 50°
In $\Delta\Delta$ ABY , ACX :
$$\begin{cases} AB = AC \\ m(\angle B) = m(\angle C) \\ BY = CX \end{cases}$$
$\therefore$ Δ ABY $\equiv$ Δ ACX $\therefore$ AY = AX
$\therefore$ Δ AXY is isosceles (First req.)
, m ($\angle$ BAY) = m ($\angle$ CAX) = 30°
$\therefore$ m ($\angle$ AYB) = 180° − (50° + 30°) = 100° (Second req.)

[b] $\because$ DA = BA , DC = BC

$\therefore$ $\overline{AC}$ is the axis of $\overline{BD}$

$\because$ $\overline{BD} \cap \overline{AC} = \{M\}$

$\therefore$ M is the midpoint of $\overline{BD}$ (Q.E.D.)

4

[a] In Δ ABC : $\because$ AB > AC

$\therefore$ m ($\angle$ ACB) > m ($\angle$ ABC)

$\therefore$ $\frac{1}{2}$ m ($\angle$ ACB) > $\frac{1}{2}$ m ($\angle$ ABC)

$\because$ $\overline{CD}$ bisects $\angle$ ACB , $\overline{BD}$ bisects $\angle$ ABC

$\therefore$ m ($\angle$ DCB) > m ($\angle$ DBC)

$\therefore$ BD > CD (Q.E.D.)

[b] $\because$ Δ ACD is an equilateral triangle.

$\therefore$ m ($\angle$ ADC) = 60°

In Δ BCD : $\because$ DB = DC

$\therefore$ m ($\angle$ B) = m ($\angle$ BCD) = 65°

$\therefore$ m ($\angle$ BDC) = 180° − 2 × 65° = 50°

$\therefore$ m ($\angle$ ADB) = 60° + 50° = 110° (The req.)

5

[a] $\because$ m ($\angle$ ABC) = 90° , $\overline{BE}$ is a median in Δ ABC

$\therefore$ BE = $\frac{1}{2}$ AC = $\frac{1}{2}$ × 12 = 6 cm.

, $\because$ $\overline{AD}$ is a median in Δ ABC

$\therefore$ M is the point of intersection of medians

$\therefore$ ME = $\frac{1}{3}$ BE = $\frac{1}{3}$ × 6 = 2 cm. (The req.)

[b] In Δ EBD : $\because$ ED + BD > BE

, $\because$ CD = ED $\therefore$ CD + BD > BE

$\therefore$ BC > BE (Q.E.D.1)

In Δ ABC : $\because$ AB + BC > AC (1)

, $\because$ CD = CE (2)

Subtracting (2) from (1) :

$\therefore$ AB + BD > AE (Q.E.D.2)

3 Cairo

1

1 (c) 2 (d) 3 (a)

4 (a) 5 (c) 6 (b)

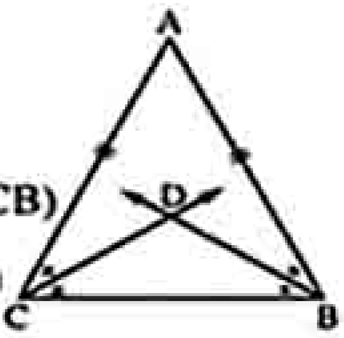
2

1 a side greater in length than that opposite to the other angle.

2 bisects the base and is perpendicular to it.

3 congruent 4 is greater than 5 5

3



[a] $\because$ AB = AC

$\therefore$ m ($\angle$ ABC) = m ($\angle$ ACB)

$\therefore$ $\frac{1}{2}$ m ($\angle$ ABC) = $\frac{1}{2}$ m ($\angle$ ACB)

$\because$ m ($\angle$ DBC) = $\frac{1}{2}$ m ($\angle$ ABC)

m ($\angle$ DCB) = $\frac{1}{2}$ m ($\angle$ ACB)

$\therefore$ m ($\angle$ DBC) = m ($\angle$ DCB)

$\therefore$ DB = DC

$\therefore$ Δ DBC is an isosceles triangle (Q.E.D.)

[b] In Δ ABC : $\because$ m ($\angle$ B) = 90° , m ($\angle$ ACB) = 30°

$\therefore$ AB = $\frac{1}{2}$ AC

, $\because$ AB = DE = 5 cm. $\therefore$ DE = $\frac{1}{2}$ AC

$\because$ $\overline{DE}$ is a median in Δ ADC

$\therefore$ m ($\angle$ ADC) = 90° (Q.E.D.)

4

[a] In Δ ABC : $\because$ AB > AC

$\therefore$ m ($\angle$ ABC) < m ($\angle$ ACB)

, $\because$ B $\in$ $\overline{AD}$, C $\in$ $\overline{AE}$

$\therefore$ 180° − m ($\angle$ ABC) > 180° − m ($\angle$ ACB)

$\therefore$ m ($\angle$ CBD) > m ($\angle$ BCE)

$\because$ $\overline{BF}$ bisects $\angle$ DBC , $\overline{CF}$ bisects $\angle$ BCE

$\therefore$ m ($\angle$ FBC) > m ($\angle$ BCF) (Q.E.D.1)

$\therefore$ CF > BF (Q.E.D.2)

[b] $\because$ $\overline{AD}$, $\overline{BE}$ are two medians in Δ ABC

$\therefore$ M is the point of intersection of the medians

$\therefore$ MB = 2 ME = 2 × 2 = 4 cm.

, MA = 2 MD = 2 × 3 = 6 cm.

, $\because$ D is the midpoint of $\overline{BC}$

, E is the midpoint of $\overline{AC}$

$\therefore$ AB = 2 DE = 2 × 4 = 8 cm.

$\therefore$ The perimeter of Δ MAB = 4 + 6 + 8 = 18 cm.

(The req.)

5

[a] From Δ ABM :

MA + MB > AB (Triangle inequality) (1)

From Δ BMC :

MB + MC > BC (Triangle inequality) (2)

From Δ AMC :

MA + MC > AC (Triangle inequality) (3)

Adding (1) , (2) and (3) :

$\therefore$ 2 MA + 2 MB + 2 MC > AB + BC + AC

$\therefore$ MA + MB + MC > $\frac{1}{2}$ the perimeter of Δ ABC

(Q.E.D.)

[b] $\because$ $\overline{AD}$ // $\overline{BC}$, $\overline{BD}$ is a transversal

$\therefore$ m ($\angle$ ADB) = m ($\angle$ DBC) (alternate angles)

, $\because$ m ($\angle$ ABD) = m ($\angle$ DBC)

$\therefore$ m ($\angle$ ABD) = m ($\angle$ ADB)

$\therefore$ In Δ ABD : AB = AD (Q.E.D.1)

, $\because$ $\overrightarrow{AE}$ bisects $\angle$ BAD

$\therefore$ $\overline{AE} \perp \overline{BD}$ (Q.E.D.2)

, E is the midpoint of $\overline{BD}$

$\therefore$ BE = ED (Q.E.D.3)

4 Giza

1

1 (c) 2 (d) 3 (a)

4 (a) 5 (d) 6 (d)

2

1 >

2 $\overline{BC}$

3 is perpendicular to it

4 3 , 11

5 18 cm.

3

[a] In Δ ABD : $\because$ BA = BD

$\therefore$ m ($\angle$ BDA) = m ($\angle$ BAD) = 70°

, $\because$ Δ ACD is an equilateral triangle

$\therefore$ m ($\angle$ ADC) = 60°

$\therefore$ m ($\angle$ BDC) = 70° + 60° = 130° (The req.)

[b] $\because$ $\overline{BE}$, $\overline{CF}$ are two medians in Δ ABC

$\therefore$ M is the point of intersection of medians

$\therefore$ MF = $\frac{1}{2}$ CM = $\frac{1}{2}$ × 6 = 3 cm.

, ME = $\frac{1}{2}$ BM = $\frac{1}{2}$ × 5 = 2.5 cm.

, $\because$ F is the midpoint of $\overline{AB}$

, E is the midpoint of $\overline{AC}$

$\therefore$ FE = $\frac{1}{2}$ BC = $\frac{1}{2}$ × 12 = 6 cm.

$\therefore$ The perimeter of Δ MEF = 3 + 2.5 + 6 = 11.5 cm.

(The req.)

4

[a] In Δ ABC : $\because$ m ($\angle$ ABC) = 90°

, $\overline{BE}$ is a median $\therefore$ BE = $\frac{1}{2}$ AC

In Δ ACD : $\because$ X is the midpoint of $\overline{AD}$

, Y is the midpoint of $\overline{CD}$ $\therefore$ XY = $\frac{1}{2}$ AC

$\therefore$ XY = BE (Q.E.D.)

[b] $\because$ m ($\angle$ DAE) = 90° − 30° = 60°

In Δ AFD :

$\therefore$ m ($\angle$ ADF) = 180° − (90° + 60°) = 30°

, $\because$ m ($\angle$ AFD) = 90°

$\therefore$ AD = 2 AF = 2 × 4 = 8 cm.

$\therefore$ The area of the square ABCD = 8 × 8 = 64 cm^2.

(The req.)

5

[a] $\because$ m ($\angle$ A) = m ($\angle$ B) $\therefore$ AC = BC

$\therefore$ $3x - 2 = x + 6$ $\therefore$ $2x = 8$ $\therefore$ $x = 4$ cm.

$\therefore$ AC = BC = 10 cm. , AB = 7 cm.

$\therefore$ The perimeter of Δ ABC = 10 + 10 + 7

= 27 cm. (The req.)

[b] In Δ ABC : $\because$ AB > AC

$\therefore$ m ($\angle$ ABC) < m ($\angle$ ACB)

, $\because$ B $\in$ $\overline{AD}$, C $\in$ $\overline{AE}$

$\therefore$ 180° − m ($\angle$ ABC) > 180° − m ($\angle$ ACB)

$\therefore$ m ($\angle$ CBD) > m ($\angle$ BCE)

$\because$ $\overrightarrow{BF}$ bisects $\angle$ DBC , $\overrightarrow{CF}$ bisects $\angle$ BCE

$\therefore$ m ($\angle$ FBC) > m ($\angle$ BCF) (Q.E.D.1)

$\therefore$ CF > BF (Q.E.D.2)

5 Giza

1

1 (b) 2 (b) 3 (c)

4 (d) 5 (c) 6 (a)

2

1 bisects the base , is perpendicular to it

2 6 cm. 3 120°

4 3 , 11 5 90°

3

[a] In Δ ABD : $\because$ AD = BD

$\therefore$ m ($\angle$ BAD) = m ($\angle$ ABD) = 40°

$\therefore m(\angle ADB) = 180° - 2 \times 40° = 100°$

$\therefore m(\angle ABD) < m(\angle ADB)$

$\therefore AD < AB$ (Q.E.D.1)

In ΔABE : $\because AD = \frac{1}{2} BE$

, $\overline{AD}$ is a median

$\therefore m(\angle BAE) = 90°$

$\therefore \overline{BC}$ is a hypotenuse of ΔBAC

$\therefore BC > AC$ (Q.E.D.2)

[b] In ΔABC : $\because AB = AC$

$\therefore m(\angle B) = m(\angle C)$

$\therefore \frac{1}{2} m(\angle B) = \frac{1}{2} m(\angle C)$

$\therefore m(\angle CBD) = m(\angle BCD)$

$\therefore BD = CD$

$\therefore \Delta DBC$ is isosceles. (Q.E.D.)

4

[a] In ΔABC :

$\because m(\angle A) + m(\angle B) + m(\angle C) = 180°$

$\therefore 6X + (4X - 9) + 3(X - 2) = 180°$

$\because 6X + 4X - 9 + 3X - 6 = 180°$

$\therefore 13X = 195°$ $\therefore X = 15°$

$\therefore m(\angle A) = 90°$, $m(\angle B) = 51°$

, $m(\angle C) = 39°$

$\therefore m(\angle C) < m(\angle B) < m(\angle A)$

$\therefore AB < AC < BC$ (The req.)

[b] $\because \overline{AC}$ is a median in ΔABD

, $AC = \frac{1}{2} BD$ $\therefore m(\angle BAD) = 90°$

$\therefore \overline{AB} \perp \overline{AD}$ (Q.E.D.)

5

[a] In ΔABC : $\because X$ is the midpoint of $\overline{AB}$

, Y is the midpoint of $\overline{BC}$

$\therefore XY = \frac{1}{2} AC = \frac{1}{2} \times 24 = 12$ cm.

In ΔXBY : $\because m(\angle XBY) = 90°$

, $\overline{BD}$ is a median in ΔXBY

$\therefore BD = \frac{1}{2} XY = \frac{1}{2} \times 12 = 6$ cm. (The req.)

[b] $\because \overline{BN}$, $\overline{CF}$ are two medians in ΔABC

$\therefore$ M is the point of intersection of medians

$\therefore MF = \frac{1}{3} CF = \frac{1}{3} \times 10.5 = 3.5$ cm.

, $MN = \frac{1}{2} BM = \frac{1}{2} \times 6 = 3$ cm.

, $\because AF = \frac{1}{2} AB = \frac{1}{2} \times 8 = 4$ cm.

, $AN = \frac{1}{2} AC = \frac{1}{2} \times 12 = 6$ cm.

$\therefore$ The perimeter of AFMN $= 3.5 + 3 + 4 + 6$

$= 16.5$ cm. (The req.)

6 Alexandria

1

1 25°	2 AC	3 120°
4 2	5 <	6 concurrent

2

1 (a)	2 (a)	3 (d)
4 (b)	5 (c)	

3

[a] $\because m(\angle C) = 180° - (90° + 75°) = 15°$

$\therefore m(\angle B) > m(\angle A) > m(\angle C)$

$\therefore AC > BC > AB$ (The req.)

[b] $\because \overline{YM}$, $\overline{ZL}$ are two medians in ΔXYZ

$\therefore$ N is the point of intersection of medians

$\therefore NL = \frac{1}{3} LZ = \frac{1}{3} \times 15 = 5$ cm.

, $YN = \frac{2}{3} YM = \frac{2}{3} \times 18 = 12$ cm.

, $YL = \frac{1}{2} XY = \frac{1}{2} \times 20 = 10$ cm.

$\therefore$ The perimeter of $\Delta NLY = 5 + 12 + 10$

$= 27$ cm. (The req.)

4

[a] In ΔABC :

$\because m(\angle ABC) = 90°$, $m(\angle ACB) = 30°$

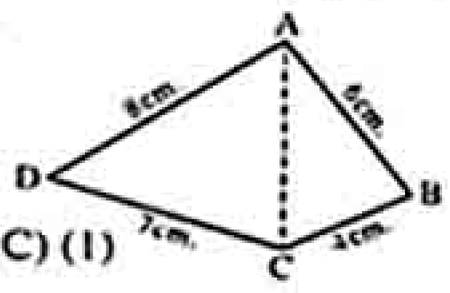
$\therefore AB = \frac{1}{2} AC$

, $\because AB = DE$ $\therefore DE = \frac{1}{2} AC$

$\therefore \overline{DE}$ is a median in ΔADC

$\therefore m(\angle ADC) = 90°$ (Q.E.D.)

[b] Construction : Draw $\overline{AC}$



Proof : In ΔABC

$\because AB > BC$

$\therefore m(\angle ACB) > m(\angle BAC)$ (1)

In ΔACD : $\because AD > CD$

$\therefore m(\angle ACD) > m(\angle CAD)$ (2)

Adding (1) , (2) :

$\therefore m(\angle BCD) > m(\angle BAD)$ (Q.E.D.)

5

[a] $\because \overline{DE} \parallel \overline{BC}$, $\overline{BD}$ is a transversal

$\therefore m(\angle CBD) = m(\angle BDE)$ (alternate angles)

, $\because m(\angle EBD) = m(\angle CBD)$

$\therefore m(\angle EBD) = m(\angle BDE)$

$\therefore BE = DE$

$\therefore \triangle EBD$ is an isosceles triangle. (Q.E.D.)

[b] $\because \triangle ABC$ is an equilateral triangle

$\therefore m(\angle BAC) = 60°$

In $\triangle ACD$: $\because AD = CD$, $m(\angle D) = 96°$

$\therefore m(\angle DAC) = m(\angle DCA) = \frac{180° - 96°}{2} = 42°$

$\therefore m(\angle DAB) = 60° + 42° = 102°$ (The req.)

7 Alexandria

1

1 (b)	2 (b)	3 (b)
4 (b)	5 (d)	6 (c)

2

1 right	2 greater than	3 8
4 3 , 11	5 40°	

3

[a] In $\triangle ABC$:

$\because X$ is the midpoint of $\overline{AB}$

, Y is the midpoint of $\overline{BC}$

$\therefore XY = \frac{1}{2} AC = \frac{1}{2} \times 20 = 10$ cm.

In $\triangle XBY$: $\because m(\angle XBY) = 90°$

, $\overline{BD}$ is a median

$\therefore BD = \frac{1}{2} XY = \frac{1}{2} \times 10 = 5$ cm. (The req.)

[b] $\because \overline{XY} \parallel \overline{AC}$, $\overline{AB}$ is a transversal.

$\therefore m(\angle A) = m(\angle ABX) = 62°$ (alternate angles)

$\therefore m(\angle ABC) = 180° - (56° + 62°) = 62°$

$\therefore m(\angle ABC) = m(\angle BAC)$

$\therefore AC = BC$ (Q.E.D.)

4

[a] $\because \overline{BE}$, $\overline{CD}$ are two medians in $\triangle ABC$

$\therefore$ M is the point of intersection of medians

$\therefore ME = \frac{1}{2} BM = \frac{1}{2} \times 4 = 2$ cm.

, $MD = \frac{1}{3} DC = \frac{1}{3} \times 9 = 3$ cm.

, $\because$ D is the midpoint of $\overline{AB}$

, E is the midpoint of $\overline{AC}$

$\therefore DE = \frac{1}{2} BC = \frac{1}{2} \times 8 = 4$ cm.

$\therefore$ The perimeter of $\triangle DME = 2 + 3 + 4 = 9$ cm.

(The req.)

[b] $\because m(\angle EAD) = 90° - 30° = 60°$

In $\triangle ADF$:

$\therefore m(\angle ADF) = 180° - (90° + 60°) = 30°$

, $\because m(\angle AFD) = 90°$

$\therefore AD = 2 AF = 2 \times 4 = 8$ cm.

$\therefore$ The area of the square ABCD $= 8 \times 8 = 64$ cm^2.

(The req.)

5

[a] $\because \overline{AD} \parallel \overline{BC}$, $\overline{AB}$ is a transversal

$\therefore m(\angle BAD) + m(\angle B) = 180°$ (interior angles)

$\therefore m(\angle B) = 180° - 120° = 60°$

$\therefore m(\angle BAC) > m(\angle B)$

$\therefore BC > AC$ (Q.E.D.)

[b] $\because \triangle ACD$ is an equilateral triangle.

$\therefore m(\angle ADC) = 60°$

, in $\triangle ABD$: $\because AB = BD$

$\therefore m(\angle BDA) = m(\angle BAD) = 70°$

$\therefore m(\angle BDC) = 60° + 70° = 130°$ (The req.)

8 El-Kalyoubia

1

1 (b)	2 (d)	3 (a)
4 (b)	5 (d)	6 (a)

2

1 concurrent	2 hypotenuse	3 an isosceles
4 >	5 3 , 15	

3

[a] $\because m(\angle B) > m(\angle C) > m(\angle A)$

$\therefore AC > AB > BC$ (The req.)

[b] In $\triangle ABC$: $\because AB = BC$

$\therefore m(\angle A) = m(\angle C)$

, $\because \overline{XY} \parallel \overline{AC}$, $\overline{AB}$ is a transversal

$\therefore m(\angle BXY) = m(\angle A)$ (corresponding angles)

, ∵ $\overline{XY}$ // $\overline{AC}$, $\overline{BC}$ is a transversal

∴ m (∠ BYX) = m (∠ C) (corresponding angles)

, ∵ m (∠ A) = m (∠ C)

∴ m (∠ BXY) = m (∠ BYX)

In Δ BXY : ∴ BX = BY (Q.E.D.)

4

[a] In Δ XYL : ∵ YX = LX , LM = MY

∴ $\overline{XM}$ is the axis of $\overline{YL}$

In Δ YZL : ∵ YZ = LZ , LM = MY

∴ $\overline{ZM}$ is the axis of $\overline{YL}$

∴ X , M , Z are on the same straight line (Q.E.D.)

[b] In Δ ADB : ∵ DB = DA

∴ m (∠ B) = m (∠ BAD) (1)

In Δ ADC : ∴ DC = DA

∴ m (∠ C) = m (∠ CAD) (2)

, in Δ ABC : ∵ AB > AC

∴ m (∠ C) > m (∠ B) (3)

From (1) , (2) , (3) :

∴ m (∠ BAD) < m (∠ CAD) (Q.E.D.)

5

[a] In Δ ABC :

∵ m (∠ ABC) = 90° , $\overline{BD}$ is a median

∴ BD = $\frac{1}{2}$ AC = $\frac{1}{2}$ × 9 = 4.5 cm.

, ∵ $\overline{AE}$ and $\overline{BD}$ are two medians in Δ ABC

∴ M is the point of intersection of medians

∴ BM = $\frac{2}{3}$ BD = $\frac{2}{3}$ × 4.5 = 3 cm.

, MD = $\frac{1}{3}$ BD = $\frac{1}{3}$ × 4.5 = 1.5 cm.

, ∵ m (∠ C) = 30°

∴ AB = $\frac{1}{2}$ AC = $\frac{1}{2}$ × 9 = 4.5 cm. (The req.)

[b] ∵ m (∠ A) + m (∠ B) + m (∠ C) = 180°

∵ 2 X + X + 40 + 3 X – 10 = 180°

∴ 6 X = 150° ∴ X = 25°

∴ m (∠ A) = 50° , m (∠ B) = 65°

, m (∠ C) = 65° ∴ m (∠ B) = m (∠ C)

∴ AB = AC (Q.E.D.)

9 El-Sharkia

1

1 (a)	2 (b)	3 (c)
4 (d)	5 (a)	6 (b)

2

1 5 2 90° 3 the hypotenuse

4 0

5 bisects it , is perpendicular to the base.

3

[a] In Δ XYZ :

∵ m (∠ YXZ) = 180° – (70° + 30°) = 80°

∴ $\overline{XL}$ bisects ∠ YXZ

∴ m (∠ LXZ) = m (∠ LXY) = 80° ÷ 2 = 40°

In Δ XLZ :

∴ m (∠ XLZ) = 180° – (70° + 40°) = 70° (First req.)

∴ m (∠ XLZ) = m (∠ Z) ∴ XL = XZ

∴ Δ XLZ is isosceles. (Second req.)

[b] In Δ ADC : ∵ AD = DC

∴ m (∠ CAD) = m (∠ ACD) (1)

In Δ ABC : ∵ BC > AB

∴ m (∠ BAC) > m (∠ ACB) (2)

Adding (1) , (2) :

∴ m (∠ BAD) > m (∠ BCD) (Q.E.D.)

4

[a] ∵ $\overline{AY}$, $\overline{BX}$ are two medians in Δ ABC

∴ M is the point of intersection of medians

∴ MY = $\frac{1}{2}$ AM = $\frac{1}{2}$ × 5 = 2.5 cm.

, MX = $\frac{1}{3}$ BX = $\frac{1}{3}$ × 6 = 2 cm.

, ∵ X is the midpoint of $\overline{AC}$

, Y is the midpoint of $\overline{BC}$

∴ XY = $\frac{1}{2}$ AB = $\frac{1}{2}$ × 8 = 4 cm.

∴ The perimeter of Δ XMY = 2.5 + 2 + 4

= 8.5 cm. (The req.)

[b] ∵ ∠ ACD is an exterior angle of Δ ABC

∴ m (∠ A) + m (∠ B) = 140°

, ∵ AC = BC

∴ m (∠ A) = m (∠ B)

∴ m (∠ B) = $\frac{140^\circ}{2}$ = 70°

, ∵ $\overrightarrow{AB}$ // $\overrightarrow{DE}$, $\overrightarrow{BD}$ is a transversal

∴ m (∠ BDE) = m (∠ B) = 70° (alternate angles)

(The req.)

5

[a] ∵ $\overline{AD}$ // $\overline{BC}$, $\overrightarrow{BD}$ is a transversal.

∴ m (∠ ADB) = m (∠ DBC) (alternate angles)

, ∵ m (∠ ABD) = m (∠ DBC)

∴ m (∠ ABD) = m (∠ ADB)

∴ In Δ ABD : AD = AB (Q.E.D.1)

, ∵ $\overrightarrow{AE}$ bisects ∠ BAD

∴ $\overrightarrow{AE}$ ⊥ $\overline{BD}$ (Q.E.D.2)

[b] ∵ The point A ∈ $\overleftrightarrow{BE}$

∴ m (∠ BAC) = 180° − (75° + 35°) = 70°

, ∵ $\overrightarrow{AD}$ // $\overline{BC}$, $\overline{AC}$ is a transversal

∴ m (∠ C) = m (∠ CAD) = 35° (alternate angles)

∴ m (∠ BAC) > m (∠ C)

∴ BC > AB (Q.E.D.)

10 El-Monofia

1

1 (a) 2 (a) 3 (b)

4 (c) 5 (c) 6 (c)

2

1 congruent 2 260° 3 3

4 < 5 is peroendicular to it

3

[a] ∵ $\overline{BE}$, $\overline{CD}$ are two medians in Δ ABC

∴ F is the point of intersection of medians

∴ FE = $\frac{1}{2}$ FB = $\frac{1}{2}$ × 6 = 3 cm.

, FD = $\frac{1}{2}$ FC = $\frac{1}{2}$ × 4 = 2 cm.

, ∵ D is the midpoint of $\overline{AB}$

, E is the midpoint of $\overline{AC}$

∴ DE = $\frac{1}{2}$ BC = $\frac{1}{2}$ × 8 = 4 cm.

∴ The perimeter of Δ DFE = 3 + 2 + 4 = 9 cm.

(The req.)

[b] In Δ ABD : ∵ AD = BD

∴ m (∠ BAD) = m (∠ ABD) = 35°

, ∵ ∠ ADC is an exterior angle of Δ ABD

∴ m (∠ ADC) = 35° + 35° = 70°

In Δ ADC : ∵ AC = AD

∴ m (∠ C) = m (∠ ADC) = 70°

∴ m (∠ CAD) = 180° − 2 × 70° = 40°

∴ m (∠ BAC) = 35° + 40° = 75° (The req.)

4

[a] In Δ ABC : ∵ AC = AB

∴ m (∠ B) = m (∠ C)

, ∵ ∠ ADB is an exterior angle of Δ ADC

∴ m (∠ ADB) > m (∠ C)

∴ m (∠ ADB) > m (∠ B)

In Δ ADB : ∴ AB > AD (Q.E.D.)

[b] In Δ ABC :

∵ m (∠ C) = 180° − (40° + 80°) = 60°

∴ m (∠ B) > m (∠ C) > m (∠ A)

∴ AC > AB > BC (The req.)

5

In Δ ABC :

∵ E is the midpoint of $\overline{AC}$

, F is the midpoint of $\overline{AB}$

∴ FE = $\frac{1}{2}$ BC (1)

In Δ BDC : ∵ m (∠ BDC) = 90°

, $\overline{DG}$ is a median

∴ GD = $\frac{1}{2}$ BC = $\frac{1}{2}$ × 10 = 5 cm. (2)

, ∵ m (∠ CBD) = 30°

∴ DC = $\frac{1}{2}$ BC = $\frac{1}{2}$ × 10 = 5 cm. (3)

From (1) , (2) , (3) : ∴ FE = DC = GD (First req.)

, ∵ CG = $\frac{1}{2}$ BC = $\frac{1}{2}$ × 10 = 5 cm.

∴ The perimeter of Δ GCD = 5 + 5 + 5 = 15 cm.

(Second req.)

11 El-Dakahlia

1

1 (a) 2 (b) 3 (d)

4 (d) 5 (c) 6 (a)

2

1 l

2 bisects the base and is perpendicular to it

3 one point 4 hypotenuse 5 100°

3

[a] In Δ ABC : ∵ m (∠ ABC) = 90°

, $\overline{BD}$ is a median

∴ BD = $\frac{1}{2}$ AC (1)

In Δ BDE : ∵ m (∠ BDE) = 90°

, m (∠ E) = 30°

∴ BD = $\frac{1}{2}$ BE (2)

From (1) , (2) : ∴ AC = BE (Q.E.D.)

[b] In Δ ABC : ∵ AB = AC

∴ m (∠ ABC) = m (∠ ACB) (1)

In Δ BCD : ∵ DC > DB

∴ m (∠ CBD) > m (∠ BCD) (2)

Adding (1) , (2) :

∴ m (∠ ABD) > m (∠ ACD) (Q.E.D.)

4

[a] ∵ $\overline{AD}$ // $\overline{BC}$, $\overleftrightarrow{AB}$ is a transversal.

∴ m (∠ B) = m (∠ BAD) = 60° (alternate angles)

In Δ ABC :

∴ m (∠ C) = 180° – (50° + 60°) = 70°

∴ m (∠ C) > m (∠ B)

∴ AB > AC (Q.E.D.)

[b] ∵ ∠ ACL is an exterior angle of Δ ABC

∴ m (∠ A) + m (∠ B) = 130°

, ∵ AC = BC ∴ m (∠ A) = m (∠ B)

∴ m (∠ B) = $\frac{130°}{2}$ = 65°

, ∵ $\overleftrightarrow{AB}$ // $\overleftrightarrow{LM}$, $\overleftrightarrow{BL}$ is a transversal

∴ m (∠ MLC) = m (∠ B) = 65° (alternate angles)

(The req.)

5

[a] In Δ ABC :

∵ X is the midpoint of $\overline{AB}$

, Y is the midpoint of $\overline{BC}$

∴ XY = $\frac{1}{2}$ AC = $\frac{1}{2}$ × 22 = 11 cm.

In Δ XBY : ∵ m (∠ XBY) = 90°

, $\overline{BD}$ is a median

∴ BD = $\frac{1}{2}$ XY = $\frac{1}{2}$ × 11 = 5.5 cm. (The req.)

[b] ∵ AB = AC

, BE = CE

∴ $\overleftrightarrow{AE}$ is the axis of $\overline{BC}$

∴ BD = CD (Q.E.D.)

12 Suez

1

1 congruent 2 equilateral 3 $\overline{BC}$

4 2 : 1 5 2

2

1 (d) 2 (d) 3 (b)

4 (a) 5 (b) 6 (a)

3

[a] In Δ ABC :

∵ m (∠ C) = 180° – (40° + 75°) = 65°

∴ m (∠ B) > m (∠ C) > m (∠ A)

∴ AC > AB > BC (The req.)

[b] In Δ ABC :

∵ AB = AC , $\overrightarrow{AE}$ bisects ∠ BAC

∴ $\overline{AE}$ ⊥ $\overline{BC}$, E is the midpoint of $\overline{BC}$

∴ $\overleftrightarrow{AE}$ is axis of $\overline{BC}$

, ∵ D ∈ $\overleftrightarrow{AE}$ ∴ BD = CD (Q.E.D.)

4

[a] In Δ ABD : ∵ AD = AB

∴ m (∠ ADB) = m (∠ ABD) = 25°

, ∵ $\overline{AD}$ // $\overline{BC}$, $\overline{BD}$ is a transversal

∴ m (∠ DBC) = m (∠ ADB) = 25°

(alternate angles)

∴ X = 25°

, in Δ BCD :

y = 180° – (25° + 63) = 92° (The req.)

[b] In Δ ABD : ∵ AB = BD = DA

∴ Δ ABD is an equilateral triangle

∴ m (∠ BAD) = m (∠ B)

∴ m (∠ CAD) + m (∠ BAD) > m (∠ B)

∴ m (∠ BAC) > m (∠ B)

∴ BC > AC (Q.E.D.)

5

[a] In Δ ABC : ∵ AB > BC

∴ m (∠ ACB) > m (∠ BAC) (1)

In Δ ACD : ∵ AD > CD

∴ m (∠ ACD) > m (∠ CAD) (2)

Adding (1) , (2) :

∴ m (∠ BCD) > m (∠ BAD) (Q.E.D.)

[b] In Δ ABC :

∵ m (∠ ABC) = 90° , m (∠ C) = 30°

∴ AB = $\frac{1}{2}$ AC = $\frac{1}{2}$ × 12 = 6 cm.

, ∵ $\overline{BD}$ is a median

∴ BD = $\frac{1}{2}$ AC = $\frac{1}{2}$ × 12 = 6 cm.

∵ D is the midpoint of $\overline{AC}$

∴ AD = $\frac{1}{2}$ AC = $\frac{1}{2}$ × 12 = 6 cm.

∴ The perimeter of Δ ABD = 6 + 6 + 6 = 18 cm. (The req.)

13 El-Beheira

1

1 $\frac{1}{2}$ 2 18 cm. 3 1

4 a side greater in length than that opposite to the other angle.

5 5 cm.

2

1 (b) 2 (c) 3 (d)

4 (c) 5 (a) 6 (b)

3

[a] ∵ $\overline{BE}$, $\overline{CD}$ are two medians in Δ ABC

∴ M is the point of intersection of medians

∴ ME = $\frac{1}{2}$ MB = $\frac{1}{2}$ × 6 = 3 cm.

, MD = $\frac{1}{2}$ MC = $\frac{1}{2}$ × 8 = 4 cm.

, ∵ D is the midpoint of $\overline{AB}$

, E is the midpoint of $\overline{AC}$

∴ DE = $\frac{1}{2}$ BC = $\frac{1}{2}$ × 12 = 6 cm.

∴ The perimeter of Δ MDE = 3 + 4 + 6 = 13 cm. (The req.)

[b] ∵ AC = $\frac{1}{2}$ BD , $\overline{AC}$ is a median in Δ ABD

∴ m (∠ BAD) = 90° (Q.E.D.)

4

[a] In Δ ABD : ∵ AB = AD

∴ m (∠ ABD) = m (∠ ADB) (1)

In Δ BCD : ∵ CD > BC

∴ m (∠ DBC) > m (∠ BDC) (2)

Adding (1) , (2) :

∴ m (∠ ABC) > m (∠ ADC) (Q.E.D.)

[b] ∵ $\overline{AD}$ // $\overline{BC}$, $\overline{AB}$ is a transversal.

∴ m (∠ ACB) = m (∠ DAC) = 35° (alternate angles)

In Δ ABC :

∴ m (∠ B) = 180° − (70° + 35°) = 75°

∴ m (∠ B) > m (∠ BAC)

∴ AC > BC (Q.E.D.)

5

In Δ ABC : ∵ AB = AC

∴ m (∠ B) = m (∠ C) = 50°

∴ In ΔΔ ABX , ACY :

$\begin{cases} AB = AC \\ m(\angle B) = m(\angle C) \\ BX = CY \end{cases}$

∴ Δ ABX ≡ Δ ACY ∴ AX = AY

∴ Δ AYX is isosceles (First req.)

, ∵ m (∠ XAB) = m (∠ YAC) = 30°

, ∠ AXY is an exterior angle of Δ ABX

∴ m (∠ AXY) = 50° + 30° = 80° (Second req.)

14 El-Menia

1

1 (a) 2 (a) 3 (b)

4 (c) 5 (b) 6 (d)

2

1 360° 2 bisects 3 equilateral

4 1 : 2 5 >

3

[a] In Δ AMC : ∵ m (∠ C) = 90°

∴ AM > CM (1)

In Δ BMD : ∵ m (∠ D) = 90°

∴ BM > DM (2)

Adding (1) , (2) : ∴ AB > CD (Q.E.D.)

[b] In Δ ABD : ∵ AB = AD

∴ m (∠ ABD) = m (∠ ADB) (1)

In Δ BCD : $\because$ BC = CD

$\therefore$ m ($\angle$ CBD) = m ($\angle$ CDB) (2)

Adding (1) , (2) : $\therefore$ m ($\angle$ ABC) = m ($\angle$ ADC) (Q.E.D.)

4

[a] In Δ ABC : $\because$ AB > BC

$\therefore$ m ($\angle$ C) > m ($\angle$ A) (1)

$\because$ $\overline{XY}$ // $\overline{BC}$, $\overline{AC}$ is a transversal

$\therefore$ m ($\angle$ AYX) = m ($\angle$ C) (corresponding angles) (2)

From (1) , (2) :

$\therefore$ m ($\angle$ AYX) > m ($\angle$ A)

$\therefore$ AX > XY (Q.E.D.)

[b] $\because$ $\overline{AD}$ // $\overline{BC}$, $\overline{AC}$ is a transversal

$\therefore$ m ($\angle$ C) = m ($\angle$ CAD) = 30° (alternate angles)

, $\because$ AC = BC

$\therefore$ m ($\angle$ BAC) = m ($\angle$ B) = $\frac{180° - 30°}{2}$ = 75° (The req.)

5

[a] In Δ ABC : $\because$ m ($\angle$ ABC) = 90°

, $\overline{BD}$ is a median $\therefore$ BD = $\frac{1}{2}$ AC (1)

In Δ BDE : $\because$ m ($\angle$ BDE) = 90°

, m ($\angle$ E) = 30°

$\therefore$ BD = $\frac{1}{2}$ BE (2)

From (1) , (2) : $\therefore$ AC = BE (Q.E.D.)

[b] In Δ ABC :

$\because$ AB = AC $\therefore$ m ($\angle$ B) = m ($\angle$ C)

$\therefore$ $\frac{1}{2}$ m ($\angle$ B) = $\frac{1}{2}$ m ($\angle$ C)

$\therefore$ m ($\angle$ DBC) = m ($\angle$ DCB) $\therefore$ BD = CD

$\therefore$ Δ DBC is isosceles. (Q.E.D.)

15 Qena

1

1 3 2 50° 3 <

4 2 : 1 5 its axis of symmetry.

2

1 (a) 2 (a) 3 (c)

4 (b) 5 (b)

3

[a] In Δ ABC : $\because$ AB = AC

$\therefore$ m ($\angle$ B) = m ($\angle$ C)

, $\because$ $\overline{XY}$ // $\overline{BC}$, $\overline{AB}$ is a transversal

$\therefore$ m ($\angle$ AXY) = m ($\angle$ B) (corresponding angles)

, $\because$ $\overline{XY}$ // $\overline{BC}$, $\overline{AC}$ is a transversal

$\therefore$ m ($\angle$ AYX) = m ($\angle$ C) (corresponding angles)

$\because$ m ($\angle$ B) = m ($\angle$ C)

$\therefore$ m ($\angle$ AXY) = m ($\angle$ AYX)

In Δ AXY : $\therefore$ AX = AY

$\therefore$ Δ AXY is isosceles (Q.E.D.)

[b] $\because$ m ($\angle$ C) = 180° − (40° + 75°) = 65°

$\therefore$ m ($\angle$ A) < m ($\angle$ C) < m ($\angle$ B)

$\therefore$ BC < AB < AC (The req.)

4

[a] In Δ ABC : $\because$ E is the midpoint of $\overline{AC}$

, F is the midpoint of $\overline{AB}$

$\therefore$ EF = $\frac{1}{2}$ BC = $\frac{1}{2}$ × 20 = 10 cm.

In Δ ADB : $\because$ m ($\angle$ ADB) = 90°

, $\overline{DF}$ is a median

$\therefore$ DF = $\frac{1}{2}$ AB = $\frac{1}{2}$ × 14 = 7 cm.

In Δ ADC : $\because$ m ($\angle$ ADC) = 90°

, $\overline{DE}$ is a median

$\therefore$ DE = $\frac{1}{2}$ AC = $\frac{1}{2}$ × 18 = 9 cm.

$\therefore$ The perimeter of Δ DEF = 10 + 7 + 9 = 26 cm. (The req.)

[b] In Δ ABD : $\because$ AD > AB

$\therefore$ m ($\angle$ ABD) > m ($\angle$ ADB) (1)

In Δ BCD : $\because$ CD > BC

$\therefore$ m ($\angle$ CBD) > m ($\angle$ CDB) (2)

Adding (1) , (2) :

$\therefore$ m ($\angle$ ABC) > m ($\angle$ ADC) (Q.E.D.)

5

[a] $\because$ $\overline{AD}$ // $\overline{BC}$, $\overline{AC}$ is a transversal

$\therefore$ m ($\angle$ ACB) = m ($\angle$ CAD) = 30° (alternate angles)

In Δ ABC :

$\therefore$ m ($\angle$ BAC) > m ($\angle$ ACB)

$\therefore$ BC > AB (Q.E.D.)

[b] 2 , 12